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NEGATIVELY CHARGED PION BEAMS

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Neutron Physics Division

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Abstract

Calculations have been carried out using the Monte Carlo radiation transport code HETC to determine the spatial distribution of the absorbed dose produced in tissue by monoenergetic negatively charged pion beams. The depth dependence of the absorbed dose has been computed for narrow (zero width) negative-pion beams incident normally on one side of a slab of tissue 60 cm thick. Beam energies of 53, 82, and 108 MeV (corresponding to ranges in tissue of approximately 10, 20, and 30 cm) were used. For the beam having a range of 20 cm, the absorbed dose as a function of the lateral distance from the beam axis has also been computed. The use of the results from monoenergetic beams to synthesize the incident-energy distribution needed to produce desired depth-dose distributions is discussed, and, as an example, the energy spectrum of the incident beam required to produce a uniform absorbed dose over the depth interval from 20 to 30 cm is computed.

1. INTRODUCTION

The use of negatively charged pions in cancer radiotherapy offers significant advantages over those radiations conventionally employed because negatively charged pions have properties which, in principle, allow a better localization of the dose and improved biological effectiveness (e.g., Fowler and Perkins 1961, Raju and Richman 1970, Rosen 1968). To obtain some basic quantitative information on the physical effects produced by negative-pion beams and to provide data to aid in the design of the biomedical pion channel for the meson facility under construction at the Los Alamos Scientific Laboratory, detailed calculations have been carried out to determine the spatial distribution of the absorbed dose produced in tissue by monoenergetic negative-pion beams.

The calculations were made for narrow (zero width) monoenergetic pion beams incident normally on one side of a slab of tissue 60 cm thick.* Beam

*For the semi-infinite slab phantom considered here, the laterally integrated dose at a given depth produced by particles incident at a single arbitrary point on the slab surface (i.e., zero-width beam) is numerically the same as the dose at the same depth produced by an infinitely broad beam. The equivalence of these two interpretations is discussed elsewhere (Turner, Zerby, Woodyard, Wright, Kinney, Snyder and Neufeld 1964).

energies of 53, 82, and 108 MeV (corresponding to ranges in tissue of approximately 10, 20, and 30 cm) were used.

The calculations were carried out using the nucleon-meson transport code HETC (Armstrong and Chandler 1972a). An important feature of this code is its detailed treatment of nuclear interactions. This is accomplished by using Monte Carlo methods in conjunction with theoretical nuclear-interaction

models to determine the energy, angle, and multiplicity of the particles produced in nucleon-nucleus and pion-nucleus (including pion capture) interactions. The ability of HETC to predict depth-dose distributions produced by pion beams has previously been checked, and good agreement with measured dose distributions has been obtained (Armstrong and Chandler 1972b).

The results obtained include the depth dependence of the laterally integrated absorbed dose for each beam energy and, for the case of the beam producing a peak dose at 20 cm, the absorbed dose as a function of the lateral distance from the beam axis. Special emphasis is placed on determining the contribution of secondary particles to the absorbed dose.

Although monoenergetic pion beams do not represent a source distribution of primary concern in practical radiotherapy, it is of interest in understanding the dependence of the absorbed dose on beam energy to have absorbed-dose distributions produced by beams of discrete energies. Moreover, the results for monoenergetic beams can be used to estimate the incident-pion-energy distribution needed to give a uniform absorbed dose over a specified depth interval. As an example, the calculated results for monoenergetic beams producing peak doses at 20 and 30 cm are used to synthesize approximately the incident-pion-energy spectrum needed to produce a uniform absorbed dose in the depth interval from 20 to 30 cm.

To obtain a quantity which reflects the LET dependence of the energy deposited, the depth dependence of the dose equivalent has also been calculated for each incident beam and is presented elsewhere (see Appendix A). It should be realized that the dose equivalent is only very approximately related to biological damage because the LET-dependent quality factors used (National Committee on Radiation Protection 1954) are designed for

radiation-protection purposes. However, the depth dependence of the energy deposited as a function of particle energy for each type of particle has been calculated and is given in Appendix B. With the information on the energy deposition, energy-dependent weighting factors other than radiation-protection quality factors can be applied to estimate biological damage.

In the next section the details of the calculational method are given, and in the last section the results are presented and discussed.

2. METHOD OF CALCULATION

A detailed description of the treatment of the various physical processes in HETC is given elsewhere (Armstrong, Alsmiller, Chandler and Bishop 1972), so only a brief account of the calculational method will be given here. The code takes into account charged-particle energy loss due to atomic ionization and excitation, multiple Coulomb scattering by the primary particles, elastic and nonelastic nucleon collisions and nonelastic pion collisions with nuclei other than hydrogen, elastic and nonelastic nucleon and pion collisions with hydrogen nuclei, pion and muon decay in flight and at rest, and negative-pion capture at rest.

