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**CREEP-FATIGUE INTERACTION
and
CUMULATIVE DAMAGE EVALUATIONS
for TYPE 304 STAINLESS STEEL**

Hold-Time Fatigue Test Program and Review of Multiaxial Fatigue

E. P. Esztergar

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and Review of Multiaxial Fatigue**

E. P. Esztergar

Consultant

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JUNE 1972

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PREFACE

The study reported herein was conducted as a part of the Oak Ridge National Laboratory program entitled: High-Temperature Structural Design Methods for LMFBR Components. The study was initiated with the basic objective of proposing a creep-fatigue test program for type 304 stainless steel that reflected the needs of designers and design code bodies and that would lead to the formulation and use of confident design criteria. E. P. Esztergar was chosen for the task because of his past experience with the high-temperature creep-fatigue design problem at Gulf General Atomic, Inc., because of his continued cognizance of the area, and because of his knowledge of design code use and needs.

To arrive at a final test program to be recommended to the U.S. Atomic Energy Commission, the test program presented in this report was reviewed and commented upon by a number of recognized authorities in the areas of creep-fatigue and high-temperature design. As a result of these reviews, minor changes were made to the test matrix, and a group of ancillary low-cycle fatigue tests, a fatigue crack growth test group, a small statistical test study group, and a set of special tensile tests, were added. These added test groups were intended to broaden the usefulness and versatility of the total program test results.

The resulting modified test program was formally submitted by ORNL to USAEC-RDT as a recommended creep-fatigue base test program for 304 stainless steel. It was recommended that the entire program first be conducted on a single heat of material with a single pretest treatment. Additional testing would be required to evaluate the effects of heat-to-heat variations and of metallurgical and environmental variables. Similar data would also be required for weldment and irradiated materials.

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CREEP-FATIGUE INTERACTION AND CUMULATIVE DAMAGE EVALUATIONS FOR TYPE 304 STAINLESS STEEL

E. P. Esztegar

ABSTRACT

Components in steam generating service are subjected to long periods of steady operation interrupted by cyclic load and temperature variations. Cyclic strain has long been recognized as the source of fatigue damage, and design methods have been developed to prevent failure. The periods of steady operation received little attention in fatigue investigations, because at low temperatures fatigue damage was found to be independent of the time elapsed between cycles. At high temperatures, however, the conditions are reversed: during hold periods creep damage is accumulated and all time-dependent effects (relaxation, creep recovery, strain-rate sensitivity) become important in designing for cyclic operation. The influence of time-related effects on cyclic endurance is collectively called creep-fatigue interaction, and this phenomenon is the subject of this study. The available data from time-controlled fatigue tests are assembled, and it is shown that hold-time data and continuous-cycle data can be presented in a form relating total time to failure and number of cycles to failure. The time-dependent effects are accurately described by equations of the general form,

$$t = k(n)^{-m},$$

where t is total time, n denotes cycles to failure, and k and m are material constants. Assuming that the total damage is separated into cycle- and time-dependent fractions (n/n_f and t/t_r), the failure criterion can be expressed as

$$\alpha(n/n_f)^u + \beta(t/t_r)^v = 1,$$

where α , β , u , and v are interaction constants.

The constants are evaluated for 304 stainless steel, based on the limited amount of available data. It is shown that the interaction is highly nonlinear and that the magnitude of interaction is dependent on the wave shape of the load cycle.

Using these relationships as a guide, a recommended test program is assembled for generating data covering a wide range of hold time and strain ranges at temperatures of 900 to 1300°F. The test program is described in detail, and diagrams of the expected results are presented.

1. INTRODUCTION

Components in steam generating service are subjected to periods of steady operation interrupted by transient temperature and load variation. The transient conditions have been recognized as a possible source of failure due to fatigue damage, and reliable methods have been developed to prevent fatigue failure from cyclic operation. The periods of steady operation received little attention in fatigue investigations because at low temperatures fatigue damage is independent of the time elapsed between periods of rapid strain variation. As interest developed in high-temperature fatigue, it was found that in the creep range fatigue endurance is significantly shorter than would be predicted by low-temperature theories, particularly when hold periods are introduced between strain cycles. The reason for the reduced fatigue life is that, in addition to fatigue damage due to strain cycling, creep damage is also accumulated during hold periods. This is quite logical, since in the creep range failure can occur even under steady loads, as in a creep rupture test. Thus, at creep temperatures endurance is dependent both on the number of strain cycles and on the elapsed time between cycles. In fact, all time-related effects (creep, relaxation, creep recovery, strain-rate sensitivity, etc.) become important factors in designing for cyclic operation.

Since the conditions during hold periods and the magnitude of cyclic strain variation both affect the endurance of a component, the designer must have sufficient knowledge of the deformation and load history of a component to account for time- and cycle-dependent effects. Even a cursory review of the literature shows that time-dependent (creep) analysis techniques are not yet developed to the point where complete account can be taken of the changes in material behavior due to cyclic effects. Many analytical treatments of creep problems have been presented in the literature, but most of these analyses have been developed for "pure" creep problems. Pure creep seldom occurs in a real structure, because in the presence of stress and temperature gradients simultaneous creep and relaxation take place.

Further, load cycles affect both creep and relaxation rates, and even the unloaded periods may be important because of recovery effects.

Similarly, the creep rupture strength, although conventionally regarded as an independent material property, is dependent on load and temperature variations and cannot be separated from fatigue. Therefore, both creep deformation and creep fatigue are strongly influenced by load history. Since both are cumulative in nature, strong interactions occur, particularly when the dominant mechanisms of creep and fatigue are of the same type; for example, void generation in creep can reduce the fatigue life by nucleating fatigue cracks, thus bypassing the crack initiation phase of the fatigue process.

As a rule the interaction is detrimental, but it can be beneficial when strain cycling results in an increase in creep strength by strain aging. In either case, the material characterization and constitutive theories, which form the basis for the analysis method, must reflect this history dependence, and the present constitutive theories can only be regarded as a first approximation to this general problem. Thus it can be said that the analytical description of the time and load history dependent behavior of structural materials has not yet been completely developed, and for the present, analysis methods must still be based on classical creep analysis techniques. Therefore, it is in the design criteria that corrections must be made by introducing design rules and factors that reflect the actual creep-fatigue effects.

In this report a method is developed for the evaluation of stress histories computed on the basis of simplified creep analysis techniques. The method uses cumulative damage relationships that take realistic account of creep-fatigue interaction effects. It will be shown, however, that the interaction effects are complex and that their magnitude is material dependent. Since the creep-fatigue evaluation method described here is based on limited test data for a few materials, similar data for other materials will be needed to extend the validity of this approach for general application.

2. BACKGROUND

2.1 Review of Time Effects on Fatigue Behavior

During the last ten years, an increasing number of investigations have been directed toward determining the effects of strain rate and hold periods on the fatigue properties of engineering materials. More recently, programs have been initiated to investigate in depth the time variables for materials of particular interest. The results of these investigations are reviewed in the following sections, with emphasis on the very recent data for stainless steel.

Strain rate

A test program conducted on 300 series stainless steel has shown that strain rate is an important fatigue variable at temperatures in the creep range.¹ The type of continuous-cycle loading used in these tests is characterized by constant strain rate during loading and unloading (Fig. 1). The test results showed that for particular combinations of strain range and temperature, fatigue life is shorter at slower strain rates than at faster strain rates (Fig. 2). Since the range of strain rates used in the creep range results in insignificant frequency effects at subcreep temperatures, the observed reductions in fatigue life at slower strain rates must be due to the creep damage accumulated during the relatively longer time spent near the peak stress of the cycle. Conversely, at faster strain rates the total test time is shorter, and a point is reached where creep damage is negligible; thus the fatigue endurance becomes independent of time.

Cyclic relaxation

Even more pronounced time effects have been observed in tests involving hold periods. The load cycle most commonly used in these tests incorporates hold periods, during which the maximum strain is maintained constant and the stress is allowed to relax (Fig. 3). Investigations conducted on several alloy steels using this type of cyclic-relaxation loading

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ϵ_e = STRAIN AMPLITUDE
 $\Delta\epsilon$ = STRAIN RANGE
 $\dot{\epsilon}$ = STRAIN RATE
 $\Delta\sigma$ = STRESS RANGE
 σ_h = MAXIMUM TENSILE STRESS

Fig. 1. Continuous-cycle loading.

Fig. 2. Continuous-cycle fatigue data for 304 stainless steel at 1200°F.

Fig. 3. Cyclic-relaxation loading.

have shown that tensile hold periods reduce fatigue life, the reduction increasing with increasing hold periods, particularly at low strain ranges.²⁻⁶ However, the data from these tests, Figs. 4 to 6, show, that the rate of reduction gradually decreases for longer and longer hold times, indicating the possibility that a saturation limit of the relaxation damage is reached at relatively short hold times. This observation is of great practical significance and will be discussed later in this report.

The data described so far have concerned only hold periods introduced in the tensile part of the cycle. In recent tests on 304 stainless steel⁷ the effects of other forms of cyclic relaxation were explored using hold periods at peak tensile strains only, at peak compressive strains only, and at combinations of the two. The data from these tests, shown in Figs. 6 and 7, demonstrate that the relative magnitudes of the hold-time effects

Fig. 4. Cyclic-relaxation data for 2 1/4 Cr-1 Mo steel at 600°F.

Fig. 5. Cyclic-relaxation data for high ductility 1 Cr-1 Mo-0.25 V steel at 1000°F.

Fig. 6. Cyclic-relaxation data for 304 stainless steel at 1200°F.

Fig. 7. Effect of waveform on cyclic-relaxation data for 304 stainless steel.

are strongly dependent on the cycle shape. That is, hold periods at peak tensile strain are the most detrimental, while compressive hold periods have no significant effect for 304 stainless steel. Holds at both tensile and compressive strains reduce the fatigue life only slightly; whereas the compressive hold period apparently has a healing effect.

Similar tests were performed with Udinet 700 material using various types of cyclic-relaxation loading.⁸ Results of these tests showed exactly the reverse effect: compressive hold periods had a detrimental effect on fatigue endurance, whereas tensile hold periods had little effect. Thus the character of the time-dependent damage during cyclic relaxation varies from material to material, and caution should be exercised in the formulation of general criteria without adequate test data.

Although the variation of strain rate has been investigated extensively in continuous-cycle tests, very few tests have been conducted to determine the effects of this variable in tests involving hold periods as well. A limited number of cyclic-relaxation tests have been conducted on 304 stainless steel for investigating the effects of various strain rates with constant hold time. It was found that for hold periods of 30 min the fatigue life determined for slow strain rates is not significantly different from that for fast strain rate.⁹ This suggests that creep damage accumulated during hold periods is much larger than the fatigue damage accumulated during periods of loading and unloading.

Cyclic creep

Another type of load cycle used in creep-fatigue interaction studies is characterized by hold periods in which the maximum tensile stress is maintained constant and the strain is allowed to increase (Fig. 8). Investigations using this form of loading have shown that hold periods of increasing creep strain under constant stress have a detrimental cumulative effect on fatigue life.^{4,10}

Cyclic-creep test data for low-carbon high-manganese steel generated by Wood are shown in Fig. 9, including a yet unpublished data point for 240 hr hold time. It can be seen that longer hold periods produce larger reductions in fatigue endurance without any indication of the saturation effect detected in the data for relaxation-type load cycles.

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$\Delta\epsilon$ = CONTROLLED STRAIN RANGE
 $\pm\epsilon$ = CONTROLLED STRAIN AMPLITUDE
 $\dot{\epsilon}$ = STRAIN RATE
 $\dot{\epsilon}_c$ = CREEP RATE
 t_h = HOLD PERIOD
 $\Delta\sigma$ = STRESS RANGE
 σ_b = MAXIMUM TENSILE STRESS

Fig. 8. Cyclic creep loading.

In view of the evidence discussed above, analysis procedures for cyclic operation in the creep range must take into account the time effects demonstrated by the wave-shape-dependent variation in the magnitudes of the creep-fatigue interaction. Modifications of the low-temperature procedures have been unsuccessful in accounting for the variety of creep behavior. Thus new fatigue analysis methods specialized for high-temperature application are needed.

2.2 Basis for High-Temperature Design Procedures

For temperatures in the subcreep range a convenient basis for design procedures is offered by fatigue data presented in the form of s-n diagrams,

Fig. 9. Cyclic-creep data for low-carbon, high-manganese steel.

relating strain range and cycles to failure. For temperatures in the creep range, an additional variable, time, is needed to describe the fatigue data. An evaluation of the available time-controlled test data discussed in the preceding section shows that hold-time data and continuous-cycle fatigue data can be presented in a new convenient form, relating total time to failure t and number of cycles to failure n . Such plots, termed t - n diagrams, can greatly simplify the design evaluation of creep-fatigue interaction effects, as will be described below.

The t-n diagram

In this method of data presentation time to failure is plotted against cycles to failure. In hold-time tests at creep temperatures, time to failure is measured as an independent test variable. For continuous-cycle tests, time to failure can be calculated from the frequency f or the cyclic period τ as

$$t = \frac{n}{f} \text{ or } t = \tau n, \quad (1)$$

where n is the number of cycles to failure. Therefore data required for the construction of t-n diagrams are readily available.

