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THE EFFECT OF STRESS STATE ON
HIGH-TEMPERATURE LOW-CYCLE FATIGUE

C. R. Kennedy

OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

for the

U.S. ATOMIC ENERGY COMMISSION

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ABSTRACT

A study of the behavior of Inconel at 1500°F under realistic dynamic stress states has been accomplished. The purpose of this work was to critically evaluate present criteria used to solve low-cycle fatigue problems and to demonstrate their applicability for the more complex situations.

The relative agreement between the static complex stress-creep-rupture results and the low-cycle fatigue data is demonstrated. From the relationships developed for static complex stress-creep-rupture data, it is demonstrated that the magnitudes of the effects of stress state and frequency on low-cycle fatigue can be determined.

INTRODUCTION

It is apparent to those concerned with determining life expectancy of component parts that the stress states are rarely simple in structural applications. The use of uniaxial data under these conditions is of little value unless the effect of the complex states is known. The study of the effect of complex stress states upon creep rupture has received some attention; however, the criterion for creep fracture has not been well established. The criterion for fracture is particularly vague when the complex stress states are dynamic in nature.

Several criteria have been formulated for evaluating low-cycle fatigue life based upon various parameters. It has been demonstrated by Coffin¹

¹L. F. Coffin, Trans. ASME 76, 931 (1954).

and others,²⁻⁵ for example, that material fatigue life can be related as proposed by Manson⁶ to the plastic strain range per cycle.

$$N^n \Delta \epsilon_p = K \quad (1)$$

where

N = number of cycles to failure,

$\Delta \epsilon_p$ = plastic strain range per cycle, and

n, K = material constants.

Since, in general, thermal-fatigue problems result from differential strains during thermal cycling, the plastic strain range per cycle provides a convenient parameter for evaluating low-cycle fatigue life. Equation (1) has recently received further support⁷ from proposed failure criteria based upon energy concepts which relate the material constants n and K to stress-strain data.

The above criterion, however, has not been demonstrated to be successful in correlating results under complex stress states. It has also been limited to temperatures below which the mechanical properties are time-dependent.

It is the purpose of this investigation to examine the criterion for failure under complex stress conditions at high temperatures. The experimental work consisted of two test series on Inconel tubes at 1500°F.

²C. R. Kennedy, "Plastic Strain Absorption as a Criterion for High-Temperature Design," p 193 in High-Temperature Materials, Their Strength Potentials and Limitations - Proceedings of the Fourth Sagamore Ordnance Materials Research, Syracuse Research Institute, New York, 1957.

³R. W. Swindeman and D. A. Douglas, Trans. ASME Ser. D 81(2), 203 (1959).

⁴R. W. Swindeman, Strain Fatigue Properties of Inconel. Part II. Isothermal Tests with Constant Hold Time, ORNL-3250 (March 29, 1962).

⁵A. E. Carden and S. H. Sodergren, "The Failure of 304 Stainless Steel by Thermal Cycling at Elevated Temperatures," paper presented at the ASME Annual Meeting, Nov. 26-Dec. 1, 1961, New York, 61-WA-200 (1961).

⁶S. S. Manson, Behavior of Materials Under Conditions of Thermal Stress, NACA TN-2933 (1953).

⁷G. R. Halford and JoDean Morrow, "Low-Cycle Fatigue in Torsion," paper presented at the ASTM Annual Meeting, June 25-30, 1961, Atlantic City, New Jersey, Preprint No. 63 (1962).

PROCEDURE

The material used is the same material used in previous investigations of creep under complex stresses⁸ and in uniaxial low-cycle fatigue.³ Specimens were machined from $\frac{3}{4}$ -in. sched-40 seamless Inconel pipe as shown in Fig. 1. The chemical analysis provided by the vendor is given below:

Element	Wt %
C	0.04
Mn	0.29
Fe	7.56
S	0.007
Cu	0.13
Ni	75.76
Cr	15.97

The specimens were reannealed at 1950°F for 2 hr in hydrogen after machining, producing the structure as shown in Fig. 2. All tests were performed at 1500°F, well into the creep range of the material.