For nonelastic collisions with nuclei other than hydrogen, the intra-nuclear-cascade-evaporation model is used for pion collisions at all energies and for nucleon collisions at energies ≥ 15 MeV. At each collision, a subroutine version of Bertini's latest intranuclear-cascade program (Bertini and Guthrie 1971) and a subroutine version of Guthrie's evaporation program (Guthrie 1970) are performed to determine the identity, multiplicity, energy, and direction of the emitted particles, and the charge, mass, and the excitation energy and recoil energy of the residual nucleus. The particles

produced in the collision may be protons, neutrons, charged pions, or neutral pions from the intranuclear cascade, and protons, neutrons, deuterons, tritons, ^3He 's, or alpha particles from the evaporation. Above 15 MeV, neutron elastic collisions with nuclei other than hydrogen are taken into account using experimental data (Coleman and Armstrong 1970), and proton and pion elastic collisions with nuclei other than hydrogen are neglected.

Elastic and nonelastic nucleon collisions with hydrogen ≥ 15 MeV and nonelastic pion collisions at all energies are treated using the same differential particle-particle cross sections as used in the intranuclear-cascade calculations.

Neutron collisions with all nuclei in the energy range from 15 MeV to thermal are treated by using experimental cross-section data (Coleman and Armstrong 1970) in conjunction with an evaporation model (Guthrie 1970) to determine the production from nonelastic collisions. Protons with energies ≤ 15 MeV and deuterons, tritons, ^3He 's, alpha particles, and recoiling residual nuclei are assumed to slow down and come to rest without undergoing nuclear interaction. Furthermore, the slowing-down distance is neglected since, for the low energies involved, the range of these particles in tissue is very small.

Positively charged pions that come to rest are assumed to decay. Negatively charged pions that come to rest are assumed to undergo nuclear capture (Guthrie, Alsmiller and Bertini 1968). An intranuclear-cascade-evaporation calculation is performed to determine the capture products, and the nucleons and charged pions produced are included in the transport calculations. The type of nucleus capturing the pion is determined using the Fermi-Teller Z-law

(Fermi and Teller 1947); i.e., the probability of final capture by a nucleus of charge Z (excluding hydrogen) is taken to be proportional to Z times the atom density of Z -type nuclei.

The electron-photon cascade initiated by the photons from neutral-pion decay and by the electrons and positrons from muon decay are included in an approximate manner (Alsmiller, Armstrong and Bishop 1971). Photons from all nonelastic nucleon-nucleus and pion-nucleus collisions are assumed to deposit their energy at their point of origin.

Each particle in the cascade is followed until it eventually escapes from the phantom, undergoes absorption, or comes to rest. A complete description of each "event" (nuclear interaction, stopped charged particle, etc.) that occurs during the transport calculation is stored on magnetic tape. These history tapes are then analyzed to obtain the results of interest. Thus, a single transport calculation provides not only the spatial dependence of the dose but also much useful ancillary information.

One type of nuclear interaction that is not included in the present version of the code HETC is the elastic scattering of pions with nuclei other than hydrogen. Experimental data on pion elastic scattering (e.g., Auerbach, Fleming and Sternheim 1967) show that the differential scattering cross section is very strongly peaked about the forward direction. Therefore, for the configuration of interest here, it is expected that the effect of pion elastic scattering with nuclei other than hydrogen is small and that the lateral spread of the cascade and the spatial distribution of the energy deposition are predominantly governed by other processes which are included in the code, i.e., multiple Coulomb scattering, nucleon and pion nonelastic collisions, neutron elastic collisions with all nuclei, and pion elastic collisions with hydrogen.

The version of HETC used for the present calculations does not include range straggling. Therefore, range straggling was taken into account here by using an approximate method whereby the incident pion energy was chosen from a distribution defined to give the same spatial distribution of stopped primary pions as would be obtained if range straggling were included. The spatial distribution of stopped primary pions caused by range straggling was taken to be Gaussian in shape with a standard deviation given by the formula of Fowler and Perkins (1961) for the range straggling of pions in water.

3. RESULTS

3.1. Depth-Dose Distributions

The laterally integrated absorbed dose as a function of tissue depth is shown in figs. 1, 2, and 3 for incident π^- energies of 53, 82, and 108 MeV, respectively. In these figures, histograms for both the total absorbed dose and the absorbed dose produced only by the primary pions are shown, where the dose from primary pions is due to atomic ionization and excitation by those incident pions that have not undergone nuclear collision. Thus, the difference between the two histograms in each figure gives the dose produced by secondary particles. The error bars shown in these and in subsequent figures correspond to statistical errors of one standard deviation.

