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Imaginary Potential in Resonating-Group

Calculation of $^3\text{He} + ^3\text{He}$ ScatteringRonald E. Brown and I. Reichstein[†]

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Abstract: The $^3\text{He} + ^3\text{He}$ differential scattering and reaction cross section data in the energy range of 1.5 to 15 MeV (c.m.) is analyzed with a one-channel resonating-group method to which is added a phenomenological imaginary potential in order to approximately account for open reaction channels. With a nucleon-nucleon potential of near-Serber exchange mixture it is found that satisfactory results can be obtained with such a formalism. The presence of the imaginary potential is shown to be important, especially at higher energies; it affects the magnitude of the differential cross section mainly, but can also accentuate the diffraction features at certain energies.

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1. Introduction

Recently a phenomenological imaginary potential has been introduced into the one-channel, resonating-group formalism to take approximate account of open reaction channels. When applied to the problems of $\alpha + \alpha^{1)}$ and ${}^3\text{He} + \alpha^{2)}$ scattering, it was found that rather satisfactory results can be obtained. Here we report briefly on a similar study of ${}^3\text{He} + {}^3\text{He}$ scattering. This is a particularly interesting case, since it is one of the few instances in light nuclear systems where a significant number of total reaction cross sections have been measured.

In the present analysis we shall consider the reaction-cross-section data of Bacher³⁾ from 1.5 to 8.95 MeV[†], the elastic-scattering, differential-

[†]All energies will be in the c.m. system unless otherwise stated.

cross-section data of Tombrello and Bacher⁴⁾ from 1.5 to 5.96 MeV, of Bacher et al.⁵⁾ from 6.94 to 8.95 MeV, and of Jenkin et al.⁶⁾ from 8.95 to 15 MeV. The scattering data of Bredin et al.⁷⁾ and of Gammel et al.⁸⁾ will not be used in this analysis, since they are in the same energy region as those just mentioned and are generally of lower precision.

2. Brief Formulation

The formulation of resonating-group calculations^{††}, one-channel

^{††}See, for example, the Proceedings of the International Conference on Clustering Phenomena in Nuclei, Bochum, Germany, 1969 (International Atomic Energy Agency, Vienna, Austria, 1969); in particular, the papers by H. H. Hackenbroich, by Y. C. Tang, and by K. Wildermuth are relevant.

calculations^{9,10)} in particular, has been thoroughly discussed elsewhere and hence will not be repeated here. With the introduction of an imaginary potential $iW(r)$ into the usual ${}^3\text{He} + {}^3\text{He}$ formalism^{11,12)}, the functions $f_{\ell s}$, which describe the relative motion of the two ${}^3\text{He}$ clusters with relative orbital angular momentum ℓ and total spin angular momentum $s^\dagger$, satisfy integro-differential equations of the form

[†]For the ${}^3\text{He} + {}^3\text{He}$ system, ℓ and s are correlated by the equation

$$s = \frac{1}{2}[1 - (-1)^\ell].$$

$$\left\{ \frac{\hbar^2}{2\mu} \left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + E - V_D(r) - V_C(r) - iW(r) \right\} f_{\ell s}(r) = \int_0^\infty k_{\ell s}(r, r') f_{\ell s}(r') dr', \quad (1)$$

where μ is the reduced mass, E is the total kinetic energy of the two clusters at large separation in the c.m. system, and $V_D(r)$, $V_C(r)$, and $k_{\ell s}(r, r')$ are, respectively, the direct nuclear potential, the direct Coulomb potential, and the partial-wave kernel functions for the nonlocal interaction between the clusters.

The quantities V_D , V_C , and $k_{\ell s}$ are derived by using the cluster wave function of refs.^{11,12)} and a nucleon-nucleon potential used in resonating-group calculations for a number of light nuclear systems^{1,13-15)}. This nucleon-nucleon potential is somewhat different from that used in refs.^{11,12)}, and hence the forms of V_D , V_C , and $k_{\ell s}$ are not the same as given there. However, the required modifications are relatively minor and can be performed in a manner similar to that indicated in the Appendix of ref.¹⁴⁾.

