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PRELIMINARY ANALYSIS OF
DEFECTS IN METALS

by

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July 1969

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ABSTRACT

This work is concerned with some preliminary analyses leading toward the determination of the response of defects to the pulsed eddy currents used in nondestructive testing. The first part involves a pulsed vertical magnetic field in air above the surface of an infinitely thick metal. The response is the voltage in a pickup coil in the air above the metal surface. The second part is concerned with the same problem except that the metal has a finite thickness. A short survey of references involving information about defects is also presented.

I. INTRODUCTION

The reliability of pulsed eddy currents for nondestructive testing of defects in metals can be improved by an analysis of the effects of the defects on current flow in the metal. Such an analysis would aid greatly in identifying defects, in deciding whether they are sufficiently severe to cause rejection of the metal specimens, and in predicting the effects of defects that are difficult to simulate experimentally, for example, subsurface defects.

The development of a defect analysis is part of a continuing investigation to improve the pulsed electromagnetic method of nondestructive testing. The pulsed magnetic field under a masked eddy-current probe has previously been analyzed¹ by using a two-dimensional model and by assuming that the magnetic field is applied at the plane surface of an infinitely thick metal specimen. An extension² of this analysis was used to explain why the pulsed eddy-current waves were attenuated much less in passing through a sheet of metal than would be indicated by the usual plane-wave analysis. An analysis of a pulsed magnetic field by means of a three-dimensional model has also been completed,^{3,4} and curves for the current densities in the metal have been obtained. An investigation⁴ has been started to obtain the current densities in the metal when the current source is suspended in the air above a metal of infinite depth.

This report is concerned primarily with the response as detected by a small pickup coil positioned in the air between an aperture driving source and a metal of infinite depth; a metal specimen of finite thickness also was considered. Although this analysis again assumed a two-dimensional model, we hope later to extend the analysis to the three-dimensional case. The investigation has been directed toward obtaining the response of various types of defects in metals. Two possible methods of obtaining the response of a defect are suggested. As a help in this direction, some discussion and references on defects are included.

II. ANALYSIS--INFINITELY THICK METAL

The idealized two-dimensional model is shown in Fig. 1, where the y-axis is perpendicular to the plane of the figure and directed into the paper. The metal is assumed to be infinitely thick, and there is no variation with y. The plane of the source is at $z = z_0$, and the Laplace transform in time of the vertical component of the magnetic field H_z at $z = z_0$ is assumed to be

$$\hat{H}_z = H_0 e^{-x/x_0} \quad (1)$$

for $x > 0$, where H_0 is a constant. Equation 1 represents a first-order impulse, in time, of the z magnetic-field component and also an exponential variation of the field with the distance x . The variation with x is an odd function of x , as shown in Fig. 2, and the distance x_0 represents the length in which the field decreases to e^{-1} . The circumflex ($\hat{}$) in Eq. 1 is used to indicate that H_z in the equation is a Laplace transform with respect to the time t .

Fig. 1. Two-dimensional Model for a Metal of Infinite Depth

Fig. 2. Variation of the Transform of H_z along the Plane of the Source $z = z_0$

The determination of the primary field follows, i.e., the field propagating from the source toward the metal. From the Helmholtz equation in region 0,

$$\nabla^2 \hat{H}_z - \mu_0 \epsilon_0 s^2 \hat{H}_z = 0$$

or

$$\frac{\partial^2 \hat{H}_z}{\partial x^2} + \frac{\partial^2 \hat{H}_z}{\partial z^2} - \mu_0 \epsilon_0 s^2 \hat{H}_z = 0. \quad (2)$$