The equipment used for the test series in which the tubes were internally pressurized and axially loaded has been described previously.² The cyclic stress states produced in the four tests of this type simulate a tube under internal pressure which is thermally cycled in the axial direction. The stress states applied must, therefore, produce an axial strain cycle about zero strain with the internal pressure held constant. This was done with the stress states as shown in Table 1.

Table 1. Stress States of Tubes Cycled with Constant Internal Pressure at 1500°F

Test Number	First Half of Cycle (psi)			Second Half of Cycle (psi)		
	Axial Stress	Tangential Stress	Radial Stress	Axial Stress	Tangential Stress	Radial Stress
	σ_z	σ_θ	σ_R	σ_z	σ_θ	σ_R
1	4,000	4,000	-285	-285	4,000	-285
2	6,000	4,000	-285	-2285	4,000	-285
3	8,000	4,000	-285	-4285	4,000	-285
4	10,000	4,000	-285	-6285	4,000	-285

⁸C. R. Kennedy et al., Trans. ASME Ser. D 81(4), 599 (1959).

Fig. 2. Photomicrograph of Inconel Annealed 2 hr at 1950°F Before Test. 100X.

As seen in Table 1, all tests were performed with the tangential stress equal to 4,000 psi for the complete cycle. The half-cycle stress states were calculated to produce principal strains equal and opposite in the axial direction; however, the radial and tangential strains were not equal and opposite. This type of testing is comparable to standard fatigue testing with a mean load except that the mean stress is not in the same stress axis.

The second test series consisted of torsion cycling of tubes. The equipment used for torsion cycling is shown schematically in Fig. 3. The specimen was induction brazed into the torque rods to eliminate "play" normally encountered in mechanical joints. The torque rods were designed to allow axial motion during test and particularly during heatup when the specimen and loading rods expanded. The force was applied by the double-acting air cylinder controlled by solenoid valves and regulators. The tests were performed by adjusting the regulators to produce the predetermined cyclic shear stress. A cyclic timer used to energize and deenergize the solenoid valves was set at a frequency of 1 cpm. The specimen was allowed to deform freely during the constant load applications and the strain was measured at frequent intervals during the test. The strain was read and monitored by the use of a one-meter galvanometer mirror placed on the torque rod below the furnace. The plastic strain was determined by releasing the torque at the end of two successive half cycles. Failure was determined by the first crack in the wall allowing a pressure of 5 psi to bleed down to 2 psi in 1 min. This pressure loss was sensed by a pressure switch. The test apparatus has the potential of performing tests on tubes with combinations of axial, tangential, and shear stresses.

RESULTS AND DISCUSSION

The objective of the first test series was to determine the effect of a creep stress imposed at 90 deg to the stress imposing the strain cycle. The results of this test series are given in Table 2.

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Fig. 3. Schematic Drawing of Torsion Cycling Test Equipment.

Table 2. Results of First Test Series

Test Number	Axial Plastic Strain Range per Cycle, ϵ_z (%)	Effective Plastic Strain Range per Cycle, $\bar{\epsilon}$ (%)	Frequency Time per Cycle (hr)	Time to Failure, t_f (hr)	Number of Cycles to Failure, N_f
1	0.37	0.74	96	758	7.5
2	0.22	0.30	10	441	44
3	0.33	0.39	2	188	94
4	0.40	0.44	1.1	99	90

The plastic strain per cycle did increase from the first half cycle, remained constant for over 75% of the cycles to failure, then increased again before fracture. The values reported are the average plastic strain ranges considering all cycles to failure. The effective plastic strain per cycle, $\bar{\epsilon}$, was calculated from the axial strain using the relationships

$$\begin{aligned}\dot{\epsilon}_z &= \frac{\dot{\bar{\epsilon}}}{\bar{\sigma}} \left(\sigma_z - \frac{\sigma_\theta + \sigma_R}{2} \right) , \\ \dot{\epsilon}_\theta &= \frac{\dot{\bar{\epsilon}}}{\bar{\sigma}} \left(\sigma_\theta - \frac{\sigma_R + \sigma_z}{2} \right) , \text{ and} \\ \dot{\epsilon}_R &= \frac{\dot{\bar{\epsilon}}}{\bar{\sigma}} \left(\sigma_R - \frac{\sigma_z + \sigma_\theta}{2} \right)\end{aligned}\quad (2)$$