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As in refs.¹⁾ and ²⁾, the potential $W(r)$ is taken to have the form

$$W(r) = -W_0 \left\{ \frac{1}{1 + \exp(\frac{r-R}{a})} + \frac{4 \exp(\frac{r-R}{a})}{[1 + \exp(\frac{r-R}{a})]^2} \right\}, \quad (2)$$

which has both a volume and a surface term. The geometry parameters R and a will be assumed to be energy-independent and chosen as

$$\begin{aligned} R &= 3.3 \text{ fm}, \\ a &= 0.5 \text{ fm}. \end{aligned} \quad (3)$$

With this choice, the mean-square radius of the volume part of $W(r)$ is 10.0 fm^2 , which is about equal to the mean-square radius of the direct potential V_D .

The adjustable parameters in our theory are the exchange-mixture parameter u in the nucleon-nucleon potential and the potential depth W_0 . These are determined by fitting the $^3\text{He} + ^3\text{He}$ scattering data in the energy range of 1.5 to 8.95 MeV in the following manner. We choose a series of values for u and adjust the energy-dependent parameter W_0 to fit the reaction-cross-section data for each of the u values. The best value for u is then the one which yields an over-all good fit to the differential-cross-section data in this energy range. As has been explained previously^{14,15)}, it is important that the resultant value for u turn out to be close to 1, which corresponds to a Serber exchange mixture for the nucleon-nucleon potential. Otherwise, we would consider our calculation as grossly inadequate and a major revision to the formulation must then be sought.

3. Results

Using the procedure described above, we find that the best value of u is equal to 1.04. With this value for u , the results we obtain for the potential depth W_0 , the calculated reaction cross section σ_R and the complex phase shifts $\delta_\ell = \delta_\ell^R + i\delta_\ell^I$ in the energy region of 1.5 to 8.95 MeV are tabulated in table 1.

In table 1 we have also listed the values of χ^2 , defined as

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N \left\{ \frac{1}{\Delta_i} [\sigma_c(\theta_i) - \sigma(\theta_i)] \right\}^2, \quad (4)$$

where $\sigma_c(\theta)$ and $\sigma(\theta)$ are the calculated and experimental differential cross sections, respectively, N is the number of data points and Δ_i is, for convenience, set equal to 5% of $\sigma(\theta_i)$. From these values of χ^2 , it is clear that the agreement between theory and experiment is good at the lower energies, but begins to become poorer at about 8 MeV. This is demonstrated in figs. 1 and 2, where a comparison between the calculated and experimental differential cross sections is shown at all eight energies.

At energies above 8.95 MeV, there are at present no reported measurements of reaction cross sections. This means that, at these energies, the potential depth W_0 cannot be determined in the same way as discussed above. In the present analysis we shall, therefore, estimate the values of W_0 at higher energies by an extrapolation procedure. This is shown in fig. 3, where we have drawn a solid line through the points representing the values of W_0 given in table 1. Guided by information obtained from our recent $\alpha + \alpha$ study¹⁾ about the trend of the variation of W_0 with energy,

we extend the solid line into the higher-energy region by the indicated dashed line. In this way, we estimate that the values of W_0 at 11, 13, and 15 MeV are approximately equal to 0.35, 0.5, and 0.6 MeV. At even higher energies, this extrapolation procedure becomes less reliable. For example, at 22 and 30 MeV, where we shall also make a brief study of the influence of the imaginary potential on the differential scattering cross section, it is only possible to say that the trend of the dashed line leads to values of W_0 lying approximately in the range of 1.0 to 1.5 MeV and 2.0 to 3.0 MeV, respectively.

With the values of W_0 so estimated, we have listed in table 2 the values of δ_ℓ in the energy range of 11 to 30 MeV. As has been mentioned previously¹⁵⁾, these will be useful as starting values in any future attempts to analyze the experimental data using complex phase shifts.

