STUDY OF NUCLEAR STRUCTURE FROM THE ANALYSIS OF γ RAYS
FOLLOWING μ^- CAPTURE IN Al, Si, Ca, and Co

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Contents

Abstract.	vii
I. Introduction.	1
II. Theory of Muon Capture	
A. The Muon Capture Reaction in Complex Nuclei	
1. The Effective Hamiltonian	8
2. Systematics of Muon Capture	11
3. Total Capture Rates for Muon Capture.	14
4. Muon Partial Capture Rates.	15
5. Allowed and Forbidden Transitions	16
B. Review of the Giant Resonances	
1. Giant Resonance Models of Photoexcitation	20
2. Application to Muon Capture	27
C. Particle Emission following Muon Capture	
1. Neutron Multiplicities.	31
2. Neutron Energy Spectrum	31
3. Charged Particles	32
III. Experimental Apparatus and Procedure	
A. Muon Telescope and Ge(Li) Detector.	35
B. Pulse Height Analysis System.	38
C. Recording of Results.	42
D. Calibration	
1. Energy Calibration.	43
2. Efficiency Calibration.	44
3. Experimental Sensitivity.	47
IV. Analysis of Data	
A. Gate Width Correction	51
B. Detector Deterioration.	53

C. Doppler Broadened Lines.	57
D. Determination of Net Peak Areas	
1. Background Subtraction	64
2. Peak Areas from Data Minus Background Spectra. . .	64
E. Line Identification.	65
F. Determination of Transition Yield per Muon Stop. . . .	66
G. Population of State Calculations	67
V. Experimental Results	
A. Aluminum	68
B. Cobalt	71
C. Calcium.	75
D. Silicon.	78
VI. Comments on the Results	
A. Population of Hole States in ^{39}K	84
B. Giant M1 Dipole Resonance in ^{28}Si	89
C. Muon Capture and the Photoproton Reaction in ^{28}Si . . .	98
D. Comparison with Published Neutron Multiplicities . . .	101
E. Comments on Charged Particle Emission.	103
F. Comparison with Radiative Pion Capture Results	106
G. Isospin Splitting of the Giant Resonance	108
VII. Conclusions.	112
Appendices	
A. Details on the Efficiency Calibrations	
1. Muon Stopping Distribution	114
2. Solid Angle Correction	118
3. Target Self Attenuation.	118
4. Correction for Muons Stopping in S_3	119
5. Ratio of $K_{\alpha}/\Sigma K$	120

6. Absolute Detector Efficiency.	121
7. Relative Efficiency of Detector	121
8. Effective Efficiency.	122
B. Doppler Broadened Gamma Peaks Following Muon Capture.	123
C. Population of State Calculations.	129
Acknowledgements.	132
Footnotes and References.	133

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ABSTRACT

An analysis of γ rays emitted following μ^- capture in Al, Si, Ca, and Co has been carried out with the intention of studying the structure of the nuclear states excited in the muon capture reaction. The results of this experiment are correlated and compared with the results of single nucleon pick-up reactions, photoexcitation and electroexcitation studies, and thermal neutron capture experiments as well as with theoretical nuclear structure calculations. These correlations reveal the similarities of the various excitation techniques and show how muon capture studies compliment and extend the information one can obtain about nuclear structure.

The close correlation of the states populated in ^{39}K following muon capture in ^{40}Ca with those populated in the single nucleon pickup reaction $^{40}\text{Ca}(t, \alpha)^{39}\text{K}$ indicates that single hole states are populated preferentially in the residual ^{39}K nucleus when a single neutron is emitted following the muon capture. An identification of the states formed in ^{28}Al following muon capture in ^{28}Si with levels thought to be configuration mixed particle-hole states is made.

The results of the photoexcitation reaction $^{40}\text{Ca}(\gamma, p\gamma')^{39}\text{K}$ are compared with the $^{40}\text{Ca}(\mu^-, \nu n)^{39}\text{K}$ results and indicate the presence of a giant resonance mechanism in the muon capture process. The relative frequency of populating $1d_{5/2}$ proton hole states to that of populating

the $2s_{1/2}$ proton hole state in ^{39}K following μ^- capture in ^{40}Ca is compared with the theoretically calculated ratio. The inference of a giant M1 resonance mechanism in muon capture drawn from a correlation with the 180 deg inelastic electron scattering data for ^{28}Si is used to identify the 2202 keV, $J^\pi = 1^+$, level in ^{28}Al as the parent of the isobaric analog state at 11.41 MeV in ^{28}Si . The strong excitation of the third excited state in ^{27}Al following muon capture in ^{28}Si when contrasted with the absence of evidence for a proton group corresponding to the excitation of this level in the $^{28}\text{Si}(\gamma, p)^{27}\text{Al}$ reaction demonstrates that additional states are excited in muon capture which are not excited in photoexcitation. The populations of states in ^{26}Mg in the $^{27}\text{Al}(\mu^-, \nu n)$ ^{26}Mg and $^{27}\text{Al}(\gamma, p\gamma)^{26}\text{Mg}$ reactions correlate closely and this may be evidence that the photoexcitation reaction is proceeding via the $\Delta T = 1$ branch.

The results of this experiment are discussed in the context of other muon capture experiments which measure neutron multiplicities, charged particle emission, and no particle emission. A comparison with other gamma spectra measurements is made for Si, Ca, and Co.

I. INTRODUCTION

With the discovery of muons in cosmic rays in the mid 1930's, it was thought that the quanta of the field responsible for nuclear forces had been found. However, as shown later¹ the mu mesons do not interact appreciably with nucleons. The pi mesons discovered by Lattes, Occhialini and Powell² in 1947 are the quanta of the nuclear force predicted by Yukawa³ in 1935. Mu mesons are subject to electromagnetic and weak interactions with nuclei, but as mentioned do not interact strongly.

Even though they are not the quanta of the nuclear force μ mesons have yielded much information about this force and about nuclear structure. The electromagnetic interaction of muons with nuclei has been exploited in the study of mu-mesic atoms⁴ to determine the nuclear charge distribution with high precision. The μ^- meson stopping in condensed material interacts electromagnetically with the nucleus and is trapped in a Bohr orbit. In a time very short compared to its decay lifetime, it falls into the 1s orbit⁵ and from there either decays or interacts with the nucleus. As a result of the interatomic transitions between Bohr orbits, Auger electrons and mu-mesic x rays are emitted. Since the muon is 200 times as heavy as an electron the radius of its innermost orbit is much smaller than that of an electron. In fact, in medium and large Z elements the muon wave function in these orbits is inside the nucleus much of the time. It is this nuclear penetration and the subsequent departure from the simple Bohr hydrogen-atom formula for describing the muon energy levels that yields information on the nuclear charge radius from measurements of mu-mesic x-ray energies.

Total capture rates^{6,7} measured for many elements are in good agreement with the predictions of Primakoff⁸ which are based on a Universal Fermi Interaction which also describes the analogous electron

K-capture reaction. However, in contrast to K-electron capture which is endoergic for stable nuclei, muon capture, because of the large rest mass of the muon, is exoergic by approximately 100 MeV for all nuclei. Thus nuclear structure in the continuum can be studied in muon capture. A dominant feature of nuclear structure in this energy region is the giant dipole resonance.⁹ This phenomenon was first observed¹⁰ in the photon cross section of all nuclei.^{9,11} In fact, it is a result of the high (γ, n) cross section in the resonance that it is possible to produce large fluxes of neutrons using bremsstrahlung from electron linacs.

In addition to the study of muonic atoms, there have been several attempts to use the muon nuclear capture reaction as a probe of nuclear structure. In particular, attempts have been made to relate neutron multiplicity measurements¹² to the nucleon velocity distributions within the target nuclei. More recently it has been shown that muon capture proceeds largely through a giant dipole excitation mechanism similar to that of photo-excitation. The existence of this resonance mechanism in muon capture was established without doubt by the neutron energy spectrum measurements of Evseev et al.¹³ which exhibited definite resonance structure.

The use of nuclear gamma rays emitted following muon capture to study this resonance capture mechanism was first demonstrated in the partial capture rates¹⁴ for muons in ^{16}O leading to excited nuclear states in ^{15}N .¹⁵ In particular, the partial capture rate to the 6.322 MeV, $J^\pi = 3/2^-$, state in ^{15}N was measured to be $\lambda_{\text{exp}} = 2.50 (\pm 0.23) \times 10^4 \text{ sec}^{-1}$. Raphael, Uberall, and Weratz¹⁶ had used a generalized Goldhaber-Teller model for nuclear giant dipole resonances and predicted that this rate should be $\lambda_{\text{theory}} = 3 \times 10^4 \text{ sec}^{-1}$.

The work described here continues in the spirit of probing nuclear

structure from the analysis of gamma rays following μ^- capture. To be specific the results of this experiment are correlated with the results of single nucleon pick-up reactions, photoexcitation and electro-excitation studies, and thermal neutron capture experiments as well as with theoretical nuclear structure calculations. These correlations then reveal the similarities of the various excitation techniques and show how much capture studies compliment and extend the information one can obtain about nuclear structure.

The muon capture reaction is shown schematically in Fig. 1a,b. The μ^- is captured by the nucleus from a 1s atomic orbit. The capture proceeds via the reaction

where M is the target nucleus and N is the product nucleus. Treating the nuclear recoil non-relativistically, conservation of energy and momentum give the neutrino energy for this reaction as

$$p_\nu c = \mu c^2 - \left[{}^A_{Z-1} N - \frac{A}{Z} M \right] c^2 - B.E._\mu - Q$$

where

$\frac{A}{Z} M$ = mass of the target nucleus

μ = rest mass of the muon

$B.E._\mu$ = K-shell binding energy of the μ^- meson

${}^A_{Z-1} N$ = mass of the product nucleus (in its ground state)

p_ν = momentum of the neutrino

Q = excitation energy of the product nucleus

c = velocity of light

A comparison of calculations made by Primakoff⁸ with experimental data indicates that the average neutrino energy following muon capture is approximately 80 MeV. The result of this reaction is shown on a nuclear

(a)

(b)

XBL 723-584

Fig. 1. Schematic representation of the μ^- capture process, a) before capture the muon is in the 1s Bohr orbit about the nucleus M, (b) the excited nucleus N^* recoils with momentum $\vec{p}_{N^*}$ following the emission of a neutrino with momentum $\vec{p}_\nu$.

energy level diagram in Fig. 2. The capture can leave the resulting nucleus in a bound state or it can excite a virtual state in the continuum. If excitation to an excited bound state occurs, gamma decay characteristic of this nucleus is observed. If the capture proceeds to a state in the continuum, particle emission occurs leaving a residual nucleus in either an excited state or its ground state. If the residual nucleus is left in an excited state, gamma rays characteristic of the residual nucleus are observed. It is these gamma transitions from bound excited states in the intermediate nucleus and residual nuclei that are detected in this experiment.

As a result of an analysis of these gamma rays, the final nuclear species formed following muon capture can be identified by identifying the spectral lines as gamma transitions between states of a particular nucleus. By comparing the probabilities of exciting particular nuclear states in other reactions such as pickup reactions the structure of states populated in muon capture can be identified (i.e., particle-hole, single-hole, etc.). Once the structure of states populated preferentially in muon capture is established, this reaction can be used in the classification of states whose structure is not known.

As a consequence of the large kinetic energy retained by the neutrino, the capturing nucleus recoils with some significant velocity. Short-lived nuclear levels may be distinguished from long-lived levels because of the Doppler broadening of lines for transitions with lifetimes shorter than the slowing down time of the recoiling nucleus. Furthermore, as subsequent particle emission effects the velocity distribution of the recoiling nucleus, the shape of Doppler broadened lines may help identify the process (for example, whether or not particle emission occurs) in ambiguous cases.

XBL 723-583

Fig. 2. Energy level diagram. Muon capture in a target nucleus A_Z can lead to bound or unbound excited (cross hatched region) states in the ${}^{A+1}_{Z-1}$ nucleus. The unbound excited states can then decay by particle emission, e.g. to excited states in the ${}^{(A-1)}_{Z-1}$ nucleus. The bold arrows represent the gamma ray transitions detected in this experiment.

Particle decay characteristics from excitation into the continuum may be determined from an analysis of gamma rays following muon capture. In particular a large fraction of proton or neutron plus proton emission may imply that the capture occurs on a nucleon cluster at the nuclear surface.¹⁷

A comparison of the muon capture results with those of photo-excitation and inelastic electron scattering experiments can reveal the type and amount of giant resonance mechanism in the muon capture process.

The results of an analysis of gamma-ray spectra following μ^- capture in Al, Si, Ca, and Co are reported here. In Chapter II the background material necessary to understand the muon capture nuclear excitation reaction is presented. A review of the theory of muon capture is given, pertinent aspects of the giant resonance mechanism are presented, and several muon capture experiments are reviewed. The experimental apparatus and procedure for this experiment are described in Chapter III. Chapter IV describes the data analysis. The experimental results are presented in Chapter V. The types of comparisons and correlations mentioned above are made in Chapter VI and in Chapter VII the conclusions drawn from this study are presented.

II. THEORY OF MUON CAPTURE

Since understanding the μ^- capture process is important in understanding and interpreting the final nuclear excitations which are a result of the capture reaction, a review of the muon capture reaction in complex nuclei is given in this chapter.¹⁸⁻²⁰ Several muon capture experiments are also reviewed and the giant resonance mechanism in nuclear reactions is discussed to lay the foundation for the presentation of our experimental results and the conclusions drawn from these results.

A. The Muon Capture Reaction in Complex Nuclei

The development of the theory of the muon capture reaction is given in several stages. First, a review of the derivation of an effective Hamiltonian for this reaction is presented. This review begins with a treatment of the simplified interaction of a muon with a single proton in the framework of the Universal Fermi Interaction (UFI) formalism and continues by discussing the steps leading from the description of this simple interaction to the Hamiltonian for the interaction of a muon with a complex nucleus. Then a formulation of the systematics of muon capture based on this Hamiltonian is presented. Finally, a review of the results of calculations (based on this Hamiltonian and specific nuclear models) of nuclear muon capture rates is given.

1. The Effective Hamiltonian

A muon (here only negative muons concern us) is a lepton. As such it can interact weakly, for example the muon decays

$$\mu^- \rightarrow e^- + \nu + \bar{\nu}$$

with a mean lifetime $\tau_m = 2.1983 (\pm 0.0008 \times 10^{-6})$ sec.²¹ The weak interactions are described in the framework of the Universal Fermi Interaction. The UFI theory is also used in describing muon capture by a nucleus, since this reaction proceeds via a weak interaction of the muon

with the nucleus. First, the theory is applied to a simplified interaction of four bare fermions

$$\mu^- + p \rightarrow n + \nu$$

as represented schematically in Fig. 3. This reaction corresponds to muon capture on ${}^1\text{H}$. The Hamiltonian for this reaction is written as a product of a nucleon current and a lepton current, where each term is the difference of a vector and an axial vector operator operating on the incident particle wave functions.²² This interaction is thus called the V-A interaction.

The muon capture reaction is entirely similar to the K-capture beta-decay process. In both K capture and muon capture the bare nucleon coupling constants must be renormalized to account for the fact that the nucleons are "dressed" in a meson cloud.²² In this picture, for example a proton may exist for a short time as a neutron and a π^+ meson. As a result of the fact that the nucleons are "dressed," an induced pseudo-scalar term and a weak magnetism term must also be included in the muon capture Hamiltonian as well as the vector and axial vector terms.

Using the Dirac equation, an effective Hamiltonian to describe the capture of a muon by an arbitrary nucleus is derived²³ from the V-A currents including the above mentioned terms. The probability of muon capture with a nuclear transition from state a to state b is then written as

$$W(a \rightarrow b) = \frac{2\pi}{\hbar} \langle b | H_\mu | a \rangle^2 \frac{dn}{dE}$$

where H_μ -- the effective Hamiltonian described above

$\frac{dn}{dE}$ -- energy density of final states for the system.

At this point we remark that the "constants" of the effective Hamiltonian are renormalized with respect to those of the muon capture on ${}^1\text{H}$ inter-

Fig. 3. Muon capture on a proton.

XBL 723-585

action. This is a consequence of the fact that nucleons inside the nucleus interact with a lepton field differently than free nucleons do. This is for two reasons: First, the "constants" in the effective Hamiltonian are really form factors dependent on the momentum of the neutrino. In the case of muon capture by a nucleus part of the energy is used up in overcoming the nucleon binding energy and part left as residual nuclear excitation, so that the neutrino receives considerably less energy than in the case of capture in hydrogen; second, the weak interaction between nucleons within the nucleus can be described by special diagrams that do not exist for the single-nucleon problem. They result when exchange meson currents within the nucleus are taken into account. In the example shown in Fig. 4 a π^+ and π^0 are being exchanged between two nucleons and the μ^- interacts with this pion pair to produce a neutrino. Bertero et al.²⁴ have estimated the contribution of this diagram to the total probability of muon capture to about 1%.

The theoretical approaches to the problem of muon capture have been generally speaking of three different types:

- a. a study of the partial capture rate in a nucleus to a specific final state in the daughter nucleus in which case one requires a reliable nuclear model to obtain wave functions for the final and initial states;
- b. a study of the total capture rate in a certain nucleus for which, again, a reliable nuclear model is required;
- c. or a study of the systematics of the total capture in an attempt to explain the differences between the capture rates for different nuclei.

2. Systematics of Muon Capture

The differences mentioned in the study of muon capture systematics

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Fig. 4. Muon interacting with a pion exchange pair.

are mainly due to two effects which may be explained quasi-classically. The capture rate is proportional to a) the number of capturing protons Z and b) the probability density for finding a muon within the nuclear volume $|\phi_\mu|^2$. For a point nucleus and non-relativistic wave functions

$$|\phi_\mu|^2 = \frac{Z^3}{\pi a_0^3}$$

where

$$a_0 = \frac{\hbar^2}{M_\mu e^2}$$

is the muon Bohr radius and hence the rate for muon capture will be proportional to Z^4 . As Z increases, the actual value of $|\phi_\mu|^2$ will not increase as rapidly as Z^3 because the root-mean-square muon orbit radius will become smaller than the nuclear radius. These effects were first included in the theory of Wheeler²⁵ who introduced the quantity

$$Z_{\text{eff}}^4 = Z^4 |\phi_\mu|^2 / \left(\frac{Z^3}{\pi a_0^3} \right)$$

to account for this behavior. The more elaborate theory of Primakoff⁸ makes use of this quantity and Primakoff shows that, in a closure approximation the capture rate will equal

$$\Lambda_T = R Z_{\text{eff}}^4 \left(\frac{\nu}{\nu_\mu} \right)^2 \left[1 - \frac{A}{2Z} F(\nu) \right]$$

where R depends upon the weak interaction coupling constants, $\nu_\mu = M_\mu c$, ν is the appropriately weighted average neutrino momentum and $F(\nu)$ is the Fourier transform of a certain two-particle nuclear correlation function. For this problem all of the nuclear physics is contained in $F(\nu)$. To see how well this scheme fits the data for various model assumptions one can consult the literature.^{8,26-30} In particular, Eckhause et al.⁶ and Sens⁷ present experimental results and compare them with the Primakoff

formula.

3. Total Capture Rates for Muon Capture

Work on the total capture rates for a particular nucleus has been directed toward the doubly magic nuclei (${}^4_2\text{He}$, ${}^{16}_8\text{O}$, ${}^{40}_{20}\text{Ca}$) and the even-even nucleus ${}^{12}_6\text{C}$ where the nuclear shell model should be reliable. Luyten, Rood and Tolhoek²⁹ considered ${}^{16}_8\text{O}$ and ${}^{40}_{20}\text{Ca}$ in a shell model calculation. Their rates were approximately 50% larger than the experimental rates. Barlow et al.³¹ used the shell model matrix elements of Luyten et al., but replaced the shell model particle-hole energies which were about 10 MeV below the giant resonance energy with the giant dipole energy from photo-excitation³² and calculated a capture rate for ${}^{16}_8\text{O}$ in good agreement with their experimental rate. Foldy and Walecka³⁰ related the nuclear matrix elements to the cross section of the giant dipole resonance in photoabsorption to calculate capture rates in ${}^4_2\text{He}$, ${}^{12}_6\text{C}$, ${}^{16}_8\text{O}$, and ${}^{20}_{10}\text{Ca}$. The agreement for ${}^{16}_8\text{O}$ and ${}^{40}_{20}\text{Ca}$ with the experimental rates was quite good. (For more detail see the section on the Giant Resonance Mechanism.) Uberall and Hill³³ apply the particle-hole model including residual interactions (i.e., the schematic model of Brown,³⁴ which is described in Sec. B of this chapter) to obtain total capture rates in ${}^{12}_6\text{C}$, ${}^{16}_8\text{O}$, ${}^{28}_{14}\text{Si}$, ${}^{32}_{16}\text{S}$, and ${}^{40}_{20}\text{Ca}$. Their results are tabulated next to the experimental rates measured by Eckhause et al.⁶ in Table I. The agreement between predicted and experimental rates is good for ${}^{12}_6\text{C}$. The discrepancy between predicted and experimental rates increases with increasing mass number to nearly a factor of two for ${}^{40}_{20}\text{Ca}$.