The contributions to the absorbed dose by various kinds of particles have also been calculated as a function of tissue depth, and these results are shown in figs. 4, 5, and 6. The sum of the individual contributions shown in figs. 4, 5, and 6 correspond to the total absorbed-dose histograms of figs. 1, 2, and 3, and the dose due to primary pions in figs. 4, 5, and 6 is the same as in figs. 1, 2, and 3. The histograms labeled "protons" give

Fig. 1. Absorbed dose vs depth in tissue for a 53-MeV incident π^- beam.

Fig. 2. Absorbed dose vs depth in tissue for an 82-MeV incident π^- beam.

Fig. 3. Absorbed dose vs depth in tissue for a 108-MeV incident π^- beam.

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Fig. 4. Contributions to the absorbed dose by various types of particles for a 53-MeV incident π^- beam.

Fig. 5. Contributions to the absorbed dose by various types of particles for an 82-MeV incident π^- beam.

Fig. 6. Contributions to the absorbed dose by various types of particles for a 108-MeV incident π^- beam.

the absorbed dose from the excitation and ionization of atomic electrons by protons produced from nonelastic nucleon-nucleus and pion-nucleus collisions and from the elastic collisions of nucleons and pions with hydrogen nuclei. The histograms labeled "heavy nuclei" give the absorbed dose from particles with mass number greater than unity produced from nonelastic nucleon-nucleus and pion-nucleus collisions and the absorbed dose from the recoiling nuclei produced from elastic neutron-nucleus collisions and from nonelastic nucleon-nucleus and pion-nucleus collisions. The histograms labeled "secondary charged pions" give the dose from atomic ionization and excitation by both positively and negatively charged pions produced from nucleon-nucleus and pion-nucleus nonelastic collisions. The histograms labeled "photons from neutral pions" give the absorbed dose from the electron-photon cascade produced by photons which arise from the decay of neutral pions. The histograms labeled "electrons, positrons, and photons" give the absorbed dose from the electrons and positrons produced by muon decay and the absorbed dose from the photons produced by nucleon-nucleus and pion-nucleus nonelastic collisions. The histograms labeled "muons" give the absorbed dose from the excitation and ionization of atomic electrons by both positively and negatively charged muons.

A summary of the primary- and secondary-particle contributions to the absorbed dose in the plateau and peak regions of the depth-dose distributions is given in table 1. This table, together with figs. 1-6, shows that, in addition to the secondary particles resulting from π^- capture (which contribute mainly to the dose near the end of the π^- range), the secondary particles arising from nuclear collisions in the plateau region are also very important in obtaining the dose distributions.

Table 1. Contribution, in percent, by various particle types to the absorbed dose in the plateau and peak dose regions

Particle Type	Plateau Region ^a			Peak Region ^b		
	$E_o = 53 \text{ MeV}$ $R \approx 10 \text{ cm}$	$E_o = 82 \text{ MeV}$ $R \approx 20 \text{ cm}$	$E_o = 108 \text{ MeV}$ $R \approx 30 \text{ cm}$	$E_o = 53 \text{ MeV}$ $R \approx 10 \text{ cm}$	$E_o = 82 \text{ MeV}$ $R \approx 20 \text{ cm}$	$E_o = 108 \text{ MeV}$ $R \approx 30 \text{ cm}$
primary pions	72.3	71.7	61.5	17.5	22.7	23.2
secondary pions	1.4	3.9	7.0	0.0	0.0	0.0
protons	15.0	13.6	16.3	31.7	33.6	34.4
heavy nuclei	5.0	4.7	7.0	42.9	35.9	33.9
π^0 photons	1.3	2.4	4.0	0.2	0.3	0.8
electrons, positrons, photons	4.8	3.6	4.0	7.5	7.3	7.3
muons	<u>0.2</u>	<u>0.1</u>	<u>0.2</u>	<u>0.2</u>	<u>0.2</u>	<u>0.4</u>
	100.0	100.0	100.0	100.0	100.0	100.0

a. Average contribution over the depth intervals 0-1, 0-2, and 0-4 cm for $E_o = 53, 82, \text{ and } 108 \text{ MeV}$, respectively.

b. Average contribution over the depth intervals 9.6-10.0, 19.5-20.0, and 30.0-30.4 cm for $E_o = 53, 82, \text{ and } 108 \text{ MeV}$, respectively.