In the case of continuous-cycle data generated for various strain rates, t-n curves can be plotted for constant strain ranges, as shown in Fig. 10a. Similarly, hold-time data generated using cyclic-relaxation and cyclic-creep loading can be plotted as t-n curves for constant strain range, as shown in Fig. 10b,c. The t-n curves have negative slopes since increasing hold periods and decreasing strain rates severely reduce the number of cycles to failure. The assembled data (Fig. 11) for continuous-cycle and cyclic-relaxation loading show: (1) t-n curves are linear for constant strain ranges, (2) t-n curves form families of straight lines intersecting at common focal points (A,B), and (3) the slopes and focal points of t-n curves are different for continuous-cycle loading and cyclic-relaxation loading. No physical significance is attached to the intersection points A and B, nor is it implied that for extreme values of strain rate and for very long hold periods the data points would still fall on these straight lines. Indications for a saturation limit have already been pointed out, and it is easy to see that a literal interpretation of the convergence of the constant strain lines would lead to the absurd conclusion that beyond a certain strain rate or hold-time value the fatigue endurance and total test time would become independent of the imposed strain range. Such an extension of the empirical observations based on a limited range of the test variables is unwarranted, and no such claim is made. The claim that is made follows from the demonstrated trend of the available data, which show a systematic variation of the total time and

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(a) CONTINUOUS CYCLE

(b) CYCLIC RELAXATION

(c) CYCLIC CREEP

Fig. 10. The t - n diagram.

(a) t - n CURVES FOR CONTINUOUS CYCLE AND CYCLIC RELAXATION DATA (b) t - n CURVES FOR CYCLIC CREEP DATAFig. 11. t - n curves for continuous cycle, cyclic relaxation, and cyclic-creep loading.

cycles to failure as hold time or strain rate is varied. The fact that the variation is linear in logarithmic coordinates makes the curve-fitting procedures simpler and permits the construction of time-dependent fatigue curves from a set of conventional (continuous-cycle) data if a few test points with hold periods and slow strain rates are also generated. Thus the significance of the "fan" of constant strain range lines is not that the intersection points A and B define some actual physical limit of the

fatigue endurance at high temperatures. Rather, at this stage of empiricism, the coordinates of these hypothetical intersection points are to be used to establish the numerical values of the curve-fitting constants, playing a role in predicting fatigue endurances similar to the role played by constants in time-temperature parameters for predicting creep rupture life. (For example, the numerical value of C in the Larson-Miller parameter is the ordinate of the intersection point of the constant-stress lines, corresponding to physically meaningless negative rupture times.)

For cyclic-creep type of loading there is only a limited amount of data available, not enough to establish the empirical correlation with certainty. The very long hold-period data points (24 and 240 hr) support, however, a modified approach based on considering the monotonic creep rupture time as the limiting case of cyclic creep of a single cycle. (Then in the limit the t - n curves do not converge to a single point but intersect the ordinate axis at the creep rupture times that correspond to the stress for the controlled strain amplitudes, as shown in Fig. 11. (The moot point of whether the conventional rupture test should be regarded as one-quarter or one-half "cycle" is of academic interest at this point. The intent is to find the best functional correlation with the available data and not to argue for an interpretation of the constants.) The necessary creep rupture times are available from stress rupture data, and, by analogy with the time-damage accumulation exhibited in cyclic-relaxation and continuous-cycle loading, the linear t - n diagram can be constructed. Such curves predict the effects of long hold periods based on continuous-cycle fatigue data only.

These regularities of the time-dependent effects on the fatigue endurance are demonstrated in the following sections by constructing t - n diagrams using the data reviewed in Section 2.1.

Continuous-cycle fatigue data

Use of the t - n diagram in presenting continuous-cycle fatigue data is illustrated in Fig. 12 for 30% stainless steel and in Fig. 13 for Incoloy 800. Since the t - n curves for this type of test are linear for logarithmic coordinates and intersect at common points, this family of data can be described by equations of the form

Fig. 12. Continuous-cycle fatigue data for 304 stainless steel at 1200°F.

Fig. 13. Continuous-cycle fatigue data for Incoloy 800 at 1200°F.

$$t = k(n)^{-m}, \quad (2)$$

where t is time to failure, n is cycles to failure, and k and m are constants. These relationships can be used in establishing t - n curves for constant values of strain rate. Using Eq. (1) and the expression for the cyclic period of continuous-cycle loading, $\tau = 2\Delta\epsilon/\dot{\epsilon}$, time to failure is eliminated from Eq. (2), giving the direct relationship for n ,

$$n = \left[\frac{k\dot{\epsilon}}{2\Delta\epsilon} \right]^{1/(1+m)}, \quad (3)$$

where $\Delta\epsilon$ is strain range and $\dot{\epsilon}$ is strain rate. Since k , m , and $\Delta\epsilon$ are known for particular t - n curves, cycles to failure can be calculated for any strain rate of interest. By performing similar calculations for several t - n curves, constant strain rate curves can be constructed as shown in Fig. 14.

One application of these diagrams is that of predicting the effects of the very slow strain rates that prevail in normal operating cycles. For example, more than six months of testing would be required for a

Fig. 14. Constant strain-rate curves for 304 stainless steel.

strain range of 0.5% and a strain rate of $4 \times 10^{-7} \text{ sec}^{-1}$. It can be seen in Fig. 14 that the cyclic life for these conditions can readily be determined using the intersection point of the t - n curves and the constant strain rate curves.

Cyclic-relaxation data

The t - n diagrams for cyclic-relaxation data are shown in Fig. 15 for 304 stainless steel and in Figs. 16 and 17 for Incoloy 800. The t - n curves are linear for this type of load cycle also and intersect at a common point, facilitating the construction of curves for constant values of hold period.

A relationship between time to failure and cycles to failure can be established by using Eq. (1) and the expression for the period of cyclic-relaxation loading, $\tau = 2\Delta\epsilon/\dot{\epsilon} + t_h$. Eliminating time to failure from Eq. (2) yields the direct relationship for fatigue cycles, n :

$$n = \left[\frac{k}{2\Delta\epsilon/\dot{\epsilon} + t_h} \right]^{1/(1+m)}, \quad (4)$$

Fig. 15. Cyclic-relaxation data for 304 stainless steel at 1200°F.

Fig. 16. Cyclic-relaxation data for Incoloy 800 at 1200°F.

Fig. 17. Cyclic-relaxation data for Incoloy 800 at 1400°F.

where n is number of cycles, $\Delta\epsilon$ is strain range, $\dot{\epsilon}$ is strain rate, t_h is hold period, and k and m are curve-fitting constants. Equation (4) can be used to calculate cycles to failure for any hold period, since k , m , $\Delta\epsilon$, and $\dot{\epsilon}$ are known for particular t - n curves. Constant hold-period curves can be constructed by performing these calculations for several t - n curves as illustrated in Fig. 18.

The constant hold-period curves can be used in conjunction with extrapolated sections of t - n curves to predict the effects of hold periods outside the range amenable to testing. For example, more than a year would be required for a test at a strain range of 0.5% and hold periods of 48 hr. In contrast, if the focal point A is determined by a few shorter hold-time tests at other strain ranges, the t - n curve can be drawn for any strain range using the conventional continuous-cycle fatigue data. For example, the intersection point of the 0.5% strain-range t - n curve and the 48-hr hold-period line marked in Fig. 18 predicts a cyclic life of about 220 cycles for these conditions without testing. A test to provide this point would last for about 11,000 hr.

Fig. 18. Constant hold-period curves for 304 stainless steel.

Cyclic-creep data

The t - n diagram for cyclic-creep data for a low-carbon, high-manganese steel is shown in Fig. 19. Although the data show considerable scatter, the available long hold-time points tend to fall about a straight line constructed using creep rupture data and continuous-cycle data. As in the case of cyclic-relaxation loading, curves can be constructed for constant values of hold period and used in conjunction with extrapolated sections of t - n curves to predict the effects of prolonged hold periods. To illustrate the differences between cyclic-creep and cyclic-relaxation loading, cyclic-creep curves were constructed for 304 stainless steel. The predicted life for 48 hr hold time at 0.5% strain range is only about five cycles, much shorter than the 220 cycles in cyclic relaxation under identical conditions (Fig. 20).

The correlation of the various kinds of creep-fatigue data reviewed in Figs. 12 to 19 and the predictions based on the t - n diagrams is very satisfactory. In Chapter 3, it will be shown how the regularities of the t - n diagram can be used for developing design criteria for cyclic operation in the creep range. The superposition of t - n curves for the three types of load cycles discussed here (Fig. 21) illustrates the fact that the

Fig. 19. Cyclic-creep data for low-carbon, high-manganese steel.

Fig. 20. Predicted cyclic-creep data and constant hold-period curves for 304 stainless steel.

Fig. 21. Comparison of the effect of wave shape.

severity of creep-fatigue interaction is dependent on the form of the wave shape. This is shown by the decreasing slope of t-n curves from continuous-cycle loading to cyclic-creep loading. Therefore cumulative damage criteria that do not recognize the wave-shape dependence of creep-fatigue interaction are inaccurate and possibly unsafe.

3. DEVELOPMENT OF CREEP-FATIGUE INTERACTION AND CUMULATIVE DAMAGE EQUATIONS

Operating conditions in high-temperature service generally involve complex load and temperature variations. The time history of the resulting stresses and strains is even more complex and may be different for each point within a structural component. Thus it would be impractical to conduct tests that would simulate the many possible forms of structural response. Instead analysis methods should be developed that can treat general loading histories using test data generated for basic wave forms of cyclic loading. This can be done if the tests are designed to isolate the effects of one variable while holding the other variables constant. The creep-fatigue test data reviewed in the preceding sections are of this type, the controlled variable being time (length of hold periods and strain rate) while the strain range is held constant. The test results were assembled in t-n diagrams that demonstrated that the time effects are systematic, thus making the derivation of general relationships for time-dependent fatigue analysis possible.

Generically, the time effects on fatigue are referred to as creep-fatigue interactions, and in this section a method is developed for determining the interaction equations. The method is based on the separation of the total damage into creep and fatigue components. Creep and fatigue damage are expressed in the form of creep life fractions and fatigue cycle fractions, for which functional relationships can be established to take account of creep-fatigue interaction for any hold time.

3.1 Damage Concepts

The basic assumption of the fractional damage approach is that "pure" fatigue damage occurs during rapid strain changes and creep damage occurs

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Fig. 22. Significance of the damage separation assumption in relation to the t - n diagram.

while the strain is constant, as in stress relaxation, or changes slowly, as in creep under constant stress. Figure 22 illustrates the ideal conditions for the separation of damage components in relation to the t - n diagram. It can be seen that for this assumption to be rigorously true, the same number of cycles to failure would have to result regardless of whether time spent in creep is concentrated at the beginning or the end of strain cycling or is distributed uniformly over the entire load history. This, of course, is not expected to hold true for very severe loadings, with a life expectancy of only a few cycles, or for the complete separation of hold time and reversed cycling as shown in Fig. 22. Rather, the damage separation assumption implies that the t - n curves are not significantly affected by the manner in which hold time is distributed.

Directly applicable program-load test data are not available, but the results of a few constant hold-time tests with various strain rates indicate that this assumption is valid for reasonably mixed loading histories.

Given the statistical nature of both fatigue and creep failure laws, the t-n diagram is expected to represent the average failure distribution.

Accepting such separation of damage sources, the fatigue damage fraction is expressed as the cycle ratio

$$d_f = \frac{n}{n_f}, \quad (5)$$

where n is the number of cycles at the strain range $\Delta\epsilon$, and n_f is the number of cycles to failure for the strain range $\Delta\epsilon$. Similarly, the creep damage fraction is expressed as the creep-time ratio,

$$d_c = \frac{t_c}{t_r}, \quad (6)$$

where t_c is the time spent at the stress σ and t_r is the time corresponding to creep rupture at the stress σ .

In cyclic-creep loading the stress is constant and the creep damage fractions can be readily determined. For cyclic-relaxation loading the determination of such fractions is not as straightforward since the stress varies during the hold periods, requiring a choice of a suitable value for t_r .¹¹ In the present work, the simplest solution is adopted, that is, t_r is assumed to correspond to the time to rupture under the maximum tensile stress achieved on loading.

3.2 Creep-Fatigue Interaction Relationships

Creep-fatigue interaction relationships can be expressed in terms of damage fractions and interaction constants, as follows:

$$d = \alpha(d_f)^u + \beta(d_c)^v, \quad (7)$$

where d denotes total damage, d_f denotes fatigue damage, d_c denotes creep damage, and α , β , u , and v are interaction constants. Assuming a failure criterion of $d = 1.0$ and substituting for d_f and d_c , Eq. (7) can be written

$$\alpha \left(\frac{n}{n_r} \right)^u + \beta \left(\frac{t_c}{t_r} \right)^v = 1.0 \quad (8)$$

Possible forms of interaction curves for various values of the constants are shown in Fig. 23. When the exponents u and v are assumed to be unity, eq. (8) becomes the linear interaction equation. Such a linear interaction assumption was proposed by Manson and Halford^{12,13} in the form

$$\frac{n}{n_f} + \kappa \frac{t_c}{t_r} = 1.0.$$

The authors suggested that a numerical value of 0.3 for κ is satisfactory for a number of materials. It will be shown, however, that the interaction constants are highly dependent on the wave shape of the loading, which excludes the possibility of finding universal constants valid for all loading conditions.