Table I

Nucleus	$\Lambda_{\text{exp}} [10^4 \text{ sec}^{-1}]$ a)	$\Lambda_{\text{th}} [10^4 \text{ sec}^{-1}]$ b)
^{12}C	3.71 ± 0.11	3.75
^{16}O	9.74 ± 0.31	11.08
^{28}Si	84.9 ± 0.3	130.048
^{32}S	133.8 ± 0.7	216.69
^{40}Ca	245.0 ± 2	435.3

a) Ref. 6.

b) Ref. 33.

4. Muon Partial Capture Rates

Early considerations of partial capture rates focused on muon capture in light nuclei, where wave functions were best known, and between levels where one type of transition (i.e., allowed, first forbidden, etc.) was dominant, so that conclusions about the strengths of the effective coupling constants could be drawn. These works included the study of muon capture in ^3He , ^6Li , and ^{12}C . Shell model calculations both with and without the inclusion of residual interaction terms have been made. Summing over the partial capture rates then gives the total capture rate as described in the previous section.

The assumed mechanism is that, depending on how much energy the neutrino carries away, the capture proceeds to either bound states of the intermediate nucleus or to quasi-stationary states above the energy for particle emission and therefore in the continuum. The states in the continuum then decay by particle emission unless this process is inhibited by angular momentum or isospin considerations to such an extent that gamma decay is competitive.

In order to relate 1) the energy spectra of particles emitted

following muon capture or 2) the population densities of final states in residual nuclei to muon capture, the widths for decay through various channels must be determined. Ueberall and collaborators³³ have used the R-matrix theory as developed by Boeker and Jonker³⁵ to determine the decay widths. Balashov et al.³⁶ have developed a "unified theory of direct and resonance processes" which they apply to muon capture to calculate partial transition rates to specific states of residual nuclei following particle emission.

The capture reaction

is shown on an energy level diagram, Fig. 5. In this diagram where only single neutron emission channels are shown, three of the levels in ${}^{16}_8\text{N}$ decay through both single neutron emission channels shown. The branching ratio is just the ratio of the widths for decay through the respective channels. By summing over all the excited states which can decay to a particular level in the residual nucleus one then has the partial capture rate to that level. The results of Raphael et al.¹⁶ and of Balashov et al.³⁶ for the partial capture rate to the $3/2^-$ state at 6.32 MeV in ${}^{15}_7\text{N}$ following muon capture in ${}^{16}_8\text{O}$ are $3.0 \times 10^4 \text{ sec}^{-1}$ and $2.65 \times 10^4 \text{ sec}^{-1}$ respectively. These rates agree well with the measured rate¹⁵ of $2.5(\pm 0.2) \times 10^4 \text{ sec}^{-1}$. Both authors have applied their models to muon capture in ${}^{40}_{20}\text{Ca}$. The situation is shown schematically in Fig. 6. Decay to the single proton hole states in ${}^{39}_{20}\text{K}$ (corresponding to the $(1d_{3/2})^{-1}$ ground state, $(2s_{1/2})^{-1}$ first excited state, and $(1d_{5/2})$ state at about 6 MeV) is considered. The calculations will be compared with the results of this experiment in Chap. VI.

5. Allowed and Forbidden Transitions

As in beta-decay, the neutrino plane wave emitted as a result of

XBL 723-607

Fig. 5. Energy level diagram showing levels populated in ^{16}N following muon capture in ^{16}O and neutron emission to the $3/2^-$ excited state and $1/2^-$ ground state in ^{15}N . (After Ref. 36.)

MUON CAPTURE IN ^{40}Ca

XBL 721-30

Fig. 6. Energy level diagram showing levels populated in ^{40}K following muon capture in ^{40}Ca and neutron emission to the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ single proton hole states in ^{39}K . (After Ref. 36.)

muon capture is expanded as a sum of spherical waves. The classification of transitions by degree of forbiddenness is also carried out in a manner analogous to that in beta decay theory.

The neutrino angular momentum ℓ determines the order of forbiddenness i.e., $\ell = 0$ "allowed," $\ell = 1$ "first forbidden," $\ell = 2$ "second forbidden," etc. For closed shell nuclei the largest contributions are due to the "first forbidden" transitions. As a result of the Pauli principle the matrix elements for allowed transitions are very small in these nuclei, (i.e., for an $\ell = 0$, no parity change transition the particle must skip one complete harmonic oscillator shell) thus the next order in the expansion becomes dominant. Because of the much larger neutrino wave number in muon capture than in K electron capture the expansion for the capture rate does not converge as rapidly for muon capture as for electron capture. So on this account it is expected that the first forbidden transitions need to be considered for all nuclei. The contributions of higher orders of forbiddenness to the capture rate become appreciable as the mass of the capturing nucleus increases as shown by Luyten, Rood, and Tolhoek.²⁹ They show that second forbidden transitions contribute considerably more to the muon capture rate in ^{40}Ca than in ^{16}O .

B. Review of the Giant Resonance

Since many of the gamma transitions observed in this experiment are from levels formed either directly in the giant resonance process or as a result of particle emission from an intermediate resonance, it is appropriate to review just what takes place in this interaction mechanism.

1. Giant Resonance Models of Photoexcitation.

The giant electric dipole resonance was first observed¹⁰ as a resonance in the photoreaction cross section occurring between 13 and 25 MeV with a width of from 3 to 10 MeV⁹ and was present in all nuclei. In the light elements (carbon through calcium), the giant resonance peaked near 20 MeV with little dependence on A.^{9,11}

Goldhaber and Teller³⁷ explained this phenomenon by assuming that "the gamma-rays excite a motion in which the bulk of the protons move in one direction while the bulk of the neutrons move in the opposite direction." The breadth of the resonance was described as "probably due to the transfer of energy from this orderly vibration into other, less orderly, modes of nuclear motion." This was an early expression of a concept that is now known as a "doorway state."⁹ Using this model the correct energy for the resonance was obtained and a width was accounted for. When experiments of higher resolution were performed³⁸ it was seen that the giant resonance had structure, which could not be accounted for by this collective oscillation model.

Wilkinson³⁹ applying the independent particle model, pictured the photonuclear reaction as taking place in three stages: first, the absorption of the photon and the excitation of a single nucleon to its excited state; second, the breakdown of that simple configuration by collisions among the nucleons and the eventual setting-up of a compound state, and third, the statistical decay of that compound state. One of

the strengths of the model is the possibility of the excited nucleon being emitted before the formation of the compound state, giving rise to a "resonance direct" effect. Figure 7 shows schematically single particle excitations of roughly the same excitation energy for a hypothetical nucleus with a partially filled $1f_{5/2}$ subshell which might contribute as an aggregate to a giant dipole resonance.⁴⁰ This model accounts for structure within the resonance and for narrower widths near closed shells, but gives a resonance energy nearly a factor of two too low.

Shell model calculations which account for residual interactions are in principle capable of a complete description of reaction processes. The calculations of Elliot and Flowers⁴¹ showed that the residual interactions effect a concentration of the collective strength into only a few excited states resulting in a width comparable to experiment and shifted the energy of these states up appreciably. These calculations, like those in many detailed descriptions, tend to lose sight of the simple systematic features that characterize the zero-order treatments.

The schematic model of Brown^{34,42} retains the physical interpretation and describes the process well for closed shell nuclei. In this model the dipole operator lifts a particle out of its initial state to some final state creating a particle-hole pair, Fig. 8a. (This treatment follows that of Brown given in Ref. 34.) The dotted line x --represents the amplitude D_{ph} for this absorption giving rise to the particle-hole state. This diagram is identified with the diagram representing the particle-hole interaction matrix elements, Fig. 8b. With this identification, the secular equation of the shell model describing the mixing of the particle-hole states due to the presence of the particle-hole interaction gives rise to the dispersion relation

GDR EXCITATIONS

XBL 721-32

Fig. 7. Energy level diagram for nucleus with a partially filled $1f_{5/2}$ subshell showing single particle-single hole excitations of approximately the same energy which can be excited by absorbing electric dipole radiation. After Ref. 40.

(a)

(b)

XBL 723-606

Fig. 8. (a) Particle-hole pair created by the absorption of a photon.
 (b) The particle and hole p' - h' interact via the residual interaction and scatter into different particle-hole states p - h .
 After Ref. 34.

$$1 = \sum_{p,h} \frac{\lambda D_{ph}^2}{E - (\epsilon_p - \epsilon_h)}$$

where ϵ_p and ϵ_h are the Hartree-Fock energies of the particle and hole. Lambda is the proportionality constant relating the gamma absorption amplitude to the particle hole matrix elements. A graphical solution to this equation is shown in Fig. 9. It is clear that one solution of this equation is at an energy significantly higher than the other solutions. Brown has shown that most of the dipole absorption strength is in this level.

For a situation where all the unperturbed eigenvalues-- $\epsilon_p - \epsilon_h$ --become degenerate the perturbed eigenvalue is given by³⁴

$$E = \epsilon_p - \epsilon_h + \sum_{m,i} \lambda D_{m,i}^2$$

Brown and Bosterli⁴² give each term in the sum as approximately 1 MeV. Thus the dipole state is shifted to a significantly higher energy than the energy of the unperturbed shell model states.

When less dramatic simplifying assumptions are made, the schematic model distributes the dipole strength for photon absorption in $^{16}_O$ as shown in Fig. 10.⁴¹⁻⁴³ The dipole strength given by the shell model without accounting for the residual interaction is shown for comparison. Also shown is the measured cross section for the $^{16}_O(\gamma, p)^{15}_N$ at 90 deg.⁴⁴

The isospin selection rules derived by Gell-Mann and Telegdi⁴⁵ allow only $\Delta T = 1$ transitions for photoexcitation of nuclei with $T = 0$ ground states. The doubly magic nuclei $^{16}_O$ and $^{40}_{Ca}$ and the even-even nucleus $^{28}_{Si}$ have $T = 0$ ground states and thus only $T = 1$ states are formed in photoexcitation of these nuclei.

Closed shell nuclei ground state spin and parity $J^\pi = 0^+$. If

ADL 71-1405

Fig. 9. Graphical solutions to the secular equation of the "schematic model." The eigenenergies in this model are the intersections of the curves and the dotted line. Note that one solution is at a significantly higher energy than the eigenenergy given by the shell model without residual interactions (the vertical lines). (After Ref. 34.)

5817-10-1

Fig. 10. Giant dipole resonance cross sections. Comparison of calculations and the experimental cross section of Ref. 44.

the nucleus also has the same number of neutrons and protons, as do ^{16}O and ^{40}Ca , the ground state isospin is $T = 0$. For a giant E1 resonance excitation the excited state has a spin and parity $J^\pi = 1^-$. Brown has shown that the giant dipole resonance, with protons vibrating against neutrons, corresponds to a $T = 1$ excited state.³⁴ To review, the schematic model predicts that the resonance state is pushed much higher in energy by the residual interaction and that most of the dipole strength is concentrated in this high energy state.

2. Application to Muon Capture

In doubly closed shell or doubly closed subshell nuclei muon capture leads to the creation of a pair¹⁸ consisting of a "particle" (neutron-n) and a "hole" (proton- p^{-1}) (Fig. 11a). As a result of the residual interaction between the particle and hole, one can be scattered by the other and the quantum numbers of the particle and hole may change during the scattering process, examples are shown in Figs. 11b and 11c. Hence the creation of a particle-hole pair, each with definite quantum numbers directly in the muon capture event does not mean (in contrast to the single-particle approach where the residual interaction between particle and hole is neglected) that these are the quantum numbers that characterize the final state of the nucleus (Fig. 11d).

Let $|p^{-1}n\rangle_{J_f}$ be a particle-hole state that is coupled to angular momentum J_f . Then the wave function ψ_{J_f} for the final nuclear state is a linear super-position of such particle-hole states as a result of the residual interaction. That is

$$\psi_{J_f} = \sum_{n,p} a(n,p^{-1}) |p^{-1}n\rangle_{J_f}$$

The particle-hole formalism has been extended by a number of methods,⁴⁶ all of which lead to the "random phase approximation" (RPA) in

Fig. 11. Muon-nucleon interactions. (a) Muon capture produces a neutron particle-proton hole pair (p^{-1}, n) . A particle-hole pair can interact via the residual interaction one (b) or more (c) times, creating different particle-hole states. (d) A muon capture produces a particle-hole pair (p^{-1}, n) which "scatters" twice resulting in the particle-hole state $[(p')^{-1}, n']$.

the lowest order. The results of these sophisticated calculations are not significantly better than those of the simpler schematic model.

The giant resonance picture for nuclear reactions has also been extended.⁴⁷⁻⁴⁹ To the giant E1 dipole vibrations already discussed, where in a $T = 1$ state protons vibrate against neutrons (shown schematically in Fig. 12a) which are called isospin or i-waves, are added spin (s) waves (Fig. 12b) and spin-isospin (si) waves (Fig. 12c). The spin waves, where nucleons with spin up vibrate against nucleons with spin down are not easily excited. The si-waves ($S = 1$), in the dipole case ($L = 1$) form states $J = 0^-$, 1^- and 2^- , whereas the i-waves ($S = 0$), appear only as one state with $J = 1^-$. In photo-absorption, only i-waves are strongly excited, in contrast to electro-excitation where both i and si-waves are excited. In muon capture, the Fermi interaction with the single particle τ_+ operator excites i-waves and the Gamow-Teller part of the interaction with the single particle operator $\tau_+ \sigma$, excites si-waves. The model includes the possibility of exciting giant quadrupole ($L = 2$) vibrations. The giant quadrupole vibrations occur at higher excitation energies than the dipole vibrations. Ubersall has also suggested that giant M1 resonances should take place via a magnetic dipole transition.⁵⁰ Fagg et al.⁵¹ have seen these 1^+ states excited strongly in the 180 deg electron scattering reaction on ^{28}Si . A selection rule due to Morpurgo⁵² strongly inhibits $T = 0$ magnetic dipole transitions in nuclei for which $N = Z$, so that only $T = 1$ states are generally excited. It should therefore be possible to excite the isobaric analog state in ^{28}Al via muon capture in ^{28}Si .

Thus it is clear that muon capture is a complementary method to photo-excitation and electro-excitation for studying high energy excitations in nuclei.

XBL 723-604

Fig. 12. Generalized nuclear resonance vibrations.

(a) isospin vibration, (b) spin vibrations, (c) spin-isospin vibrations. See text for detailed discussion.

C. Particle Emission following Muon Capture

Neutrons are the most likely candidates for particle emission following muon capture. The reasons for this are both the excited neutron character of the capture process and the fact that protons, deuterons, and alpha particles have to overcome the Coulomb barrier to be emitted from a nucleus.

1. Neutron Multiplicities

A compound nucleus model was used to interpret the neutron multiplicity results of MacDonald et al.¹² In this model the neutron spectrum following muon capture is the result of boil off neutrons, resulting when the neutron formed during the μ^- capture quickly shares its kinetic energy with the other nucleons to form a compound nucleus. There was not quantitative agreement between the multiplicities predicted by the model and those observed experimentally. The inclusion of direct emission processes, as calculated by Singer⁵³ improved the agreement for Ag, I, Au, and Pb, and brought the predicted multiplicities for Ag into agreement, within statistics, with the measured value, but did not improve the agreement for Al, Si, Ca, and Fe. The multiplicities measured by MacDonald et al. are compared with the limits set by this experiment in Chap. VI.

2. Neutron Energy Spectrum

The energy spectrum of neutrons emitted following muon capture has been studied in several nuclei. Evseev¹³ has measured the neutron spectra from ^{16}O , ^{32}S , ^{40}Ca and natural Pb. Plett and Sobottka⁵⁴ have results in ^{16}O and ^{12}C . Both authors looked in the energy range where the neutron kinetic energy was below 15 MeV. Resonance structure is clear in their spectra. Sundelin and Edelstein⁵⁵ show neutron spectra from ~ 10 to ~ 45 MeV in Si and Ca. A composite curve of the neutron spectra following muon

capture in Ca is shown in Fig. 13. A direct reaction model as considered by Bogan⁵⁶ fits the high energy (above 20 MeV) portion of this spectrum well. The boil off spectrum of a compound nucleus model corresponding to a temperature of 1.5 MeV fits part of the low energy range well.¹³ The resonance models with calculations by Balashov et al.³⁶ and Uberall³³ and collaborators predict resonances such as those near 10 MeV.

3. Charged Particles

The spectrum of charged particles emitted following μ^- capture has been determined by analyzing stars formed in emulsions as a result of the muon capture.⁵⁷ The boil off spectrum for an initial Fermi momentum distribution with a temperature of 9 MeV fits the alpha spectrum well,⁵⁸ but grossly underestimates the proton distribution. Singer⁵³ has suggested an additional emission mechanism: direct muon absorption by correlated nucleon pairs, which is analogous to the well-known "quasi-deuteron" mechanism of high-energy gamma absorption. The calculations show that the pair absorption mechanism makes a notable contribution to the total capture probability and that it can explain the high proton yield.

We note here that another source of fast protons in the muon-capture process might be a peculiar muon-absorption mechanism produced by an interaction between the muon and the exchange currents in the nuclei. Bertero²⁴ and co-workers have shown that for a Fermi gas model in the closure approximation that this process has about a .01 probability in ^{40}Ca .

The resonance capture theory also provides a mechanism for proton emission. The formation of intermediate quasi-stationary nuclear (i.e., the giant resonance) states in muon capture leads to strong coupling between the neutron and proton decay channels. (See the section describing the giant resonance model for more detail.) One may therefore

NRL 70-104

Fig. 13. Spectrum of neutrons emitted following muon capture in Ca, synthesized from data of Refs. 13 and 55.

expect an effect that is entirely forbidden in direct capture theory: the transfer of nuclear excitation energy to the proton rather than to the neutron. As in the Singer theory, the transfer of excitation to a proton that has not participated directly in the muon absorption event may be the result of correlations between the nucleons.

In this model the nucleon correlations are imagined to result in the excitation of two particle-two hole states where both holes are proton holes and one particle is a proton and one particle is a neutron. The particle decay is then that of the proton particle. A distinguishing feature is that in this model the proton spectrum would contain resonances just as the neutron spectrum does. This model has also been applied as an additional mechanism to explain the resonance structure and high yield of protons following photoexcitation.³⁶

III. EXPERIMENTAL APPARATUS AND PROCEDURE

A. Muon Telescope and Ge(Li) Detector

The experiment was performed at the Berkeley 184-inch cyclotron. The physical layout of the meson cave is shown in Fig. 14. A beam of protons was accelerated in the cyclotron and then collided with an internal Be target. The fringe field of the cyclotron bent a beam of the negatively charged reaction products (π^- , μ^- , and e^-) into the slot in the meson wheel and this beam then passed through a bending magnet, which selected particles with an average momentum of 180 MeV.c, and was focused on the target with a double quadrupole magnet. Coincidence signals from the counters of a muon telescope consisting of four scintillation counters (S_1 , S_2 , S_3 , and A), a water Cerenkov counter (C) and a variable thickness CH_2 degrader were used to signal a muon stop in the target. The target was placed at a 45 deg angle to the beamline and sandwiched between counters S_3 and A. The quadrupole magnet currents were adjusted to give the maximum stopping rate in the target when the degrader thickness was set at the muon range. (See the differential range curve shown in Fig. 41.) With the degrader set at this thickness, most of the pions were stopped in the degrader since the range of pions is shorter than the range of muons of the same momentum. The electrons in the beam produced Cerenkov radiation in the Cerenkov counter, so that electrons stopping in the target could be eliminated by putting signals from counter C in anti-coincidence for the muon stop signature. Signals from counter A were also put in anti-coincidence for the stop signature since a particle causing a scintillation in counter A could not have stopped in the target. The signature for a muon stop in the target was then $S_1 S_2 S_3 \overline{AC}$, that is a coincidence of signals from each of the counters S_1 , S_2 , and S_3 and no signal from either counter A or C.

