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THE PHYSICS OF RF HEATING

BY

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The Physics of RF Heating *

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ABSTRACT

Electrical power passes through six identifiable stages in radiofrequency plasma heating. The first three are (1) rf generation, (2) transmission of rf power to the plasma surface, and (3) coupling of rf energy to the plasma. An efficient form for Stage (3) is to match the frequency and parallel wave-length of the driving field to those of a natural mode in the plasma, thereby exciting a "coupling resonance". Stage (4) is the modification of the rf wave or perturbation in the plasma, which may take place due to either linear and nonlinear processes. Stage (5) is the absorption of the wave in the plasma, which must compete with eddy current dissipation in the walls, and Stage (6) is the effective thermalization of the energy added to the plasma. These stages are discussed in turn.

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The aim of this paper is to give a general outline of the entire area of radiofrequency plasma heating, to define some of the terms, to discuss in an elementary way the physical concepts, and to break up the overall process into six components each of which can reasonably well be examined independently. The components, or stages, are blocked out in Fig. 1, which shows the generation of rf power on the left in Stage I, and on the right, in Stage VI, the addition of thermal energy to the ions and electrons of the plasma. During transmission between these two stages, the power adopts several different forms. After some preliminary comments, we will discuss the different stages in more detail.

The first decision we are faced with in radiofrequency plasma heating is the choice of frequency. The low-frequency end of our possible heating spectrum is zero-frequency, or more reasonably, the reciprocal of the plasma lifetime. For finite frequencies, however, difficulty can arise with respect to the penetration of $E_{\parallel}$ into a plasma. The skin depth, which is the imaginary component of k_r , can be determined by the formula from cold-plasma theory,

$$k_r^2 = \frac{\omega^2}{c^2} - \frac{4\pi n_e e^2}{m_e c^2} \cdot \frac{\omega}{\omega + i\nu_{ei}}.$$

The first term on the right is due to displacement current, and represents a small effect for low frequencies.

If the plasma were a perfect conductor, its skin depth would

be zero. The skin depth is finite for $\vec{E} \parallel \vec{B}$ only because the electron mobility is limited — at low frequencies by electron-ion collisions, and at high frequencies by electron inertia. Now the low-frequency $\vec{E} \parallel \vec{B}$ mode of heating is just exactly Ohmic heating, and the problem of skin penetration for Ohmic heating is already well known — even if not well understood. The skin depth formula shows that raising the frequency makes the skin layer even thinner — until displacement current starts to play a meaningful role, which occurs around the electron plasma frequency. But at this frequency, we have problems with obtaining sufficient amounts of rf power. Which brings us to a discussion of Stage I.

I. RF GENERATION

The electron density in a typical laboratory plasma today might be around $3 \times 10^{13} \text{ cm}^{-3}$. The plasma frequency for this density is 50 gigahertz, corresponding to a wavelength in vacuo of 6 mm. If we look now at a graph prepared by A. Scott in 1969 showing single-vacuum-tube power capabilities (Fig. 2), we see that available rf power at 50 gigahertz is down by a factor of about twenty-five from the plateau. Electron densities in a fusion reactor will be about six times higher, and to heat these plasmas we would require microwave power around 100 gigahertz.

Now we recall from the paper on plasma heating that powers in the range of 1-10 megawatts are needed for effective supplementary heating in today's tokamak plasmas, and 10-100 megawatts will be needed for supplementary heating in bringing

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reactor plasmas to ignition. Therefore, for engineering reasons alone, it is reasonable to restrict our thinking to the frequency range below a few gigahertz, which is to say to frequencies typically of the order of the ion plasma frequency, or the geometric mean of the ion and electron cyclotron frequencies, or below.

II. RF TRANSMISSION

The problems of rf transmission at high microwave frequencies are similar to those of rf generation. Transmission lines and waveguides become lossier at the high frequencies, and maintaining good electrical insulation for waveguides and feedthroughs become increasingly difficult. The problem of high-power transmission at very high microwave frequencies therefore supports the argument above favoring the use of frequencies of a few gigahertz and below for effective plasma heating.