WHERE: 1. CREEP DAMAGE ONLY, $\alpha=0$
 2. FATIGUE DAMAGE ONLY, $\beta=0$
 3. NONLINEAR CREEP-FATIGUE INTERACTION $u, v \leq 1$
 4. LINEAR CREEP-FATIGUE INTERACTION $u, v = 1$

Fig. 23. Creep-fatigue interaction curves.

3.3 Interaction Laws Based on the t-n Diagram

The constants in Eq. (8) are determined using the equations for t-n curves. It has been shown that t-n curves can be written in the form

$$t = k(n)^{-m} . \quad (9)$$

For particular curves, the minimum time to failure t_f and the maximum number of cycles to failure n_f are given by continuous-cycle fatigue data (Fig. 10a). These values can be used to eliminate k from Eq. (9) to give

$$\left(\frac{t}{t_f}\right) = \left(\frac{n}{n_f}\right)^{-m} . \quad (10)$$

The time for creep rupture t_r can be introduced on the left-hand side to form creep life fractions:

$$\left(\frac{t_r}{t_f}\right) \left(\frac{t}{t_r}\right) = \left(\frac{n}{n_f}\right)^{-m} . \quad (11)$$

Total time to failure t is the sum of transient time and hold time:

$$t = t_t + t_c . \quad (12)$$

The effect of the transient time is already included in the number of cycles to failure under continuous cycling (n_f). Thus in Eq. (11), t can be expressed as the creep time t_c . Substituting t_c and rearranging Eq. (11) yields

$$\left(\frac{n}{n_f}\right)^{-m} - \left(\frac{t_r}{t_f}\right) \left(\frac{t_c}{t_r}\right) = 0 . \quad (13)$$

To satisfy the condition $n/n_f = 1.0$ when $t_c/t_r = 0$, Eq. (13) can be written with a small error,

$$\left(\frac{n}{n_f}\right)^{-m} - A \left(\frac{t_c}{t_r}\right) = 1.0 . \quad (14)$$

Here, n is the number of cycles to failure in a hold-time test for strain range $\Delta\epsilon$ and hold period t_h , n_f is the number of cycles to failure in continuous-cycle fatigue for strain range $\Delta\epsilon$, $t_c = n \times t_h$ is the time spent in creep, t_r is the creep rupture time corresponding to maximum tensile stress σ , and m and A are interaction constants.

The constants for interaction relationships of this type are given for 304 stainless steel, Incoloy 800, and low-carbon, high-manganese steel in Table 1. It is interesting to note that the exponent m is primarily dependent on the strain range, while the constant A is mainly temperature dependent.

The interaction curves computed using these equations are shown with linear scales in Figs. 24 and 25 and with logarithmic scales in Figs. 26 and 27 for 304 stainless steel and low-carbon low-manganese steel. The interaction curves fit the experimental data as well as they fit the corresponding t - n curves. This is hardly surprising since the interaction diagram is a normalized form of the t - n diagram. The normalized interaction diagram, however, has the advantage of showing clearly the magnitude of the creep effect. Thus it allows comparison between actual material behavior and the linear interaction assumption that has been generally

Table 1. Interaction constants for the materials tested

Load cycle	Temperature (°F)	Strain range	Interaction constant	
			m	A
<u>304 Stainless Steel</u>				
Cyclic relaxation	1200	2.0	2.84	63.3
		1.0	2.23	51.1
		0.5	1.71	38.1
<u>Incoloy 800</u>				
Cyclic relaxation	1400	2.0	3.88	56.6
		1.0	3.2	53.9
		0.55	2.67	43.6
<u>Low-Carbon High-Manganese Steel</u>				
Cyclic creep	660	1.3	1.59	2.87 × 10 ⁴

Fig. 24. Interaction curve for 304 stainless steel.

Fig. 25. Interaction curve for low-carbon, high-manganese steel.

Fig. 26. Interaction curve for 304 stainless steel.

accepted in the past. It can be seen that for cyclic-relaxation loading, the linear interaction relationship can be overconservative for long hold periods and unconservative for hold times less than about 30 min (Fig. 24). For cyclic-creep loading, the linear interaction assumption is unconservative throughout the entire range.

The major advantage of this approach is that it gives interaction relationships that fit available data and that can be used to extrapolate interaction effects outside the range amenable to testing. The disadvantage is the relative complexity of the relationship, which does not allow

Fig. 27. Interaction curve for low-carbon, high-manganese steel.

the direct evaluation of the cumulative damage for an arbitrary load history. It will be shown, however, that this difficulty can be overcome by the use of equivalent damage factors.

3.4 Development of Cumulative Damage Relationships

To be useful in design applications, the creep-fatigue analysis method has to account for differences between the complex load history and the simple wave shapes of loading used in laboratory tests. For example, in cyclic-relaxation tests the maximum strain is held constant during the hold period, as shown in Figs. 3 and 22. In actual service the hold periods correspond to steady-state operation, during which the loads (and strains) are considerably reduced from the maximum values reached in the transient periods.

Typical service cycles with intermediate strain and load conditions during the hold periods are shown in Fig. 28. Clearly, the creep damage

Fig. 28. Thermal and mechanical load changes, giving approximate cyclic relaxation and cyclic-creep loading.

is less in such a cycle than in one where the maximum load remains constant during the hold period. In many service applications where the transient periods are short in comparison with the length of steady operations, the creep damage would be grossly overestimated if the method of evaluation depended solely on the peak strains reached during the transients.¹⁴ For subcreep temperatures, where the fatigue damage depends largely on the strain amplitude, cumulative damage rules were developed to evaluate loading sequences of varying strain amplitudes. For creep temperatures, more comprehensive methods are needed to account for the variation in the time-dependent parameters (hold time, strain rate) as well as for the strain amplitude variations.

3.5 Cumulative Damage Based on Cycle Fractions

If the service loading corresponds closely to that used in the fatigue test determining the t-n curves, a modified form of Miner's law can be used in conjunction with the t-n diagram for design analysis. Continuous-cycle loading (no hold periods), with strain range and strain rate varying between cycles, is in this category for which the linear cumulative damage relationship can be used:

$$\sum_{i=1}^I \left(\frac{n_s}{n} \right)_i \leq K, \quad (15)$$

where n_s is the number of service cycles at particular combinations of strain range and strain rate, n is the number of cycles to failure from the t-n diagram, and K is a factor accounting for sequence effects.

Since the t-n diagrams include the interaction effects, this cumulative damage rule can also be used for the analysis of cyclic-creep and cyclic-relaxation type loading with varying hold periods and strain ranges if the service loading cycles can be idealized into combinations of simple load shapes. For these types of loading the values of cycles to failure, determined from t-n diagrams for the particular hold period and strain range combination of interest, already include the time effects, and the sum of the cycle ratios expresses the total damage.

3.6 Cumulative Damage Based on Creep and Fatigue Fractions

The alternative approach is to use the interaction relationships in establishing failure criteria for cumulative damage laws. This is achieved by using equivalent damage factors, which permit the inclusion of damage from creep periods not associated with strain cycles (e.g., a number of relaxation cycles followed by a long period of creep strain accumulation). The creep- and fatigue-damage fractions for a particular strain range and hold period are combined into an equivalent damage factor D ,

$$\frac{n}{n_f} + \frac{t_c}{t_r} = D. \quad (16)$$

The equivalent damage factor is expressed as a function of the interaction equation by eliminating the time fraction between Eqs. (14) and (16), giving

$$D = \frac{n}{n_f} + \frac{1}{A} \left[\left(\frac{n}{n_f} \right)^m - 1 \right]. \quad (17)$$

Equation (17) is used for constructing curves relating D to n/n_f for particular values of strain range. Curves of this type are shown in Fig. 29 for cyclic-relaxation loading and in Fig. 30 for cyclic-creep loading.

Equation (17) can be expressed in the form of a failure criterion by observing that in the simplest loading case (involving a single value of strain range) failure occurs when the number of service cycles is equal to the number of cycles to failure in hold-time tests; that is, when $n_s = n$,

$$\frac{1}{D} \left(\frac{n_s}{n_f} + \frac{t_c}{t_r} \right) = 1.0. \quad (18)$$

Using Eq. (18), the failure criterion is brought in coincidence with the actual hold-time data, functionally expressed by the interaction equation, Eq. (14). Partial damage is computed by Eq. (18) when $n_s < n$, and the summation of the partial damages yields the cumulative damage relationship:

$$\sum \frac{1}{D_i} \left(\frac{n_s}{n_f} + \frac{t_c}{t_{r,i}} \right) \leq K. \quad (19)$$

This form of the damage equation for hold-time effects permits the inclusion of creep damage due to periods of strain accumulation, not represented by the t-n curves. The complete cumulative damage equation then is

$$\sum \frac{1}{D_i} \left(\frac{n_s}{n_f} + \frac{t_c}{t_{r,i}} \right) + \sum \left(\frac{t_a}{t_{r,j}} \right) \leq K, \quad (20)$$

where

n_s = number of service cycles at strain range $\Delta\epsilon$,

n_f = cycles to failure for $\Delta\epsilon$ (from s-n curves or t-n curves for zero hold time),

Fig. 29. Equivalent interaction constants for 304 stainless steel at 1200°F for cyclic relaxation.

$t_c = n_s \times t_h$ = total hold time at stress σ_1 (initial stress in case of relaxation),

t_a = time spent at stress σ_k ,

t_r = rupture time for stress σ_1 or σ_k ,

D_1 = equivalent damage factor,

K = factor accounting for sequence effects.

In developing Eq. (19) it was assumed that the interaction constants m and A and the equivalent damage factor D_1 are independent of strain rate. The basis for this assumption is the very limited data discussed earlier, suggesting that strain-rate effects are negligible in comparison with the hold-time effects, provided hold periods are longer than about 30 min. Clearly, the possibility exists that the effects of strain rates slower than those investigated may not be entirely masked by hold-period effects. Until additional test data become available, strain-rate effects may not be considered in using Eq. (19) for design analysis, and the values of n_f should be determined using s - n curves generated at strain rates rapid enough for time effects to be negligible.

Fig. 30. Equivalent interaction constants for 304 stainless steel at 1200°F for cyclic creep.

For continuous-cycle loading, Eq. (19) reduces to Miner's law, since $t_c/t_r = 0$ and $D = 1.0$. However, for this type of loading, account can be made for strain-rate effects by substituting n for n_f and using the t - n diagrams in determining the time-dependent n values, as was described in connection with Eq. (15).

3.7 Sequence Factors

It has long been recognized that Miner's law can be seriously in error for certain types of loading sequences. For example, it has been shown that a few cycles of loading involving large strain amplitudes at the beginning of a load history (a "high-low" sequence) can have a marked effect on subsequent fatigue behavior. For this type of loading, Miner's law often proves unconservative since the failure occurs at cumulative damage values much less than unity. At temperatures in the creep range, the situation is further complicated by the effects of creep on material properties. It has been shown¹⁵ that long hold periods at the beginning of programmed load tests can be either beneficial or detrimental, depending upon the material being tested. It seems improbable, therefore, that a single cumulative damage relationship will be applicable for all high-temperature material and all possible types of loading. For this reason, material-dependent sequence factors were introduced in Eqs. (15) and (19).

Two approaches may be adopted for establishing these factors. A series of exploratory tests could be conducted to determine the type of programmed loading that has the most detrimental effect on fatigue endurance of the particular material. Then, tests using this type of loading could be conducted to determine the minimum value for K to be used in the cumulative damage equation. Alternatively, service loading types could be categorized into typical sequences and minimum K values assigned to each category. For these cases Eq. (19) can be modified by normalizing the equivalent damage factor:

$$\sum_{i=1}^i \frac{1}{D_i K_i} \left(\frac{n_s}{n_f} + \frac{t_c}{t_r} \right) \leq 1.0 . \quad (21)$$

Clearly, the latter approach requires an extensive series of programmed loading tests. Since strain-rate-controlled hold-time tests with programmed loading are not yet available, numerical values have not been established for the sequence factors. A limited number of programmed loading test sequences of the type shown in Fig. 22 should be planned following the completion of the basic test program described in Chapter 4.

3.8 Summary

Use of the t-n diagram

The conventional method for presenting fatigue data, the s-n diagram, cannot be used to isolate the effects of time on fatigue behavior. Therefore, a new method was developed to replace the s-n diagram for creep-fatigue analysis. By plotting time to failure against number of cycles, t-n curves can be constructed for constant strain ranges using data generated for various strain rates and hold periods. Since fatigue damage is constant for a constant strain range, the t-n curves show directly the effects of time on fatigue endurance. Comparisons of Figs. 6 and 9 with Figs. 18 and 20 clearly show the advantage of this method in presenting high-temperature fatigue data.

Service in the creep temperature range involves combinations of strain range, strain rate, and hold periods. Since it is impractical to generate data for all possible combinations of the variables, methods are required for predicting creep-fatigue life based on limited test data. The regularities of the t-n diagram can be used to limit the number of tests to those necessary for the definition of intersection points of the t-n curves.