NBL 721-15

Fig. 1^b. Experimental configuration of the meson cave at the Berkeley 184-in. cyclotron for the muon gamma ray experiment.

A 15 cm³ Ge(Li) detector with planar geometry was set at 90 deg to the beamline to detect photons emitted from the target. The detector was collimated using Pb bricks to reduce the photon background and surrounded with boric acid bricks to reduce the neutron flux on the detector. Figure 14 also shows the Ge(Li) detector, the degrader and the muon telescope. A delayed anti-coincidence of signals from counter A with the Ge(Li) signals prevented muon decay electrons from being counted.

B. Pulse Height Analysis System

The photon spectra were accumulated in a pulse height analyzer (PHA). The PHA system consisted of an analog to digital converter (ADC) of the successive approximations type,⁵⁹ a PDP-5 computer and an M-6 data disc. The PDP-5 program for this system has been described elsewhere.⁶⁰ The pulse output of the Ge(Li) detector was put in coincidence with one of four time gates and stored in one of four 4096 channel pulse height spectra by the PHA system. The Ge(Li) prompt pulse was delayed with respect to the counter telescope muon stop trigger to allow the accumulation of a "negative time" spectrum corresponding to photons uncorrelated with a stopping muon. A schematic diagram of the electronic circuitry is shown in Fig. 15.

This "negative time" spectrum was stored as G1. Mu-mesic x rays and prompt gamma rays occurring within a 50 nsec band about a muon stop were recorded in a second spectrum G2. Two additional spectra corresponding to photons detected in two consecutive time intervals following a muon stop were taken (G3, G4). The time relationship of the muon stop trigger pulse and the four routing gates with a Ge(Li) signal is shown in Fig. 16. The Ge(Li) pulses γ_1 , γ_2 , γ_3 , and γ_4 are stored respectively in the negative time, prompt time, short delay and long delay spectra. Figure 17 shows the first 1600 channels of the data accumulated in four hours for the Ca target. The Ca mu-mesic K x-ray series is evident in G2. The positron annihilation line at 511 keV, the $^{10}\text{B}(n,\alpha)^7\text{Li}$ line at 478 keV, and the peak due to inelastic neutron scattering in the Ge(Li) detector, $^{72}\text{Ge}(n,n')^{72}\text{Ge}$, are background lines labelled on the G1 spectrum. To varying degrees both the x-rays and these background lines occur in G3 and G4. Gamma-ray peaks at energies corresponding to nuclear transitions in ^{39}K , ^{39}Ar and ^{38}Ar identified in G3 appear less intensely in G4.

SBL 72-1114

Fig. 15. Electronics set-up for muon gamma ray experiment.

XBL 723-617

Fig. 16. Time relationship of Ge(Li) signals to the muon stopping trigger. The Ge(Li) signals were delayed electronically as shown to allow a segmentation into four different spectra according to their time of arrival relative to a muon stop. For example, a pulse γ_1 was routed to the background spectrum G1, γ_2 to a prompt spectrum G2, γ_3 to a short delay spectrum G3, and γ_4 to a long delay spectrum G4.

Fig. 17. First 1600 channels of data accumulated in four hours for the Ca target. From bottom to top the spectra are minus time (G1), prompt (G2), short delay (G3), and long delay (G4).

C. Recording of Results

The data was collected over a time span of approximately three months. Many runs of approximately two hour duration were made with each target. The pulse height data from each of these runs was written onto magnetic tape. Later the data for each target was summed as described in the chapter on data analysis. Energy calibration runs were taken frequently throughout the experiment and also recorded on tape. Elementary data analysis, such as summing counts in groups of channels, was done using the PDP-5 computer, mainly as preliminary consistency checks on the data.

D. Calibrations1. Energy Calibration

Energy calibrations were taken frequently during the experiment using ^{60}Co and ^{24}Na sources and the ^{16}O γ rays from the β decay of ^{16}N formed in the reaction $^{16}\text{O}(n,p)^{16}\text{N}$ on the cyclotron platform. A digital gain stabilizer set on a pulser at an amplitude corresponding to 6 MeV compensated for system drift throughout the course of data collection. Agreement of energy assignments for well known gamma transitions in our data which are tabulated in Table II indicate an uncertainty of less than 1 keV over an energy range which extends from about 250 keV to 4000 keV.

Table II. Energy Accuracy.

Line (reference)		Published Energy (keV)	This experiment energy (keV)
Al K_{α} x ray	4	347.2 ± 0.5	347.3 ± 0.5
Si K_{α} x ray	4	400.4 ± 0.5	401.1 ± 0.5
Ca K_{α} x ray	4	783.85 ± 0.15	784.9 ± 0.5
Ni K_{α} x ray	4	1427.4 ± 0.5	1426.7 ± 0.5
^{26}Mg 1809 $\rightarrow$ 0	98	1808.8 ± 0.5	1808.1 ± 0.5
^{26}Mg 3941 $\rightarrow$ 2938	98	1129.5 ± 0.3	1129.9 ± 0.5
^{26}Mg 3941 $\rightarrow$ 1809	98	2131.3 ± 2.0	2130.7 ± 0.5
^{26}Mg 2938 $\rightarrow$ 0	98	2937.7 ± 1.5	2941.4 ± 0.5
^{38}Ar 2168 $\rightarrow$ 0	124	2167.7 ± 0.14	2166.6 ± 0.5
^{38}Ar 3810 $\rightarrow$ 2168	124	1642.3 ± 0.14	1642.4 ± 0.5
^{38}Ar 3936 $\rightarrow$ 0	124	3936.1 ± 0.5	3938.3 ± 0.5
^{39}Ar 1267 $\rightarrow$ 0	82	1266.5 ± 1.0	1266.8 ± 0.5
^{39}Ar 1516 $\rightarrow$ 0	86	1516.5 ± 1.0	1517.4 ± 0.5
^{56}Fe 847 $\rightarrow$ 0	125	846.8 ± 0.1	847.3 ± 0.5
^{56}Fe 2085 $\rightarrow$ 847	125	1238.3 ± 0.1	1237.9 ± 0.5
^{57}Fe 367 $\rightarrow$ 14	80	352.5 ± 0.1	352.4 ± 0.5

2. Efficiency Calibration

Recording the prompt spectrum for several elements in the same experimental configuration under which the nuclear gamma ray data was taken provided data for an absolute detector efficiency determination.^{15,61} It was assumed that each stopping muon produce a K x ray. Cascade calculations by Eisenberg and Kessler⁶² show that this leads to an error of at most 3% for Al and less for heavier elements. Ratios of the yield of K_{α} x rays to the sum of all K x rays were determined for several elements. These ratios are presented in Table III and compared with other published values. This ratio was assumed to be 0.9 for the Ag, Sn and Pb targets. (The ratio for Pb has been calculated by M. K. Sundareson⁶³ to be 0.913. This number is based on the calculation of a cascade in the field of a finite nucleus.) Corrections made during the efficiency calculations were as follows: target self attenuation and detector solid angle weighted by the muon stopping distribution; fraction of muons which decay in orbit; and accounting for muon stopping triggers $S_1 S_2 S_3 \overline{AC}$ which correspond to muons stopping in scintillator S_3 . (For a detailed description of how these corrections were made see Appendix A.) The full energy peak (FEP) absolute efficiency obtained in this manner covered an energy range from 345 keV to 5960 keV and was close to an exponential up to 3950 keV as can be seen in Fig. 18. To extend the energy range and determine the response shape in more detail an FEP relative efficiency curve with measurement at 37 energies from 59 keV to 4072 keV was made using standard IAEA sources⁶⁷ and other radioisotopes.⁶⁸ This curve was then scaled to the raw efficiency at the K_{α} x-ray energy for the target as described in Appendix A and target self attenuation corrections as a function of energy and geometry corrections weighted by the muon stopping distribution were made relative to this value. A relative double escape

XBL 721-28

Fig. 18. Absolute efficiency of Ge(Li) detector.

Table III

Target	$K_{\alpha}/\Sigma K$ (%)					
	My measure- ment	Eisenberg and Kessler a)	Suzuki b)	Quitman c)	Johnson d)	Pratt e)
¹³ Al	80±2.7				81±10%	
¹⁴ Si	79±2.7	79	79±2.2			
²⁰ Ca	81±2.0	83	81±1.7	83±6	84±10	87±3.0
²⁶ Fe	73±2.2				75±10	72±9.0
²⁷ Co	74±1.9			71±10	75±10	
²⁸ Ni	77±1.7			72±12		
²⁹ Cu	76±2.5			68±8		
⁴² Mo	82±3.1					87±5
a) Ref. 62		c) Ref. 65	e) Ref. 61			
b) Ref. 64		d) Ref. 66				

peak (DEP) efficiency was scaled to the absolute DEP efficiency at the Mo K_{α} x-ray energy. A best fit of the DEP efficiency response for a 15 cm^3 planar detector as calculated by Euler and Kaplan⁶⁹ was made to these points plus the absolute DEP efficiencies as measured for the Ag, Sn, and Pb targets. Self attenuation corrections as a function of energy were then made for each target. The effective efficiency curve for Ca showing both the FEP and DEP efficiency is seen in Fig. 19.

3. Experimental Sensitivity

An experimental sensitivity as a function of energy has been determined for each target. This sensitivity is the yield of a transition which will be seen above the background with a 90% confidence level and is therefore a function of both the detector efficiency and the background. As a function of the background that must be contended with, it depends upon the peak width, determined by the detector resolution (for sharp peaks) or a combination of resolution and characteristic broadening (for Doppler broadened peaks). One therefore has an FEP sensitivity and a DEP sensitivity, as well as a sensitivity for the detection of sharp peaks which is different than the sensitivity for the detection of broad peaks.

The target nucleus always recoils from the emission of a neutrino following muon capture. If particle emission occurs, the nucleus recoils again. If the lifetime of the nuclear state from which a de-excitation gamma originates is short compared to the slowing down time of the nucleus, the spectral line of the gamma transition is Doppler broadened. This phenomenon is discussed in more detail in a following section.

The sensitivity was determined from the expression

XBL 723-100

Fig. 19. Ge(Li) detector-system efficiency.

$$S(E) = \frac{N \sqrt{2} \sum_{i=\text{start}}^{\text{stop}} \text{bkg}_i}{\mu_S W (\lambda_n / \lambda_m)} \text{eff}(E)$$

where bkg_i = background in the i th channel

$N = 1.658$ for 90% confidence

start = channel corresponding to lower limit for background

stop = channel corresponding to upper limit for background

$\text{eff}(E)$ = detector efficiency at energy E

μ_S = number of muon stops in the target

W = ratewidth efficiency (described in a following section)

λ_n = nuclear capture rate

λ_m = total muon disappearance rate

The start and stop channels in the sum were determined as follows. For the sharp peak sensitivity: The detector resolution (FWHM) at energy E was determined from a linear least squares fit to the experimental resolution data points. The start and stop channels were then taken to correspond to energies $E \pm \sigma$, where $\sigma = \text{FWHM}/2.345$. For the broad peaks: Broad peak sensitivities were based on broadening due to isotropic nuclear recoils from an 80 MeV neutrino and a 3 MeV neutron. The start and stop channels included the full base width of the resulting triangular shaped line. (For details see the section in Chap. IV on Doppler broadening.)

The detection sensitivity for the Ca target is shown in Fig. 20. A sensitivity of 0.003, for example, implies that the yield of a transition of 0.003 per nuclear muon capture will be seen above background with 90% confidence. As mentioned above, the reduced sensitivities for peaks that are Doppler broadened because of decay in flight are due to the greater background areas under the broader peaks.

ADL 112-1917

Fig. 20. Detection sensitivity for the Ca target.

IV. ANALYSIS OF DATA

This section describes how the data was reduced. Several complications are discussed. The derivation of an effective gatewidth which corrects for the finite counting time is given. The background subtraction procedure is described. The fast neutron damage to the detector is described. The significant Doppler broadening of many spectral lines is pointed out. The method used for determining net peak areas is described. The technique for determining the energy of a gamma transition and criteria used for the identification of such transitions are discussed. A formula giving the yield of a particular gamma transition is presented. Finally, the population of state calculations are described.

The data was collected in many runs per target, each of approximately two hours duration. Energy calibrations were taken at least every eight hours during data collection. For the data reduction these short runs for each target were summed. Table IV displays the number of muon stop triggers and physical characteristics for each target.

Table IV

Target	Thickness (gm/cm ²)	Density a) (gm/cm ³)	Net stops (10 ⁹)
Al	6.87	2.70	4.22
Si	6.66	1.31	3.85
Ca	9.78	1.55	4.31
Co	5.18	3.41	2.31

a) Density for Si and Co actual density of packed powder.

A. Gate Width Correction

The muon disappearance rate in Al, Si and Ca is sufficiently slow that data in both the G3 and G4 spectra was used for these targets. The shorter muon lifetime in Co reduced the signal to background ratio in G4 for this target considerably and thus yields in Co were determined from the G3 spectrum. A comparison of the yield for a line determined from

the G⁴ spectrum distinguishes between gammas which are associated with a muon capture and background gammas such as those from thermal neutron capture which are equally intense in the G₁, G₃ and G⁴ spectra.

Since the time gate during which Ge(Li) pulses were counted in G₃, or in G₃ + G⁴ was finite, a gatewidth correction must be made in the determination of the yield of a gamma ray per muon stop. The gatewidth efficiency W is the integral over the gatewidth of the probability of a muon disappearing per unit time once it is captured into the 1s atomic orbit of a mu-mesic atom. The equation for W is

$$W = \lambda_m \int_{t_1}^{t_2} e^{-\lambda_m t} dt$$

where λ_m = total disappearance rate in target

t_1 = time after muon stop when gate opens

t_2 = time after muon stop when gate closes

Table V gives the disappearance rates in Al, Si, Ca, and Co; and W for G₃ + G⁴ in Al, Si, and Ca; and W for G₃ in Co.

Table V

Target	Disappearance Rate ^{a)} (10 ⁶ sec ⁻¹)	Gatewidth efficiency ^{b)}
Al	1.156±0.005	0.434±0.013
Si	1.304±0.003	0.471±0.014
Ca	2.915±0.025	0.716±0.028
Co	5.400±0.044	0.654±0.048

a) Ref. 6.

b) Includes G₃ and G⁴ for Al, Si, and Ca but only G₃ for Co.

B. Detector Deterioration

During the experiment the shape of the detector response to single sharp lines deteriorated noticeably. Figure 21 displays this response, near the beginning and end of the experiment, to the 2754 keV nuclear transition following the β decay of a ^{24}Na source. It is known^{70,71} that fast neutrons can cause this kind of detector damage. The changes in the peak shapes are similar to those described by Chartrand and Malm.⁷¹

An estimated integrated neutron flux to which the detector was exposed is 10^9 to 10^{10} neutrons. When the beam was directed into the meson cave and onto our target the detector was exposed to a large neutron flux. These neutrons came directly from the target, approximately one neutron per stopped muon,¹² and from the degrader, approximately three neutrons per stopped pion.⁷² The pion intensity in the beam is approximately ten times the muon intensity. (See the differential range curve, Fig. 41 in Appendix A.)

The detector was well shielded from neutrons produced at the location of the stopping pions, so that only the very high energy neutrons from this source reached the detector. Other factors which tend to reduce the significance of neutrons produced by the stopping pions are the CH_2 attenuation of these neutrons and the relatively small solid angle the detector sees for this source. The total number of muons stopped was approximately 2.5×10^{10} . Assuming a point source and a fractional solid angle subtended by the detector of 9×10^{-3} an estimated 2.3×10^8 neutrons strike the detector. Perhaps half as many muons stopped in the target during the beam spike as during the beam spill, so that possibly as many as 3.5×10^8 neutrons struck the detector as a result of muon capture in the targets. The detector was in the meson cave for five months. When other experiments were taking beam the telescope and

XBL 72-1-51

Fig. 21. (a) Detector response for the 2754 keV ^{24}Na line near the beginning of the experiment. (b) Response to the same line near the end of the experiment.

detector apparatus were rolled away from the beamline. However, the neutron flux over a large portion of the meson cave when the beam is directed into the meson cave is approximately $200 \text{ n/cm}^2\text{-sec}$. Assuming the beam was directed into the meson cave 25% of the time the total flux incident on the detector over the five month period is approximately

$$\begin{aligned} & (200 \text{ n cm}^{-2} \text{ sec}^{-1})(5/12)(\pi \times 10^7 \text{ sec}) \\ & \approx 8.3 \times 10^8 \text{ n cm}^{-2} \end{aligned}$$

from background alone.

This estimated integral flux on the detector is the same order of magnitude as that determined by Kraner, Chasman and Jones⁷⁰ to cause significant detector damage. They observed a degradation from 3.6- to 6.5-keV FWHM of the peak shape of the 1.33 MeV ^{60}Co line after irradiating a detector with 5.75 MeV neutrons with an integral flux of $6.7 \times 10^9 \text{ n/cm}^2$. The deterioration of the 1.33 MeV peak was more severe than that of the 1.17 MeV peak. They also showed that higher energy neutrons do more damage per unit flux than lower energy neutrons. This effect is presumably due to the cascade of the struck Ge atoms causing dislocations as they slow down.

The energy spectrum of neutrons incident on the detector while this experiment was taking beam extends to the many tens of MeV region. The energy spectrum of neutrons following muon capture extends up to 60 MeV.⁵⁵ The average neutron energy following the star formation in π^- capture measured in ^{12}C is 20 MeV⁷² and the spectrum extends up to 100 MeV.⁷³ Thus it is clear that the estimated neutron flux caused the peak shape deterioration observed over the course of the experiment, and in an experiment such as this such damage must be accepted to some degree.

Data for each target was collected over an extended period and in

many segments. These separate segments were then summed for the data analysis. But each segment of the sum had a slightly different detector response as described above. An effective resolution vs. energy plot, for the summed data (see Fig. 25 in sec. C) was obtained by summing energy calibration runs over the same time span and taking the FWHM of peaks on this spectrum as the effective resolution.

C. Doppler Broadened Lines

Many lines in our spectra were Doppler broadened. Figure 22 shows four double escape peaks (DEP) of gamma rays near 6 MeV which have been identified as transitions in ^{39}K following muon capture in ^{40}Ca . The bars are the effective detector resolution, full-width at half-maximum (FWHM), of the detector for sharp peaks. All four lines are broadened considerably.

The target nucleus always recoils from the emission of a neutrino following muon capture. If particle emission occurs, the nucleus recoils again. If the lifetime of the nuclear state from which a de-excitation gamma originates is short compared to the slowing down time of the recoiling nucleus, the spectral line of the gamma transition is Doppler broadened.

Examples of Doppler broadening arising from recoils due to the emission of both a neutrino and a neutron were shown above (i.e., the 6 MeV transitions in ^{39}K). An example of broadening without particle emission is the 2175 keV gamma transition in ^{28}Al following muon capture in ^{28}Si . (Fig. 23) The particular significance of this transition, that is, as evidence for a giant M1 dipole resonance mechanism in muon capture, is discussed in the comments on the results.

The slowing down time of recoiling nuclei in targets for nuclei in the $27 \leq A \leq 40$ mass region is 10^{-12} to 10^{-13} sec.⁷⁴ As evidence that many Doppler broadened lines should be expected, known lifetimes⁷⁵ of several levels in ^{40}K are listed in Table VI. In fact, these lifetimes were measured using the Doppler Shift Attenuation Method. It can be seen that many lines corresponding to transitions in ^{40}K could be broadened. Thus one should expect gamma transitions from levels with lifetimes less than 10^{-12} sec to be Doppler broadened for the nuclei

DOPPLER BROADENED LINES FROM REACTION $^{40}\text{Ca}(\mu^-, \nu n)^{39}\text{K}$

XBL 719-1340

Fig. 22. Doppler broadened lines in the Ca spectrum.

BROAD LINES NEAR 2 MeV in SILICON TARGET

XBL 719-1341

Fig. 23. Doppler broadened lines in the Si spectrum.