3.2. Radial Dose Distributions

For the case of 82-MeV incident pions, the radial distribution of the absorbed dose has also been calculated. Fig. 7 shows the absorbed dose as a function of the radial distance from the beam axis for four different depth intervals: 0-2 cm, 12-14 cm (near the end of the plateau region and the onset of the π^- capture region), 19.8-20.2 cm (near the maximum of the π^- capture region), and 28-30 cm (approximately 10 cm past the π^- capture region). Both the total dose and the dose due to ionization and excitation by primary pions are shown to indicate the secondary-particle contribution. The radial spread of the dose due to primary pions increases with increasing depths because of multiple Coulomb scattering, and at depths less than the ranges of the incident pions, the radial distribution of the total dose is governed by the spreading of the primary pions. At depths greater than the ranges of the incident pions, the dose is due mainly to neutrons and high-energy protons produced in the π^- capture region, and since these particles are produced approximately isotropically, the radial dose distribution becomes relatively uniform. It should be noted that at the smaller depths the energy deposition is concentrated near the beam axis and varies rapidly with radius. Therefore, the average value for the dose in the first radial interval (0-0.5 cm) and the 0- to 12-cm depth interval shown in fig. 7 is particularly dependent upon the size of the volume element chosen for computing the dose. Information on the radial distribution of the absorbed dose is presented in another way in fig. 8, where the fraction of the total (over all radii) dose deposited in cylinders of various radii centered about the beam axis is plotted.

The radial dependence of the secondary-particle contribution to the absorbed dose is given in table 2. Curtis and Raju (1968) have calculated "central-axis" depth-dose curves for pion beams in water and neglected the

Fig. 7. Radial dependence of the absorbed dose in various depth intervals for an 82-MeV incident π^- beam.

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Fig. 8. Absorbed dose in cylinders of various radii centered about the beam axis for an 82-MeV incident π^- beam.

Table 2. Contribution, in percent, of secondary particles to absorbed dose for an 82-MeV incident pion beam

Radial Interval (cm)	Depth Interval (cm)			
	0 - 2	6 - 8	12 - 14	19.8 - 20.2
0 - 0.5	10.4	8.7	8.4	75.2
0 - 1.0	12.5	10.6	8.4	74.5
0 - 2.5	15.8	14.5	12.1	76.3
0 - 5.0	19.9	18.0	16.1	77.3
0 - 10.0	23.0	22.8	22.0	77.7
0 - 15.0	23.9	25.3	25.3	77.8
0 - 25.0	25.9	28.7	28.6	78.0
0 - ∞	29.3	33.6	33.3	78.3

dose from secondary particles in the plateau region. Table 2 gives an indication of the magnitude of this approximation. In the plateau region, table 2 shows that secondary particles contribute $\sim 10\%$ to the dose close to the beam axis. Therefore, the neglect of secondary particles in computing the dose along the beam axis in the plateau may be an adequate approximation for some purposes. However, the secondary-particle contribution in the plateau becomes increasingly important the greater the distance from the beam axis, amounting to $\sim 30\%$ of the radially integrated dose.

3.3. Source Synthesis for Uniform Absorbed Dose

In planning the treatment of patients using negatively charged pions, it is necessary to be able to accurately predict the energy or range distribution for the incident pion beam needed to produce a uniform absorbed dose or biological effect over a specified volume (representing a tumor) at a given position in the body. A general computational method for this purpose has not yet been developed. However, by using the depth-dose distributions for monoenergetic incident pions shown in figs. 1-3, the incident beam can be determined approximately for the special case of producing a uniform absorbed dose over a specified depth interval in tissue.

As an example, the depth-dose distributions for monoenergetic incident pions producing peak doses at 20 and 30 cm (figs. 2 and 3) have been used to predict the energy distribution of the incident pion beam required to give a uniform absorbed dose in the depth interval from 20 to 30 cm. The algorithm used (see Appendix C) requires the depth-dose distributions for monoenergetic beams producing peak doses between 20 and 30 cm, and these were estimated using a linear combination of the depth-dose distributions peaking at 20 and 30 cm. The source spectrum was obtained by solving an integral equation using an iterative procedure. Since the method used for estimating the source spectrum involves several approximations, the predicted source spectrum was used as input to the code HETC, and the depth

dependence of the dose was calculated to check the uniformity of the dose in the 20- to 30-cm interval.

Fig. 9 shows the predicted energy spectrum for the incident pion beam needed to give a uniform dose in the 20- to 30-cm-depth interval. The corresponding range distribution for the incident pions is also shown in fig. 9 for comparison.

Fig. 10 shows the depth dependence of the absorbed dose computed with HETC for the incident energy spectrum shown in fig. 9. Except for very near 20 and 30 cm, the absorbed dose over the 20- to 30-cm-depth interval varies by about $\pm 5\%$, which is approximately the same magnitude as the fractional standard deviation of the statistical fluctuations in this depth interval. Improvement in the uniformity of the dose near the boundaries of the depth interval could probably be obtained by slightly modifying the source-synthesis algorithm to allow some incident pions with ranges slightly greater than 30 cm and slightly less than 20 cm.