It is also impractical to generate data at conditions typical of service, because of the excessive time required for tests involving slow strain rates or long hold periods. It has been shown that extrapolations of t-n curves in conjunction with constant hold periods or constant-strain-rate curves can be used for predicting cyclic life. A minimum requirement for such an approach is that it should give conservative predictions. The data shown in Figs. 12 to 16 indicate that this requirement is satisfied.

Modified form of Miner's law

As noted in Section 3.5, a modified form of Miner's law can be used in conjunction with t-n diagrams for computing a cumulative fatigue damage. In this approach a complex service loading is reduced for analysis purposes to a sequence of equivalent load cycles similar to the load types used in the laboratory fatigue tests. Possible errors resulting from such simplification are eliminated by using sequence factors to modify the failure criteria, as described in Section 3.7. The modified form of Miner's law has several advantages. Cycle ratios determined from the t-n diagrams are used to measure creep and fatigue damage without making the assumption of damage separation. The resulting cumulative damage law is easy to use in analyzing loading cycles with and without hold periods. It is also important that the analytical complications due to cyclic hardening or softening are avoided (these will be discussed in the following section). The disadvantage is a certain lack of generality, which makes some loading sequences difficult to evaluate.

Equivalent damage factor methods

It was shown in Section 3.6 that more general cumulative damage relationships can be developed based on separate cycle and time fractions. This approach depends on the assumption that damage occurring during a complex loading can be separated into creep and fatigue components. The validity of this assumption can be tested by determining the effects of hold-time distribution on t-n curves. Three of the many possible forms of loading which can be used for this purpose were shown in Fig. 22. If differences between t-n curves for the various types of loading are insignificant, then the total damage is independent of the distribution of hold time and average hold-period values can be used in the analysis. Inaccuracies due to this assumption are compensated for by introducing the sequence factors. The procedures for determining the numerical values of sequence factors are outlined in Section 3.7.

The equivalent damage factor approach suffers some limitations due to the use of monotonic creep rupture data for determining creep damage

fractions. The choice of a reference creep rupture time is not straightforward if the stress varies during hold periods, as in cyclic-relaxation types of loading. For such cases two approaches can be adapted to calculate time fractions: (1) the maximum tensile stress achieved upon loading can be assumed to apply for the entire hold period, or, (2) the hold period can be divided into time increments during which the stress is assumed constant and the creep fractions for each increment summed for the entire hold period. Although the second approach might appear to have more physical significance, this is only true for the first load cycle. Engineering materials exhibit either cyclic hardening or cyclic softening, and after the first few cycles the monotonic stress-strain relationships no longer apply, and their use results in creep fractions with no physical significance. On the other hand, the effects of cyclic variations in the material properties cannot be completely accounted for just by substituting cyclic stress-strain curves in place of the conventional stress-strain relationships for the virgin material. This is because cycling in the plastic range also affects the creep rupture properties. Without rupture data for cyclically hardened material it is questionable whether or not meaningful creep fractions for a material that has been cycled extensively can be developed.

One solution to the problem would be to generate creep rupture data for fully hardened or softened material. However, such attempts to lend physical significance to creep fractions introduce complications in the design analysis as well and would require extensive creep rupture testing. At the present, the best solution is to use the nominal creep rupture values but insure that the same simplifying assumptions are made about the material behavior in both the criteria and in the design analysis. Such an approach is similar to the low-temperature fatigue analysis procedures in which the elastic modulus is used to compute fictitious elastic stresses in place of plastic strains as input to the fatigue evaluation. Although derived from strain cycling data, the design fatigue curves are given in terms of stresses that include the elastic modulus as a multiplier. Using the same value of modulus in the analysis and in the fatigue curves cancels out the multiplier, which was introduced only for computational convenience. Much work is needed in developing simplified methods for

creep-fatigue analysis and defining corresponding design criteria. The approach described in this chapter is hopefully a step in that direction.

4. DESCRIPTION OF THE FATIGUE TEST PROGRAM FOR TYPE 304 STAINLESS STEEL

4.1 Introduction

On the basis of the review of the existing data, a test program has been developed for type 304 stainless steel. The t-n diagram was used to guide the program planning, with the aim of minimizing the total time and the cost of the program. The t-n diagram served a similar purpose to the creep-stress-temperature parameter (e.g., Larson-Miller parameter) in developing an economic and fast creep test matrix. In the creep range the fatigue data must describe three types of time effects: (1) strain-rate sensitivity, (2) hold times in cyclic relaxation, and (3) hold times in cyclic creep.

Since these three types of time effects accumulate creep damage at different rates, a creep-fatigue test program would require essentially three times the number of test specimens needed for establishing fatigue design curves in the subcreep range (below approximately 800°F). The t-n diagrams reduce the test program to a set of spot checks to determine the shape of the diagrams at strategic points. Such spot checks confirm the established trends and supply the minimum amount of data required for the numerical evaluation of the interaction constants in the creep-fatigue equations.

The t-n diagrams for the existing data revealed saturation limits for hold time in cyclic-relaxation tests and for strain rate in continuous-cycle-type tests. The saturation limits are important for design analysis because relaxation hold periods in excess of the saturation limits do not contribute to further damage. The number of cycles corresponding to the saturation limits forms a fatigue curve that includes the maximum relaxation hold-time effect. Therefore the saturation limits represent time endurance limits that can be used for design in a manner analogous to the use of fatigue endurance limits at subcreep temperatures. Since at the saturation limit the cyclic life is significantly reduced, it is important

to define the asymptotic lines accurately and avoid overly conservative fatigue curves.

In a manner analogous to that for hold-time effects, strain rates above a critical value do not affect the number of cycles to failure. This fact can be used to establish allowables for low strain ranges, which if tested by low-frequency, feedback-controlled equipment would require excessive testing time.

4.2 Test Equipment Requirements

The available cyclic-relaxation data for 304 stainless steel were generated at General Electric-Nuclear Systems Programs (GE-NSP) on strain-rate feedback-controlled hydraulic test equipment. Since this test program relies heavily on the existing data and specifies spot checks for the established trends instead of relying on completely new data sets, the same type of test equipment is required:

Loading	Push-pull
Control	Diametral strain feedback control and hold with constant strain or constant stress
Specimen	Hourglass type
Temperature	$\pm 5^{\circ}\text{F}$

The diametral strain feedback control system includes a mechanical extensometer that permits only relatively low-frequency operation. To generate data for the range above 10^5 cycles, such low frequencies lead to very long test durations. Therefore, a method was devised for the use of conventional high-frequency load control testing equipment with the following description:

Loading	Push-pull
Control	Displacement and stress control
Specimen	Hourglass type
Temperature control	$\pm 5^{\circ}\text{F}$
Cyclic frequency	2000 to 6000 cpm

Very high frequency (e.g., resonant ultrasonic) equipment could also be used if the temperature control compensates for the internal heating of the specimen.

4.3 Data Required

For each data point specified in this test program the following information should be reported:

1. strain range;
2. elastic and plastic strain components (axial and diametral where applicable);
3. frequency;
4. strain rate;
5. time to failure, cycles to failure, and cycles for 5% drop in load;
6. hysteresis loops and wave shape during initial shakedown period.

4.4 Test Matrix

The test program is presented in Tables 2-10 and in the corresponding figures (31-45). The elevated temperature properties used in determining the test points are summarized in Table 11. The supporting creep rate and creep rupture data are shown on Figs. 46-59, and the optimized creep rupture parameter is shown on Fig. 48. Short time stress-strain curves for two strain rates are shown on Fig. 46. A description of the test matrix, indicating the number of specimens and expected duration of the tests, is given in Table 12.

The specimens noted in the tables as optional are expected to fail in excess of 3000 hr. Since these tests would delay the whole program, the optional tests should be scheduled depending on the availability of test machines. In general, the test results should be monitored continuously and the results compared with the predictions as shown in Figs. 31 to 45, which have been included for this purpose. The interaction constants in the creep-fatigue damage equations are based on the slope of the constant strain range lines and on the coordinates of focal points A and B. If the early results indicate large variations from the expected endurance, the corresponding t-n diagrams for the same group should be readjusted and the test matrix modified to optimize the location of the remaining test points. The monitoring is particularly advisable for test groups 2, 4, and 11 to 14, for which prior data points were not available.

Table 2. Hold-time fatigue tests for 304 stainless steel at 900°F

Load type: cyclic relaxation

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
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Figure 31

1.1	0.6	4×10^{-3}	0	23,000	17
1.2	0.6	4×10^{-4}	0	1,400	96
1.3	1.0	4×10^{-3}	0	4,000	6
1.4	2.0	4×10^{-3}	0	1,200	3
1.5	2.0	4×10^{-5}	0	680	170
1.6	4.0	4×10^{-3}	0	400	2
1.7	4.0	4×10^{-5}	0	270	89

Figure 32

2.1	0.6	4×10^{-4}	10	5,200	910
2.2	1.0	4×10^{-5}	45	970	1020
2.3	2.0	4×10^{-5}	45	400	430
2.4	2.0	4×10^{-5}	600	250	2600
2.5	4.0	4×10^{-5}	45	180	200
2.6	4.0	4×10^{-5}	600	120	1300

Table 3. Hold-time fatigue tests for 304 stainless steel at 1000°F

Load type: cyclic relaxation

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 33</u>					
3.1	0.5	4×10^{-4}	0	12,000	85
3.2	1.0	4×10^{-5}	0	1,300	180
3.3	2.0	4×10^{-3}	0	930	2.5
3.4	2.0	4×10^{-4}	0	620	16
3.5	2.0	4×10^{-5}	0	450	90
3.6	4.0	4×10^{-3}	0	250	1.5
3.7	4.0	4×10^{-4}	0	180	9
3.8	4.0	4×10^{-4}	0	140	48
<u>Figure 34</u>					
4.1	0.5	4×10^{-4}	10	3,500	630
4.2	1.0	4×10^{-5}	60	600	640
4.3	2.0	4×10^{-5}	55	240	260
4.4	2.0	4×10^{-5}	300	170	660
4.5	4.0	4×10^{-4}	10	140	22
4.6	4.0	4×10^{-5}	60	100	104
4.7	4.0	4×10^{-5}	300	80	310

Table 4. Hold-time fatigue tests^a for 304 stainless steel at 1100°F

Load type: cyclic relaxation

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 35</u>					
5.1	0.6	4×10^{-3}	0	12,000	10.0
5.2	1.0	4×10^{-3}	0	2,600	3.5
5.3	2.0	4×10^{-3}	0	780	2.0
5.4	4.0	4×10^{-3}	0	210	1.0
5.5	0.6	4×10^{-3}	15	2,200	420
5.6	0.6	4×10^{-3}	180	1,050	3100 ^a
5.7	1.0	4×10^{-3}	15	620	110
5.8	1.0	4×10^{-3}	180	350	980
5.9	2.0	4×10^{-3}	15	260	50
5.10	2.0	4×10^{-3}	180	150	480
5.11	4.0	4×10^{-3}	15	110	24
5.12	4.0	4×10^{-3}	300	60	620
<u>Figure 36</u>					
6.1	0.6	4×10^{-5}	0	3,400	500
6.2	0.6	4×10^{-5}	60	1,100	1400
6.3	0.6	4×10^{-5}	600	500	6200 ^a
6.4	1.0	4×10^{-5}	0	1,000	310
6.5	1.0	4×10^{-5}	60	420	640
6.6	1.0	4×10^{-5}	600	220	3000 ^a
6.7	2.0	4×10^{-5}	0	370	160
6.8	2.0	4×10^{-5}	60	180	310
6.9	2.0	4×10^{-5}	600	100	1200 ^a

^aOptional tests.

Table 5. Hold-time fatigue tests for 304 stainless steel at 1200°F

Load type: cyclic relaxation

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 37</u>					
7.1	0.30	4×10^{-5}	0	11,000	750
7.2	0.30	4×10^{-3}	10	6,000	1000
7.3	0.30	4×10^{-3}	30	3,200	2400
7.4	0.30	4×10^{-3}	180	2,400	8000 ^a
7.5	0.50	4×10^{-3}	180	880	2700
7.6	1.00	4×10^{-3}	180	260	900
7.7	4.00	4×10^{-3}	10	90	15
7.8	4.00	4×10^{-3}	30	70	50
7.9	4.00	4×10^{-3}	180	55	120
7.10	4.00	4×10^{-5}	0	150	80
<u>Figure 38</u>					
8.1	0.30	4×10^{-5}	170	1,600	5800 ^a
8.2	0.50	4×10^{-5}	175	600	1900
8.3	1.00	4×10^{-5}	180	200	750
8.4	2.00	4×10^{-5}	185	110	300
8.5	2.00	4×10^{-5}	600	100	1100
8.6	4.00	4×10^{-5}	170	55	120
8.7	4.00	4×10^{-5}	680	55	780

^aOptional tests.