Table VI. Mean lifetimes of nuclear levels in ^{40}K . a)

Excitation energy (keV)	τ_m (fs)
800	500±150
892	1500±450
1959	1100±350
2047	560±170
2070	700±300
2104	1000±300
2261	80±30
2290	110±40
2291	250±80
2397	50±20
2419	> 1000
2576	100±30
2626	370±110

a) Ref. 75

studied in this experiment.

Line shapes which result from 1) nuclear recoil due to the emission of a neutrino, and 2) nuclear recoils from the emission of a neutrino and then particle emission of a neutron, are derived in Appendix B. Case 1 results in a square distribution, such as shown in Fig. 24a. The trapezoidal distribution shown in Fig. 24b arises in case 2.

In a previous paper⁷⁶ an analysis of the Doppler broadened peak at 6.323 MeV in ^{15}N following muon capture in ^{16}O showed that the data is consistent with the emission of an 80 MeV neutrino and a 3 MeV neutron. The nuclear recoil from both of these processes has the same magnitude. For this condition the trapezoidal distribution of case 2 above becomes triangular. This distribution is shown in Fig. 24c. These same conditions were assumed in estimating the widths of Doppler broadened

(a)

(b)

(c)

XBL 723-1220

Fig. 24. Calculated Doppler broadened line shapes for (a) a nucleus moving with velocity v resulting from the recoil due to the nuon-neutrino, (b) a nucleus recoiling from the neutrino to the emission of a neutron, and (c) same as (b) with $v_v = v_n$.

peaks due to neutrino plus neutron emission in the present experiment.

We note that the direction of the emitted neutrino is correlated with the residual longitudinal muon polarization,^{77,78} and therefore the assumption of an isotropic nuclear recoil is not strictly justified. However, as just noted, a treatment which neglects this correlation, describes the Doppler broadened peak in ¹⁵N following muon capture in ¹⁶O well. These correlations are therefore neglected in the approximations developed here.

To estimate total widths for Doppler broadened lines the estimated Doppler broadened FWHM and the detector resolution were added in quadrature. Figure 25 shows the detector resolution and the estimated total FWHM for Doppler broadened lines corresponding to gamma transitions in ²⁷Al and ²⁸Al, and ³⁹K and ⁴⁰K following muon capture in ²⁸Si and ⁴⁰Ca. For the case of no neutron emission the neutrino energy was taken to be

$$E = p_{\nu}c = \mu c^2 + \frac{A}{Z-1} M^* c^2 - \frac{A}{Z} M c^2 - B \cdot E_{\mu}$$

where the symbols are as defined earlier.

XBL 721-33

Fig. 25. Comparison of Doppler broadened peak widths with the detector resolution.

D. Determination of Net Peak Areas

1. Background Subtraction

Gates for the negative time G1, short delay G3, and long delay G4 spectra were of equal duration in time. This arrangement allowed a direct subtraction of the background (i.e., $G3 - G1$, $G4 - G1$, or $G3 + G4 - 2G1$). Net peak areas were then determined from these "data minus background" spectra.

2. Peak Areas from Data Minus Background Spectra

Since many lines could be expected to be broader than the detector resolution, a fitting routine utilizing expected peak shapes using parameters determined from the detector response to isolated sharp peaks proved to be unfeasible in the data analysis. Instead a simple procedure of choosing a region under the peak and a region on each side of the peak which determined a linear background was adopted. The total number of counts under the peak minus the linear background in that region was then taken as the net peak area.

This procedure precluded the separation of lines lying very close in energy, but in most cases such as this the corresponding parent levels had another branch so that reasonable estimates on the relative yield assignments could be made.

E. Line Identification

Lines in the summed spectra were assigned as due to nuclear transitions on the basis of the following criteria:

- a. agreement with energy of a known transition within combined errors;
 - b. the presence of branching transitions in the data when a particular nuclear level decays through two or more branches;
 - c. An assignment to a transition in a nucleus that might reasonably result following muon capture in the target; and
 - d. the presence of both a FEP and DEP in energy regions where the experimental sensitivity is large enough for both peaks to be seen.
- (See Table VII for references used for known energy levels.)

Table VII

Nucleus	Reference	Nucleus	Reference
^{25}Mg	98	^{58}Fe	126
^{26}Mg	98	^{59}Fe	127
^{27}Mg	98	^{38}Cl	98
^{26}Al	98	^{38}Ar	98, 124
^{27}Al	98	^{39}Ar	82, 98
^{28}Al	91, 98, 101	^{38}K	98
^{56}Fe	125	^{39}K	98, 128
^{57}Fe	79, 80	^{40}K	98, 129

F. Determination of Transition Yield per Muon Stop

The yield of a transition of energy E per stopped muon is given by

$$Y(E) = \frac{1}{\mu_S W (\lambda_n / \lambda_m)} \frac{A}{\epsilon(E)}$$

where

$\epsilon(E)$ = the detector efficiency interpolated linearly between
calibration points for effective efficiency for the target

A = net area under peak

μ_S = net muon stops in target

W = gatewidth efficiency

λ_n = nuclear capture rate in target

λ_m = total muon disappearance rate in target

When both FEP and DEP lines were observed an average transition yield was computed.

G. Population of State Calculations

The probability of populating a level per muon capture (that is the excitation probability/capture) was calculated as follows. All transitions in a residual nucleus were identified. Cascades from higher levels were subtracted in the calculation. When a level decays through two or more branches, level populations were determined by averaging the yield of a transition divided by its branching ratio weighted by the errors in this ratio, over all branches seen. Table VII lists the references used for the energies and branching ratios in these calculations. See Appendix C for a detailed discussion of these calculations.

V. EXPERIMENTAL RESULTS

The basic results of the experiment are presented in this chapter. A nuclear level diagram which displays the states excited following muon capture which were detected by observing the decay gamma rays is presented for each target. The results are also given in tabular form for each target. The tables include population of state probabilities, which were calculated as described in the previous chapter. The results of other muon capture gamma-ray experiments are listed for comparison when such results are available. Where there are ambiguities in the assignments of gamma transitions a discussion of the alternatives is presented.

A. Aluminum

A nuclear level diagram showing schematically the intensities and identifications of observed peaks in the summed G3 and G4 spectra for the Al target is shown in Fig. 26. Transitions from levels corresponding to residual excitations, as high as 22 MeV above the ^{27}Al ground state, are observed in ^{25}Mg . No transitions implying proton emission were observed. The ^{26}Mg 1809 keV transition was quite intense. Table VIII displays the same information in tabular form as well as entries for the number of times a level is populated and the number of times each isotope is formed per 100 nuclear captures. Seventy percent of the time a muon is captured in ^{27}Al an excited state in ^{26}Mg is observed while excited states in ^{25}Mg and ^{27}Mg are observed 7% and 5% of the time respectively. These values constitute lower limits for production of these final states.

ABL7112-4937

Fig. 26. Transitions following muon capture in ^{27}Al .

Table VIII. Muon capture in $^{27}\text{Al}(5/2^+ \text{ g.s.})$.

Residual nucleus	Excited State keV, 1^π		Observed γ -ray decay modes		Transition intensities	Population of state	Formation of isotope
$^{27}\text{Mg}, 1/2^+$	984.1	$3/2^+$	985.5	$3/2^+ \rightarrow 1/2^+$	4.3 ± 0.4	2.9 ± 0.5	6.8 ± 0.7
	1692	$5/2^+$	1698.3	$5/2^+ \rightarrow 1/2^+$	1.9 ± 0.4	1.9 ± 0.4	
	1936	$5/2^+$	1936.6 ± 1.6	$5/2^+ \rightarrow 1/2^+$	0.7 ± 0.4	2.0 ± 0.3	
			956.8 ± 0.4		1.4 ± 0.3		
$^{26}\text{Mg}, 0^+$	1808.9	2^+	1808.0	$2^+ \rightarrow 0^+$	65.0 ± 3.0	37.0 ± 4.0	66.2 ± 6.0
	2938.0	2^+	2939.0	$2^+ \rightarrow 0^+$	1.4 ± 0.6	14.4 ± 4.0	
			1129.9	$2^+ \rightarrow 2^+$	17.1 ± 1.5		
	3940.5	$(3)^+$	1003.8	$(3)^+ \rightarrow 2^+$	2.4 ± 0.4	3.8 ± 0.3	
			2130.7	$(3)^+ \rightarrow 2^+$	1.4 ± 0.5		
	4331.0	$(4)^+$	2910.5	$(4)^+ \rightarrow 2^+$	9.8 ± 0.9	10.7 ± 1.4	
$^{25}\text{Mg}, 1/2^+$	585.2	$1/2^+$	584.6	$1/2^+ \rightarrow 5/2^+$	3.1 ± 0.3	1.7 ± 0.4	4.5 ± 0.5
	974.7	$3/2^+$	975.5	$3/2^+ \rightarrow 5/2^+$	1.4 ± 0.3	2.8 ± 0.2	
			390.8	$3/2^+ \rightarrow 1/2^+$	1.4 ± 0.2		
$\Sigma = 78 \pm 6$							

E. Cobalt

The level diagram for the Co target is shown in Fig. 27. Here the $E_\gamma = 1322.3$ keV transition identified as $^{58}\text{Fe } 2133.4 \text{ } 3^+ \rightarrow 810.5 \text{ } 2^+$ is consistent also with $^{56}\text{Mn } 1321 \rightarrow 0$. However, the branching to the 1674.1 level is consistent with the current assignment. Although, the Nuclear Data sheets⁷⁹ show no branch of the $^{57}\text{Fe } 1265$ keV state to the ground state, it is here identified as such, based on Groshev's⁸⁰ work. He shows the line twice, once as $1265 \rightarrow 0$ and once as $1627 \rightarrow 366.8$. The total yield of the $366.8 \rightarrow 14.4$ transition is only 0.9 ± 0.1 , whereas the $E_\gamma = 1265$ intensity is approximately 11% indicating that most of the observed $E_\gamma = 1265$ intensity is not accounted for in this manner. The Co results are shown in Table IX and compared with transition intensities measured by Backenstoss et al.⁸¹ Yields for the 4550 and 1674 keV levels in ^{58}Fe are consistent with at most only 0.3 of the 4.5% yield of the $E_\gamma = 1674.1$ keV gamma being due to the $4550 \rightarrow 2876$ transition. Agreement between the two experiments is very good with the single exception that Backenstoss report a 1.4% yield to the first excited state in ^{55}Fe , which is not seen here. Such a final state requires the emission of four neutrons and is at an excitation energy 37 MeV above the ground state of ^{59}Co . We should have easily observed a yield this large. The observed transitions show no proton emission, 1.7% no nucleon emission, 46% single neutron emission, 27% emission of two neutrons, 5.4% emission of three neutrons and account for 71% of the muon captures in Co.

Backenstoss et al.⁸¹ report a transition intensity of 8% from the 136.3 keV $5/2^-$ state to the 14 keV $3/2^-$ state in ^{57}Fe . This transition is below the energy threshold in our experiment. The 136 keV level is fed with a 2.4% intensity from the 707 keV $5/2^-$ level as shown in Fig. 27.⁸⁰ Thus this state is directly populated 5.6% of the time. If the

Table IX. Muon capture in ^{59}Co ($7/2^-$ g.s.).

Resultant nucleus	Excited state keV, I^π	Observed γ -ray decay modes	Transition intensities	Transition intensities ^a Backscatoss	Population of state	Formation of isotope
^{59}Fe , $3/2^-$	289 $1/2^-$	Below cutoff				
	473 $5/2^-$	472.7 $5/2^- \rightarrow 3/2^-$	1.7 ± 0.3	1.9 ± 0.3	1.7 ± 0.3	1.7 ± 0.3
^{58}Fe , 0^+	610.5 2^+	811.3 $2^+ \rightarrow 0^+$	40.5 ± 4.5	44.0 ± 5.0	30.1 ± 4.6	45.0 ± 4.8
	1674.1 2^+	864.6 $2^+ \rightarrow 2^+$	5.5 ± 0.6	5.1 ± 0.7	7.5 ± 7.0	
		1674.2 $2^+ \rightarrow 0^+$	4.5 ± 0.7	5.0 ± 3.0		
	2133.4 3^+	458.8 $3^+ \rightarrow 2^+$	1.9 ± 0.2	2.2 ± 0.4	6.5 ± 0.5	
		1322.3 $3^+ \rightarrow 2^+$	4.4 ± 0.5			
	2876.0 2	2064 $2 \rightarrow 2^+$	0.5 ± 0.2		$0.2^{+0.5}_{-0.2}$	
	4550 $1,3$	3743.7 $1,3 \rightarrow 2^+$	0.3 ± 0.3		0.7 ± 0.7	
		2888.8 $1,3 \rightarrow 2^+$	0.3 ± 0.4			
^{57}Fe , $1/2^-$	14.4 $3/2^-$	Below cutoff				13.0 ± 1.4
	136.3 $5/2^-$	Below cutoff		8.0 ± 1.5		
	366.8 $3/2^-$	352.4 $3/2^- \rightarrow 3/2^-$	0.9 ± 0.1	1.2 ± 0.3	1.0 ± 0.1	
	706.4 $5/2^-$	692.5 $5/2^- \rightarrow 3/2^-$	3.2 ± 0.4	3.6 ± 0.7	3.5 ± 0.4	
	1190 $3/2^-$	1190.4 $3/2^- \rightarrow 3/2^-$	1.5 ± 0.2		1.5 ± 0.2	
	1265 $3/2^-$	1264.9 $3/2^- \rightarrow 3/2^-$	11.7 ± 1.3	15.7 ± 2.5	11.7 ± 1.3	
	1627 $3/2^-$	1627.5 $3/2^- \rightarrow 3/2^-$	0.3 ± 0.2		0.3 ± 0.2	
^{56}Fe , 0^+	846.75 2^+	847.3 $2^+ \rightarrow 0^+$	5.4 ± 0.6	7.1 ± 1.5	4.1 ± 0.7	5.4 ± 0.8
	2085.1 4^+	1237.8 $4^+ \rightarrow 2^+$	1.3 ± 0.3	2.0 ± 0.7	1.3 ± 0.3	
						$\Sigma = 70 \pm 6$
^a Ref. 7.						

population of this level is included a total of 77% of the muon captures are accounted for in Co.

C. Calcium

The observed transitions in Ca are shown in Fig. 28. The gamma transition from the 3935 keV state to the ground state of ^{39}K (shown dotted) is within the energy resolution consistent with the gamma identified here as a transition in ^{38}Ar from the 3936 keV state to the ground state. The identification in this work is made on the basis of branching ratios measured by Bass and Saleh-Bass⁸² for the ^{39}K level. A 1% branch of the 3935 keV ^{39}K level to the ground state would imply approximately 0.2% yields for transitions with energies of 332- and 1118-keV which were not seen. With our experimental sensitivity (see Fig. 20) this large a yield at these energies should have been easily visible.

Table X gives our results for the Ca target and shows the results of Igo-Kemenes et al.⁸³ and Vuilleumier⁸⁴ for comparison.

x8L7112-4878

Fig. 28. Transitions following muon capture ^{40}Ca .

Table X. Muon capture in ^{40}Ca .

Resultant nucleus	Excited state keV, 1^a	Observed γ -ray decay modes	Transition intensities	Transition intensities, Vulliamier ^b	Population of state	Population of state, Vulliamier ^b	Population of state, 160-KeV group ^c	Formation of isotope		
$^{40}\text{K}, 4^+$	30	3^+	Below cutoff							
	800	2^-	771.3	$2^- \rightarrow 3^+$	7.4±0.6	5.87±0.37	5.0±0.7	4.19±0.45	3.6±1.01	10.2±1.0
	890	3^-	899.5	$3^- \rightarrow 4^+$	0.3±0.3	0.3±0.3	0.3±0.3			
	1644	0^+	1611	$0^+ \rightarrow 3^-$	0.8±0.2	0.89±0.15	1.0±0.3 ^d	0.94±0.19		
			84.1	$0^+ \rightarrow 2^-$	1.1±0.2 ^e	0.23±0.1				
	1959	2^-	1160 ^f	$2^- \rightarrow 2^-$	1.1 ^d	0.4±0.1	1.3±0.4	0.56±0.13	0.42±0.17	
	2047	2^-	2030	$2^- \rightarrow 3^-$	0.4±0.2	0.39±0.14	1.3±0.12	0.93±0.22	0.45±0.15	
			1251	$2^- \rightarrow 2^-$	0.54±0.08					
	2103	1^-	2073	$1^- \rightarrow 3^-$	0.5±0.2	0.4±0.1	0.4±0.1	0.36±0.17		
	2290	1^+	643 ^g	$1^+ \rightarrow 0^+$	0.22±0.10	0.18±0.05	0.35±0.16	0.41±0.13		
	2419	$2,3$	1620	$2,3 \rightarrow 2^-$	0.2±0.2	0.35±0.12	0.2±0.2	0.4±0.2		
	2525	0^+	522 ^h	$0^+ \rightarrow 1^+$	0.29±0.10	0.39±0.1	0.29±0.10	0.29±0.10		
$^{39}\text{K}, 3/2^+$	2526	$1/2^+$	2521.3	$1/2^+ \rightarrow 3/2^+$	7.1±1.8	5.9±0.5	7.1±1.8	5.9±0.5	6.6±0.4	18.1±1.5
	2817	$7/2^-$	2813.5	$7/2^- \rightarrow 3/2^+$	4.0±0.5	3.2±0.3	3.8±0.2	3.2±0.3	3.3±0.3	
	3021	$3/2^-$	3018.1	$3/2^- \rightarrow 3/2^+$	2.9±0.5	2.1±0.4	2.9±0.5	2.1±0.4	1.6±0.3	
	3719	$7/2^-$	3870.7	$7/2^- \rightarrow 3/2^+$	0.3±0.1	0.2±0.5	0.3±0.1	0.2±0.5	0.76±0.2	
	5830	$(3/2^+)(5/2^+)$	5873±10	$5800 \rightarrow 0$	0.8±0.2	0.9±0.6	0.8±0.2	0.9±0.6	0.4±0.2	
	5620	$(3/2^+)(5/2^+)$	5612±10	$5600 \rightarrow 0$	1.3±0.2	0.7±0.5	1.3±0.2	0.7±0.5	0.4±0.2	
	5960	$(1/2^+)(3/2^+)$	6002±10	$5960 \rightarrow 0$	0.2±0.1	0.2±0.1	0.2±0.1	0.2±0.1	0.4±0.2	
	6350	$(3/2^+)(5/2^+)$	6335±10	$6390 \rightarrow 0$	1.7±0.2	0.8±0.6	1.7±0.2	0.8±0.6		
$^{38}\text{K}, 3^+$	130	0^+	Below cutoff							
	458	1^+	Not observed		0.28±0.08 ⁱ	0.28±0.08	0.28±0.08	0.28±0.08	0.28±0.08	
	1700	1^+	1572	$1^+ \rightarrow 0^+$	0.6±0.2	0.5±0.1	0.6±0.2	0.5±0.1	0.9±0.2	
$^{39}\text{Ar}, 7/2^-$	1267		1266.5	$3/2^- \rightarrow 7/2^-$	4.0±0.4	2.8±0.2	1.0±0.1	0.35±0.33	0.99±0.35	5.9±1.5
	1516		1516.5	$3/2^- \rightarrow 7/2^-$	0.7±0.2	1.01±0.14	1.5±0.6	1.99±0.26	2.64±0.56	
			249	$3/2^- \rightarrow 3/2^-$	1.07±0.11	1.17±0.2				
	2091		2091 ^j	$2091 \rightarrow 0$	0.3±0.2	0.73±0.16	0.4±0.2	0.9±0.19	0.69±0.23	
	2341		2341 ^k	$0 \rightarrow 0$	0.1±0.1	0.38±0.14	0.4±0.2	0.38±0.14	0.53±0.2	
	2357	$1/2^+$	1095 ^l	$1/2^+ \rightarrow 3/2^-$	0.4±0.2	0.2	0.1±0.1	0.38±0.14	0.53±0.2	
	2432		2432	$0 \rightarrow 0$	0.3±0.2	0.2	1.2±0.8	1.13±0.18	0.76±0.13	
			1166	$2432 \rightarrow 1267$	0.9 ^d	0.85±0.12				
	2502		904	$2502 \rightarrow 1516$	0.5±0.1	0.19±0.1	0.5±0.1	0.19±0.1		
	2631	$7/2^-$	911 ^l	$2631 \rightarrow 2091$	0.4±0.1	0.2±0.1	0.4±0.1	0.2±0.1		
	3260		Not observed						0.32±0.19	
$^{38}\text{Ar}, 0^+$	2167.7	2^+	2166.6	$2^+ \rightarrow 0^+$	7.1±1.1	5.22±0.35	4.8±1.1	3.3±0.4	3.1±0.4	6.1±1.4
	3376.8	0^+	3211.9	$0^+ \rightarrow 2^-$	0.4±0.2	0.32±0.08	0.4±0.2	0.32±0.08	0.9±0.16	
	3810.0	3^-	1642.4	$3^- \rightarrow 2^-$	2.0±0.4	1.59±0.14	1.3±0.5	0.89±0.22	1.5±0.15	
	3936.1	$(2)^+$	3930.3	$(2)^+ \rightarrow 0^+$	1.0±0.3	1.8	1.0±0.3	1.8	1.2±0.1	
	4473.6	4^-	669.9	$4^- \rightarrow 3^-$	0.6±0.2	0.7±0.1	0.6±0.2	0.7±0.1		
$^{38}\text{Cl}, 2^-$	1311	4^-	1316	$4^- \rightarrow 2^-$	0.4±0.2	0.5±0.1	0.4±0.2	0.5±0.1	1.05±0.11	0.4±0.1
									$\Sigma = 43.35 \pm 1.0$	

7 = 43.3±0.1

^aRef. 17.^bRef. 8.^cSee addendum.^dThis line is not clearly separable from ^{39}Ar $2432 \rightarrow 1267$ transition, with $E = 1165$ keV. Intensities taken to be consistent with ^{39}Ar $2432 \rightarrow 0$ branch.^ePopulation of this state determined from ^{40}K $1644 \rightarrow 30$ transition intensity.^fSee d above.