where

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_z - \sigma_\theta)^2 + (\sigma_z - \sigma_R)^2 + (\sigma_\theta - \sigma_R)^2 \right]^{\frac{1}{2}} \text{ and} \quad (3)$$

$$\dot{\bar{\epsilon}} = \frac{\sqrt{2}}{3} \left[(\dot{\epsilon}_z - \dot{\epsilon}_\theta)^2 + (\dot{\epsilon}_z - \dot{\epsilon}_R)^2 + (\dot{\epsilon}_\theta - \dot{\epsilon}_R)^2 \right]^{\frac{1}{2}} . \quad (4)$$

The second test series was that of torsion cycling which yielded the following results (Table 3).

Table 3. Results of Torsion Testing

Test Number	Alternating Shear Stress, τ , or $\left(\frac{\sigma_1 - \sigma_3}{2}\right)^a$ (psi)	Shear Strain per Cycle, γ , or $(\epsilon_1 - \epsilon_3)^b$ (%)	Effective Stress per Cycle $\bar{\sigma}$ (psi)	Effective Strain per Cycle $\bar{\epsilon}$ (%)	Time to Failure t_f (hr)	Number of Cycles to Failure, N_f
5	15,000	8.6	26,000	4.9	1.30	80
6	13,500	8.05	23,400	4.6	2.1	140
7	8,800	2.85	15,250	1.6	30	1,804
8	7,000	1.2	12,120	0.69	82	4,940
9	5,000	0.073	8,660	0.042	925	54,390

^a σ_1 and σ_3 are maximum and minimum principal stresses, respectively.

^b ϵ_1 and ϵ_3 are maximum and minimum principal strains, respectively.

The effective stress $\bar{\sigma}$ and strain $\bar{\epsilon}$ were again calculated using Eqs. (2), (3), and (4). The shear strain per cycle γ also followed the same increases throughout the lifetime and again the reported values are averages. It should be noted that the increases in strain per cycle resulted from the onset of a primary stage of creep that was nonexistent in the first one-half cycle. The linear rate obtained after the primary stage, during all except the final cycles, was essentially the same or slightly less than the initial creep rate. A comparison of all of the cyclic strain-rate data to the static creep-strain rate data is given in Fig. 4. Also included in this figure are uniaxial test results for this material as given in previous papers.^{2,8} The cyclic strain rates were obtained by taking the total plastic strain per cycle and dividing by the time per cycle, thus including the primary stage. As expected, all of the cyclic data calculated in this manner fell to the right of the curve except for those obtained at very high stresses and strain rates. The contribution of the plastic strain in the primary stage for the high stress tests was essentially nil in comparison to the large total plastic strain per cycle. Unfortunately, complete strain-time recordings were not obtained for these tests and the indicated strain hardening could not be verified.

Fig. 4. Creep Rate vs Stress for Reannealed Inconel at 1500°F.

It is significant, however, that the apparent strain softening or hardening appears to be singly dependent upon the strain per cycle and independent of the stress state and frequency. Thus, as is shown in Fig. 4, cyclic strain-rate data from all tests can be represented fairly well by a single smooth curve. In correlating service failure with material properties, it is important to recognize this dynamic instability of the deformation per cycle. It is easily seen from Fig. 4 that the calculated strain per cycle obtained from static creep data can be in error by as much as a factor of 2. If the plastic strain per cycle as given by Eq. (1) is used as the criterion for failure, the error is magnified since the exponent n is less than one.

Previous results⁸ obtained from testing materials under complex creep stresses tended to indicate that fracture was controlled by a maximum principal stress criterion rather than a maximum shear criterion. However, the data did not show this conclusively. It is certain that the principal stress criterion cannot explain notch strengthening and fracture initiation in other specimen geometries where the shear stress is greatest and the principal stress is a minimum.