Table 6. Hold-time fatigue tests for 304 stainless steel at 1300°F

Load type: cyclic relaxation

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 39</u>					
9.1	0.3	4×10^{-3}	150	1800	4400 ^a
9.2	0.5	4×10^{-3}	10	1250	210
9.3	0.5	4×10^{-3}	60	720	740
9.4	0.5	4×10^{-3}	150	520	1320
9.5	1.0	4×10^{-3}	150	190	450
9.6	2.0	4×10^{-3}	600	80	810
9.7	2.0	4×10^{-3}	150	90	220
9.8	4.0	4×10^{-3}	600	40	400
9.9	4.0	4×10^{-3}	150	46	110
9.10	4.0	4×10^{-3}	60	50	51
9.11	4.0	4×10^{-3}	10	66	1.2
<u>Figure 40</u>					
10.1	0.3	4×10^{-5}	0	9500	700
10.2	0.3	4×10^{-5}	140	1600	4000 ^a
10.3	0.5	4×10^{-5}	0	2300	210
10.4	0.5	4×10^{-5}	130	450	1100
10.5	1.0	4×10^{-5}	0	600	100
10.6	1.0	4×10^{-5}	150	170	400
10.7	2.0	4×10^{-5}	0	240	65
10.8	2.0	4×10^{-5}	130	90	200
10.9	2.0	4×10^{-5}	450	80	600
10.10	4.0	4×10^{-5}	0	100	70
10.11	4.0	4×10^{-5}	120	50	100
10.12	4.0	4×10^{-5}	630	40	480

^aOptional tests.

Table 7. Hold-time fatigue tests for 304 stainless steel at 1000°F

Load type: cyclic creep

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 41</u>					
11.1	0.5	4×10^{-3}	60	490	500
11.2	1.0	4×10^{-3}	120	120	240
11.3	1.0	4×10^{-3}	10	400	64
11.4	2.0	4×10^{-3}	120	50	100
11.5	2.0	4×10^{-3}	600	28	280
<u>Figure 42</u>					
12.1	0.5	4×10^{-4}	10	2200	400
12.2	1.0	4×10^{-4}	10	600	100
12.3	2.0	4×10^{-4}	120	80	160

Table 8. Hold-time fatigue tests for 304 stainless steel at 1200°F

Load type: cyclic creep

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
<u>Figure 43</u>					
13.1	0.4	4×10^{-3}	60	200	202
13.2	0.4	4×10^{-3}	600	35	350
13.3	0.5	4×10^{-3}	30	120	61
13.4	0.5	4×10^{-3}	180	31	92
13.5	1.0	4×10^{-3}	60	30	30
13.6	1.0	4×10^{-3}	600	7	70
13.7	2.0	4×10^{-3}	30	26	14
13.8	2.0	4×10^{-3}	180	10	30
<u>Figure 44</u>					
14.1	0.5	4×10^{-4}	30	200	100
14.2	1.0	4×10^{-4}	60	50	51
14.3	1.0	4×10^{-4}	600	11	660
14.4	2.0	4×10^{-4}	30	55	27
14.5	2.0	4×10^{-4}	180	14	42

Table 9. High-frequency fatigue tests for 304 stainless steel at 1200°F

Load type: stress controlled, continuous cycle

Group No.	Strain range (%)	Initial stress amplitude (psi)	Stabilized stress amplitude, approx. (psi)	Frequency (cpm)	Expected number of cycles	Expected failure time (hr)
15.1	0.25	17,200	24,600	2400	1.5×10^7	104.0 ^a
15.2	0.25	17,200	24,800	6000	1.8×10^7	51.0
15.3	0.30	18,000	25,700	2400	1.4×10^6	10.0
15.4	0.30	18,000	25,900	6000	1.8×10^6	5.0
15.5	0.40	18,500	27,000	2400	1.2×10^5	0.8
15.6	0.40	18,500	27,500	6000	1.4×10^5	0.4
15.7	0.50	19,300	28,200	2400	4.2×10^4	0.2

^aOptional test.

Table 10. Low-frequency fatigue test for 304 stainless steel at 1200°F

Control data for group 15
Load type: strain controlled, continuous cycle

Group No.	Strain range (%)	Strain rate (in. in. ⁻¹ sec ⁻¹)	Hold time (min)	Expected number of cycles	Expected failure time (hr)
16.1	0.4	4×10^{-1}	0	1.1×10^5	0.60
16.2	0.4	4×10^{-2}	0	1.0×10^5	5.50
16.3	0.5	4×10^{-1}	0	3.0×10^4	0.20
16.4	0.5	4×10^{-2}	0	2.8×10^4	1.30

Table 11. Elevated temperature properties of 304 stainless steel

Short time stress-strain properties (Fig. 46).

Stress-rupture properties (Fig. 48).

A time-temperature-stress-strain parameter was derived using an optimization procedure. The best fit was found by the linear (Manson-Haferd) parameter modified in the form:

$$P = \log t_r + AT,$$

where, t_r = rupture time (hr), T = temperature (°F), and A = constant, 1.50×10^{-2}

Temperature (°F)	Yield strength (psi)	Strain (%)	Stress at strain rate -		Rupture time (hr)
			4×10^{-3}	4×10^{-5}	
1000	23,000	0.4	23,000	23,500	210,000
		0.5	23,600	24,200	112,000
		1.0	26,000	26,600	71,000
		2.0	28,500	29,200	24,000
1200	21,000	0.25	19,200	19,600	1,200
		0.30	19,700	19,900	960
		0.4	20,100	20,200	890
		0.5	22,200	22,300	234
		1.0	23,500	23,500	220
		2.0	25,000	25,300	213
1300	19,000	0.4	19,000	18,500	75
		0.5	19,700	19,200	60
		1.0	22,300	22,000	25
		2.0	24,500	24,300	18

Table 12. Description of test matrix

Temperature (°F)	Test group	Cycle type	Number of specimens		Test time (hr)	
			Minimum	Optional	Minimum	Optional
900	1, 2	Relaxation	13		7,800	
1000	3, 4	Relaxation	15		2,600	
1100	5, 6	Relaxation	17	4	6,000	13,500
1200	7, 8	Relaxation	15	2	12,700	13,800
1300	9, 10	Relaxation	21	2	3,900	8,400
1000	11, 12	Cyclic creep	8		1,800	
1200	13, 14	Cyclic creep	13		1,700	
1200	15, 16	Continuous cycle	10	1	73	104
		Totals	112	9	36,573	35,804

Fig. 31. Fatigue test for 304 stainless steel at 900°F.

Fig. 32. Hold-time fatigue test for 304 stainless steel at 900°F.

Fig. 33. Fatigue test for 304 stainless steel at 1000°F.

Fig. 34. Hold-time fatigue test for 304 stainless steel at 1000°F.

Fig. 35. Hold-time fatigue test for 304 stainless steel at 1100°F.

Fig. 36. Hold-time fatigue test for 304 stainless steel at 1100°F.

Fig. 37. Hold-time fatigue test for 304 stainless steel at 1200°F.

Fig. 38. Hold-time fatigue test for 304 stainless steel at 1200°F.

Fig. 39. Hold-time fatigue test for 304 stainless steel at 1300°F.

Fig. 40. Hold-time fatigue test for 304 stainless steel at 1300°F.

Fig. 41. Hold-time fatigue test for 304 stainless steel at 1000°F.

Fig. 43. Hold-time fatigue test for 304 stainless steel at 1200°F.

Fig. 44. Hold-time fatigue test for 304 stainless steel at 1200°F.

Fig. 45. High-frequency load-controlled fatigue: tests at 1200°F.

Fig. 46. True stress-strain curves for 304 stainless steel.

Fig. 47. Stress-strain curve for 304 stainless steel at 1200°F.

Fig. 48. Linear parameter master curve for rupture, 304 stainless steel.

Fig. 49. Isochronous rupture data for 304 stainless steel.

Fig. 50. Isobar lines for rupture, 304 stainless steel.

Fig. 51. Rupture time vs reciprocal of the absolute temperature for 304 stainless steel.

Fig. 52. Rupture time vs stress for 304 stainless steel at 1000°F.

Fig. 53. Rupture time vs stress for 304 stainless steel at 1050°F.

Fig. 54. Rupture time vs stress for 304 stainless steel at 1100°F.

Fig. 55. Rupture time vs stress for 304 stainless steel at 1200°F.

Fig. 56. Rupture time vs stress for 304 stainless steel at 1300°F.

Fig. 57. Creep rate vs stress for 304 stainless steel at 1050°F.

Fig. 58. Creep rate vs strain for 304 stainless steel at 1100°F.

Fig. 59. Creep rate vs strain for 304 stainless steel at 1200°F.

4.5 Test Groups

In Chapter 2 of this report, a review of the existing creep-fatigue data for 304 stainless steel (and similar materials), it was shown that the damage accumulation in the creep range is not only time dependent but is strongly influenced by the wave shape of the loading cycle. The variation of time dependence due to the various wave shapes is best illustrated by a composite t-n diagram showing hold-time test results and continuous-cycle data. The test data are fitted by three families of curves that correspond to the three basic load cycle types: cyclic creep, cyclic relaxation, and continuous cycle. Since the slope of the failure curve is proportional to the severity of the creep-fatigue interaction, the test program is designed to define these families of curves with the minimum number of test points and shortest test duration. The location of focal points A and B and the monotonic creep rupture data will define the cycles to failure for any combination of strain rate and hold-time wave shapes. Ideally, the focal points could be determined using the continuous-cycle data as the base line and six additional test points, two for each basic load cycle type at each temperature. The inevitable scatter, even in conventional fatigue test results, requires multiple test points, increasing the test matrix from this ideal minimum size. It is customary in fatigue testing to generate duplicate or triplicate data points for each setting and to determine the best-fit failure curve by statistical methods. However, this is not the approach followed in this program. Instead of using identical test conditions to provide the scatter band width, single tests covering the range of the strain amplitudes that correspond to the anticipated service conditions in LMFBR type reactor systems are specified. This approach was adopted because an additional variable, the time endurance or saturation limit, also needs to be evaluated. The duplication of test points in this case would lead to an intolerably large and time-consuming test program.

The tests are arranged in groups, within which only one time variable is explored at each temperature. At the lower temperatures (900 to 1000°F) strain-rate-controlled continuous-cycle data were not available; thus groups 1 and 3 are included to provide the base line for the t-n diagrams.

At the higher temperatures (1200 to 1300°F) a few additional strain rate data points are needed, which were included in groups 7 and 10. The available data shown in Figs. 60 and 61 for the temperature range of 800 to 1500°F, indicate that the values for the mid-temperature of 1100°F can be interpolated. The test points specified for 1100°F are placed in such an arrangement that the interpolation technique can be verified with the fewest number of specimens.

Extensive cyclic-relaxation data were available at 1200°F for a strain rate of $4 \times 10^{-3} \text{ sec}^{-1}$. The tests in groups 7 and 8 extend the long hold-time data to a slower strain rate region. These tests are designed to compare the validity of the t-n diagram for any combination of hold time and strain rate. Groups 9 and 10 specify similar tests at 1300°F.

It can be seen in Figs. 37 to 40 that at low strain amplitudes (less than 0.5%) even relatively short hold times (2 to 3 hr) lead to expected failure times in the thousands of hours. Since such low strain amplitudes are important in design, tests for 0.3% strain range have been included in these test groups. The corresponding failure times range from 4000 to 8000 hr. These tests are optional, as noted previously.

Fig. 60. Isothermal fatigue life of 304 stainless steel.

Fig. 61. Temperature dependence on fatigue life of 304 stainless steel.

Cyclic-creep data are virtually nonexistent for 304 stainless steel. The monotonic properties of the material (Table 11) were used to anchor t-n curves for cyclic-creep conditions. The tests in groups 11 to 14 were designed to generate such data for 1000 and 1200°F, representing low and high creep activation rates. These tests require constant attention under manual control or special automatic equipment with the capacity of switching from strain to stress control. Since these tests will be difficult to perform, the test matrix should be extended to the entire temperature range only when these initial data points confirm the validity of the approach.

A similar situation exists in the range of very high cycle fatigue. The strain-rate feedback-controlled machinery can operate only at slow rates because of the low natural frequency of the diametral strain sensor; to generate data in the regime of 10^7 to 10^8 cycles to failure would tie up the equipment for months just for one data point. The solution proposed here is to utilize high-frequency, load-controlled fatigue machines. The tests are to be performed at various frequencies but at constant

stress amplitude. At low strain amplitudes cyclic hardening has only a small effect, and in a few cycles the stress-strain relationship stabilizes. During subsequent cycles either stress (or displacement) control can function as an effective strain and strain-rate control. By generating data for a number of different frequencies with the same strain range, isobar lines can be assembled and extrapolated to the strain-rate values that were used in the low-cycle tests. Group 16 tests were specified to accomplish this extrapolation, as shown in Fig. 45. The test frequencies used in this group may be different from those specified so long as a large enough range of strain rates are tested to make the extrapolation between strain-controlled and load-controlled tests meaningful.