D. Silicon

The nuclear level diagram for the Si target is shown in Fig. 29. The results for this target are tabulated in Table XI. Transition intensities reported by Eckhause et al.⁸⁵ and Pratt⁸⁶ are shown for comparison.

The 985.7 keV gamma ray is consistent with a transition in ^{28}Al between levels at 1014.5- and 30.6-keV and also with a transition from the first excited state at 984 keV to the ground state in ^{27}Mg . The formation of ^{27}Mg requires the emission of a proton which must overcome the Coulomb barrier. The first excited state in ^{27}Mg has a spin and parity of $J^\pi = 3/2^+$. Bunatyan⁸⁷ postulates the existence of a 1^+ level in ^{28}Al near 8 MeV which should be intensely populated directly from muon capture in ^{28}Si . Particle decay of this level to form ^{27}Mg is not energetically possible. If a proton is emitted from a 1^- resonance level in ^{28}Al one unit of orbital angular momentum has to be carried away to form the positive parity $3/2^+$ state in ^{27}Mg . For this situation the proton must overcome an angular momentum barrier as well as the Coulomb barrier. Finally the neutron multiplicity measurements of MacDonald et al.¹² and the no particle emission results of Bunatyan et al.⁸⁷ and Eckhause et al.⁸⁵ show that ^{28}Al is formed without subsequent particle decay at least 28% of the time following μ^- capture in ^{28}Si .

The points made above argue against the assignment of this gamma ray as a transition in ^{27}Mg . However, other information argues in favor of such an assignment. In fact, Eckhause et al.⁸⁵ make this assignment. Vil'gel'mova et al.⁸⁸ have measured the probability of the occurrence of the reaction $^{28}\text{Si}(\mu^-, p)^{27}\text{Mg}$ by using an activation method. They find this probability to be 0.053 ± 0.010 . It is also noted that single proton emission was observed 7% of the time for the Ca target in this experiment.

Fig. 29. Nuclear transitions following muon capture in ^{28}Si .

Table XI Muon capture in ^{23}Si .

Resultant nucleus	Excited state (keV), J^π	Observed γ -ray decay modes	Transition intensities (%)	Transition intensities Eckhauser ^a (%)	Transition intensities Pratt ^b (%)	Population of state (%)	Formation of isotope (%)
$^{25}\text{Al}, 3^-$	30.6 2^+ 972.6 1^- 1372.6 1^+ 1622.5 (2,3) ⁺ 2202.0 1^+	Below cutoff 942.3 1342 $1^+ \rightarrow 2^+$ 1627 (2,3) ⁺ $\rightarrow 3^+$ 1593 (2,3) ⁺ $\rightarrow 2^+$ 2175 $1^+ \rightarrow 2^+$ 1224 $1^+ \rightarrow 2^+$	 2.1 $\pm$ 0.3 2.0 $\pm$ 0.4 ^c 2.5 $\pm$ 0.4 ^d 1.0 $\pm$ 0.4 3.2 $\pm$ 0.6 0.9 $\pm$ 0.4	13.1 $\pm$ 1.3 2.0 $\pm$ 0.3 1.7 $\pm$ 0.2 1.8 $\pm$ 0.3 1.7 $\pm$ 0.3 4.6 $\pm$ 0.3 1.1 $\pm$ 0.2	 1.2 $\pm$ 1.0 1.5 $\pm$ 2.1 1.7 $\pm$ 0.3 4.6 $\pm$ 0.3 1.1 $\pm$ 0.2	0.1 $\pm$ 0.5 3.6 $\pm$ 0.7 3.4 $\pm$ 0.8 4.2 $\pm$ 1.4	11.3 $\pm$ 1.8
$^{27}\text{Al}, 5/2^+$	842.9 $1/2^+$ 1013.0 $3/2^+$ 2212 $7/2^+$ 2732 $5/2^+$ 2950 $3/2^+$ 3674 $1/2^+$ 4409 (5/2)	845.0 $1/2^+ \rightarrow 5/2^+$ 1015.1 $3/2^+ \rightarrow 5/2^+$ 2213 $7/2^+ \rightarrow 5/2^+$ 2732 $5/2^+ \rightarrow 5/2^+$ 1720 $5/2^+ \rightarrow 3/2^+$ 2985 $3/2^+ \rightarrow 5/2^+$ 2837 $1/2^+ \rightarrow 5/2^+$ 4405 (5/2) $\rightarrow 5/2^+$	13.8 $\pm$ 1.2 12.0 $\pm$ 1.2 5.0 ^{+0.7} _{-1.3} 0.4 $\pm$ 0.3 2.1 $\pm$ 0.4 2.3 $\pm$ 1.0 0.7 $\pm$ 0.5 ^f 3.0 $\pm$ 1.5	10.4 $\pm$ 0.8 10.3 $\pm$ 0.8 7.0 $\pm$ 3.0 0.4 $\pm$ 0.3 0.8 $\pm$ 0.3 2.6 $\pm$ 0.8 0.6 $\pm$ 0.2 3.0 $\pm$ 1.5	7.8 $\pm$ 1.4 5.5 $\pm$ 1.4 6.0 $\pm$ 5.0 2.6 $\pm$ 1.4 2.0 $\pm$ 3.0 2.3 $\pm$ 1.0 0.8 $\pm$ 0.2 3.0 $\pm$ 1.5	13.1 $\pm$ 1.3 9.9 $\pm$ 1.3 4.9 ^{+0.7} _{-1.3} 2.6 $\pm$ 1.4 2.3 $\pm$ 1.0 0.8 $\pm$ 0.2 3.0 $\pm$ 1.5	36.6 $\pm$ 3.3
$^{26}\text{Al}, 5^+$	229 0^+ 416 3^+	Below cutoff 417 $3^+ \rightarrow 5^+$	 0.69 $\pm$ 0.13	0.7 $\pm$ 0.2 0.9 $\pm$ 0.2	 0.7 $\pm$ 0.1	 0.7 $\pm$ 0.1	0.7 $\pm$ 0.1
$^{25}\text{Mg}, 1^-$	1305.9 2^+ 2933.0 2^- 3534.7 0^-	1305.2 $2^+ \rightarrow 0^+$ 1129.0 $2^- \rightarrow 2^+$ 1779 $\pm$ 1 $0^- \rightarrow 2^+$	9.2 $\pm$ 1.0 2.2 $\pm$ 0.4 6.0 ^{+1.2} _{-6.0}	10.0 $\pm$ 1.0 3.2 $\pm$ 0.5 3.8 $\pm$ 1.6	 1.8 $\pm$ 1.1 3.8 $\pm$ 1.6	1.0 $\pm$ 1.6 2.2 $\pm$ 0.4 6.0 ^{+1.2} _{-6.0}	9.2 $\pm$ 1.0
$^{25}\text{Mg}, 5/2^+$	595.5 $1/2^+$ 974.7 $3/2^+$	555 $1/2^+ \rightarrow 5/2^+$ 350 $3/2^+ \rightarrow 1/2^+$	0.91 ^{+0.26} _{-0.90} 0.67 $\pm$ 0.12 ^h	0.6 $\pm$ 0.3 0.8 $\pm$ 0.3	 1.4 $\pm$ 0.2	0.9 ^{+0.2} _{-0.9} 1.4 $\pm$ 0.2	2.3 $\pm$ 1.0 $\Sigma = 50\pm 4$

^aL. Eckhauser, P. Martin, G. H. Miller and R. E. Welch, Proc. 4th Inter. Conf. on High Energy Physics and Nuclear Structure, Dubna, USSR, Sept. 1971
b Ref. 17

^c $E_\gamma = 600$ keV, $13^- \rightarrow 973$ transition masked by K_α x ray.

^dAssigned to 1622.5 keV level rather than 1620.1 keV level on basis of energy resolution and branching ratio.

^eSignificant $^{27}\text{Al}(n,n')^{27}\text{Al}$ background.

^f $E_\gamma = 600$ keV 3674 $13^- \rightarrow 2212$ transition not seen.

^gThe $63/61$ ratio for this line indicates it may be largely background.

^h 975 $512 \rightarrow 0$ ^{25}Mg transition not seen.

The assignment of this gamma ray is thus at best ambiguous and the two possibilities are shown as dotted lines in our energy level diagram. The intensity of this unassigned transition is 0.008 ± 0.003 per μ^- capture.

Figure 30a shows the two broad peaks at 2175- and 2212-keV in the Si target spectrum and two nearby sharp peaks for comparison. This spectrum is the sum of the G3 and G4 spectra. In the background spectrum (Fig. 30b) a strong line at 2223 keV is identified as the neutron capture transition in deuterium. Presumably this capture is on hydrogen in the CH_2 degrader and in the boric acid surrounding the detector. Capture in the lucite target cases for the Si and Co targets increases this background in these targets. The sharp line in the background spectrum at 1778 keV is due to a transition in ^{28}Si following the β^- decay of ^{28}Al which has a 2.3 min. half life. The difference spectrum (G3 + G4 - 2G1) is shown in Fig. 30c. Both the 2175- and 2212-keV lines are broad. The 2212 keV level in ^{27}Al has a width of 15.8 milli-eV⁸⁹ corresponding to a mean lifetime of 4.2×10^{-14} sec and the line should be Doppler broadened.

The target holder and the detector housing were aluminum. Due to the neutron background and the significant inelastic neutron scattering cross section of ^{27}Al ,⁹⁰ background lines corresponding to excitations in ^{27}Al are expected in the data for all targets.

Transitions from the first three excited states in ^{27}Al are seen in the background for each target. They are seen most intensely in the aluminum target. To determine if the total intensity of lines at the energies of these transitions is due to inelastic neutron scattering on Al, ratios of the line intensity for the G3 spectrum to that in the G1 spectrum were computed. For lines such as those due to the reactions

Fig. 30. Si spectra near 2 MeV; (a) sum of short delay and long delay spectra, (b) background spectrum (i.e. minus time spectrum), and (c) sum of short delay and long delay spectra minus twice the background spectrum.

XBL 723-019

$^{10}\text{B}(n,\alpha)^7\text{Li}$, $^{208}\text{Pb}(n,n'\gamma)^{208}\text{Pb}$, and $^{72}\text{Ge}(n,n')^{72}\text{Ge}$, i.e., known neutron background lines, this ratio is 1.4 for Si and Al, 1.2 for Ca, and 2.0 for Co. The variation in this ratio reflects the variation in the average neutron multiplicity. By this criterion about 25% of the 2212 keV line intensity in the Si target data might be attributed to neutron background.

The G3/G1 ratio for the 1779 keV gamma ray identified as the transition between the 3585- and 1809-keV levels in ^{26}Mg indicates a large background contribution. This background is seen in Fig. 30b. The assigned intensity is that obtained from the difference spectrum. A large yield for the formation of ^{26}Mg corresponding to the emission of a neutron and a proton or of a deuteron following muon capture in ^{28}Si is consistent with the trend reported by Budyashov et al. Thus in spite of the large background the difference spectrum intensity is assigned to ^{26}Mg , but with a large negative error.

The assignments to transitions in ^{25}Mg are rather uncertain. The G3/G1 ratio for the 586 keV line indicates a large background contribution. A transition from the 975 keV state to the ground state should be seen with an intensity equal to the 985- to 586-keV transition. This gamma ray was not observed. However, Eckhause et al.,⁸⁵ report this transition. It is noted that the formation of ^{25}Mg following muon capture in ^{28}Si requires an excitation of more than 31 MeV.

The assignments listed in Table XI account for 60% of the muon captures. Observed transitions in ^{28}Al occur 11% of the time. Single neutron emission forming ^{27}Al occurs 37% of the time. An appreciable amount of proton plus neutron emission is seen (approximately 9%).

VI. COMMENTS ON THE RESULTS

In this chapter detailed comments are made on the results presented in the previous chapter. A comparison of the $^{40}\text{Ca}(\mu^-, \nu n)^{39}\text{K}$ results is made with the results of the photo-excitation reaction $^{40}\text{Ca}(\gamma, p\gamma')^{39}\text{K}$ and with the results of the pickup reaction $^{40}\text{Ca}(t, \alpha)^{39}\text{K}$. Our results for the Si target for the reaction $^{28}\text{Si}(\mu^-, \nu)^{28}\text{Al}$ are compared with inelastic electron scattering data on ^{28}Si and we conclude that a giant M1 dipole mechanism is present in muon capture. A correlation of the states populated in ^{28}Al following muon capture in ^{28}Si is made with states identified as configuration mixtures of single particle-single hole configurations by Boerma and Smith.⁹¹ The lower limits on neutron multiplicities set by this experiment are compared with the measurements of MacDonald et al.¹² Similarly the limits set on charged particle emission by this experiment are discussed in the framework of other evidence on charged particle emission following muon capture. The results of recent experiments using the analogous radiative pion capture reaction are presented. Finally, the Al results for the $^{27}\text{Al}(\mu^-, \nu n)^{26}\text{Mg}$ reaction are compared to the results of the photo-excitation reaction $^{27}\text{Al}(\gamma, p\gamma')^{26}\text{Mg}$ and discussed as a probe of the isospin splitting of the giant resonance.

A. Population of Hole States in ^{39}K

The existence of a "giant resonance" mechanism in the muon capture process is well established (e.g. resonance structure in neutron spectra observed by Evseev et al.¹³ and Plett and Sobottka⁵⁴).

It is of interest in the study of nuclear structure to correlate where possible the events following muon capture with those following photoexcitation since this process has been under investigation for some time. It was pointed out in Chapter II that some of the initial

steps in theoretical calculations of muon capture rates (Foldy and Walecka³⁰) related the integrated photoabsorption cross section to the nuclear matrix elements in muon capture and thereby achieved good agreement with measured capture rates at least for ^{16}O and ^{40}Ca .

In attempting to understand exactly how nuclear structure influences the muon capture reaction or conversely what nuclear structure information may be obtained from studying the muon capture reaction a comparison of levels populated following muon capture with those populated in particle transfer reactions which are well understood is useful.

Figure 31 shows the probability of populating states in ^{39}K for three kinds of reactions. The results of this experiment for the reaction $^{40}\text{Ca}(\mu^-, \nu n)^{39}\text{K}$ are shown next to the column showing the relative positions of energy levels which are populated significantly in the reactions shown. In each case the probabilities are shown relative to that for populating the first excited state. The results of Ullrich and Krauth⁹¹ who detected the decay gammas from the reaction $^{40}\text{Ca}(\gamma, p\gamma)^{39}\text{K}$ are shown next to muon capture results. On the far right the results of the cross section measurements made by Hinds and Middleton⁹³ for the pickup reaction $^{40}\text{Ca}(t, \alpha)^{39}\text{K}$ are displayed. On the basis of angular distributions Hinds and Middleton conclude that most of the $1d_{5/2}$ proton hole strength is shared by the three levels at 5.28, 5.62 and 6.35 MeV. The probabilities of populating states in the muon capture reaction correlates well with those in the (t, α) reaction indicating a preference for leaving the final nucleus in a simple shell model state. The correlation between muon capture and photo-excitation does not seem as strong although the same states are populated in both reactions.

Fig. 31. Comparison of relative probabilities of exciting states in ^{39}K via three different reaction mechanisms.

It is emphasized once again that muon capture is expected to excite a wider variety of states than does photoexcitation and that it can excite the same states in more than one way. For example Balashov et al.³⁶ have calculated partial transition rates to states in ^{39}K following muon capture in ^{40}Ca . They include allowed, first forbidden, and second forbidden transitions in the calculations. A schematic of this model is shown in Fig. 6. Hill and Uberall³³ have also calculated these rates considering only first forbidden transitions. Note the isospin first forbidden transitions in muon capture are analogous to photoexcitation. The total capture rate calculated by Hill is 1.75 times the experimental rate and Balashov accounts for more than the total capture rate with transitions to ^{39}K . Thus comparison of absolute rates is not particularly interesting. However, a comparison of the relative probabilities for populating the $1d_{5/2}$ hole state and the $2s_{1/2}$ hold state can be made. For this comparison the summed yields for the 5.28, 5.62, and 6.35 MeV states was taken as the probability of populating the $1d_{5/2}$ hole state. This comparison is made in Table XII. The present results are in some-

Table XII

$1d_{5/2}^{-1} / 2s_{1/2}^{-1}$			
Balashov ^{a)}	Uberall ^{b)}	This Experiment	Vuilleumier ^{c)}
0.57	0.35	0.54 ± 0.14	0.41 ± 0.14
a) Ref. 36	b) Ref. 33	c) Ref. 84	

what better agreement with the calculations of Balashov et al., suggesting that perhaps allowed and second forbidden transitions must be included even for the doubly magic Ca nucleus. It should be noted however, that Vuilleumier's⁸⁴ results while in statistical agreement with the present experiment are closer to the predictions of Hill

and Uberall. At any rate, an unambiguous determination of this problem requires the coincidence measurement of the corresponding neutrons and gamma rays.

B. Giant M1 Dipole Resonance in ^{28}Si

As mentioned in the chapter on theory Ubersall⁵⁰ has suggested that a giant M1 dipole resonance mechanism might contribute to the muon capture process in mid-shell nuclei. That is, a transition for which $\Delta J = 1$ and the parity is not changed. This situation corresponds to an allowed transition and as described earlier such transitions are not expected for muon capture in closed shell nuclei. However, they are indeed possible within the framework of the shell model for mid-shell nuclei. Fagg et al.⁵¹ have measured the 180 deg inelastic electron scattering cross section of ^{28}Si (shown in Fig. 32). They see a "resonance" in the cross section at 11.41 MeV. From the behavior of this resonance as the momentum transfer is varied it is identified as a magnetic dipole transition. Figure 33 is taken from Fagg and shows the level diagram for ^{28}Si and ^{28}Al where the ^{28}Al levels have been shifted to account for the Coulomb energy difference. Fagg identifies the 2.207 MeV (1^+) level in ^{28}Al as the "parent" of the 1^+ isobaric analogue state at 11.41 MeV in ^{28}Si .

The (d,p) results of Boerma and Smith⁹¹ show a 1^+ state with an excitation energy of 2202 ± 1 keV in ^{28}Al . We see this state populated 4.2% of the time following muon capture in ^{28}Si . We suggest that this is positive evidence for the identification of the 2202 keV, 1^+ state in ^{28}Al as the "parent" of the 11.41 MeV isobaric analog state in ^{28}Si .