It is certain that the maximum principal stress criterion is inconsistent with observed fracture characteristics of specimens having stress gradients which produce the maximum shear stress and maximum principal stress in different regions. Examples of the failure of this criterion in this respect are those involving notch strengthening and the initiation of failure on the inside surface of tubes internally pressurized. The notched bar in comparison to a smooth bar has a much larger maximum principal stress; however, the shear stress is less for the notched bar and its rupture life is extended. A tube under internal pressure loading results in a stress distribution where the maximum shear stress exists at the inside surface and a maximum principal stress exists at the outside surface. The creep failure of moderately heavy to thin-walled pressurized tubes is always found to initiate on the inside surface.

Similar considerations also indicate that the shear stress alone cannot be used as the sole criterion for failure in view of the large evidence

of data for torsional loads^{9,10} which show rupture lives much larger than predicted. In addition, consider that in two investigations^{9,11} no evidence of a failure under uniaxial compressive creep loading could be found. In both of these two investigations metallographic examination revealed that no evidence of damage, void formation at grain boundaries, was introduced by the uniaxial compressive stresses.

Hull and Rimmer¹¹ have reported that void formation under the same shear stress is a simple function of the maximum tensile stress. It was apparent from this study and the other evidence cited that a shear stress and its accompanying deformation are responsible for the nucleation of voids, and that a tensile stress is required for their growth in directions normal to the tensile stress axis. The following expression suggested by the work of Hull and Rimmer was developed to satisfy the requirements of considering both the shear and tensile stresses:

$$t_r = \left(\frac{B}{\bar{\sigma}} \right)^\beta \left(\frac{\bar{\sigma}}{\sigma_1} \right) \quad (5)$$

where

t_r = time to rupture,

σ_1 = maximum principal tensile stress,

$\bar{\sigma}$ = effective stress, and

B, β = material constants.

This equation is the shear-stress criterion modified by the ratio $\frac{\bar{\sigma}}{\sigma_1}$ for stress states other than uniaxial tension having a principal stress not equal to the effective stress. This equation states that under compressive uniaxial loading the time to rupture is infinite since $\sigma_1 = 0$.

⁹A. E. Johnson, S. Henderson, and V. D. Mathus, The Engineer 196, 261 (1956).

¹⁰A. E. Johnson and S. Henderson, Complex Stress Creep, Relaxation, and Fracture of Metallic Alloys, Her Majesty's Stationery Office, Edinburgh, 1962.

¹¹D. Hull and D. E. Rimmer, Phil. Mag. [8]4, 673-87 (1959).

It also predicts failure on the inside surface of internally pressurized tubes. For the tube geometry used in this investigation, the time required to rupture at the inside surface should be 0.86 of the rupture life of the outside surface.

The fit of Eq. (5) to complex stress-rupture test data is shown in Figs. 5 and 6 where rupture time is multiplied by $\frac{\sigma_1}{\bar{\sigma}}$ and plotted vs the effective stress. Figure 5 gives results of tubes internally pressurized and axially loaded, whereas Fig. 6 gives Hull and Rimmer's results on copper wires tested under tension and hydrostatic pressure. Equation (5) does fit the data with sufficient accuracy, although there is some scatter resulting from not considering the greater tangential stress increase with deformation.

The problem of comparing fatigue test results to static constant load creep-rupture data is that the stress changes under the constant load tests. Unless a true stress-rupture curve is available, a comparison cannot be made. The true stress-rupture curve can, however, be approximated from constant load tests. It can be assumed that small segments of the uniaxial time stress-rupture curve can be represented by

$$t_f = \left(\frac{B'}{\bar{\sigma}'} \right)^{\beta'} \left(\frac{\bar{\sigma}'}{\sigma_1'} \right), \quad (6)$$

where

t_f = time to fracture,

$\bar{\sigma}'$ = true effective stress,

σ_1' = true maximum principal stress, and

B', β' = material constants.