5. REVIEW OF FATIGUE UNDER MULTIAXIAL STRESSES

In tests at elevated temperature most structural components experience multiaxial stress variation by virtue of geometry and temperature distribution. When a nonuniform temperature distribution is the predominant cause of the stresses, the relative magnitude and direction of the stress components may also change during the course of thermal transients. The duplication of such complex stress and strain histories for purposes of fatigue tests is difficult; so the bulk of the available fatigue data was generated under uniaxial conditions. It is important, therefore, to define the correlation of multiaxial fatigue with uniaxial fatigue test results and to develop procedures for applying uniaxial data to multiaxial structural design problems. Considering the importance of the problem, it is surprising how little research has been directed to determining such procedures. During the past decade hundreds of papers have appeared on uniaxial fatigue, while hardly more than 40 articles and reports were published in the open literature on multiaxial fatigue. Only a fraction of these are concerned with the comparison of the effects of the different stress states. Multiaxial fatigue in the creep range is even more neglected; there is only one report that includes a set of data with hold-time effects. The dearth of reliable data reflects the difficulties encountered in testing many aspects of multiaxial fatigue, such as the effects of load path, strain rate, rotating principal stresses, phase

relationships, etc. Recent advances in the design of servocontrolled automatic fatigue testing equipment offer new techniques for complex stress-fatigue studies. The results of such test programs will hopefully substantiate the tentative conclusions reached in this report.

5.1 Failure Theories for Multiaxial Stress Conditions

Early theories of material strength concentrated on the description of the conditions at failure defined by the plastic instability of actual rupture. Engineering structures seldom fail in this manner since permanent distortion or cracking terminates their usefulness long before complete rupture would occur. Since contemporary design philosophy is more concerned with the prevention of gross deformation, and the description of the conditions for yielding became synonymous with failure theory. The interpretation of uniaxial test data for use in design for multiaxial stress conditions prompted the development of a number of yield theories. The maximum shear stress theory of Tresca and the maximum distortion energy theory of von Mises gained general acceptance for defining the onset of plastic flow. The two theories are identical for uniaxial and equibiaxial stress states but differ at other biaxial conditions, as shown by the shaded areas in Fig. 62. The maximum distortion energy theory fits closely the experimental data and was also confirmed by two independent approaches: Eichinger and Nadai showed that the yield criterion using the octahedral shear stress is identical to the distortion energy theory, and Sachs gave a physical interpretation of the theory based on the statistical treatment of the shear behavior of randomly oriented crystals of body-centered cubic materials. Many complex plastic stress problems became solvable by using these yield criteria. As a consequence, the reliance on the ultimate strength in establishing design rules shifted to limiting the average stress in a cross section to values less than the yield strength of the material.

In modern engineering problems the primary concern is nonsteady and cyclic loading. As prevention of fatigue failure became an important consideration, the same problem of interpretation of uniaxial test data for multiaxial stress conditions arose in fatigue design. The usefulness of

Fig. 62. Tresca and von Mises yield criteria.

the multiaxial yield theories for fatigue conditions were not noticed immediately because of the dissimilarities in the appearance of the static and fatigue failure surfaces. Static failures show signs of distortion and metal flow, while fatigue failures are associated with little flow and have the appearance of cracks with negligible gross deformation. Therefore fatigue failure surfaces are more similar to what would be called brittle failure in static tests. The nature of fatigue failures was further clouded by the fact that the alternating stresses in engineering structures are nominally elastic, and yielding was not an obvious factor to consider. It became clear, however, that the fatigue failures initiate at highly stressed local areas of stress concentration, where the stresses are almost always above the yield strength. The relationship between the average stresses and the strains at these critical locations are not linear; therefore, the local strains are not proportional to the applied loads even though the average stresses are below the proportional limit. Early attempts at fatigue theories were hindered by the lack of

the recognition of the local nature of fatigue failures and also because the fatigue data were presented as a function of the nominal cycle stresses, computed from the load and the average area of the specimen. This method proved to be inadequate in the low-cycle region, where the shallow slope of the characteristically S-shaped fatigue curves showed inexplicably large reduction in fatigue life due to even trivial increases in the nominal stresses.

The recognition of the local aspects of the fatigue problem led to the development of failure criteria based first on the plastic strain range (Coffin relationship) and later expanded by Manson to the total strain (Manson's universal slopes) as the governing variable, replacing the nominal stresses in interpreting fatigue data. These fatigue theories, using the strain range as the failure criterion, were developed by correlating uniaxial push-pull test results but were demonstrated to be also applicable to alternating bending, cyclic torsion, and uniaxial thermal loading conditions. The success of the methods based on the strain range in correlating fatigue life for a variety of simple stress states suggests that fatigue is indeed similar to yielding and that failure is the result of a process of accumulating deformation due to minute local yielding on preferred planes of the crystal structure. It is quite natural, therefore, to apply the proven concepts of multiaxial static yield theories to fatigue failure under complex multiaxial stress conditions. The difficulties inherent in performing multiaxial fatigue tests restricted the early investigations to relatively simple biaxial stress conditions under combined bending and torsional loads. Test results from a number of sources, summarized in Fig. 63, indicate that fatigue endurance under these biaxial stress states can be correlated with uniaxial data satisfactorily on the basis of the equivalent stress of the von Mises criterion¹⁶⁻¹⁹ and less accurately by the Tresca yield condition. However, even if all the possible combinations of torsion and bending are exhausted, the biaxial stress ratios cover only the range 0 to 1.0; thus the data provided only limited proof of the validity of the application of these theories for fatigue conditions.

The yield theories cannot be accepted for fatigue conditions without exploring the effects of various combinations of the principal stresses in

Fig. 63. Combined torsion and bending fatigue (Ref. 16).

all four quadrants of the biaxial field. More general stress conditions in the first and second quadrant were tested by Sawert²⁰ by applying reversed axial loading on variously shaped test pieces and developing a wider range of stress ratios. The results shown in Fig. 64 indicate that the von Mises theory is in good agreement with observed fatigue failures. Following the encouraging results of these early experiments, increasing effort was devoted to generating multiaxial fatigue data for stress ratios common in engineering structures. A variety of testing techniques were developed to simplify the process of multiaxial fatigue data generation, using some characteristics of the variation of the equivalent stress and strain.

Frequent references will be made to the yield theories and the corresponding equivalent stress and strain quantities. These theories are briefly summarized below.

Tresca maximum shear stress theory

The maximum shear stress theory predicts the onset of yielding when the shear stress on any plane reaches the yield strength in uniaxial loading. The shear stress is the difference of the principal stresses,

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3) = \text{constant} .$$

In uniaxial tension at yield the nominal principal stresses are $\sigma_1 = \sigma_y$ and $\sigma_3 = 0$; therefore $\tau_{\max} = \sigma_y/2$. The locus of all combinations of biaxial principal stresses is a hexagon, as shown in Fig. 62. The equivalent stress and strain are defined by

$$\bar{\sigma} = \sigma_1 - \sigma_3 = 2\tau ,$$

$$\bar{\epsilon} = k(\epsilon_1 - \epsilon_3) .$$

von Mises distortion energy theory

In this theory the condition of yielding is reached when the energy of distortion in any combination of stresses equals the value at yield in uniaxial tension. This is expressed as

Fig. 64. Fatigue strength under biaxial stress (Ref. 20).

$$(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = \text{constant} .$$

From uniaxial tension, the constant is $2\sigma_y^2$. For the biaxial case the locus of yield limit is an ellipse circumscribing the hexagon in Fig. 62. The equivalent stress and strain are defined as

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + \dots} ,$$

$$\bar{\epsilon} = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + \dots} .$$

Octahedral shear stress theory

Yielding is predicted when the maximum octahedral shear stress reaches the value that corresponds to the uniaxial condition

$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} = c .$$

The constant is $\sqrt{2/3} \sigma_y$. Thus the equivalent stress and strain are

$$\bar{\tau} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + \dots} ,$$

$$\bar{\gamma} = \frac{2}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + \dots} .$$

Therefore the relationship between the octahedral shear and von Mises equivalent stress is simply a constant:

$$\bar{\sigma} = \frac{3}{\sqrt{2}} \bar{\tau}_{\text{oct}} ,$$

$$\bar{\epsilon} = \frac{1}{\sqrt{2}} \bar{\gamma} .$$

5.2 Mechanical Strain Fatigue in Biaxial Stress States

Test requirements may be simplified when it is recognized that fatigue failures generally originate at a point on the surface of a structure. With the exception of some unusual applications, the surface of a pressure vessel type of structure under mechanical loading is free of sizable tractions. The state of stress is biaxial, or close to biaxial, since in comparison with the other two principal stresses the third stress component is small (i.e., radial stress due to pressure). Therefore, a variety of simple plate and cylinder test specimen geometries were developed for generating biaxial data that are representative of the stress state of real structural components under mechanical loading. These simplified biaxial test techniques using plate-type specimens (e.g., bulging plate, wide cantilever plate, and cruciform plate) and cylinder-type specimens (e.g., open- and closed-end cylinder, rotating disk, etc.) are reviewed below.

Flat-plate-type specimens

In a wide sheet under pure bending, the theory predicts a biaxial state of stress at the center of the plate. If the biaxiality ratio is defined as

$$\alpha = \frac{\text{minimum principal stress}}{\text{maximum principal stress}},$$

then, in case of a plate, the ratio is

$$\alpha = \frac{\sigma_T}{\sigma_L},$$

where σ_L and σ_T are the longitudinal and transverse principal stresses. In the plastic range a maximum biaxiality ratio of 0.5 develops at the center of the sheet when the width-to-thickness ratio is sufficiently large to eliminate the effect of the stress-free boundaries along the length of the plate and the condition of plane strain is approached due to the suppression of the antielastic curvature. By decreasing the specimen

width the biaxiality ratio decreases (as shown in Fig. 65), and fatigue data for a range of biaxiality conditions can be generated by using successively narrower specimens. Results of this type of fatigue tests are shown in Fig. 66, which demonstrates clearly the effects of biaxiality.²¹

Fig. 65. Biaxiality of a bend plate (Ref. 21).

Fig. 66. Biaxial fatigue strength of aluminum (Ref. 21).

For comparison, data for push-pull type uniaxial tests are also shown, indicating good agreement with the bending tests on narrow, essentially uniaxial specimens. The decreasing fatigue endurance with increasing biaxiality ratios in these tests can be correlated with the proportional loss in the static fracture ductility (ϵ_f) as shown in Fig. 67. For the wider specimens the ductility is considerably reduced from the uniaxial value. These data appear to fit well the Coffin relationship,

$$\Delta \epsilon_p N^a = C = \frac{\epsilon_f}{2}, \quad (22)$$

for biaxiality conditions if the static biaxial fracture ductility is used at $N = 1/4$ cycle as shown in Fig. 66. More pronounced effects of biaxiality were reported by McClaren and Best.²²

Although the materials investigated in this test program were very high-strength alloys used in aerospace applications, the carefully documented test data show trends that are of general interest. In this program biaxial cross-shaped specimens were tested under 0:1, 1:1, and 1:2 stress ratios, both in fatigue and static tensile tests. Fatigue data for

Fig. 67. Fracture ductility in bend test (Ref. 21).

the alloy closest in composition to common pressure vessel materials (304 stainless steel) are shown in Fig. 68 and Table 13. The same data are replotted as a function of the ultimate strength in Fig. 69, showing a correlation of static biaxial ductility and biaxial fatigue endurance. The data indicate that the high-strength materials are very sensitive to the loss of apparent ductility because of the increasing restraint on the plastic flow.

Wide-beam-type specimens were used in a series of tests on pressure vessel steels (A-201, A-302, T-1) at Lehigh University. The results were reported²³ and compared with equibiaxial (1:1) fatigue data for the same materials on the basis of von Mises equivalent strain. The trend is similar to the one described in the foregoing: the degree of biaxiality has only a slight effect on the fatigue endurance of the lower-strength carbon steels (Fig. 70), while for the high-strength alloy (T-1) the reduction in fatigue life is more pronounced. The biaxial fatigue lives reported here correspond to crack initiation rather than to complete failure. Unfortunately, uniaxial test data for crack initiation are not available for the same materials. However, a comparison of the data in Figs. 66 to 71 shows a consistent reduction of fatigue life with increasing biaxial constraint, and it is reasonable to assume that this also applies to complete failure conditions.

Zamrik and co-workers²⁴ further developed the plate specimen. Using rhombic plates, they tested stress states in the first and fourth quadrants by imposing antielastic bending. Stress ratios from 0:1 to 1:0 were correlated on the basis of octahedral shear strain range in the form of Eq. (22):

$$\Delta \gamma \bar{\epsilon}^2 = C . \quad (23)$$

The results are shown in Fig. 71. The maximum strain range plot shows a large scatter, while the octahedral shear strain fits the data well for all the stress ratios except for torsional loading, which exhibited higher fatigue strength than predicted from uniaxial data. The explanation of this anomaly is likely to be connected to the degree of axial restraint imposed on the torsion specimens. This will be discussed in the following section.

Fig. 68. Biaxial fatigue endurance of 301 stainless steel (Ref. 22).

Fig. 69. Fatigue strength and ductility of 301 stainless steel (Ref. 22).

Table 13. Biaxial fatigue data for 301 stainless steel

Biaxiality (percent of ultimate)	0:1			1:1			1:2		
	σ_u	$\bar{\epsilon}_f$	N	σ_u	$\bar{\epsilon}_f$	N	σ_u	$\bar{\epsilon}_f$	N
100	190	32		200	5.5		220	9.5	
95	180	3.5	9,800	190	2.0	2200	208	1.4	5,500
90	171	2.4	21,000	180	1.4	3600	198	1.0	8,800
85	161	1.4	41,000	170	0.95	5100	187	0.90	11,000
80	152	1.0	85,000	160	0.8	7500	176	0.70	20,000

Note: σ_u = tensile strength (ksi), $\bar{\epsilon}_f$ = strain range and static fracture ductility, and N = fatigue endurance.