III. COUPLING RF ENERGY TO THE PLASMA

In the discussion above, we managed to eliminate the use of moderate frequencies for plasma heating on the basis of a skin-depth argument. And we eliminated heating at high frequencies — above the plasma frequency, where skin depth is not a problem — because adequate rf power is not available. It appears at first as though we have eliminated all the possibilities for rf plasma heating. But re-examining our skin-depth considerations, we see that they applied only to the $\vec{E} \parallel \vec{B}$ mode. The physical origin of the small skin depth in this mode is that electrons can move along the magnetic lines of force and short out the $E_{\parallel}$ field. Let us look now at the polarization where $\vec{E}$ is approximately

perpendicular to $\vec{B}$. Electrons now move in the $\vec{E} \times \vec{B}$ direction, and even if they do produce some current by this motion, this current is not able to short out the driving $E_{\perp}$ field. It is therefore these modes, with approximately $\vec{E} \perp \vec{B}$, which give us penetration at frequencies below the electron plasma frequency, and these will be the important modes for plasma heating. The best known modes here are the following:

Electromagnetic Waves [$\vec{E}$ not parallel to $\vec{k}$, $|\omega/k_{\parallel}| \gg (2kT_{\parallel}^{(e)}/m_e)^{1/2}$]

Shear Alfvén Wave (or Slow Hydromagnetic Wave)

Compressional Alfvén Wave (or Fast Hydromagnetic Wave)

Ion Cyclotron Wave

Whistler Mode

Slow Electrostatic Waves ($\vec{E}$ parallel to $\vec{k}$, $|\omega/k_{\parallel}| \ll (2kT_{\parallel}^{(e)}/m_e)^{1/2}$]

Quasi-Perpendicular Acoustic Wave (or Electrostatic Ion Cyclotron Wave)

Drift Wave

Analysis of these waves is based on self-consistent solutions of a set of equations describing the plasma together with Maxwell's equations. The plasma can be described either by the fluid equations for the ions and electrons, or by kinetic equations, that is, the Boltzmann equations for the particles. Small amplitude motions are the easiest to study — in this limit the set of equations describing the plasma may be expanded in terms of some parameter such as E/B , or v/v_{phase} , or v/v_{thermal} . The zero-order equations in this expansion describe the unperturbed plasma equilibrium. The first-order equations will describe the

wave. Second-order solutions, which we will come to later, will describe wave-wave interactions, and will introduce new concepts such as amplitude saturation and non-linear instabilities. But returning now to the discussion of the first-order set of equations, we introduce Fourier analysis in space and time. The set of "linearized" equations then consists of n linear equations with n unknowns, the unknowns being the Fourier field amplitudes such as $\tilde{E}_x(\omega, \vec{k})$ or $\tilde{B}_y$ or moments of the distribution function such as $\tilde{n}$ or $\langle \tilde{v} \rangle$. The condition that this set of n equations has a non-trivial solution is that its determinant be zero. Setting the determinant equal to zero then leads to a specific relationship between ω and $\vec{k}$ which is called the "dispersion relation". The dispersion relation for light waves, as a familiar example, is just $\omega^2 = k^2 c^2$. Our skin depth formula, given above, is actually the dispersion relation for the so-called "quasi-transverse ordinary" mode. The dispersion relations for the electromagnetic plasma waves listed just above can be found in standard texts on plasma physics. The dispersion relations for the two low-frequency electrostatic waves listed above also appear frequently in the literature, but we might put down a combined form here just as an example. It reads

$$\omega^2 \left(1 - \frac{\omega_D}{\omega}\right) = \Omega_i^2 \left(1 - \frac{\omega_D}{\omega} - \frac{k_z^2 v_{AC}^2}{\omega^2}\right) + (k_x^2 - k_z^2) v_{AC}^2$$

where

$$\Omega_i \equiv \frac{ZeB_0}{m_i c}$$

$$v_{AC}^2 \equiv \frac{k_z^2 Z k T_e}{m_i}$$

$$\omega_D \equiv \frac{k_x k T_e c}{e B_0} \frac{1}{n} \frac{dn}{dy}$$

$$\vec{B}_0 \equiv \hat{z} B_0$$

Choosing the parameters such that different terms in this dispersion relation are dominant, one can pick out the familiar forms:

$$\omega = \omega_D \quad \text{Drift waves}$$

$$\omega^2 = k_z^2 v_{AC}^2 \quad \text{Ion acoustic waves}$$

$$\omega^2 = \Omega_i^2 + k_x^2 v_{AC}^2 \quad \text{Quasi-perpendicular acoustic waves}$$