The true stress ($\bar{\sigma}' = \sigma'$ for uniaxial tests), which at zero time is equal to the engineering stress, increases as a function of the strain rate as given below:

$$\sigma' = \sigma_0 (1 + \epsilon) = \sigma_0 (1 + \dot{\epsilon}t), \quad (7)$$

where

σ_0 = stress at zero time,

ϵ = strain, and

$\dot{\epsilon}$ = strain rate.

Fig. 5. Stress-Rupture Properties of Reannealed Inconel Under Complex Stresses at 1500°F.

Using Minor's hypothesis of damage accumulation under variable stresses

$$B \beta' = \int_0^{t_f} \sigma' \beta' dt \quad (8)$$

Combining Eqs. (7) and (8)

$$B \beta' = \sigma_0 \beta' \int_0^{t_f} (1 + \dot{\epsilon} t) \beta' dt \quad (9)$$

which integrates to

$$B \beta' = \sigma_0 \beta' \left[\frac{1}{\dot{\epsilon}(\beta' + 1)} (\dot{\epsilon} t_f + 1)^{\beta' + 1} \right] \quad (10)$$

The true fracture stress under constant load is

$$\sigma_f' = \sigma_0 (1 + \dot{\epsilon} t_f) \quad (11)$$

and

$$\sigma_0 \beta' = \left(\frac{\sigma_f'}{1 + \dot{\epsilon} t_f} \right)^{\beta'} \quad (12)$$

Combining Eqs. (10) and (12) and reducing terms

$$B \beta' = \sigma_f'^{\beta'} \frac{(\dot{\epsilon} t_f + 1)}{\dot{\epsilon}(\beta' + 1)} \quad (13)$$

The true stress-rupture curve developed from Eq. (13) is shown in Fig. 7.

By use of Eq. (5) the cyclic data under complex stresses can now be compared to static data developed by Eq. (13) in Fig. 7. In this plot the time scale again has been modified by the ratio $\frac{\sigma_1}{\bar{\sigma}}$ so that direct comparisons can be made. The time plotted for the cycle data is only that time accumulated under the principal tensile stress causing failure in the specimens. The uniaxial cyclic creep-rupture tests have tensile stresses acting on the specimen for only one half of the total rupture life; thus, the time available to produce damage is only one half of the total time. For the complex stress conditions where tensile stresses are acting in more than a single principal stress axis, the damage accumulations in each axis have to be considered separately. The damage produced and accumulated in a specimen under creep conditions consists essentially of grain-boundary voids and cracks normal to the tensile stress axis. Thus, the damage accumulated in one stress axis will be parallel to other stress

Fig. 7. True Stress-Rupture Properties of Reannealed Inconel at 1500°F.

axis and not influence the life expectancy in these axis. Damage accumulation in the complex stress tests is then determined for each stress axis independent of the other axis. The torsion-fatigue life is determined as in the uniaxial fatigue tests by only considering one half of the actual life. The damage accumulation for the internally pressurized tubes axially cycled is no different; however, the tangential stress is constantly applied and the damage accumulation in this direction is the total time under test. The damage in the axial direction is only accumulated for one half of the time since the tensile stress is only applied for this period. The two axes have to be compared to determine which axis will first accumulate sufficient damage for fracture. Table 4 lists the ratios of $\frac{\bar{\sigma}}{\sigma_{\theta}}$ and $2\frac{\bar{\sigma}}{\sigma_z}$, the smallest value of which will determine the axis normal to the fracture.

Table 4. Determination of Direction of Fracture

Test Number	$\frac{\bar{\sigma}}{\sigma_{\theta}}$	$2\frac{\bar{\sigma}}{\sigma_z}$	Direction Normal to Fracture
1	1.07	2.14	Tangential
2	1.39	1.85	Tangential
3	1.79	1.70	Axial
4	2.21	1.77	Axial

As is shown in Table 4, the agreement in direction was correct in each test. Thus, two of the tests failed due to static creep and two failed as a result of a cyclic axial stress. Tests 1 and 2 are given in Fig. 7 with the other fatigue tests; however, since failure was a result of a static load, they should be compared with the data in Fig. 5. If this is done, these two points fall in the middle of the scatter. Tests 3 and 4, however, did fail due to the axial tensile stress and, as shown in Fig. 7, the rupture times are comparable to those based on the axial fatigue data. Here, then, are four tests in which the direction and time of failure were correctly determined by assessing and comparing damage accumulation in each axis independently.