Fig. 11 shows the contributions made by various kinds of particles to the absorbed-dose histogram shown in fig. 10. The contribution of secondary particles varies considerably over the 20- to 30-cm interval. This suggests that the LET spectra, and consequently the biological damage, may vary substantially over the interval.

Using radiation-protection quality factors, the depth dependence of the dose equivalent has also been calculated* for the source spectrum shown in

*The incident-pion spectrum for producing a uniform dose equivalent over the 20- to 30-cm-depth interval has been determined, and this source spectrum has been used to calculate the depth dependence of the dose equivalent and absorbed dose. These results are presented elsewhere (Armstrong, Alsmiller and Chandler 1973).

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Fig. 9. Range and kinetic energy distribution of an incident π^- beam designed to produce a uniform absorbed dose in tissue over the depth interval from 20 to 30 cm.

Fig. 10. Absorbed dose vs depth for an incident π^- beam designed to produce a uniform absorbed dose over the depth interval from 20 to 30 cm.

Fig. 11. Contributions by various types of particles to the absorbed dose shown in fig. 10.

fig. 9 (see Appendix D). The dose equivalent varies by a factor of ~ 2 over the 20- to 30-cm interval where the absorbed dose is approximately constant. Although the dose equivalent is only a very approximate estimate of biological damage, these calculations suggest that a pion beam designed to give a constant absorbed dose over a specified volume may produce a spatial distribution for the biological damage that varies considerably over the volume. For radiotherapy purposes, the determination of the energy spectrum of the incident beam to obtain a constant biological effect, such as cell survival, over the volume of interest would appear to be a more realistic criterion than using absorbed dose as a basis for specifying the incident beam. Calculations using charged-particle spectra computed with HETC, together with the cell-inactivation model of Katz (1971), to estimate the spatial dependence of cell inactivation in tissue produced by π^- beams are in progress.

APPENDIX A

DOSE EQUIVALENT VS DEPTH

The depth dependence of the dose equivalent for negatively charged pion beams having energies of 53, 82, and 108 MeV are shown in figs. A1-A3, respectively. For comparison, the absorbed-dose results given in figs. 1-3 of the text are shown again in figs. A1-A3.

The dose equivalent for protons, charged pions, and muons was calculated using the quality factor vs linear energy transfer relation given by Turner *et al.* (1964), which is based on recommendations of the National Committee on Radiation Protection (1954). A quality factor of 20 was assigned to the energy deposited by all nuclei with mass number greater than unity, and a quality factor of unity was assigned to the energy deposition due to electrons, positrons, and photons.

Figs. A4-A6 show the contributions to the dose equivalent by various types of particles. The particle-type definitions here are the same as those described in the text for the analogous absorbed-dose results.

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Fig. A1. Absorbed dose and dose equivalent vs depth in tissue for a 53-MeV incident π^- beam.

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Fig. A2. Absorbed dose and dose equivalent vs depth in tissue for an 82-MeV incident π^- beam.

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Fig. A3. Absorbed dose and dose equivalent vs depth in tissue for a 108-MeV incident π^- beam.

Fig. A4. Contributions by various kinds of particles to the dose equivalent for a 53-MeV incident π^- beam.

Fig. A5. Contributions by various kinds of particles to the dose equivalent for an 82-MeV incident π^- beam.

Fig. A6. Contributions by various kinds of particles to the dose equivalent for a 108-MeV incident π^- beam.

APPENDIX B

ENERGY-DEPOSITION SPECTRA

Figs. B1-B12 show the energy deposited by various charged particles as a function of energy and tissue depth for each of the three monoenergetic π^- beams considered. As described below, this information on energy deposition allows weighting factors other than the particular set of quality factors used in computing the dose equivalent given in Appendix A to be applied for estimating biological damage.

Consider a charged particle losing energy $\Delta E = E_1 - E_2$ by atomic ionization and excitation in some volume ΔV . The absorbed dose D , in rads, deposited by this particle is

$$D = \frac{c}{\Delta V} \int_{E_2}^{E_1} dE' ,$$

where energy is in MeV, $c = 1.6 \times 10^{-8} / \rho \text{ rad} \cdot \text{cm}^3 \cdot \text{MeV}^{-1}$, ρ = density of stopping medium in $\text{g} \cdot \text{cm}^{-3}$, and ΔV is in units of cm^3 . The corresponding effective dose obtained by weighting the absorbed dose by a factor for biological damage is

$$ED = \frac{c}{\Delta V} \int_{E_2}^{E_1} W(E') dE' ,$$

where $W(E')$ is the biological weighting factor at energy E' for the type of charged particle under consideration. (The dose-equivalent results shown in Appendices A and D were obtained using radiation-protection quality factors for W .)