The modes described here are, with the exception of the drift wave, modes for a homogeneous uniform plasma. However, they can usually also be used to describe the modes of a plasma column of finite radius, even in the presence of a density gradient. It will generally turn out that the plasma column must be larger than a certain critical size in order that a mode fit in, just as ordinary rectangular waveguides must be at least a half-wavelength

across to allow microwave propagation. But provided that a mode does indeed fit into the plasma column, it becomes an eigenmode of the system, and we may consider trying to excite it with externally-supplied rf power. If we design the antenna structure to have the same parallel wavelength as the plasma eigenmode, we can speak of a "coupling resonance", which is a highly efficient way to transfer energy to the plasma, minimizing both the required voltages and the copper losses on the antenna structure. A mechanical analogy for setting up motion at a coupling resonance is shown in Fig. 3. In plasma physics, an example of the use of a coupling resonance is the excitation of ion cyclotron waves with an antenna structure that produces a B_z perturbation which is periodic in z , and which may extend typically a couple of wavelengths in the z direction. An example of plasma excitation without a coupling resonance is the classical configuration for magnetic pumping.

IV. MODIFICATION OF THE WAVE IN THE PLASMA

An actual plasma column is generally quite nonuniform, with both density and temperature falling off sharply near the plasma edge. Therefore the simple picture of a single eigenmode in the plasma may be incorrect. We can suggest some of the alternative possibilities.

(a) Linear mode conversion. In the two dispersion relations written out above, we saw that the dependence on frequency can be quite complicated. Similarly, the introduction of the effects of finite plasma pressure or, equivalently, of finite Larmor radii, can give the dispersion relation a similarly-complicated

dependence on wavenumber. Now we can transform a dispersion relation back into a wave-equation in an approximate fashion by replacing ik_x where it appears by the operator ∇ . Let us imagine that we then have a wave equation which looks like

$$a(y) \frac{d^4 E}{dy^4} + b(y) \frac{d^2 E}{dy^2} + c(y) E = 0$$

where the density and temperature gradients of the plasma are in the y direction, and where the coefficients a , b , and c are functions of the local plasma parameters, $B(y)$, $n(y)$ and $T(y)$ as well as functions of k_x , k_z , and ω . Now if the middle term is sufficiently large, we have two approximate solutions in the WKB or optical approximation sense,

$$k_I^2(y) = \frac{b(y)}{a(y)}, \quad k_{II}^2(y) = \frac{c(y)}{b(y)}.$$

But it may happen that $b(y)$ goes through zero at a "resonance layer" in the plasma, $y = y_{res}$, that is,

$$b(y_{res}) = 0, \quad b(y) = (y - y_{res})b'(y_{res}) + \dots$$

We may substitute this value for $b(y)$ into the wave equation. A careful analysis of the equation then shows that if the variation of physical parameters with distance is sufficiently slow, a type I mode approaching the resonance layer will be reflected or transmitted as a type II mode. Strong attenuation of the mode may also take place, depending on the strength of the damping processes at the resonance layer. The process will take place for waves of

arbitrarily small amplitude, and is called "linear mode conversion"

(b) Wave-wave interaction. There may be present in the plasma a finite-amplitude wave even in the absence of any external excitation. Acoustic turbulence or drift waves are frequently found, driven by inhomogeneous or non-Maxwellian velocity distributions. It is then to be expected that any incoming wave will be scattered by the indigenous wave, producing sidebands at the sum and difference frequencies and sum and difference wave-numbers.

(c) Parametric Decay Instabilities. It is also possible that an incoming wave can decay spontaneously into two daughter waves. This process is known as a "parametric decay instability". It is a relative newcomer on the scene in plasma physics, and some brief mathematical description seems appropriate. Let us consider that the wave equation for a plasma can be written in terms of a single field variable, E , as

$$D(n, T, B, \nabla, \partial/\partial t, E) = 0$$

in a local approximation where n , T , and B are the zero-order local values for these quantities. The relation includes nonlinear terms in E . We now put in a trial function for E ,

$$E = \sum_{n=1}^3 \psi_n \exp(i\vec{k}_n \cdot \vec{r} - i\omega_n t) + \text{complex conjugate},$$

where

$$\begin{aligned} \omega_3 &= \omega_1 + \omega_2 \\ \vec{k}_3 &= \vec{k}_1 + \vec{k}_2 \end{aligned}$$