The discussion thus far has dealt only with stress-rupture correlations. It would be profitable to analyze these results with respect to their strain-cycle behavior. The uniaxial data given previously³ for this material are shown in Fig. 8, where they are compared with data from the complex stress tests using a parameter based on effective strain range per cycle. These data demonstrate that both the effect of frequency and stress state must be considered before a direct comparison can be made.

Although it has been demonstrated that the cyclic creep test results do not follow the static creep results implicitly, static creep-rupture relationships can be used to indicate relative magnitudes of the effects of frequency and stress states. At 1500°F, the test temperature, the material creep behavior can be closely approximated by

$$\bar{\epsilon}_c = \left(\frac{\bar{\sigma}}{A} \right)^{\alpha} t \quad (14)$$

where

$\bar{\epsilon}_c$ = effective creep strain,

t = time, and

A, α = material constants.

From Fig. 4 it is apparent that the fatigue data do not follow the static creep data exactly; however, the lack of agreement does not appear to be associated with frequency or stress state. The cyclic creep tests can then be represented by

$$2\Delta\bar{\epsilon}_p = \left(\frac{\bar{\sigma}}{A} \right)^{\alpha} t_n \quad (15)$$

where

$\Delta\bar{\epsilon}_p$ = effective plastic strain range, and

t_n = time per cycle.

Equation (5) is solved for the effective stress to obtain

$$\bar{\sigma} = B \left(\frac{1}{t_r} \right)^{\frac{1}{\beta}} \left(\frac{\bar{\sigma}}{\sigma_1} \right)^{\frac{1}{\beta}} \quad (16)$$

Fig. 8. Strain Cycle Test Results.

The time, frequency, and number of cycles are related depending upon the type of stress cycle. For those cases where the specimen fails as a result of a tensile stress acting on the specimen for half of the cycle, the relation is

$$\frac{N_f}{2\eta} = t_r \quad (17)$$

where

η = frequency.

For those cases where the failure is a result of a static tensile stress acting on the specimen, the relation is

$$\frac{N_f}{\eta} = t_r \quad (18)$$

Considering the first case, that of cyclic stresses, Eqs. (16) and (17) are combined to obtain

$$\bar{\sigma} = B \left(\frac{2\eta}{N_f} \right)^{\frac{1}{\beta}} \left(\frac{\bar{\sigma}}{\sigma_1} \right)^{\frac{1}{\beta}} \quad (19)$$

This equation can now be substituted into Eq. (15)

$$N_f^{\frac{\alpha}{\beta}} \Delta \bar{\epsilon}_p = \left(\frac{B}{A} \right)^{\alpha} (2\eta)^{\frac{\alpha - \beta}{\beta}} \left(\frac{\bar{\sigma}}{\sigma_1} \right)^{\frac{\alpha}{\beta}} \quad (20)$$

This equation is of the same form as Eq. (1) and, if the cyclic creep tests agreed explicitly with the static creep results, n should be equal to $\frac{\alpha}{\beta}$. This is not the case as $\frac{\alpha}{\beta} = 1.45$ and n from Fig. 8 is equal to 0.762. However, regardless of the agreement of the cyclic creep to the static creep constants, α and β are almost equal to one another and the values of $\frac{\alpha}{\beta}$ and $\frac{\alpha - \beta}{\beta}$ will not vary appreciably. Thus, the effects of stress state and frequency can be generalized.