Dividing the energy interval into I subintervals,

$$\begin{aligned}
 ED &= \frac{c}{\Delta V} \sum_{i=1}^I \int_{E_{i-1}}^{E_i} W(E') dE' \\
 &= c \sum_{i=1}^I \left[\frac{\int_{E_{i-1}}^{E_i} W(E') dE'}{\int_{E_{i-1}}^{E_i} dE'} \right] \left[\int_{E_{i-1}}^{E_i} dE' / \Delta V \right] \\
 &\equiv c \sum_{i=1}^I \bar{W}_i \epsilon_i ,
 \end{aligned}$$

where $\bar{W}_i$ is the average weighting factor over the i th energy interval and ϵ_i , which will be referred to as the energy-deposition spectrum, is the energy deposited per unit volume in the i th energy interval. Generalizing to a mixed-radiation field where there is a variety of charged-particle types depositing energy in volume element ΔV about $\bar{r}$,

$$ED(\bar{r}) = c \sum_j \sum_i \bar{W}_{ij} \epsilon_{ij}(\bar{r}) , \quad (B1)$$

where j denotes particle type.

Figs. B1-B12 show the energy-deposition spectra averaged over various depth intervals for each type of charged particle for incident pion beams of energies of 53, 82, and 108 MeV. The quantity plotted is

$$\epsilon_{ijk} = \frac{\int_{\Delta z_k} dz \int_0^\infty dr 2\pi r \epsilon_{ij}(r, z)}{\int_{\Delta z_k} dz} ,$$

where ϵ_{ijk} is normalized to one incident π^- . The points shown on the figures at 20 MeV are the values for ϵ_{ijk} in the energy interval from 20 MeV to infinity. Values for ϵ_{ijk} are not shown for pions in the 0- to 1-MeV interval because π^+ slowing down to 1 MeV are assumed to decay and π^- are assumed to undergo nuclear capture at 1 MeV. The contribution to the energy

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DEPTH INTERVAL : 0.0 TO 7.0 CM.

Fig. B1. Energy deposition per unit depth interval vs energy in the depth interval from 0.0 to 7.0 cm for a 53-MeV incident π^- beam.

ORNL-DWG 72-14297

DEPTH INTERVAL : 7.0 TO 9.4 CM.

Fig. B2. Energy deposition per unit depth interval vs energy in the depth interval from 7.0 to 9.4 cm for a 53-MeV incident π^- beam.

ORNL-DWG 72-14298

DEPTH INTERVAL : 9.4 TO 10.2 CM.

Fig. B3. Energy deposition per unit depth interval vs energy in the depth interval from 9.4 to 10.2 cm for a 53-MeV incident π^- beam.

ORNL-DWG 72-14299

DEPTH INTERVAL : 10.2 TO 60.0 CM.

Fig. B4. Energy deposition per unit depth interval vs energy in the depth interval from 10.2 to 60.0 cm for a 53-MeV incident π^- beam.

ORNL-DWG 72-14300

DEPTH INTERVAL : 0.0 TO 17.0 CM.

Fig. B5. Energy deposition per unit depth interval vs energy in the depth interval from 0.0 to 17.0 cm for an 82-MeV incident π^- beam.

ORNL-DWG 72-14301

DEPTH INTERVAL : 17.0 TO 19.4 CM.

Fig. B6. Energy deposition per unit depth interval vs energy in the depth interval from 17.0 to 19.4 cm for an 82-MeV incident π^- beam.

ORNL-DWG 72-14302

DEPTH INTERVAL : 19.4 TO 20.2 CM.

Fig. B7. Energy deposition per unit depth interval vs energy in the depth interval from 19.4 to 20.2 cm for an 82-MeV incident π^- beam.

ORNL-DWG 72-14303

DEPTH INTERVAL : 20.2 TO 60.0 CM.

Fig. B8. Energy deposition per unit depth interval vs energy in the depth interval from 20.2 to 60.0 cm for an 82-MeV incident π^- beam.

ORNL-DWG 72-14304

DEPTH INTERVAL : 0.0 TO 28.0 CM.

Fig. B9. Energy deposition per unit depth interval vs energy in the depth interval from 0.0 to 28.0 cm for a 108-MeV incident π^- beam.

ORNL-DWG 72-14305

DEPTH INTERVAL : 28.0 TO 29.6 CM.

Fig. B10. Energy deposition per unit depth interval vs energy in the depth interval from 28.0 to 29.6 cm for a 108-MeV incident π^- beam.

ORNL-DWG 72-14306

DEPTH INTERVAL : 29.6 TO 30.4 CM.

Fig. B11. Energy deposition per unit depth interval vs energy in the depth interval from 29.6 to 30.4 cm for a 108-MeV incident π^- beam.

ORNL-DWG 72-14307

DEPTH INTERVAL : 30.4 TO 60.0 CM.