Then, for $0 \leq z \leq z_0$,

$$\hat{H}_z = \int_0^\infty C_0(\lambda) (\sin \lambda x) e^{K_{z0}(z-z_0)} d\lambda, \quad (3)$$

where

$$K_{z0} = \sqrt{\lambda^2 + \mu_0 \epsilon_0 s^2}.$$

From Maxwell's equations,

$$\nabla \times \hat{\vec{H}} = \epsilon_0 s \hat{\vec{E}} \text{ and } \nabla \times \hat{\vec{E}} = -\mu_0 s \hat{\vec{H}},$$

where the bar (-) represents a vector,

$$\hat{E}_x = 0, \quad \hat{E}_z = 0, \quad \hat{H}_y = 0,$$

$$\hat{E}_y = \mu_0 s \int_0^\infty \frac{C_0(\lambda)}{\lambda} (\cos \lambda x) e^{K_{z0}(z-z_0)} d\lambda, \quad (4)$$

and

$$\hat{H}_x = \int_0^\infty \frac{K_{z0} C_0(\lambda)}{\lambda} (\cos \lambda x) e^{K_{z0}(z-z_0)} d\lambda. \quad (5)$$

To determine $C_0(\lambda)$ by the use of Eq. 1, at $z = z_0$,

$$\hat{H}_z = \int_0^\infty C_0(\lambda) (\sin \lambda x) d\lambda = H_0 e^{-x/x_0}, \quad \text{for } x > 0.$$

By the use of a Fourier transform,

$$C_0(\lambda) = \frac{2}{\pi} \int_0^\infty H_0 e^{-x/x_0} (\sin \lambda x) dx = \frac{2H_0 \lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)}. \quad (6)$$

The three components of the primary field then are, for $0 \leq z \leq z_0$,

$$\hat{H}_z = \int_0^\infty \frac{2H_0\lambda}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} (\sin \lambda x) e^{K_{z_0}(z-z_0)} d\lambda, \quad (7)$$

$$\hat{H}_x = \int_0^\infty \frac{2H_0K_{z_0}}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} (\cos \lambda x) e^{K_{z_0}(z-z_0)} d\lambda, \quad (8)$$

and

$$\hat{E}_y = \int_0^\infty \frac{2H_0\mu_0 s}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} (\cos \lambda x) e^{K_{z_0}(z-z_0)} d\lambda. \quad (9)$$

In a similar manner, the components of the secondary field, i.e., the field in the air propagating from the metal and moving in the positive z direction, for $0 \leq z \leq z_0$, are

$$\hat{H}_z = \int_0^\infty D_0(\lambda) (\sin \lambda x) e^{-K_{z_0}(z+z_0)} d\lambda, \quad (10)$$

$$\hat{H}_x = - \int_0^\infty \frac{K_{z_0}D_0(\lambda)}{\lambda} (\cos \lambda x) e^{-K_{z_0}(z+z_0)} d\lambda, \quad (11)$$

and

$$\hat{E}_y = \mu_0 s \int_0^\infty \frac{D_0(\lambda)}{\lambda} (\cos \lambda x) e^{-K_{z_0}(z+z_0)} d\lambda. \quad (12)$$

From Eqs. 7 and 10, the total vertical magnetic field in the air for $0 \leq z \leq z_0$ is

$$\hat{H}_z = \int_0^\infty \left[\frac{2H_0\lambda}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} e^{K_{z_0}(z-z_0)} + D_0(\lambda) e^{-K_{z_0}(z+z_0)} \right] (\sin \lambda x) d\lambda. \quad (13)$$

Similarly,

$$\hat{H}_x = \int_0^\infty \left[\frac{2H_0 K_{z0}}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} e^{K_{z0}(z-z_0)} - \frac{K_{z0} D_0(\lambda)}{\lambda} e^{-K_{z0}(z+z_0)} \right] (\cos \lambda x) d\lambda \quad (14)$$

and

$$\hat{E}_y = \mu_0 s \int_0^\infty \left[\frac{2H_0}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} e^{K_{z0}(z-z_0)} + \frac{D_0(\lambda)}{\lambda} e^{-K_{z0}(z+z_0)} \right] (\cos \lambda x) d\lambda. \quad (15)$$

In region I (the metal), the Helmholtz equation is

$$\nabla^2 \hat{H} - \sigma_1 \mu_1 s \hat{H} = 0,$$

and Maxwell's equations are

$$\nabla \times \hat{H} = \sigma_1 \hat{E} \text{ and } \nabla \times \hat{E} = -\mu_1 s \hat{H}.$$