Moreover, we choose to excite the system at $\omega_3, \vec{k}_3$. Then, taking advantage of the orthogonality of the different components of our trial function, and keeping terms of order E and E^2 , the equations for the system can be written in the form

$$\text{At } \omega_1: \quad F_1 \Psi_1 + G_1 \Psi_3 \Psi_2^* = 0$$

$$\text{At } \omega_2: \quad F_2 \Psi_2 + G_2 \Psi_3 \Psi_1^* = 0$$

$$\text{At } \omega_3: \quad F_3 \Psi_3 + G_3 \Psi_1 \Psi_2 = \mu$$

where the linear and quadratic terms in E are displayed separately. The quantity μ represents the driving term at $\omega_3, \vec{k}_3$. With a little algebra, the solution for this set of equations can be written in the form

$$\Psi_1 \Psi_1^* = \frac{F_2 F_3}{G_2 G_3} + \mu \frac{e^{i\phi}}{G_3} \left| \frac{F_2 G_1}{F_1 G_2} \right|^{1/2},$$

$$e^{i\phi} \equiv \frac{\Psi_1^*}{|\Psi_1|} \frac{|\Psi_2|}{\Psi_2}$$

The left hand side of this equation is real and positive. The coefficients F and G , on the other hand, are generally complex, so the right hand side is the sum of two complex numbers. Simple vector diagrams, Figure 4, clarify the nature of the possible solutions. The phase, ϕ , is freely varied as the phase of daughter #1 is changed with respect to the phase of daughter #2. Solutions appear, therefore, when the magnitude of μ is sufficient to

enable the imaginary portions on the right hand side to cancel. This value of the excitation, μ , is its threshold value, and is given by

$$|\mu_{\text{threshold}}| = \left| \frac{F_1 F_2 F_3^2}{G_1 G_2} \right|^{1/2} \theta$$

$$\theta \equiv 1 \text{ for } \text{Re} \frac{F_2 F_3}{G_2 G_3} < 0$$

$$\theta \equiv \frac{\left| \text{Im} \frac{F_2 F_3}{G_2 G_3} \right|}{\left| \frac{F_2 F_3}{G_2 G_3} \right|} \text{ for } \text{Re} \frac{F_2 F_3}{G_2 G_3} > 0$$

When any of the modes is a natural mode or eigenmode of the linearized system, its corresponding F value is zero, or in the presence of some damping, very small. We now see minimum threshold values will occur for parametric decay instability when all three modes are eigenmodes of the linearized system. Moreover, from our vector diagram, we note three possible modes of behavior for the amplitudes of the daughter waves as the driving amplitude is varied. The graph of $\psi_1 \psi_1^*$ vs $|\mu|$ is a hyperbola — when $\text{Re} (F_2 F_3 / G_2 G_3) > 0$, two solutions are possible. The amplitude $\psi_1 \psi_1^*$ jumps immediately to a finite value as threshold is reached, and may then follow either the upper branch or the lower branch of the hyperbola.

A relevant question at this point concerns the nonlinear stability of the solutions just obtained. This question can be investigated within the same formalism. One may simply add a small growth decrement to ω_1 and look at the above equation for

values of $\psi_1 \psi_1^*$ which do not quite lie on the hyperbola. If, for example, $\psi_1 \psi_1^*$ is chosen slightly below the hyperbola, the system has nonlinear stability if the decrement in ω_1 necessary to cause the equation to be satisfied is such as to cause $\psi_1 \psi_1^*$ to grow with time.

V. WAVE ABSORPTION

Waves can be absorbed in the plasma by both collisional and collisionfree processes. They may also dissipate energy by radiation (not an important process in this context) and by eddy current losses in the walls. Even for that fraction of wave energy dissipated in the plasma, it can be important whether it is dissipated at the plasma edge or in the plasma interior, or whether the absorption might be concentrated around the magnetic field minima in such a way as to create, for instance, trapped particle instabilities.