Differential scattering cross sections at 11, 13, and 15 MeV are shown in fig. 4. Here it is seen that the agreement between theory and experiment becomes somewhat better as the energy increases, and at 15 MeV the fit to the experimental data is in fact rather fair. Also, it should be mentioned that the inclusion of an imaginary potential in the analysis is quite important at these energies. For example, at 15 MeV the value of χ^2 is reduced from 195 to 39 when W_0 is increased from 0 to 0.6 MeV.

The observation that the fit to the experimental data begins to become relatively poorer at about 8 MeV and to improve again at about 15 MeV can be readily understood. In our calculation, we have for simplicity employed a central nucleon-nucleon potential. As a consequence, the phase shifts in the odd- ℓ ($s=1$) states are not split, and it is

therefore expected that, in the energy region where there are relatively sharp, odd- ℓ , resonance levels, the agreement between the calculated and the experimental differential cross sections should be less satisfactory. As has been shown previously¹²⁾ by a one-level analysis, there is in fact an $\ell=3$ resonance level predicted by the calculation somewhere in the energy region between 10 and 15 MeV, the presence of which has also been verified by an experimental observation⁶⁾. Therefore, if one wishes to obtain improved agreement with experiment in this particular energy region, it will be necessary to use in the formulation a more complicated nucleon-nucleon potential with noncentral components. To test the validity of this assertion, we are performing a calculation for the ${}^3\text{He} + {}^3\text{He}$ system in which is included a two-nucleon, spin-orbit potential of the type which has recently been successfully applied in the case of $N + \alpha$ scattering¹⁴⁾.

Finally, in fig. 5, we show the differential cross sections at 22 MeV with $W_0 = 0, 1.0, \text{ and } 1.5$ MeV, and at 30 MeV with $W_0 = 0, 2.0, \text{ and } 3.0$ MeV. The purpose here is to see in what way the presence of an imaginary potential can influence the results at higher energies. From this figure it is seen that the inclusion of such a potential is indeed very important; it not only affects the magnitude of the differential cross section significantly, but also accentuates the diffraction features.

4. Conclusions

In this investigation, the ${}^3\text{He} + {}^3\text{He}$ scattering data is analyzed with a one-channel resonating-group method in which a phenomenological imaginary

potential is introduced to take approximate account of open reaction channels. The result shows that, with such a formalism, the experimental data in the energy range of 1.5 to 15 MeV can be described satisfactorily with the use of a nucleon-nucleon potential of near-Serber exchange mixture.

It is also shown that, especially at higher energies, the introduction of an imaginary potential into the resonating-group formalism is essential for a satisfactory description of the differential scattering cross section. The presence of such a potential is found, as expected, to affect mainly the magnitude of the differential cross section, but at certain energies, its presence also accentuates significantly the diffraction features of the differential cross section.

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Table 1

Results for W_0 , σ_R , δ_ℓ , and χ^2 with $u=1.04$
in the energy range of 1.5 to 8.95 MeV

E(MeV)	1.515	2.95	3.955	4.96	5.96	6.94	7.945	8.95
W_0 (MeV)	0.07	0.08	0.09	0.095	0.095	0.10	0.115	0.20
σ_R (mb) ^{a)}	55	107	128	136	137	147	176	318
δ_0 (deg)	-19.96+i0.18	-43.85+i0.35	-56.99+i0.45	-68.10+i0.52	-77.70+i0.54	-86.05+i0.57	-93.76+i0.68	-100.77+i1.21
δ_1 (deg)	2.91+i0.14	8.55+i0.58	10.24+i0.89	9.94+i1.07	8.32+i1.13	6.01+i1.20	3.25+i1.37	0.30+i2.35
δ_2 (deg)	-0.18+i0.02	-0.87+i0.09	-1.50+i0.18	-2.20+i0.28	-3.02+i0.36	-3.96+i0.44	-5.09+i0.57	-6.37+i1.10
δ_3 (deg)	0.03+i0	0.44+i0	1.26+i0.03	2.75+i0.07	5.03+i0.15	8.14+i0.28	12.27+i0.52	17.29+i1.33
δ_4 (deg)	0	0.04+i0	0.17+i0	0.44+i0	0.92+i0.01	1.67+i0.03	2.78+i0.06	4.27+i0.16
δ_5 (deg)	0	0	0.01+i0	0.03+i0	0.07+i0	0.14+i0	0.26+i0.01	0.44+i0.02
δ_6 (deg)	0	0	0	0	0	0.02+i0	0.05+i0	0.09+i0
χ^2	4.10	1.21	0.77	0.16	0.72	1.34	6.58	22.6