In a digression we now review what is known about the structure of this state. It is known that nuclei near mass number 28 are deformed.⁹⁴ Fu and Yost⁹⁵ have employed a unified model for deformed odd-odd nuclei to calculate energy levels and gamma transition probabilities in ^{28}Al . Their results are shown in Fig. 34. The order and spacing of the first three levels is in good agreement with experiment. The spacing of the

XBL 7112-1806

Fig. 32. Differential cross section for 180 deg. inelastic electron scattering on ^{28}Si . (After Ref. 51.)

XBL 7112-1807

Fig. 33. Energy level diagram for ^{28}Al adjusted for the Coulomb energy difference and shown next to the energy level diagram for ^{28}Si . (After Ref. 51.)

XBL 723-615

Fig. 34. Experimental and theoretical energy levels in ^{28}Al .
(After Ref. 95.)

fourth through sixth levels is not in as good agreement with experiment. However the predicted gamma branching ratios are in good agreement with the measured branching ratios for all levels. Unfortunately the calculations do not extend high enough in energy to include the 2202 keV, 1^+ level which is of particular interest here.

Boerma and Smith⁹¹ have extended the Oak Ridge⁹⁶ work by considering a truncated basis of $(d_{5/2}^{-1} s_{1/2})$ and $(d_{5/2}^{-1} d_{3/2})$ configurations as well as the basis $(d_{5/2} s_{1/2})$ and $(d_{5/2}^{-2} s_{1/2}^{-2})$ used by Glaudemans et al.⁹⁷ They then compare the calculated level schemes with their $^{27}\text{Al}(d,p)^{28}\text{Al}$ results. The comparison is shown in Fig. 35. They point out that since configuration mixing was only taken into account between states calculated in each scheme separately, only qualitative agreement is to be expected.

The following observations are made by these authors. "Both theories predict a $J^\pi = 2^+$ ground state and a $J^\pi = 3^+$ first excited state, both principally with a $(d_{5/2}^{-1} s_{1/2})$ configuration. The experimentally observed order of these levels is reversed, but the observed large $\ell_n = 0$ strength⁹⁸ for the $^{27}\text{Al}(d,p)^{28}\text{Al}$ reaction to these levels is in agreement⁹⁹ with the predicted structure if it is assumed that the wave function of the ground state of ^{27}Al has $d_{5/2}^{-1}$ as main component. Making the same assumption, it can be shown that the $\ell_n = 2$ strength for the (d,p) reaction to levels of predominantly $(d_{5/2}^{-1} d_{3/2})$ configuration is large. Experimentally, large $\ell_n = 2$ reduced widths⁹⁸ have been observed for levels at 1.01 (3^+), 2.20 (1^+), 2.27 (5^+), and 2.66 (5^+) MeV. This fact, and the reasonable agreement between the observed and the calculated excitation energies, lead to the conclusion that the levels at 1.01 and 2.20 MeV have large $(d_{5/2}^{-1} d_{3/2})$ components. Experimental information is lacking to establish the correspondence between

XBL 723-609

Fig. 35. Identification of states in ^{28}Al as simple configuration mixed particle-hole or particle-hole and two particle-two hole states. (After Ref. 91.)

observed levels and the predicted $J^\pi = 4^+$ and 2^+ levels with large $(d_{5/2}^{-1}d_{3/2})$ configuration at 1.58 and 1.62 MeV. In addition, the effects of the inclusion of configuration mixing will be important for the two theoretical levels with $J = 2^+$ near 1.6 MeV (see Fig. 35), because the calculated difference in the excitation energies is small.

The correspondence between observed levels and levels predicted to have predominantly $(d_{5/2}^{-2}s_{1/2}^{-2})$ configurations is indicated in Fig. 35 by solid or dashed lines. This comparison is in agreement with the fact that no appreciable $\ell_n = 2$ reduced widths⁹⁸ have been measured for the levels at 0.97, 1.37, and 2.14 MeV, and with the observed large $\ell_n = 0$ width for the 2.14-MeV level. The proposed structure of the $J^\pi = 1^+$ levels at 1.37 and 1.62 MeV is in accordance with the observed¹⁰⁰ small $\log(ft)$ values for allowed β^- transitions to these levels if the ground state of ^{26}Mg has a large $(d_{5/2}^{-2})$ component in the wave function."

Figure 36 shows the experimental levels of ^{28}Al separated according to the configuration identifications suggested by Boerma and Smith. Also shown are the relative probabilities of populating these levels in muon capture determined in this experiment. Except for the 1.37 MeV 1^+ state all the states populated in muon capture can be interpreted as mainly configuration mixed states of the simple $(d_{5/2}^{-1}s_{1/2}^1)$ and $(d_{5/2}^{-1}d_{3/2}^1)$ single particle-single hole configurations. The dotted line at 1.01 MeV in the muon capture column corresponds to assigning all the 986 keV gamma as a transition in ^{28}Al . (See the discussion of the Si results in Chap. V.)

As a point of general interest we mention that the 2202 keV 1^+ level is not reported by Hardell et al.¹⁰¹ who looked at capture gamma rays from the reaction $^{27}\text{Al}(n,\gamma)^{28}\text{Al}$. Note this level is populated strongly in muon capture and clearly identified in the (d,p) reaction.⁹¹

XBL 723-610

Fig. 36. Relative probabilities of exciting states in ^{28}Al following muon capture in ^{28}Si shown next to the configuration mixed identifications of Ref. 91 for these levels.

Attention is again focussed on the population of 1^+ states following muon capture. Bunatyan et al.⁸⁷ have suggested that a 1^+ state near 8 MeV in ^{28}Al should be populated about 22% of the time following muon capture in ^{28}Si . Eckhause et al.⁸⁵ report a gamma transition of 7725 keV that occurs 5.4% of the time. The energy of this transition is indistinguishable from the energy of thermal neutron capture direct transition to the ground state. A 5% population of such a high energy level exactly at the neutron binding energy would indicate a highly preferred final state structure for that level. This energy is above our high energy cut-off and such a transition was therefore not detected in this experiment. Nichol et al.¹⁰² suggest that the thermal neutron capture state is a $(2,3)^+$ state. If one of these possibilities is the proper spin assignment, then this level is clearly not the 1^+ level predicted by Bunatyan. Even if the level is a 1^+ level, the intensity is significantly lower than that predicted by Bunatyan.

C. Muon Capture and the Photoproton Reaction in ^{28}Si

The photoproton spectrum for the reaction $^{28}\text{Si}(\gamma, p)^{27}\text{Al}$ has been measured by Cannington et al.¹⁰³ Their spectrum is shown in Fig. 37. They see proton groups corresponding to the formation of ^{27}Al in the ground state (p_0), first, second, fourth, and fifth and sixth excited states (p_1 , p_2 , p_4 , and p_{5-6}). There is no evidence in the (γ, p) data for exciting the third excited state in ^{27}Al . (See arrow in figure.) The probabilities for exciting states in ^{27}Al following muon capture in ^{28}Si are shown relative to the probability for exciting the first excited state in the same figure. (The vertical lines.) The most striking difference in the (γ, p) and muon capture results is the significant probability for populating the third excited state in muon capture. Also, the relative probabilities for populating excited states are different in the two reactions. In summary, there is a similarity in the states populated in the (γ, p) reaction and those populated in μ^- capture, however the correlation is far from perfect.

An identification of "hole" states is not as clear in ^{27}Al as in ^{39}K . This is because the nuclei near mass number 28 are deformed. In the weak coupling model, the low-lying ^{27}Al states are generated by coupling a $1d_{5/2}$ hole to the ^{28}Si ground and first excited states.¹⁰⁴ In the strong coupling model of ^{27}Al , a static prolate deformation of the nucleus is assumed and the low-lying states are explained, following the predictions of the Nilsson model,¹⁰⁵ in terms of a $K^\pi = 5/2^+$ ground state rotational band and a $K^\pi = 1/2^+$ band.¹⁰⁶ Each model has been successful in describing particular aspects of ^{27}Al . The weak coupling model fails to give the spectroscopic factors derived from one-nucleon transfer reactions between ^{27}Al and ^{28}Si , while the strong coupling model does not agree with the inelastic scattering data.

XBL 7112-1805

Fig. 37. Relative probabilities of exciting levels in ^{27}Al following muon capture in ^{28}Si (bars) shown superposed on the charged particle spectrum of Ref. 103 for the $^{28}\text{Si}(\gamma, p)^{27}\text{Al}$ reaction. Note the absence of a proton group corresponding to the muon excited level indicated by the arrow.

Ropke et al.¹⁰⁷ have compared ^{27}Al with ^{25}Al and ^{25}Mg to suggest the existence of two γ -vibrational bands. They then adapt the theory of Fassler¹⁰⁸ to treat ^{27}Al with a rotation-vibration interaction. Their results exhibit features of strong coupling and weak coupling simultaneously. Overall this model describes ^{27}Al well, but there is still a problem with the spectroscopic factors from one-nucleon transfer reactions.

All of these models include an interaction of the core with the single nucleon "hole" and predict the energy levels up to 3 MeV correctly. From the strong population of these levels in the muon capture reaction it is apparent that the ^{28}Si core is excited in the reaction.

D. Comparison with Published Neutron Multiplicities

It was mentioned in the review of muon capture experiments that neutron multiplicity measurements following muon capture in several nuclei have been made by MacDonald et al.¹² With the formation of isotope calculations (see table for each target) made using the data of the present experiment, lower limits can be set on neutron multiplicities. The reason only lower limits are set is that the present experiment is not sensitive to direct populations of ground states. We also are limited to gamma transitions with energies between our upper and lower energy cutoff points, and we cannot detect weakly populated levels.

A comparison of the MacDonald results with those of this experiment is made in Table XII. The limit set for no-neutron emission includes the probability that a single proton is emitted. Similarly the limit set for single neutron emission includes the probability that a proton plus a neutron is emitted. For the aluminum target, where most of the captures are accounted for in the present experiment, the agreement is quite good. The results of the two experiments are also consistent for Si and Ca, although the differences in the quoted multiplicities and the lower limits set here are significantly larger than for Al.

Table XII

Target	Neutron multiplicity distributions (%)				
	P(0)	P(1)	P(2)	P($\nu > 1$)	P($\nu > 2$)
Al (1)	7.7 $\pm$ 3.7	82.2 $\pm$ 7.7	-6.8 $\pm$ 15.6	10.1 $\pm$ 5.6	16.9 $\pm$ 11.7
(2)	6.8 $\pm$ 0.7	66.2 $\pm$ 6.3	4.5 $\pm$ 0.9		
Si (1)	36.3 $\pm$ 5.7	45.7 $\pm$ 0.8	24.5 $\pm$ 21.1	18.0 $\pm$ 7.6	-6.5 $\pm$ 15.8
(2)	11.3 $\pm$ 1.8	45.8 $\pm$ 3.4	3.0 $\pm$ 1.0		
Ca (1)	37.0 $\pm$ 2.9	54.1 $\pm$ 5.6	9.3 $\pm$ 7.3	9.0 $\pm$ 3.6	-0.3 $\pm$ 4.6
(2)	17.2 $\pm$ 2.0	26.6 $\pm$ 2.4	0.6 $\pm$ 0.2		

(1) Measured multiplicities of MacDonald et al.¹²

(2) Limits set in this experiment.

The probability of no particle emission following muon capture in ^{28}Si has been determined by Bunatyan et al.⁸⁷ and Eckhause et al.⁸⁵ Their measured probabilities are 0.28 ± 0.04 and 0.26 ± 0.03 respectively, and are consistent with the 0.36 ± 0.06 probability of no neutron emission quoted by MacDonald. Since only 13% of the muon captures lead to excited states within the range detectable in this experiment it appears that there is a significant yield directly to the ground state.

We note here that Bunatyan⁸⁷ predicts a 0.06 probability for no neutron emission following muon capture in Al. Using an activation technique Bunatyan et al.⁸⁷ measure a 0.10 ± 0.01 probability for no particle emission. The prediction agrees well with the results of this experiment and with those of MacDonald. Bunatyan's measurement is a little higher than these.

E. Comments on Charged Particle Emission

We first note that significant charged particle emission is seen in the Si and Ca targets, whereas there is no evidence in our data for this occurring in Al and Co. In fact, the nuclear level diagrams showing gammas observed following muon capture in Al and Co are strikingly similar.

Balashov³⁶ has formulated some general principles for the description of the disintegration of light nuclei in the 2s-1d shells following muon capture, with special reference to charged particle emission. According to these principles, charged particle emission should be more prominent in nuclei with half-filled shells and less likely for closed shell nuclei. In our experiment no charged particle emission was observed in the Al target. However, little is known about the excited states of ²⁶Na which is the residual nucleus following proton emission. At any rate, the absence of unidentified lines of significant yield implies that charged particle emission to excited states of residual nuclei is infrequent. On the other hand, nearly 10% of the time a neutron and a proton, perhaps as a deuteron, were emitted following μ^- capture in ²⁸Si. However, with the present assignment no single proton emission was observed. (Assigning the full intensity of the 985 keV line to ²⁷Mg gives less than 1% single proton emission.) Whereas, in Ca single proton emission was observed about 7% of the time and neutron plus proton emission occurred about 9% of the time, nearly as frequently as in Si. No charged particle emission in ⁵⁹Co is consistent with the 27 protons being a nearly closed shell. It is clear then, that the "principles" proposed by Balashov do not hold consistently for the cases studied here.

We note that Balashov based his formulation on the analogy of the muon capture process with photoexcitation. He qualifies the "principles"

by noting that the analogy holds only for the first forbidden transitions in muon capture. The lack of agreement between the "principles" and our results indicates once again that allowed and second forbidden transitions play a significant role in muon capture in the nuclei studied here.

An experiment which determined the energy spectrum and branching ratio for charged particle emission following muon capture in ^{28}Si using a $\text{Si}(\text{Li})$ target-detector system has been performed by Sobottka and Wills.¹⁰⁹ They give the branching ratio as 0.15 ± 0.02 charged particle per muon capture. This is consistent with the lower limit set by our experiment of 0.115 ± 0.012 .

We note that the charged particle energy spectrum of Sobottka and Wills does not exhibit resonances such as the neutron energy spectra of Evseev et al.¹³ and Plett and Sobottka.⁵⁴

As was mentioned in the section reporting our Si results, Vil'gel'mova et al.⁸⁸ have measured the probabilities of the muon capture in Si proceeding via the $^{28}\text{Si}(\mu^-, \nu p)^{27}\text{Mg}$ channel. Their result is that this reaction occurs $5.3 \pm 1.0\%$ of the time. As was noted earlier, the assignment of the full 985 keV gamma seen in our experiment as a transition in ^{27}Mg would result in a population of ^{27}Mg of less than 1%.

At the intermediate excitation energies observed in our experiment alpha emission corresponding to the reaction

is energetically possible. Lines corresponding to transitions from excited states in ^{24}Na were searched for but none were found. Similar searches for gamma transitions in residual nuclei which would imply alpha particle emission were conducted for each of the other targets with no indication of this event occurring.

Budyashov et al.¹¹⁰ have reported probabilities for charged particle

emission following muon capture in several targets. Their results are for protons with energies greater than 15 MeV and deuterons with energies greater than 18 MeV. The measurements were performed for ^{28}Si , ^{32}S , ^{40}Ca , and ^{64}Cu . They simultaneously measured the ionization loss dE/dx and the total energy E and thus separated particles of the same charge but different mass.

A comparison of the charged particle yields of Budyashov et al.¹¹⁰ and the lower limits on these yields set in this experiment is made in Table XIV. The yield for deuteron emission quoted for this experiment assumes the neutron and proton leave the intermediate nucleus as a deuteron. Since the deuteron binding energy is only 2.2 MeV, many deuterons may dissociate and thus be counted as protons in Budyashov's experiment.

Table XIV

Target	Proton (%)		Deuteron (%)	
	Budyashov	This experiment	Budyashov	This experiment
Si	0.88±0.06	9.2±1.0	0.33±0.03	2.3±1.0
Ca	1.30±0.11	6.7±1.3	0.26±0.03	8.5±1.4

F. Comparison with Radiative Pion Capture Results

As was described for Ca and Si. and in the introductory theory, aspects of muon capture are analogous to photoexcitation and inelastic electron scattering. Another analogous reaction is radiative pion capture. This reaction occurs as

$$\pi^- + N(A, Z) \rightarrow N^*(A, Z - 1) + \gamma$$

The analogy arises as a consequence of the similarity of the interaction Hamiltonians for the two processes.¹¹¹

We note that since the pion has negative intrinsic parity that pion capture from the atomic 2p orbit populates the same parity nuclear states as muon capture from the 1s orbit. However, due to the fact that pions interact strongly most of the nuclear pion captures occur from orbits further out than the 1s orbit. In fact even for ¹²C, 92% of the captures occur from the 2p orbit.¹¹² Consequently, for light nuclei nuclear states of the same parity are populated in pion and muon capture.

Lam et al.¹¹³ have measured the neutron energy spectra from radiative pion capture in ¹²C, ¹⁶O, and ⁴⁰Ca as well as the angular distribution of photons emitted in coincidence with these neutrons. They see structure in the neutron spectra of ¹²C and ¹⁶O, but do not see this structure for ⁴⁰Ca. They interpret the asymmetries in the gamma angular correlation data as evidence that a resonance process is involved in the capture mechanism only 67%, 74% and 71% of the time for the ¹²C, ¹⁶O, and ⁴⁰Ca targets respectively. They attribute the balance of the captures to a direct process and show the results of calculations¹¹³ based on such a process which describe the high energy portions of their neutron spectra well.

Holland et al.¹¹⁴ also used a time of flight technique to measure the neutron spectra from radiative pion capture in ¹⁶O and ²⁸Si. They see

structure in the spectra for both nuclei.

Alder et al.¹¹⁵ have measured the neutron and gamma ray spectra in coincidence following radiative pion capture in ^{16}O . They see structure in both spectra.

The Crowe group,¹¹⁶ at this Laboratory, has used a pair spectrometer to measure the gamma energy spectrum from radiative pion capture in ^4He , ^{12}C , ^{16}O , ^{24}Mg , and ^{40}Ca . They see structure in the spectra of ^4He , ^{12}C and ^{16}O but see no significant structure in the ^{24}Mg and ^{40}Ca spectra.

The evidence, from the radiative pion capture experiments, for a resonance mechanism in the capture process for capture in ^{16}O and ^{28}Si is reassuring, but the lack of evidence for such a process in capture in ^{40}Ca is surprising.

The similarity of the Hamiltonians for the muon capture and radiative pion capture reactions is in the axial vector and pseudoscalar nuclear form factors which appear for both reactions. Muon capture has additional vector terms. Part of the explanation for the different results may be in the different Hamiltonians. However, a more likely explanation is that pion capture in medium and heavy nuclei occurs predominantly from atomic orbits further out than the 2p orbit and the close analogy for the two processes then breaks down.

G. Isospin Splitting of the Giant Resonance

As described in the section on the giant resonance mechanism, the isospin selection rules⁴⁵ allow only $T = 1$ state formation by the absorption of electric dipole radiation by a nucleus with ground state isospin $T = 0$. However, for nuclei with ground state isospin $T_g \neq 0$ excited states with isospin $T = T_g$ or $T = T_g + 1$ may be formed.

Attempts have been made to see this isospin splitting of the giant dipole resonance.¹¹⁷ The results of these attempts are thus far inconclusive. Since the states $|T_g + 1, T_g + 1\rangle$ formed in muon capture are the parent states of the $|T_g + 1, T_g\rangle$ states formed in photoexcitation, Ubersall²⁰ has suggested that information of the isospin splitting to the $T_g + 1$ state in photoexcitation can be obtained from a study of the muon capture reaction. He suggests in particular that a comparison of the two modes of excitation in ^{27}Al could be made since photoexcitation studies have been made on this nucleus.

A correlation of the probabilities for populating states in ^{26}Mg in the two reactions $^{27}\text{Al}(\mu^-, \nu n)^{26}\text{Mg}$ and $^{27}\text{Al}(\gamma, p\gamma')^{26}\text{Mg}$ is made in Fig. 38. The photoexcitation results are those of Thompson et al.¹¹⁰ Clearly, the same states are populated strongly in the two reactions.