Generally, the details of wave absorption require individual investigations for each case. We may, however, venture some general remarks. The collisional dissipative processes in the fluid regime are viscosity and resistivity. We note, in particular, that the simple occurrence of collisions does not necessarily imply the dissipation of wave energy. In Alfvén waves, for instance, ions and electrons both move with $\vec{E} \times \vec{B}$ velocities, so that the wave components of the ion and electron motions are identical at each point. Alfvén waves are therefore dissipated only very slowly by charged-particle collisions. On the other hand, ion-neutral collisions can produce strong Alfvén wave

damping. Also, there is another mode of collisions which is important even in long mean-free-path plasmas. That is, collisions between ions with different charge-to-mass ratios. Ions of two q/m species will have very different coherent velocity components for motion along the magnetic field, and also for motion across the magnetic field at frequencies of the order of their cyclotron frequencies or above.

Collisionless damping takes place when certain particles in the plasma called the "resonant particles", see, in their own rest frames, a static E field. The Doppler shift will cause this to take place for particles with $v_{||}$ such that

$$\omega - k_{||}v_{||} - n\omega_c = 0 ,$$

where n is any integer, positive or negative or zero. The case for $n = 0$ corresponds to Landau damping, for $n = \pm 1$ to cyclotron damping, and $|n| > 2$ corresponds to cyclotron harmonic damping, a process which varies in strength roughly as $e^{-\lambda} I_n(\lambda)$, where $\lambda \equiv k_{\perp}^2 k T_{\perp} / m \omega_c^2$. The cyclotron terms arise because the "rest frame" for a particle in a magnetic field is its helical orbit.

The resonant particles are able to extract the energy from the wave, and the macroscopic characteristics of the wave such as density and velocity oscillations disappear. Sample calculations of the rate of wave attenuation by Landau and cyclotron damping are given in many texts on plasma physics.

VI. EFFECTIVE THERMALIZATION OF THE ADDED ENERGY

The final question which arises concerns the actual heating

of the plasma. In the case of collisional wave absorption processes, it is clear that wave energy is immediately put into random motions of the plasma particles. On the other hand, in the case of collisionfree wave attenuation, no stochastic processes are invoked. What occurs is that the zeroth and first moments of the perturbed velocity distribution disappear, that is, the density and velocity moments both go to zero. However, the wave energy is still retained in a coherent fashion by the resonant particles. That this process can be at least partially reversed has been shown in the famous plasma wave echo experiments. Therefore, an essential part of the heating process is that resonant particles given coherent wave energy through Landau and cyclotron damping be subject to at least one randomizing collision. For this purpose collisions between like particles are completely effective because the coherent component of the resonant particle velocity is 90° out of phase and of different magnitude than the coherent velocity for non-resonant particles. Mechanical analogies for collisionfree damping and for thermalization are given in Figs. 5 and 6. Further collisions are required, of course, to distribute the energy evenly over the entire velocity distributions for ions and electrons.

SUMMARY

We have made an attempt to trace rf energy as it goes from the rf generator until it appears as a temperature increase in the plasma. It has been seen that a number of fundamental concepts in plasma physics are touched upon including both plasma wave theory and plasma kinetic theory.

With regard, now, to the future role of rf heating, the situation may be characterized in the following manner. If a single means for giving supplementary heat to a reactor plasma had to be chosen today on the basis of proven laboratory experiments, that means would have to be rf heating. The effectiveness of rf heating has been shown again and again in laboratory experiments which are now well known and part of the plasma physics literature. Moreover, the indication in the higher frequency experiments is that the rate of particle loss is unaffected by the rf heating process. And finally, as we have seen today, the basic theoretical elements of the rf plasma heating process are reasonably well understood. A continuation, then, of intensive experimental and theoretical research in this field is therefore both warranted and justified, in order to give us the means, when we need it, for heating a reactor plasma to ignition.

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Fig. 1. Six stages in radiofrequency plasma heating.

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Fig. 2. Radiofrequency and microwave power available from several different sources. From A. Scott, EDN, Oct 1969. Plasma references by W. M. Hooke.

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Fig. 3. Mechanical analogy. Man on right knocks himself out trying to energize swing with less-than-optimum phasing.

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Fig. 4. Vector diagrams to illustrate solutions for parametric instability equation.

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Fig. 5. Landau-damped wave has lost wave momentum, but retains energy.

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Fig. 6. Collisional process.

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