Fig. 70. Biaxial fatigue life of (a) A-302 carbon steel and (b) T-1 alloy (Ref. 23).

The foregoing examples show that plate-type specimens are useful for biaxial fatigue tests in the low-cycle (plastic) range. The wide-beam-type specimen is particularly practical, because it offers the advantages of simplicity and low cost. However, the wide-beam specimen has the disadvantage that the biaxiality ratio developed is strongly dependent on the effective Poisson's ratio. This method should not be used without performing careful calibration tests to determine the transverse strain for the given width-to-thickness ratio to establish the numerical values of Poisson's ratio in the plastic range. Accurate assessment of the lateral restraint and Poisson's ratio are essential for consistent interpretation of the results in terms of equivalent strain. A reasonable method

Fig. 71. Comparison of low-cycle fatigue data based on (a) maximum strain range and (b) octahedral shear strain range (Ref. 24).

based on an "effective" Poisson's ratio was worked out by Shewchuk, Zamrik, and Marin.²⁵ They included the effect of anisotropy and obtained good correlation for the aluminum alloy tested.

Cylinder-type specimens

Fatigue tests using pressurized closed-cylinder type specimens reproduce the biaxiality conditions of the gross membrane stresses in a pressure vessel or pipe (i.e., a stress ratio of 2:1). Other biaxiality

ratios can be induced by additional (independently controlled) longitudinal loading. Without external loading the biaxiality can be varied from 2:1 to close to uniaxial conditions by changing the end restraint from a closed to a free end by imposing a radial pressure only. With external longitudinal load the stress combinations in the second quadrant can be tested by overriding the pressure load with compressive axial loads. When torsional loading can also be added, stress states in the second and fourth quadrant can be tested. Tests for stress combinations in the third quadrant require a test apparatus capable of reversing the pressure from internal to external pressure loading. Theoretically, such alternating pressure loading would provide a simple means of testing at consistent biaxiality ratios. However, the difficulties due to buckling make this type of test hard to interpret, as will be discussed later.

The versatility of the cylindrical test specimen in producing a variety of biaxiality conditions was exploited by many investigators. Morrison, Crossland, and Parry²⁶ used closed-end pressurized cylinders to determine the endurance limit of a number of materials. In their tests the biaxiality was controlled by the radius-to-thickness ratio of the cylinder. At the bore the stress ratio for the thick-wall specimens was approaching uniaxial conditions (neglecting the small radial stress component), while for thin-wall cylinders the biaxiality is close to 2:1. However, these early tests are not entirely consistent with present strain-controlled fatigue testing techniques, and the numerical values are not comparable with more recent fatigue data. The trend, however, of reduced fatigue endurance with increasing biaxiality is clearly discernible. This is shown in Fig. 72 as a function of the maximum shear stress at the inside diameter for the endurance limit at 10^6 cycles. These values are only qualitative since cycles to crack initiation were not reported and the large amount of autofrettage affected the crack propagation through the thick walls, strongly influencing the number of cycles for complete failure.

A similar but more recent study by Eisenstadt et al.²⁷ utilized open-end cylinders to compare the results with pulsating tension (pull-pull) uniaxial tests. The results are shown in Fig. 73. The "dimensionless stress" ($\sigma_{\max}$) for the cylinder specimen is computed from the equivalent

Fig. 72. Endurance limit for two alloys (Ref. 26).

shear stress based on the Tresca criterion. The results can be cross plotted in a form similar to Fig. 69a, demonstrating the trend of the biaxiality effects (Fig. 74). It can be seen that the reduction in fatigue life is large at short endurance and gradually diminishes at 10^5 cycles to failure. This is similar to the vanishing biaxiality effect in 2040 aluminum found by Weiss et al.²¹ as shown in Fig. 66. The numerically lower number of cycles to failure of the cylinder specimen are attributed

Fig. 73. Comparison of uniaxial tension and pressurized cylinder (Ref. 27).

Fig. 74. Fatigue life of thick-walled cylinder (Ref. 27).

to the difference in the crack propagation process in a cylinder and in a solid axial load specimen. The presence of a part-through crack increases the circumferential stress locally, and the reduced hoop restraint causes a tendency for bulging as the hoop force is transmitted by oblique tension action at a large radius of curvature. The result is a greatly amplified stress concentration at the crack tips, and the fatigue life is reduced due to the accelerated propagation of the crack. This process is a peculiarity of the pressurized cylinder specimen, and the reduction in fatigue life should be regarded as a geometry effect in the crack propagation phase rather than a consequence of the biaxiality of the stress field.

For this reason recent studies concentrated on defining the fatigue life by the appearance of a small crack as a more reliable index of the material properties instead of relying on the complete failure of the specimen, which is strongly dependent on the specimen geometry. This will be discussed in more detail in Section 4.3. An additional factor in causing the deviation of thick-walled pressurized cylinder test data from uniaxial results is the effect of the pressurized crack. The increase of the inside "radius" during the crack propagation phase increases the nominal hoop stress in the net cross section. This results in an apparent reduction in fatigue life if interpreted on the basis of the initial nominal hoop stress. Thin-walled-cylinder specimens are not so sensitive to these geometry effects, because the crack penetrates the specimens (and is detected) before it can develop a sizable length in the longitudinal direction. Thus the cycles to failure at the outside diameter signaled by the loss of internal pressure are numerically not much different from the cycles to cracking at the inside surface.

McKenzie et al.²⁸ compared uniaxial and biaxial (shear) data from tests on thin-walled cylinders at room temperature. The results are closely correlated on the basis of hysteresis loop energy per cycle. However, a small but distinct difference in fatigue life becomes apparent if the comparison is based on the octahedral plastic shear strain as shown in Fig. 75. Similar results were reported by Pascoe and de Villiers,²⁹ but with a reversed order of shear and uniaxial fatigue strength, with

Fig. 75. Comparison of uniaxial and biaxial fatigue life of 2 1/2 Ni-Cr-Mo (Ref. 28).

slightly higher fatigue lives under shear (Fig. 76). A closer examination of the test setup indicates that the difference in the relative magnitude under shear and uniaxial loading is possibly due to the absence of longitudinal restraint in the first group of tests. Ronay³⁰ and Coffin³¹ showed analytically and experimentally that axial strain accumulates in reversed torsion. In case of the longitudinal restraint, the axial strain accumulation is prevented. Correspondingly, the specimen develops axial compressive stresses, which result in higher cycle life. This effect was shown independently by Yokobori et al.³² and by Zamrik²⁴ using torsional test data of Halford and Morrow.

Fig. 76. Comparison of biaxial and uniaxial fatigue life of 1 Ni-Cr-Mo (Ref. 29).

Thus comparisons of uniaxial and multiaxial fatigue tests can lead to erroneous conclusions if the test conditions are not evaluated rigorously. Attention must be extended even to second-order effects such as those discussed above.

Kennedy³³ developed a fatigue testing apparatus using thin-walled cylinder specimens with capabilities of combined axial, internal pressure, and torsional loading. Since these carefully designed tests are among the few that were conducted at elevated temperatures, the results are of particular interest in this study. The summary of all test data for Inconel at 1500°F, including uniaxial and torsional loading under constant internal pressure, are shown in Fig. 77. The results, displayed in terms of effective plastic strain range computed on the basis of von Mises yield theory, show good correlation. The torsional fatigue endurance is shown to be somewhat higher than the uniaxial data for the same strain range. This is attributed to the axial restraint as discussed above. A particularly useful test sequence in this program concerned the evaluation of crack initiation and the direction of crack propagation by changing the magnitude of axial stress while maintaining the internal pressure constant. By controlling the ratio of the axial and tangential stresses

Fig. 77. Comparison of fatigue life of Inconel at 1500°F under complex states of stress (Ref. 33).

the direction of fracture surface could be changed from tangential to axial. This is in consonance with the postulated failure theory that the shear strain nucleates cracks and the maximum tensile stress propagates the failure surface normal to the maximum principal stress axis. Kennedy also deduced from the test data that fatigue damage accumulates independently in each principal stress direction. (This is an important conclusion which deserves more extensive experimental verification.)

Another high-temperature biaxial test series was conducted by Reynolds.³⁴ Thin tubular specimens were expanded and compressed between two rigid concentric mandrels, the nominal strain range being controlled by the dimension of the annulus between the mandrels. Hold time was introduced at the extremes of deformation by maintaining the pressure for a predetermined length of time. In spite of the apparent simplicity of this biaxial testing technique, the fatigue results are difficult to interpret because the application of external pressure does not result in a uniform compressive deformation. At pressures in excess of the buckling

pressure the cylinder collapses on the inner mandrel in a multiple-lobe configuration, deforming by bending rather than by simple tension and compression. Therefore the strain range and biaxiality are governed by the curvature of the wrinkles, which are difficult to measure accurately. Correspondingly, the test data show a scatter far in excess of that currently obtained in uniaxial testing. Nonetheless, the results for 304 stainless steel and Incoloy 800 at 1200°F are shown in Fig. 78 because these are virtually the only elevated-temperature biaxial hold-time data available in the literature. For comparison, recent uniaxial data³⁵ for 15 min holding time are shown in the same figure. The biaxial hold-time lives of the two alloys are nearly identical, since all data fall in the same scatter band. The mean value of the biaxial data is displaced to the left of the uniaxial results, indicating qualitatively the same order of magnitude reduction in fatigue endurance found in ambient temperature tests under an equibiaxial state of stress.

Rotating-disk-type fatigue specimens were developed for testing a special class of biaxial stress states due to high-speed inertia loading in electric generator and turbine rotors. The degree of biaxiality in a rotor is largely governed by the geometry of the outer rim of the rotor, the depth and shape of blade roots, and by the relative mass of the blades.

Fig. 78. Comparison of uniaxial and biaxial fatigue data for 304 stainless steel and Incoloy 800 at 1200°F (Refs. 34 and 35).

In general, the fatigue life of such disk-type test specimens depends on the very local notch conditions at the blade roots rather than on the gross biaxiality of the stress distribution. Therefore the considerable volume of disk-fatigue data is difficult to include in this survey of the general problem of biaxiality effects on fatigue endurance. Among the few exceptions, the work of Mattavi may be cited.³⁶ The disks tested in this program had a biaxiality ratio of 0.42, reasonably close to the conditions in a pressure vessel. By the nature of the start-stop type of testing the cyclic strains were restricted to the elastic range; consequently the fatigue results are dominated by the mean strain effects. The conclusion was reached that the biaxial effects can be accounted for by using the equivalent stress and strain based on the von Mises theory and that the linear damage summation is applicable for cyclic mean strain conditions.

5.3 Multiaxial Thermal Fatigue

Thermal stresses arise when nonuniform temperature or external restraints inhibit free thermal expansion. Therefore the magnitude and distribution of thermal stresses depend on the heat transfer conditions as well as on the geometry of the structure. It was pointed out in Section 5.2 that under mechanical loading the stress ratios depend largely on the geometry and in most cases the multiaxial conditions are reducible to simpler biaxial states of stress. In contrast, thermal stresses are biaxial only as an exception, since the process of heating a structure inevitably results in thermal transients during which the stress distribution is more dependent on the direction of the heat flow than on the shape of the structure. Since the boundary conditions of geometry restraints seldom coincide with those governing the heat transfer process, the thermal stress distribution as a rule is triaxial, and the magnitude of the stress ratios are neither predetermined by the geometry nor stay constant during the loading cycle. Consequently, it is extremely difficult to perform thermal fatigue tests with accurately controlled biaxiality. Most of the early thermal fatigue investigations concentrated on enforcing a gross uniaxiality by using a standard fatigue specimen

restrained longitudinally so that the axial thermal expansion or contraction of the specimen due to the temperature change (ΔT) is prevented by the frame of the testing machine. By measuring the load on the frame and the temperature change of the specimen, the thermal stress and strain can be computed by assuming that the total axial strain is zero, as expressed by Eq. (24).

$$\Delta \epsilon + \alpha \Delta T = 0 , \quad (24)$$

$$\Delta \sigma = \frac{P}{A} . \quad (25)$$

Vast amounts of thermal fatigue test data were generated using this technique. In spite of the basic simplicity of this type of test, the results often lead to conflicting interpretation when compared with mechanical strain cycling data. The same material could be shown superior or inferior compared with isothermal mechanical strain cycling fatigue data, depending on whether the thermal stress or the thermal strain was deemed to be equivalent to those in the mechanical fatigue tests. Coffin and Manson showed that the plastic strain range ($\Delta \epsilon_p$) is related to the cycles to failure by

$$\Delta \epsilon_p = \frac{\epsilon_f}{2N^a} , \quad (26)$$

where the plastic strain range can be derived from Eq. (24) by separating the elastic and plastic strain components as follows:

$$\Delta \epsilon_p + \Delta \epsilon_e + \alpha \Delta T = 0 , \quad (27)$$

$$\Delta \epsilon_e = \frac{P}{EA} . \quad (28)$$

By using these relationships it was shown that for reversed cycling conditions the fatigue life is governed by the plastic strain range regardless of the thermal or mechanical origin of the strain.³⁷ Since the same constants (a , C) in Eq. (22) fit approximately both types of fatigue data (within measuring accuracy), the equivalence of thermal and mechanical fatigue in a nominally uniaxial state of stress has been generally

accepted. However, the accuracy of control in thermal fatigue tests is much lower than can be achieved in mechanical fatigue tests. Although it became clear that thermal fatigue is caused by a mechanism that is similar to that operating in isothermal plastic straining, the degree of proportionality between the two remained uncertain. Nonuniform temperature distribution, thermal lag of the specimen interior, and the temperature dependence of the material properties (thermal expansion coefficient, conductivity, specific heat, and heat transfer coefficient) make the stress and strain distribution nonuniform, multiaxial, and time dependent. Therefore the accuracy of the relationship between uniaxial and multiaxial thermal fatigue can be improved only if the data are compared on common bases. A better definition of the test conditions requires a consistent standard of failure and a measure of the plastic strain in a multiaxial state of stress, where the principal stress directions may change during the cycles.