An energy level diagram (Fig. 39) shows schematically the isospin states populated in the two reactions and the particle decay of these states to an excited level of ^{26}Mg . The states are labelled with kets $|T, T_z\rangle$, where T is the total isospin and $T_z = 1/2(N - Z)$ is the projection of the isospin T on the z -axis in isospace. From this picture it is clear that the same final states can be populated. And to the degree that the first forbidden, isospin resonance mechanism dominates in the muon capture reaction, an implication that much of the photoexcitation proceeds through the $\Delta T = 1$ mode may be drawn. The reason is,

XBL 723-611

Fig. 38. Relative probabilities of exciting states in ^{26}Mg following muon capture in ^{27}Al (solid lines) and in the $^{27}\text{Al}(\gamma, p)^{26}\text{Mg}$ reaction (dotted lines). The photo-excitation data from Ref. 110.

EXB 700-11

Fig. 39. Possible intermediate excitations in the photoexcitation and muon capture reactions on ^{27}Al leading to excited states in ^{26}Mg where the states are labeled with their isospin quantum numbers.

that the upper isospin $|3/2, 1/2\rangle$ state in ^{27}Al is the analog of the $|3/2, 3/2\rangle$ parent state in ^{27}Mg . However, the picture that has been presented here is much over simplified. For example, even within the first forbidden type of muon capture transitions one has the spin flip possibility which results in $\Delta J = 0, 2$ in addition to $\Delta J = 1$. These transitions are not analogous to the electric dipole photoexcitation reaction. Also, as was shown for ^{28}Si , there is evidence for significant allowed transition strength in the muon capture reaction for mid shell nuclei. And as mentioned earlier, because of the large mass of the muon and the consequently large energy carried away by the neutrino, higher order terms in the expansion of the outgoing neutrino wave are expected to be significant in the muon capture process. Thus it is expected that a significant amount of transitions of the "second forbidden" type occur. Evidence for these is the very high intermediate excitation energies implied by the levels seen in the residual nuclei in this experiment.

In spite of all these qualifications, the correlation shown in Fig. 38 is quite good.

VII. CONCLUSIONS

From an analysis of gamma rays following μ^- capture in Al, Si, Ca, and Co the following conclusions are made: the direct population of "hole" states in ^{39}K following muon capture in ^{40}Ca indicates that the muon capture reaction might be used in a manner similar to the single nucleon pick up reactions to study simple shell model structure; the interpretation of the ^{28}Al levels populated following muon capture in ^{28}Si as single particle-single hole configurations adds strength to this argument, however the strong population of states in ^{27}Al which are known to involve collective excitations of the Si core cloud this interpretation somewhat in the mass number 28 region; a comparison of the muon capture results with the results of photoexcitation experiments supports the existence of a giant resonance mechanism in muon capture, but the deviations in the relative intensities for populating final states in the two reactions shows clearly that the mechanisms are not identical; a correlation with electroexcitation data on ^{28}Si suggests the presence of a giant resonance mechanism in muon capture which is analogous to the giant M1 dipole resonance seen in electroexcitation; the consistency of our data with published results on neutron multiplicities and charged particle emission following muon capture shows that lower limits on these phenomena may be set using this technique; and finally, the existence of Doppler broadened peaks in our data is consistent with known lifetimes of nuclear levels.

In an attempt to study nuclear structure from the analysis of gamma rays following μ^- capture the results of our experiment have been compared with the theoretically calculated rates where such exist and an attempt to relate the results to simple models and thus extract structure information has been made. These efforts were limited to a discussion of the

levels populated in ^{39}K following capture in ^{40}Ca and the levels populated in ^{28}Al and ^{27}Al following capture in ^{28}Si . Similar discussions for all the residual nuclei formed in the capture reaction might be helpful in determining what nuclear structure information can be learned using this technique. To make such discussions quantitative further calculations of the type made by Uberall and collaborators and Balashov et al. are in order.

An experiment which measured the gamma energy and emitted-neutron energy in coincidence would be of great importance in providing detailed information on the intermediate states and thus the giant resonance levels in the intermediate nucleus (the parent levels of those produced in photo-excitation). An initial attempt at such a coincidence measurement has been made by Shroeder¹¹⁹ who measured the neutron energy spectrum for neutrons in coincidence with any gamma ray for Ca, Tl, Pb, and Bi. A resonance structure was observed for Ca, but no such structure was evident for the heavier targets. An experiment to measure the complementary neutron and gamma ray in coincidence will surely be performed at one of the meson factories which will soon be operational.

Finally, we note that it has been suggested¹¹⁷ that the isospin splitting of the giant dipole resonance may be studied via the muon capture reaction in such nuclei as ^{40}Ca , ^{48}Ca , ^{88}Sr , ^{140}Ce , and ^{208}Pb .

APPENDICES

A. Details on the Efficiency Calibration

A detailed description of the corrections made for the efficiency calibration calculations is given in this Appendix. As discussed in the text in the section titled "Efficiency Calibration," the key to the technique used was a measurement of the K_{α} x-ray yield per muon stop for several targets.

The steps leading to the FEP absolute efficiency curve included the following corrections: target self attenuation; solid angle subtended by the detector; and a subtraction of the muon-stop triggers- $S_1 S_2 S_3 \overline{AC}$ -- which correspond to muons stopping in scintillator S_3 .

1. Muon Stopping Distribution

The first two corrections listed depend on the location in the target at which the muon stops. This dependence was folded into these corrections by the inclusion of a weighting factor given by the muon stopping distribution in the target.

A muon stopping distribution was determined for each target from a differential range curve for muons stopping in CH_2 . In order to do this an effective CH_2 thickness must be used for each stopping material. The CH_2 equivalent thickness for element x was taken to be

$$\Delta t_{CH_2 \text{ eq.}} = \Delta t_x \left[\frac{(dE/d\xi)_x}{(dE/d\xi)_{CH_2}} \right]$$

where Δt_x is the target thickness in gm/cm^2 and $\left[\frac{(dE/d\xi)_x}{(dE/d\xi)_{CH_2}} \right]$ is an average over three energies of the ratio of the stopping power of the element x to the stopping power of CH_2 . This relation requires that the energy lost by a muon in traversing Δt_{CH_2} gm/cm^2 of CH_2 equal that lost in traversing Δt_x gm/cm^2 of element x, i.e.,

$$(\Delta\xi)_{\text{CH}_2} \frac{dE}{d\xi}_{\text{CH}_2} = (\Delta\xi)_x \frac{dE}{d\xi}_x$$

The stopping power $dE/d\xi$ at energies of 10-, 26-, and 50-MeV, for several elements and CH_2 , as given in Barkas and Berger¹²⁰ are listed in Table XV.

Table XV

Table of $dE/d\xi$ (MeV $\cdot$ cm ² $\cdot$ gm ⁻¹) ^{a)}							
E (MeV)	CH_2	Al	Fe	Cu	Ag	Pb	Lucite
50	3.07	2.29	2.08	2.00	1.80	1.51	2.85
26	4.37	3.24	2.92	2.81	2.51	2.09	4.05
10	8.52	6.22	5.55	5.32	4.70	3.91	7.88

a) Ref. 120.

Ratios $(dE/d\xi)_{\text{CH}_2} / (dE/d\xi)_x$ computed at these same energies for the same elements are also listed. The average values for these ratios over the three energies was then plotted vs. atomic number (Fig. 40). Average values for these ratios for targets not listed in the tables of Barkas and Berger were then interpolated from a linear least squares fit to these points.

The measured differential range curve for muons stopping in CH_2 is shown in Fig. 41. This curve was obtained by scaling the stop triggers-- $S_1 S_2 S_3 \overline{\text{AC}}$ --for a fixed number of incident particles-- S_1 --at several degrader thicknesses. This was a no-target range curve, i.e., the particle-stop trigger $S_1 S_2 S_3 \overline{\text{AC}}$ corresponds to a stop in the 1/4-in. scintillator S_3 .

The large peak at 6.5 in. CH_2 corresponds to the π^- range. The lower peak centered at 11 in. CH_2 corresponds to stopping muons. The solid line is a Gaussian with FWHM = 3 in. centered at and scaled to the

XBL 721-29

Fig. 40. Effective CH₂ thickness of various absorbers. For example, 1 in. of iron is equivalent to 1.5 in. of CH₂.

XBL 721-34

Fig. 41. Differential range curve for negative mesons.

maximum stops at the π^- range. The estimated π^- contamination with the degrader thickness set at 9.75 in. is less than 3% and was neglected in our calculations. A muon stopping distribution, i.e., $S(t)$ - the number of muon stops per 1/4-in lucite at t inches into the target, can be obtained from the differential range curve. This is just the CH_2 differential range curve beginning at a degrader thickness of 9.75 in. and extending t inches where

$$t = \frac{\Delta t_{\text{CH}_2}}{2.54 \rho_{\text{CH}_2}}$$

Δt_{CH_2} is given above and the density of CH_2 is, $\rho_{\text{CH}_2} = 0.9 \text{ gm/cm}^3$.

As an example, 0.25-in. Pb would extend 1.53 inches.

The targets were placed at 45 deg to the beam line, so the thickness used in stopping distribution calculations is 1.414 times the actual thickness.

2. Solid Angle Correction

Measurements of the K_{α} x-ray yield from a 0.25-in. thick Pb target placed at positions in the target holder nearest to and furthest from the Ge(Li) detector showed a 15% variation due to the difference in the solid angle subtended by the detector at the two target positions. A solid angle correction as a function of position was made as a linear interpolation of the Pb values normalized to the position nearest the detector.

3. Target Self Attenuation

The amount of photon attenuation in escaping the target depends on how much material the photon must pass through and the photon energy. For computing an absolute efficiency using the K_{α} x rays from several targets, attenuation coefficients at the energy of the x ray for each target were used. Distances from the point of production to the point

of exit from the target were computed with the assumption that all interactions occurred on the beam centerline. For the effective detector efficiency determined for each of the four targets for which gamma spectra were measured, the same geometrical treatment was used, but attenuation coefficients were interpolated as a function of energy from those listed by Hubbell and Berger.¹²¹

An empirical test of the attenuation of the Pb μ -mesic x rays by the Ni target was made. For this test the Ni target was placed between the Pb target and the Ge(Li) detector. The measured attenuation of the K_{α} (5.8 MeV) and L_{α} (2.6 MeV) x-ray lines is compared below with the calculated attenuation.

Energy (MeV)	Experimental attenuation	Theoretical attenuation
2.6	0.659 $\pm$ 0.052	0.689
5.8	0.763 $\pm$ 0.039	0.737

Interpolated attenuation coefficients from Hubbell and Berger¹²¹ were used in the calculations. The agreement is good, thus the use of the analytic expression for self absorption in the targets was assumed to be justified.

4. Correction for Muons Stopping in S_3

A muon-stop trigger $S_1 S_2 S_3 \overline{AC}$ means that the muon gave a scintillation signal in counters S_1 , S_2 , and S_3 but not A and that it did not give off Cerenkov radiation in C. Such a muon must stop either in the target or counter S_3 . To correct for muons stopping in S_3 a no-target measurement of the ratio R of the number of muon-stop triggers ($S_1 S_2 S_3 \overline{AC}$) to the number of muons incident upon counter S_3 ($S_1 S_2 S_3$) was made. Since $S_1 S_2 S_3$ was scaled during the experiment the net number of muon stops in the target is given by

$$\text{Net Stops} = S_1 S_2 S_3 \overline{AC} - S_1 S_2 S_3 \cdot R$$

The Si and Co targets were compressed powders encased in plexiglass boxes. A correction for muons stopping in the plexiglass boxes was made for these two targets.

5. Ratio of $K_\alpha/\Sigma K$

The ratio $K_\alpha/\Sigma K$ was determined experimentally for the Al, Si, Ca, Fe, Co, Ni, Cu and Mo targets.

As a first step this ratio was determined from just the raw yields of the K x-ray series in each target assuming the efficiency for detector of each line in the series to be the same. An efficiency curve was plotted using these values. Then the ratio was recalculated in an iterative fashion using the relative efficiency at the energy of each line in the K series.

That is

$$\frac{K_\alpha}{\Sigma K} = \frac{K_\alpha}{K_\alpha + \frac{\epsilon_\alpha}{\epsilon_\beta} K_\beta + \frac{\epsilon_\alpha}{\epsilon_\gamma} K_\gamma + \frac{\epsilon_\alpha}{\epsilon} K_{\text{others}}}$$

where

K_i = number of counts in the i th line of K series

ϵ_i = efficiency at energy of i th line

The reduction of the ratio from the first to second iteration was 10% for Al and 5% for Mo. The values are tabulated and compared with other measured values in Table III. The errors quoted are statistical and the error in the efficiency ratios is neglected.

The $K_\alpha/\Sigma K$ ratio was taken to be 0.91 for Pb. This is the result of a calculation by Sundaresan.⁶³ The ratio was taken as 0.98 for both Ag and Sn. A plot of the absolute efficiency on semilog paper compares well with an efficiency response reported by Cline¹²² for such a detector over

an energy range including these x rays and this indicates that these assumptions are reasonable.

6. Absolute Detector Efficiency

The absolute efficiency ϵ at the K_{α} energy for a particular target is then given by

$$\epsilon = \frac{\text{number of photons detected}}{\text{effective number of photons produced}}$$

that is

$$\epsilon = \frac{\text{net } K_{\alpha} \text{ area}}{(\text{net muon stops})(K_{\alpha}/EK)(SAG)}$$

where SAG is the self-attenuation and geometry correction described above

$$SAG(E_{\gamma}) = \frac{\int S(t)G(t)A(t, E_{\gamma})dt}{\int S(t)}$$

and

$S(t)$ = the muon stopping distribution

$G(t)$ = the solid angle correction

$A(t, E)$ = the photon attenuation in the target correction factor

The integrals are over the target thickness along the beamline. The error in this efficiency was determined in the standard way with an estimated error of 3% for the SAG term.

The absolute efficiency determined in this manner is shown in Fig. 18. The exponential character of this plot over most of the energy range is to be expected for a detector such as the one used in this experiment.¹²²

7. Relative Efficiency of Detector

As mentioned in the text an FEP relative efficiency was determined for the detector. This was done to extend the energy range and to determine the efficiency response of the detector in more detail.

Measurements at 37 energies from 59 keV to 4072 keV were made using IAEA standard sources and other radioisotopes.^{67,68}

3. Effective Efficiency

This relative efficiency was then scaled at the K_{α} x-ray energy for each of the four targets studied to a value

$$\epsilon_{\alpha} = \frac{\text{Net } K_{\alpha} \text{ Area}}{(\text{Net Muon Stops})(K_{\alpha}/\Sigma K)}$$

A self-attenuation correction for that target was then made as a function of energy to obtain the effective detector efficiency at each energy, $\text{eff}(E)$.

The errors for the effective efficiency were then treated as multiplicative, that is

$$\Delta_{\text{eff}}(E) = \sqrt{\Delta A^2 + \Delta(K_{\alpha}/\Sigma K)^2 + \Delta \epsilon_R^2}$$

where

ΔA = fractional error in net K_{α} area

$\Delta(K_{\alpha}/\Sigma K)$ = fractional error in ratio

$\Delta \epsilon_R$ = fractional error in relative efficiency

This effective efficiency is shown for Ca in Fig.19 .

B. Doppler Broadened Gamma Peaks Following Muon Capture

The first order expression for the energy seen in the lab of a nuclear gamma ray emitted from a nucleus while it is recoiling with a speed v is¹²³

$$E = E_0(1 - \beta\mu)$$

where

E_0 = the nuclear transition energy

$\beta = v/c$

c = the speed of light

$\mu = \cos \theta$

θ = angle between nuclear momentum vector and the direction of the detector

Figure 42 shows such a process. If the nuclear recoils are isotropic the probability of observing a gamma from a nucleus recoiling in $d\mu$ about μ is

$$P(\mu)d\mu = 1/2 d\mu$$

The energy distribution of gammas is then

$$P(E) = P(\mu) \left| \frac{d\mu}{dE} \right| = \frac{c}{2E_0 v}, \quad -\beta \leq \frac{E - E_0}{E_0} \leq \beta$$

This distribution is shown in Fig. 24a. If no particle emission follows muon capture and the lifetime of a populated level is short compared to the slowing down time of the nucleus from the neutrino recoil this distribution applies to the emitted gamma rays. (See the comment on the validity of assuming an isotropic neutrino distribution made in the section on Doppler broadened lines in Chap. IV.) For example, gamma emission from short lived levels in ^{40}K and ^{28}Al following muon capture in ^{40}Ca and ^{28}Si fit this description.

The energy distribution of gamma rays emitted from a nucleus following two isotropic recoils is of interest. This is assumed to describe a

XBL 721-26

Fig. 42. Gamma ray emitted from a moving nucleus.

situation when neutron emission follows muon capture. Figure 43 shows the process schematically. The distribution of the z-component of the nuclear velocity from each recoil is

$$P_1(v_{vz}) = \frac{1}{2v_v}, \quad -v_v \leq v_{vz} \leq v_v$$

and

$$P_2(v_{nz}) = \frac{1}{2v_n}, \quad -v_n \leq v_{nz} \leq v_n$$

where

v_v = nuclear recoil velocity from the emitted neutrino

v_n = nuclear recoil velocity from the emitted neutron

The z-component of the total nuclear recoil velocity is

$$v_z = v_{vz} + v_{nz}$$

Conservation of probability gives the relation

$$\int_{-v_z}^{v_z} P(v_z) dv_z = \int_{-v_v}^{v_v} \int_{-v_n}^{v_n} P_2(v_{nz}) P_1(v_{vz} = v_z - v_{nz}) dv_{nz} dv_{vz}$$

Since $P(v_{vz}) = \frac{1}{2v_v}$, and is independent of v_z a change of variables gives

$$P(v_z) dv_z = \frac{1}{4v_v v_n} \left[\int dv_{nz} \right] dv_z$$

The regions of interest are shown in v_z - v_{nz} space in Fig. 44, in this diagram $v_n > v_v$. Carrying out the integration in regions I, II, and III gives after converting to energy

$$\text{Region I: } E_o \left(1 - \frac{v_n + v_v}{c} \right) \leq E_\gamma \leq E_o \left(1 + \frac{|v_n - v_v|}{c} \right)$$

$$P(E_\gamma) = \frac{c}{E_o} \frac{1}{4v_n v_v} \left[v_v + v_n + \frac{c(E_\gamma - E_o)}{E_o} \right]$$

$$\text{Region II: } E_o \left(1 - \frac{|v_n - v_v|}{c} \right) \leq E_\gamma \leq E_o \left(1 + \frac{|v_n - v_v|}{c} \right)$$

$$P(E_\gamma) = \frac{c}{2E_o v_n}$$

XBL 72j-613

Fig. 43. Gamma ray emitted from a moving nucleus following two nuclear recoils.

INTEGRATION REGIONS

XBL 721-31

Fig. 44. Diagram of integration region for two nuclear recoil problem after change of variables.

$$\text{Region III: } E_0 \left(1 + \frac{|v_n - v_v|}{c} \right) \leq E_\gamma \leq E_0 \left(1 + \frac{v_n + v_v}{c} \right)$$

$$P(E_\gamma) = \frac{c}{E_0} \frac{1}{4v_n v_v} \left[v_v + v_n - \frac{c(E_\gamma - E_0)}{E_0} \right]$$

For the case $v_v > v_n$ the distribution in regions I and III remain the same and in region II

$$P(E_\gamma) = \frac{c}{2E_0 v_v}$$

This distribution is shown in Fig. 24b.

C. Population of State Calculations

The probability for the direct population of a particular state in a residual nucleus was calculated using the measured transition intensity for the gamma lines identified as transitions from that state and branching ratios from published decay schemes. (See Table VII for references used for branching ratios.) Where transitions from higher energy states were seen, appropriate account was taken for known cascade feeding of levels.

These population of state calculations are now described in detail. Consider a state $|A\rangle$ of a residual nucleus which decays via two branches to states $|B\rangle$ and $|C\rangle$ with branching probabilities λ_B and λ_C (Fig. 45). Say the branch to state $|B\rangle$ is observed with intensity $I_B \pm \Delta I_B$ and similarly the branch to state $|C\rangle$ is observed with intensity $I_C \pm \Delta I_C$. The number of photon transitions T_B originating from state $|A\rangle$ as calculated from the intensity of the observed transition to state $|B\rangle$ is given by

$$T_B = \frac{I_B}{\lambda_B}$$

$$\Delta T_B = \frac{\Delta I_B}{\lambda_B}$$

where the error in the branching probability is taken to be negligible. A similar relation obtains for the branch to state $|C\rangle$.