The solution for the second case, that of a static stress causing failure, is obtained by combining Eqs. (15), (16), and (17) in a similar manner to obtain

$$N_f^{\frac{\alpha}{\beta}} \Delta \bar{\epsilon}_p = \left(\frac{B}{A} \right)^{\alpha} \frac{1}{2} \eta^{\frac{\alpha - \beta}{\beta}} \left(\frac{\bar{\sigma}}{\bar{\sigma}_1} \right)^{\frac{\alpha}{\beta}} \quad (21)$$

Thus, to compare the results from tests 1 and 2, where fracture resulted from the static tangential stress, to uniaxial cyclic data, both Eqs. (20) and (21) must be used. A comparison is accomplished by considering the effect of differing frequencies and stress states on tests with the same plastic strain range. The comparison for tests 1 and 2 with the uniaxial fatigue data is

$$\frac{N_{fI}}{N_{fII}} = 2 \left(\frac{2\eta_I}{\eta_{II}} \right)^{\frac{\alpha - \beta}{\alpha}} \left(\frac{\sigma_1}{\bar{\sigma}} \right)_{II} \quad (22)$$

(Subscript I for uniaxial data and subscript II for tests 1 and 2).

The value of the exponent for Inconel at 1500°F was determined to be

$$\frac{\alpha - \beta}{\alpha} = 0.31 \quad (23)$$

Since the exponent is less than 1, the number of cycles to failure should not be seriously affected by small changes in frequency. The factors obtained from Eq. (22) for tests 1 and 2 are

Test Number	$2 \left(\frac{2\eta_I}{\eta_{II}} \right)^{0.31} \left(\frac{\sigma_1}{\bar{\sigma}} \right)_{II}$	
1	32.4	× 0.94 = 30.4
2	18	× 0.64 = 11.5

The same type of comparison is done for tests 3 and 4 to yield the factors

Test Number	$\left(\frac{\eta_I}{\eta_{III}} \right)^{0.31} \left(\frac{\sigma_1}{\bar{\sigma}} \right)_{III}$	
3	4.4	× 1.11 = 4.94
4	3.7	× 1.13 = 4.18

(Subscript III for tests 3 and 4).

The comparisons can now be made and the results are shown in Fig. 8 where the factors determined above are multiplied by the number of cycles to failure. The data do not correlate exactly and were not expected to in view of the discrepancies between the fatigue and static data. This method of adjusting data with respect to stress state and frequency does, however, demonstrate the magnitude of these effects on life expectancy.

The torsional results can be compared in a similar manner using Eq. (20) to obtain

$$N_{f_I} = N_{f_{IV}} \left(\frac{\sigma_1}{\bar{\sigma}} \right)_{IV} \quad (24)$$

(Subscript IV for torsion test data)

or

$$N_{f_I} = N_{f_{IV}} (0.577) \quad (25)$$

The dashed line in Fig. 8 is the strain fatigue data expected from testing under cyclic torsion. As seen in this figure, the data all fall slightly above this line on the conservative side. The relative agreement adds further support to the applicability of this method to determine the magnitude of the effect of stress state upon the strain cycle behavior.

SUMMARY AND CONCLUSIONS

Two test series were performed on Inconel at 1500°F to furnish failure test data under complex dynamic stresses at high temperatures. The results obtained from these series have been correlated with the results of previous static and dynamic stress tests. This investigation has furnished a general method of solving complex stress creep problems involving both static and dynamic stress states. The general solution is based upon the developed relationship,

$$t_r = \left(\frac{B}{\bar{\sigma}} \right)^{\beta} \left(\frac{\bar{\sigma}}{\sigma_1} \right),$$

and upon Minor's damage accumulation hypothesis. The damage accumulation, however, was shown to be independently additive in each principal stress axis.

The specific conclusions of this investigation are:

1. An expression has been developed for creep rupture under complex stress states.
2. A method for generating true stress-rupture data from constant load data is given.
3. The effect of cyclic stresses upon strain rate is demonstrated.
4. Results of the two test series demonstrate the independence of damage accumulation in each principal stress axis.
5. A method for determining the magnitude of the frequency effect on the number of cycles to failure is given when the plastic strain range is used as a criterion for fracture.
6. A method is given for determining the magnitude of the stress-state effect on the cycles to failure when the plastic strain range is used as a criterion for fracture.

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