A useful concept can be developed by comparing the actual test conditions to the assumed conditions which led to Eqs. (24) to (28). The nonuniformity of the temperature distribution and even limited flexibility of the test frame can alter the assumed complete restraint condition. A measure of the effective strain can be defined by the constraint factor,

$$\Phi = \frac{\Delta \epsilon}{\alpha \Delta T}, \quad (29)$$

which permits the evaluation of the variations of α and ΔT within the test specimen. Carden has shown³⁸ that a variety of thermal fatigue results can be correlated with isothermal strain cycling data if evaluated on the basis of the effective restraint factor as shown in Fig. 79.

A standard for the definition of cyclic life to failure is also important for a consistent presentation of test results. In mechanical strain cycling complete rupture is often used in the definition of cycles to failure. Complete rupture is preceded by a period of crack propagation which gradually alters the test conditions even in mechanical strain cycling. In thermal fatigue this is more important since large cracks can change the effective restraint and alter the stress state to such a large extent that the crack initiation and crack propagation phases should be

Fig. 79. Effect of restraint variation on uniaxial thermal fatigue life of 18 N 9T steel (Ref 38).

regarded as two separate tests. For this reason cycles to crack initiation is gaining recognition as the definition of failure, particularly in thermal fatigue.

For multiaxial conditions the same considerations are applicable. The constraint factor is expressed by the ratio of the equivalent strain and thermal strain, computed on the basis of one of the failure theories,

$$\Phi = \frac{\bar{\Delta\epsilon}}{\alpha \Delta T} \quad (30)$$

The choice of the failure theory is often made on the basis of convenience in the formulation of an elasto-plastic analysis. It was shown in Section 5.2 that both Tresca and von Mises criteria are reasonably accurate for multiaxial fatigue conditions. The experimental verification, however, has been restricted to synchronous (in-phase) variation of the principal stresses. For general multiaxial stress cycling with rotating stress axes and varying stress ratios (as is the case in most thermal

fatigue applications), some conceptual problems appear with the von Mises theory. The theory is based on the shear strain energy variation, which is a scalar, directionless quantity; thus it is incapable of distinguishing between phase variations of the principal stresses. The maximum shear theory of Tresca is superior in this respect (as the shear stress is a vector with direction as well as magnitude), but the correlation with fatigue results is less favorable. The octahedral shear stress theory combines the advantages of both. It has only a minor inconsistency, in that the octahedral shear is not the maximum shear stress in a cross section. However, the difference is small, and, considering the customary scatter in fatigue results, it is insignificant in all but the most theoretical investigations. Using the octahedral shear theory, Miller, Ohji, and Marin³⁹ described a method for evaluating nonsynchronous stress variation and rotating principal stress axes. Fatigue data for this type of loading (limited to a few mechanical bending moment and torsion combinations) evaluated by Miller et al. showed reasonable correlation.

More recently Zamrik⁴⁰ developed a servocontrolled test apparatus for studying the effects of phase angle variations of the principal stresses. The limited amount of test data generated for an aluminum alloy for phase angles between 0 and 90° were correlated on the basis of Coffin's relationship. The constants are numerically different ($a \approx 0.3$, $C = \epsilon_f \sqrt{1.5}$) from the uniaxial values, but the reasonable fit for the entire range of the test variables indicates that the damage mechanism in multiaxial fatigue is not fundamentally different from the uniaxial case even when the principal stress axes rotate (Fig. 80). This implies that the fundamental fatigue concepts are likely to be applicable to general cases of multiaxial thermal stresses.

The most conclusive evidence for the equivalence of thermal and mechanical straining under general triaxial stress conditions has been presented by Taira and his co-workers.⁴¹ In a comprehensive series of tests, multiaxial thermal stress, uniaxial thermal stress, and uniaxial isothermal strain cycle tests were performed and the results compared on the basis of the thermal constraint factor defined by Eq. (30). As a departure from the more conventional testing techniques, the cyclic stress and strain values were not deduced from the measured temperature variation of

Fig. 80. Maximum total strain vs cycles to failure using experimental data from various phase angle tests. Both coordinates are logarithmic (Ref. 40).

the test specimen; rather the nonsteady heat transfer problem was solved numerically using accurately determined values of the heat transfer coefficients. The results are shown in Figs. 81 and 82. On the basis of the good correlation, the conclusion is reached that fatigue life to crack initiation under cyclic multiaxial thermal stresses can be predicted from uniaxial thermal fatigue tests if the constraint factors are equivalent. In a more recent report,⁴² these conclusions were reconfirmed for several cases of three-dimensional heat flow conditions. Experimental evidence for the character of the crack propagation process in uniaxial and multiaxial conditions is also given. The crack propagation history in uniaxial thermal fatigue shows a slow growth of the crack up to a critical length at which instability sets in and the specimen fractures in a few cycles (Fig. 83). In multiaxial conditions the crack growth is rapid after nucleation and decreases gradually as the crack length increases (Fig. 84). The reason for this is the changing stress distribution, as shown in Fig. 85, which substantiates the intuitive reasoning previously discussed for selecting the crack initiation as the

Fig. 81. Comparison of uniaxial and multiaxial fatigue (Ref. 41).

Fig. 82. Constraint factor in uniaxial and multiaxial fatigue (Ref. 41).

Fig. 83. Crack growth in uniaxial thermal fatigue (Ref. 42).

Fig. 84. Crack growth in multiaxial thermal fatigue (Ref. 42).

Fig. 85. Redistribution of stress due to crack growth (Ref. 42).

definition of fatigue life. As the crack length increases, the thermal stresses are relieved and the effective restraint factor decreases. This process may reach an equilibrium and the crack may become dormant, making the "cycles to fracture" criterion completely misleading since such dormant cracks can become points of severe stress concentrations for cyclic strains due to mechanical loadings. Mechanical straining, concurrent with or following thermal cycles, could restart the crack growth, terminating the test in a complete fracture at a number of cycles that cannot be predicted using uniaxial thermal fatigue data. More experimental evidence would be needed to define the relationships between mechanical and thermal fatigue in the crack propagation phase, which could be provided by test sequences of constant and cyclic mechanical loading in various phases with the thermal cycling. At higher temperatures in the creep range, additional time variables are introduced with tests of this type. Hold periods and wave-shape effects are expected to cause pronounced creep-fatigue interactions similar to those discussed in Chapter 3 under uniaxial conditions.

Unfortunately the total lack of multiaxial hold-time data precluded the evaluation of the creep-fatigue interaction constants even in the

most approximate manner. The experience gained in uniaxial creep-fatigue tests of the type described in Chapter 4 will hopefully provide guidance in the development of multiaxial hold-time tests as the next step in this program.

5.4 Summary

The foregoing review encompasses most of the literature on multiaxial fatigue available in English. The papers and reports cited in this study represent the major trends and testing techniques and also include the results of previous investigations. For example, Ref. 23 summarizes the results of PVRC-sponsored multiaxial fatigue testing by combining the data published in a series of reports in the Welding Journal. In many of the early reports the data were presented in forms that are no longer used in the literature on fatigue. For this reason these references are not cited individually but the interested reader may find these as Refs. 1-9 in Ref. 23. Some of the investigations were conducted as doctoral research projects. Such reports may be found in Refs. 24, 25, and 40; these were not included in the list of references presented here.

The entire body of multiaxial fatigue literature revealed that a generally valid theory has not yet emerged. The available data were generated by using a variety of testing techniques and specimen design, with materials ranging from aluminum to maraging steel. Lacking a unifying theory, the experimental data are fitted with empirical relationships that are valid only for the specific conditions and materials tested. Consistent sets of data for a single material from different specimen geometries and a wide range of stress states are unavailable, and tests with nominally similar biaxiality ratios often result in different fatigue lives.

In spite of the numerous ambiguities encountered in the evaluation and interpretation of the multiaxial fatigue results, four major conclusions can be supported with reasonable certainty.

1. The biaxial fatigue strength of ductile, medium-strength pressure vessel steels (and aluminum alloys) tested with flat-plate-type specimens can be correlated with uniaxial data on the basis of the equivalent strain of the von Mises (or octahedral shear) yield theory. The

biaxial fatigue strength is lower than the uniaxial, the loss of fatigue strength being proportional to the decrease in biaxial static fracture ductility. For these materials the Coffin relationship can be used to generate a family of fatigue curves similar to Fig. 66. The maximum reduction occurs under equibiaxial (1:1) states of stress.

2. High-strength materials suffer a greater loss of fatigue strength than the decrease in biaxial static fracture ductility would predict. For these materials the Coffin-type relationship needs modification. Tests are needed for determining the effective fracture ductility, the constant C , and the exponent α in Eq. (22).

3. Biaxial results from cylinder-type specimens have also been successfully correlated to uniaxial conditions on the basis of the von Mises theory. Zamrik and his co-workers have also shown that the theory can be modified to include the effect of rotating principal stress axes by adjusting the octahedral shear stress values.

4. Multiaxial thermal stress fatigue life to crack initiation is comparable with uniaxial thermal fatigue strength, if the constraint conditions in the two tests are equivalent. The constraint factor is defined as the ratio of the total mechanical strain range to the effective thermal strain,

$$\Phi = \frac{\Delta \bar{\epsilon}}{\alpha \Delta \bar{T}},$$

where $\Delta \bar{\epsilon}$ is the von Mises equivalent strain, $\Delta \bar{T}$ is the effective temperature difference during the thermal stress cycle, and α is the instantaneous thermal expansion coefficient at the average temperature of the cycle.

These conclusions are reasonable if one assumes that damage occurs by nucleation of dislocations (or voids) and subsequent development of cracks. The cracks cause discontinuities in the solid which result in a local multiaxial stress distribution even when the applied stress is nominally uniaxial. Therefore, following crack nucleation, the damage accumulation is expected to be similar if the applied stress is multiaxial from the beginning. The factor of proportionality, however, is expected to be

different, since in the case of multiaxial applied stress the damage depends on the total stress state and not on only one of the stress components as was demonstrated by Zamrik.²⁴ The success of the multiaxial stress theory then depends on finding the combination of the stresses or invariant quantities that correlates best the total state of stress with the measured fatigue life. (This reasoning is valid only for ductile and homogeneous materials since the failure of brittle or granular materials is governed by the maximum tensile stress.) Under monotonic loading conditions the von Mises (or octahedral shear) stress theory has been proven to be the best in correlating yielding; so it is not surprising that the same theory is applicable also for cyclic conditions.

5.5 Biaxial Fatigue Design Procedures

The tentative conclusions given below indicate that a reasonably simple interim design procedure can be found for multiaxial fatigue problems without resorting to cumbersome multiaxial fatigue testing for a multitude of biaxial stress states.

1. Static fracture ductility should be measured under a 1:1 biaxial condition with cruciform flat-plate specimens. This form of testing induces the maximum restraint condition, which results in the lowest biaxial fracture ductility.

2. Using the biaxial fracture ductility in Coffin's relationship, a reduced biaxial fatigue curve can be constructed for each material, as shown schematically in Fig. 86.

3. Perform thermal fatigue tests using uniaxial test specimens under various restraint conditions. The effective restraint can be changed by inserting elastic elements of different flexibility in series with the thermal fatigue specimen.

4. Develop fatigue curves with the restraint factor as the parameter in a form similar to Fig. 87.

5. Determine the constraint factor Φ from the stress analysis results using the computed principal strains in the von Mises equivalent strain equation to calculate the equivalent strain range $\Delta\bar{\epsilon}$.

6. Compute separate fatigue damage fractions (n/n_f): (a) for the

Fig. 86. Biaxial design fatigue curves.

Fig. 87. Thermal fatigue design curves.

mechanical strain range $\Delta\bar{\epsilon}$, using the biaxial fatigue design curve, and (b) for the thermal strain range $\alpha \Delta\bar{T}$, using the thermal fatigue design curve corresponding to the constraint factor Φ .

7. Use the higher of damage factor (a) and (b) in the cumulative damage summation

$$\sum \frac{n}{n_f} \leq 1.0 .$$

It is possible that similar modifications of the hold-time fatigue design curves can be developed in the range of creep-fatigue interactions. However, experimental evidence has not been found to substantiate such a procedure for multiaxial creep conditions.

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