Fig. B12. Energy deposition per unit depth interval vs energy in the depth interval from 30.4 to 60.0 cm for a 108-MeV incident π^- beam.

deposition due to photons, electrons, and positrons is not shown in figs. B1-B12, but since a biological weighting factor of unity is usually associated with these particles, their contributions to the effective dose can be taken directly from figs. 4, 5, and 6 (in the text) which give the depth dependence of the absorbed dose.

Biological weighting factors are often expressed as a function of LET. These LET-dependent factors can be converted to energy-dependent factors for use in Eq. B1 by using the appropriate stopping power for each particle type. Conversely, the energy deposition spectra in figs. B1-B12 can be expressed as a function of LET by using the appropriate stopping powers, and these spectra can be used in conjunction with LET-dependent weighting factors to compute the effective dose. The energy-deposition data in figs. B1-B12 can also be expressed as a function of z^2/β^2 , where z is the effective charge of the particles and β is the speed of the particle relative to the speed of light, and this is the form needed in applying the model of Katz *et al.* (1971) to compute cell-survival probabilities.

APPENDIX C

SOURCE-SYNTHESIS ALGORITHM

The procedure used in determining the energy spectrum for the incident pion beam required to produce the uniform absorbed dose (discussed in the text) and the uniform dose equivalent (discussed in Appendix D) is described below.

Suppose it is desired to produce a uniform dose over a depth interval from x_a to x_b . It is assumed that the depth-dose distributions for monoenergetic incident pions producing peak doses at x_A and x_B are available, where $x_A \leq x_a$ and $x_B \geq x_b$. In the interval from x_a to x_b , the depth-dose distribution due to incident monoenergetic pions producing a peak dose at x' ($x_a \leq x' \leq x_b$) is taken to be a linear combination of the distributions peaking at x_A and x_B ; that is,

$$\tilde{D}(x, x') = f(x') D_A(x_A + x - x') + [1 - f(x')] D_B(x_B + x - x') ,$$

where

$\tilde{D}(x, x')$ = the dose at x due to a monoenergetic pion beam producing a peak dose at x' ,

D_A, D_B = the depth-dose distributions peaking at x_A and x_B , respectively, and

$$f(x') = 1 - (x' - x_A)/(x_B - x_A) .$$

The relative dose distribution produced by an incident pion beam having $N(x'_1)$ pions with range x'_1 is then approximately

$$D(x) = \sum_{i=0}^n N(x'_1) \tilde{D}(x, x'_1) , \quad (C1)$$

where the interval from x_a to x_b has been divided into n equal subintervals and $x'_i = x_a + i\Delta x$, $\Delta x = (x_b - x_a)/n$. The problem then is to determine that range distribution, say, $\tilde{N}$, such that when $\tilde{N}$ is used for N in Eq. C1, $D(x)$ will be large and constant over the interval x_a to x_b and small outside this interval. $\tilde{N}$ was determined by using an iterative procedure with $\tilde{N} = 1$ as an initial estimate. Thus, after j iterations, the estimate for $\tilde{N}$ is

$$\tilde{N}_j(x) = \frac{D_{j-1}(x_a)}{D_{j-1}(x)}, \quad (C2)$$

where

$$D_j(x) = \sum_{i=0}^n \tilde{N}_j(x'_i) \tilde{D}(x, x'_i)$$

with

$$\tilde{N}_0(x) = 1.$$

In the equation for $\tilde{N}_j(x)$, the factor $D_{j-1}(x_a)$ is included as an arbitrary normalization factor. In determining $\tilde{N}$ to produce a uniform dose equivalent, one iteration was used; for the absorbed-dose case, five iterations were used. The number of iterations needed was determined by considering the uniformity of $D_j(x)$ after each iteration. For the calculations here, $\tilde{N}_j(x)$ was set equal to zero for all x outside the interval from x_a to x_b . With this constraint, the $D_j(x)$ curve tends to decrease near the boundaries x_a and x_b . Subsequent experience indicates that $D_j(x)$ can be made more nearly uniform over the entire interval by allowing incident pions with ranges slightly outside the interval from x_a to x_b .

The relative energy spectrum for the incident pions corresponding to the desired range distribution, $\tilde{N}(x)$, is

$$S(E) = \tilde{N}(x) (dE/dx)^{-1} ,$$

where dE/dx is the stopping power. The energy spectrum input to the code HETC, $S^*(E)$, was normalized to one incident pion over the allowed energy range; that is,

$$S^*(E) = \frac{S(E)}{\int_{E_a}^{E_b} S(E) dE} ,$$

where E_a and E_b are kinetic energies corresponding to ranges in tissue of x_a and x_b , respectively.