The mean number of photon transitions originating from state S determined from the weighted average of T_B and T_C is then

$$T = \frac{T_B / \Delta T_B^2 + T_C / \Delta T_C^2}{1 / \Delta T_B^2 + 1 / \Delta T_C^2}$$

with root mean square error

$$\Delta T = \sqrt{(T_B - T)^2 + (T_C - T)^2}$$

XBL 723-612

Fig. 45. Hypothetical decay scheme.

Treating the errors in this manner leads to a conservative estimate of the error. That is, due to the method in which the efficiency curve was normalized, the contribution of the normalization error is counted twice. This leads in the worst case, for intense lines to an overestimate of the error of perhaps 30%. For weakly populated lines this contribution to the error is small.

The probability of directly populating the state $|A\rangle$ -- P -- was then taken as T if no transitions from higher energy states populated state $|A\rangle$ via cascades. If cascade populations were identifiable they were subtracted from T to obtain P. For example, if a higher energy state $|A'\rangle$ decays via a branch to state $|A\rangle$, using the notation introduced above

$$P = T - \lambda_A T' .$$

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FOOTNOTES AND REFERENCES

1. M. Conversi, E. Pancini, and O. Piccioni, Phys. Rev. 71, 209 (1947).
2. Lattes, Occialini, and Powell, Nature 160, 453 (1947).
3. H. Yukawa, Proc. Phys. Math. Soc. Japan 17, 48 (1935); Revs. Mod. Phys. 21, 474 (1949).
4. Two recent reviews on muonic atoms are C. S. Wu and L. Wilets, in Annual Reviews of Nuclear Science, v. 19, E. Segre, editor (Annual Reviews, Inc., Palo Alto, California, 1969) p. 527; and S. Devons and J. Duerdoth in Advances in Nuclear Physics v. 2, M. Baranger and E. Vogt, editors (Plenum Press, New York, 1969) p. 295.
5. E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).
6. M. Eckhause, R. T. Siegal, and R. E. Welsh, Nucl. Phys. 81, 575 (1966).
7. J. C. Sens, Phys. Rev. 113, 679 (1958).
8. H. Primakoff in Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics (Interscience Publishers, Inc. New York, 1955) p. 174; Revs. Mod. Phys. 31, 802 (1959).
9. B. M. Spicer in Advances in Nuclear Physics v. 2, M. Baranger and E. Vogt, editors (Plenum Press, New York, 1969) p. 1.
10. G. C. Baldwin and G. S. Klaiber, Phys. Rev. 73, 1156 (1948).
11. There are many reviews of the Giant Dipole Resonance. An early review was given by Evans Hayward, Revs. Mod. Phys. 35, 324 (1963). A recent review with many references is given by F. W. K. Firk, Annual Review of Nuclear Science, E. Segre, editor (Annual Reviews, Inc., Palo Alto, California 1970) p. 39.
12. B. MacDonald, J. A. Diaz, S. N. Kaplan and R. V. Pyle, Phys. Rev. 139B, 1253 (1965).

13. V. Evseev, T. Kozlowski, V. Roganov, and J. Woitkowska in High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 157.
14. As used here a partial capture rate is defined as the probability per unit time that a muon in a $1s$ atomic orbit will be captured by the nucleus and produce an excited state in the intermediate nucleus, or an excited state in a residual nucleus following particle decay of the intermediate nucleus, which then decays by the emission of a gamma ray. The total nuclear capture rate then is the sum over all the partial capture rates.
15. S. N. Kaplan, R. V. Pyle, L. E. Temple and G. F. Valby, Phys. Rev. Letters 22, 795 (1969).
16. R. Raphael, H. Uberall, and C. Erntz, Phys. Letters 24B, 15 (1967).
17. For example, if the muon is captured on a quasi-deuteron consisting of a correlated proton pair ($p-p$), one of the protons would be changed to a neutron and the resulting quasi-deuteron consisting of a correlated neutron-proton pair ($n-p$) might receive enough kinetic energy to leave the nucleus. The emission of a single proton following muon capture might be explained by a capture on a $p-p$ quasi-deuteron plus a process such that significant kinetic energy is transmitted to the proton of the resulting $n-p$ pair which is then emitted from the nucleus leaving the neutron behind.
18. Much of this review follows the review by V. V. Balashov and R. A. Eramzhyan, Atomic Energy Review 5, 3 (1967).
19. A current review is by J. D. Walecka "Semi-Leptonic Interactions in Nuclei" to be published in Muon Physics, V. W. Hughes and C. S. Wu, editors.

20. A recent review on the study of nuclear structure by muon capture is by H. Uberall, Proceedings of the Muon Physics Conference (to be published by Marcel Dekker, Inc., New York, 1972).
21. This value is the average quoted in the Particle Data Group report in Rev. Mod. Phys. 42, 87 (1970).
22. Jeremy Bernstein, Elementary Particles and Their Currents (W. H. Freeman Co., San Francisco, California, 1968). This reference gives a complete treatment of the Universal Fermi Interaction based on the modern current algebra theory.
23. A. Fujii and H. Primakoff, Nuovo Cimento 12, 327 (1959).
24. M. Bertero, G. Passatore, and G. A. Viano, Nuovo Cim. 38, 1669 (1965).
25. J. A. Wheeler, Revs. Mod. Phys. 21, 133 (1949).
26. H. A. Tolhoek and J. R. Luyten, Nucl. Phys. 3, 679 (1957);
H. A. Tolhoek, Nucl. Phys. 10, 327 (1959).
27. R. Klein and L. Wolfenstein, Phys. Rev. Letters 2, 408 (1962).
28. J. S. Bell and J. Lovseth, Nuovo Cimento 17, 408 (1964).
29. J. R. Luyten, H. P. Rood, and H. A. Tolhoek, Nucl. Phys. 41, 236 (1963).
30. L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964)
31. J. Barlow, J. C. Sens, P. J. Duke, and M. A. R. Kemp, Phys. Letters 2, 84 (1964).
32. S. A. E. Johansson and B. Forkman, Arkiv Fys. 12, 359 (1957).
33. H. Uberall (Catholic University of America, Washington, D. C.), private communication; L. L. Hill (Thesis), Catholic University of America, 1967 (unpublished).
34. G. E. Brown, Unified Theory of Nuclear Models and Forces (John Wiley and Sons, New York, 1967) p. 29.

35. E. Boeker and C. C. Jonker, Phys. Letters 6, 80 (1963).
36. V. V. Balashov, R. A. Eramzhyan, N. M. Kabachnick, G. Ya. Korenman and V. L. Korotkih, High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 174.
37. M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).
38. R. Montalbetti, L. Katz, and J. Goldemberg, Phys. Rev. 91, 659 (1953); R. Nathans and J. Halpern, Phys. Rev. 94, 437 (1954).
39. D. H. Wilkinson, Physica 22, 1039 (1956); Philosophical Magazine 3, 567 (1958).
40. D.J. Rowe, Nuclear Collective Motion; Models and Theory, (Methuen, 1970) p. 62.
41. J. P. Elliot and B. H. Flowers, Proc. Roy. Soc. A242, 57 (1957).
42. G. E. Brown and M. Bosterli, Phys. Rev. Letters 3, 472 (1959); G. E. Brown, L. Castillejo, and J. A. Evans, Nucl. Phys. 22, 1 (1961).
43. E. Hayward, in Nuclear Structure and Electromagnetic Interactions, N. MacDonald, editor (Plenum Press, New York, 1965) p. 141.
44. M. N. Thompson and J. E. E. Baglin, Phys. Letters B25, 256 (1967).
45. M. Gell-Mann and V. L. Telegdi, Phys. Rev. 91, 169 (1953).
46. V. Gillet, Proc. of the International School of Physics Enrico Fermi, v. 36 (Academic Press, Inc., New York, 1966) p. 43.
47. E. P. Wigner, Phys. Rev. 51, 106 (1937). The supermultiplets discussed in this paper encompass all of the states discussed in this section.
48. A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. of Phys. 6, 1 (1959); S. Fallieros, R. Ferrell, and M. K. Pal. Nucl. Phys. 15, 363 (1960).

49. H. Uberall, Phys. Rev. 137, B502 (1965); Nuovo Cimento 41, 25 (1966); Phys. Rev. 139, B1239 (1965).
50. H. Uberall (private communication).
51. L. W. Fagg, W. L. Bendel, E. C. Jones, Jr., and S. Numrich, Phys. Rev. 187, 1378 (1969).
52. G. Morpugo, Phys. Rev. 110, 721 (1968).
53. P. Singer, Nuovo Cimento 23, 669 (1962); Phys. Rev. 124, 1602 (1961).
54. M. E. Plett and S. E. Sobottka, Phys. Rev. C3, 1003 (1971).
55. R. M. Sundelin and R. M. Edelman, High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 150.
56. A. Bogan, Phys. Rev. Letters 22, 71 (1969).
57. H. Morinaga and W. F. Fry, Nuovo Cimento 10, 308 (1953).
58. C. Ishii, Prog. Theor. Phys. 21, 663 (1959).
59. L. B. Robinson, F. Gin, F. S. Goulding, Nucl. Instrum. Methods 62, 237 (1968). Also Lawrence Berkeley Laboratory Report UCRL-17419.
60. D. W. Jenson, Proceedings of the Fall Symposium of Digital Equipment Computer Users Society, Las Vegas, Nevada (Nov. 1969); University of California Lawrence Radiation Laboratory Report UCRL-19402, preprint (1969).
61. T. A. E. C. Pratt, Nucl. Instr. Methods 66, 348 (1968).
62. Y. Eisenberg and D. Kessler, Nuovo Cimento 19, 1195 (1961).
63. M. K. Sundareson (Carlton University, Ottawa, Canada) private communication.
64. A. Suzuki, Phys. Rev. Letters 19, 1005 (1967).
65. D. Quitmann, R. Engfer, U. Hegel, P. Brix, G. Backenstoss, K. Goebel, and B. Stadler, Nucl. Phys. 51, 609 (1964).
66. C. S. Johnson, E. P. Hincks, and H. L. Anderson, Phys. Rev. 125,

2102 (1962).

67. L. J. Jardine, University of California Lawrence Radiation Laboratory report UCRL-20476, 1971 (unpublished).
68. L. J. Jardine, Nucl. Instr. Methods 96, 71 (1971).
69. B. A. Euler and S. N. Kaplan, I. E. E. E. Trans. Nucl. Sci. NS-17, 4, 81 (1970).
70. C. Chasman, K. W. Jones, R. A. Ristinen, and J. T. Sample, Phys. Rev. 154, 239 (1966); H. W. Kraner, C. Chasman, and K. W. Jones, Nucl. Instr. Methods 62, 173 (1968).
71. M. G. Chartrand and H. L. Malm, Atomic Energy of Canada, Limited Report No. 2764, 1967 (unpublished).
72. F. H. Fowler and V. M. Mayes, Proc. Phys. Soc. 92, 377 (1967); M. P. Guthrie, R. G. Alsmiller, Jr., and H. W. Bertini, Nuc. Instr. Methods 66, 29 (1968).
73. H. L. Anderson, E. P. Hincks, C. S. Johnson, C. Rey and A. M. Negar, Phys. Rev. 133, B392 (1964).
74. E. K. Warburton, J. W. Olness, and D. E. Alburger, Phys. Rev. 140, B1202 (1963).
75. R. Wechsung, W. Strassheim, and R. Bass, Nucl. Phys. A170, 557 (1971).
76. S. N. Kaplan, R. V. Pyle, L. E. Temple and G. F. Valby, High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 163.
77. A. P. Bukhvostov and N. P. Popov, Sov. J. Nucl. Phys. 6, 589 (1967); Sov. J. Nucl. Phys. 6, 903 (1967); Phys. Letters 24B, 497 (1967); Z. Oziewicz and N. P. Popov, Phys. Letters 15, 273 (1965).
78. L. Grenacs, J. P. Deutsch, P. Lipnik, and P. C. Macq., Nucl. Instr. Methods 58, 164 (1967).

79. Nuclear Data Sheets, B3-3, 4-105 (Academic Press, New York).
80. L. V. Groshev, A. M. Demidov, G. A. Kotelnikov, and V. N. Lutsenko, Nucl. Phys. 58, 465 (1964).
81. G. Backenstoss, S. Charalambus, H. Daniel, W. D. Hamilton, U. Lynen, Ch. von der Malsburg, G. Poelz, and H. V. Povel, Nucl. Phys. A162, 541 (1971); also in High Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 160.
82. R. Bass and Fatma M. Saleh-Bass, Nucl. Phys. A131, 673 (1969).
83. P. Igo-Kemenes, J. P. Deutsch, D. Favart, L. Grenacs, P. Lipnik, and P. C. Macq, Phys. Letters 34B, 4, 286 (1971).
84. Jean-Luc Vuilleumier, (Thesis) Ecole polytechnique federale de Zurich, 1971 (unpublished).
85. M. Eckhause, P. Martin, G. H. Miller, and R. E. Welsh, IV Inter. Confr. on High-Energy Physics and Nuclear Structure, 1971 (proceedings to be published); G. H. Miller, M. Eckhause, P. Martin, and R. E. Welsh, 1972 (to be published)
86. T. A. E. C. Pratt (Thesis) Carnegie-Mellon University, 1968 (unpublished).
87. G. G. Bunatyan, V. S. Evseev, L. N. Nikityuk, V. N. Pokrovskii, V. N. Rybakov, and I. A. Yutlandov, Sov. J. Nucl. Phys. 11, 444 (1970); also High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1970) p. 182; G. G. Bunatyan, Sov. J. Nucl. Phys. 3, 613 (1966).
88. L. Vil'gel'mova, V. S. Evseev, L. N. Nikityuk, V. N. Pokrovskii, and I. A. Yutlandov, Sov. J. Nucl. Phys. 13, 310 (1971).
89. S. J. Skorka, D. Evers, J. Hertel, J. Morgenstern, T. W. Retz-Schmidt, and H. Schmidt, Nucl. Phys. 81, 370 (1966).

90. Neutron Cross Sections, v. 1, Brookhaven National Laboratory Report No. 325 (1964) p. 13-0-13.
91. D. O. Boerma and Ph. B. Smith, Phys. Rev. 24, 1200 (1971).
92. H. Ulrich and H. Krauth, Nucl. Phys. A123, 641 (1969).
93. S. Hinds and R. Middleton, Nucl. Phys. 24, 651 (1966).
94. A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, Phys. Rev. 102, 208 (1956); I. M. Naquib and G. W. Farwell, Bull. Am. Phys. Soc. 8, 318 (1963); J. S. Blair, Proc. Nuclear Spectroscopy with Direct Reactions, F. E. Throw, editor (Argonne National Laboratory Report No. 6878, 1964) p. 143.
95. C. Y. Fu and K. J. Yost, Nucl. Sci. Engin. 41, 193 (1970).
96. J. B. French, Advances in Nuclear Physics, v. 3, M. Baranger and E. Vogt, editors (Plenum Press, New York, 1969) p. 193.
97. P. W. M. Glaudemans, P. J. Brussard, and B. H. Wildenthal, Nucl. Phys. A102, 593 (1967).
98. P. M. Endt and C. van der Leun, Nucl. Phys. A105, 1 (1967); 34, 1 (1962).
99. M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).
100. D. E. Alburger and W. R. Harris, Phys. Rev. 185, 1495 (1969).
101. R. Hardell, S. O. Idetjarn, and H. Ahlgren, Nucl. Phys. A126, 392 (1960).
102. L. W. Nichol, A. H. Colenbrander, and T. J. Kennett, Canadian J. Phys. 47, 953 (1969).
103. P. H. Cannington, R. J. J. Stewart, G. R. Hogg, K. H. Lokan, and D. G. Sargood, Nucl. Phys. 72, 577 (1965).
104. For a recent review of the weak coupling model see, A. Arima and I. Hamamoto, Annual Review of Nuclear Science, E. Segre, editor (Annual Reviews, Inc., Palo Alto, Calif., 1971) p. 55. The

- results of weak coupling calculations for ^{27}Al are given in the following papers, G. M. Crawley and G. T. Garvey, Phys. Letters 19, 228 (1965); Phys. Rev. 167, 1070 (1968); J. Kokame, K. Fukunaga, and N. Nakamura, Phys. Letters 14, 234 (1965); H. Niewodniczanski et al., Nucl. Phys. 55, 386 (1964).
105. S. G. Nilsson, Mat. Fys. Medd. Dan Vid. Selsk. 29, no. 16, (1955).
 106. P. J. Smulders, C. Broude and F. Sharpey-Shafer, Can. J. Phys. 46, 261 (1968); I. Kelson and C. A. Levinson, Phys. Rev. 134, B269 (1964); I. Kelson, Phys. Rev. 132, 2189 (1963); C. A. Levinson, Phys. Rev. 132, 2184 (1963); and G. R. Bishop, Nucl. Phys. 14, 376 (1960).
 107. H. Ropke, V. Glattes, and G. Hammel, Nucl. Phys. A156, 477 (1970).
 108. A. Fassler, Nucl. Phys. 59, 177 (1964).
 109. S. E. Sobottka and E. L. Wills, Phys. Rev. Letters 20, 596 (1968).
 110. Yu. G. Budyashov, V. G. Zinov, A. D. Konin, A. I. Mukhin, and A. M. Chatrachyan, Sov. Phys. 33, 11 (1971).
 111. J. Delorme and T. E. O. Ericson, Phys. Letters 21, 98 (1966); H. Pietschmann et al., Phys. Rev. Letters 19, 1259 (1967); Griffiths and C. W. Kim, Nucl. Phys. B13, 573 (1970).
 112. W. W. Sapp, Jr., M. Eckhause, G. H. Miller, and R. E. Welsh, preprint (1971); W. W. Sapp, Pionic X-Ray Yields and Level Broadening in Low Z Atoms, College of William and Mary (Thesis) 1971 (unpublished); H. Koch, M. Krell, Ch. V. D. Malsburg, G. Poelz, H. Schmidt, L. Tauscher, G. Backenstoss, S. Charalambus, and H. Daniel, Phys. Letters 29B, 140 (1969).
 113. W. C. Lam, K. Gotov, B. MacDonald, W. P. Trower, and D. K. Anderson, Phys. Rev. Letters 28, 108 (1972).

114. M. M. Holland, R. C. Mineheart, and S. E. Sobottka, Nucl. Phys. 147A, 249 (1970).
115. J. C. Alder, R. Jerrefsky, C. Joseph, B. Vaucher, T. Bressani, E. Chiavassa, J. Favier, D. Schinzel, and C. Zupanic, Lett. Nuovo Cimento 4, 256 (1970).
116. J. A. Bistirlich, K. M. Crowe, A. S. L. Parsons, P. Skarek, and P. Truoeel, Phys. Rev. Letters 25, 689 (1970); Lawrence Berkeley Laboratory Report LBL-374, 1971 (submitted to Phys. Rev.).
117. See B. Coulard, J. Joseph and F. Ledoyen, Phys. Rev. Letters 27, 1238 (1971) and references cited in this paper.
118. J. E. M. Thompson, A Study of De-excitation Gamma Rays Following Disintegration of ^{32}S and ^{27}Al in the Giant Resonance Region, Progress Report of School of Physics, University of Melbourne (1970).
119. W. U. Schroeder (Thesis) Darmstadt Institute for Nuclear Physics 1971 (unpublished).
120. W. H. Barkas and M.J. Berger, National Aeronautics and Space Administration document SP-3013 (1964).
121. J. H. Hubbell and M. J. Berger, National Bureau of Standards Report No. 8681 (1966).
122. J. E. Cline, I. E. E. E. Trans. Nucl. Sci. NS-15, 3, 198 (1968).
123. This result follows from special relativity. See for example, J. D. Jackson, Classical Electrodynamics (John Wiley and Sons, Inc., New York, 1963) p. 364.
124. G. A. P. Engelbertnik, H. Lindeman, and M. N. J. Jacobs, Nucl. Phys. A107, 305 (1968).
125. Nuclear Data Sheets, B3-3, 4-45 (Academic Press, New York).
126. Nuclear Data Sheets, B3-3, 4-148 (Academic Press, New York).
127. Nuclear Data Sheets, B2-5-2 (Academic Press, New York).

- 128. S. Maripuu, Nucl. Phys. A151, 465 (1970).
- 129. P. J. Twin, W. C. Olsen, and D. M. Sheppard, Nucl. Phys. A143,
481 (1970).