a) The calculated and experimental reaction cross sections agree with each other to within about 5%.

Table 2

Results for δ_ℓ with $u=1.04$ and extrapolated
values of W_0 in the energy range of 11 to 30 MeV

E (MeV)	11	13	15	22		30	
W_0 (MeV)	0.35	0.5	0.6	1.0	1.5	2.0	3.0
σ_R (mb)	591	833	949	1133	1499	1478	1878
δ_0 (deg)	-113.41+i2.14	-124.06+i3.06	-133.44+i3.64	-159.45+i5.80	-159.46+i8.70	-180.91+i10.76	-180.75+i16.18
δ_1 (deg)	- 5.75+i3.92	-11.34+i5.27	-16.52+i5.94	-31.66+i7.87	-31.44+i11.77	-44.92+i12.52	-44.41+i18.50
δ_2 (deg)	-9.39+i2.16	-12.69+i3.26	-16.14+i4.00	-28.05+i6.53	-28.07+i9.81	-39.81+i11.85	-39.54+i17.83
δ_3 (deg)	29.54+i4.08	42.19+i7.70	53.55+i9.94	75.18+i11.57	76.13+i17.51	81.83+i15.13	83.30+i22.28
δ_4 (deg)	8.70+i0.63	14.86+i1.72	22.58+i3.31	51.46+i10.44	50.97+i15.90	69.88+i17.28	71.26+i26.20
δ_5 (deg)	1.01+i0.06	1.88+i0.18	3.10+i0.38	9.69+i2.28	9.54+i3.44	18.87+i9.31	17.71+i13.82
δ_6 (deg)	0.26+i0.01	0.55+i0.03	1.00+i0.07	4.18+i0.63	4.16+i0.94	10.30+i3.63	9.94+i5.40
δ_7 (deg)	0.03+i0	0.08+i0	0.16+i0.01	0.90+i0.14	0.90+i0.21	2.79+i1.01	2.74+i1.51
δ_8 (deg)	0	0	0.05+i0	0.35+i0.04	0.35+i0.05	1.35+i0.31	1.34+i0.47

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Figure Captions

- Fig. 1 Comparison of ${}^3\text{He} + {}^3\text{He}$ differential scattering cross sections calculated using $u=1.04$ with experimental data at 1.515, 2.95, 3.955, and 4.96 MeV(c.m.). The values of W_0 used are given in table 1.
- Fig. 2 Comparison of ${}^3\text{He} + {}^3\text{He}$ differential scattering cross sections calculated using $u=1.04$ with experimental data at 5.96, 6.94, 7.945, and 8.95 MeV(c.m.). The values of W_0 used are given in table 1.
- Fig. 3 Potential depth W_0 as a function of energy.
- Fig. 4 Comparison of ${}^3\text{He} + {}^3\text{He}$ differential scattering cross sections calculated using $u=1.04$ with experimental data at 11, 13, and 15 MeV(c.m.). The values of W_0 used are given in table 2.
- Fig. 5 Calculated differential scattering cross sections at 22 MeV (c.m.) with $W_0=0, 1.0, \text{ and } 1.5$ MeV, and at 30 MeV(c.m.) with $W_0=0, 2.0, \text{ and } 3.0$ MeV. The value of u used is 1.04.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5