APPENDIX D

UNIFORM DOSE EQUIVALENT

The dose-equivalent-vs-depth results given in Appendix A for mono-energetic pion beams and the algorithm discussed in Appendix C have been used to estimate the energy and range distributions for the incident pion beam needed to produce a uniform dose equivalent over the depth interval from 20 to 30 cm. The predicted range distribution for the beam needed to produce a uniform dose equivalent is shown in fig. D1. For comparison, the range distribution shown in fig. 9 of the text for computing the uniform absorbed dose over the same depth interval is shown again in fig. D1. The range distribution for producing a uniform dose equivalent was used as input to the code HETC, and the results from the transport calculation are shown in fig. D2. The contributions to the absorbed dose and dose equivalent by various types of particles are shown in figs. D3 and D4, respectively.

The dose-equivalent distribution for the source spectrum designed to give a uniform absorbed dose over the 20- to 30-cm depth interval has also been calculated and is shown in fig. D5. The absorbed-dose histograms in fig. D5 are the same as those shown in fig. 10 of the text.

Fig. D2 shows that for a source spectrum designed to give a uniform dose equivalent over the 20- to 30-cm depth interval in tissue, the absorbed dose varies by a factor of ~ 2 over the same interval. Conversely, fig. D5 shows that for an incident π^- beam that produces a uniform absorbed dose over the 20- to 30-cm depth interval, the dose equivalent varies by a factor of ~ 2 over the interval. Although the dose equivalent is only a very approximate indicator of biological damage, these results suggest that pion

beams designed to give a uniform absorbed dose over some specified volume may result in a very nonuniform biological damage over the volume, and therefore, for radiotherapy purposes, the beam parameters should be determined on the basis of biological damage rather than absorbed dose.

Fig. D1. Predicted range distributions for incident π^- beams to produce uniform dose equivalent and uniform absorbed dose over the depth interval from 20 to 30 cm.

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Fig. D2. Dose equivalent and absorbed dose vs depth in tissue for an incident π^- beam designed to produce a uniform dose equivalent over the depth interval from 20 to 30 cm.

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Fig. D3. Contribution of various particle types to the absorbed dose shown in fig. D2.

Fig. D4. Contribution of various particle types to the dose equivalent shown in fig. D2.

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Fig. D5. Dose equivalent and absorbed dose vs depth in tissue for an incident π^- beam designed to produce a uniform absorbed dose over the depth interval from 20 to 30 cm.

REFERENCES

- ALSMILLER, R. G., Jr., ARMSTRONG, T. W., and BISHOP, Barbara L., 1971, Nucl. Sci. Eng. 43, 257.
- ARMSTRONG, T. W., ALSMILLER, R. G., Jr., CHANDLER, K. C., and BISHOP, B. L., 1972, Nucl. Sci. Eng. 49, 82.
- ARMSTRONG, T. W., and CHANDLER, K. C., 1972a, Nucl. Sci. Eng. 49, 110.
- ARMSTRONG, T. W., and CHANDLER, K. C., 1972b, Radiat. Res. 52, 247.
- AUERBACH, E. H., FLEMING, D. M., and STERNHEIM, M. M., 1967, Phys. Rev. 162, 1683.
- BERTINI, Hugo W., and GUTHRIE, Miriam P., 1971, Nucl. Phys. A169, 670.
- COLEMAN, W. A., and ARMSTRONG, T. W., 1970, ORNL-4606, Oak Ridge National Laboratory.
- CURTIS, Stanley B., and RAJU, Mudundi R., 1968, Radiat. Res. 34, 239.
- FERMI, E., and TELLER, E., 1947, Phys. Rev. 72, 399.
- FOWLER, P. H., and PERKINS, D. H., 1961, Nature 189, 524.
- GUTHRIE, Miriam P., 1970, ORNL-TM-3119, Oak Ridge National Laboratory.
- GUTHRIE, M. P., ALSMILLER, R. G., Jr., and BERTINI, H. W., 1968, Nucl. Instr. Meth. 66, 29; with erratum, 1971, Nucl. Instr. Meth. 91, 669.
- KATZ, R., ACKERSON, B., HOMAYOONFAR, M., and SHARMA, S. C., 1971, Radiat. Res. 47, 402.
- NATIONAL COMMITTEE ON RADIATION PROTECTION, 1954, Permissible Dose from External Sources of Ionizing Radiation, Handbook 59, Natl. Bureau of Standards.

RAJU, M. R., and RICHMAN, C., 1970, in Proc. XIIth Intl. Congress of Radiology, Tokyo, 1969, GANN Monograph No. 9, 105.

ROSEN, Louis, 1968, Nucl. Appl. 5, 379.

TURNER, J. E., ZERBY, C. D., WOODYARD, R. L., WRIGHT, H. A., KINNEY, W. E., SNYDER, W. S., and NEUFELD, J., 1964, Health Phys. 10, 783.

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