Then,

$$\hat{H}_z = \int_0^\infty C_1(\lambda) (\sin \lambda x) e^{K_{z1} z} d\lambda, \quad (16)$$

$$\hat{H}_x = \int_0^\infty \frac{K_{z1} C_1(\lambda)}{\lambda} (\cos \lambda x) e^{K_{z1} z} d\lambda, \quad (17)$$

and

$$\hat{E}_y = \mu_1 s \int_0^\infty \frac{C_1(\lambda)}{\lambda} (\cos \lambda x) e^{K_{z1} z} d\lambda, \quad (18)$$

where

$$K_{z1} = \sqrt{\lambda^2 + \sigma_1 \mu_1 s}.$$

The boundary conditions at $z = 0$ may be applied to $\hat{H}_x$ from Eqs. 14 and 17,

$$\frac{K_{z1} C_1(\lambda)}{\lambda} = \left[\frac{2H_0}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} - \frac{D_0(\lambda)}{\lambda} \right] K_{z0} e^{-K_{z0} z_0}, \quad (19)$$

and to $\hat{E}_y$ from Eqs. 15 and 18,

$$\mu_1 \frac{C_1(\lambda)}{\lambda} = \left[\frac{2H_0}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} + \frac{D_0(\lambda)}{\lambda} \right] \mu_0 e^{-K_{z0}z_0}. \quad (20)$$

Equations 19 and 20 were solved for D_0 and C_1 ,

$$D_0(\lambda) = R_{01} \frac{2H_0\lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)}, \quad (21)$$

where the reflection factor R_{01} between the air and the metal is

$$R_{01} = \frac{\mu_1 K_{z0} - \mu_0 K_{z1}}{\mu_1 K_{z0} + \mu_0 K_{z1}} = -1 + T_{01}. \quad (22)$$

Similarly,

$$C_1(\lambda) = T_{01} \left(\frac{\mu_0}{\mu_1} \right) \left[\frac{2H_0\lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} \right] e^{-K_{z0}z_0}, \quad (23)$$

where the transmission factor T_{01} between the air and the metal is

$$T_{01} = \frac{2\mu_1 K_{z0}}{\mu_1 K_{z0} + \mu_0 K_{z1}} = R_{01} + 1. \quad (24)$$

The vertical magnetic field in the air is obtained from Eqs. 13 and 21:

$$\hat{H}_z = \int_0^\infty \frac{2H_0\lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} \left[e^{K_{z0}(z-z_0)} + R_{01}e^{-K_{z0}(z+z_0)} \right] (\sin \lambda x) d\lambda. \quad (25)$$

The horizontal magnetic field in the air, as determined from Eqs. 14 and 21, is

$$\hat{H}_x = \int_0^\infty \frac{2H_0 K_{z0}}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} \left[e^{K_{z0}(z-z_0)} - R_{01}e^{-K_{z0}(z+z_0)} \right] (\cos \lambda x) d\lambda. \quad (26)$$

Fig. 3. Position of the Pickup Coil

Suppose now that a small pickup coil of N turns and area A is located at the point (x, z) with $0 \leq z \leq z_0$, and that the plane of the coil makes an angle θ with the x -axis, as shown in Fig. 3. The flux linkages in the coil are

$$NA\mu_0(H_z \cos \theta - H_x \sin \theta),$$

and the voltage e_c generated in the coil is

$$\begin{aligned} e_c &= \frac{d}{dt} [NA\mu_0(H_z \cos \theta - H_x \sin \theta)] \\ &= NA\mu_0 \left[(\cos \theta) \frac{dH_z}{dt} - (\sin \theta) \frac{dH_x}{dt} \right]. \end{aligned} \quad (27)$$

The Laplace transform of the coil voltage is, from Eq. 27,

$$L(e_c) = NA\mu_0 s [(\cos \theta) \hat{H}_z - (\sin \theta) \hat{H}_x]. \quad (28)$$

The impulse from the plane of the source at $z = z_0$ will be reflected back and forth many times between the plane of the source and the face of the metal, with the waves traveling at the speed of light in air, $V_0 = 1/\sqrt{\mu_0 \epsilon_0}$. This velocity, V_0 , is much larger than the velocity of the waves in the metal, so much so that to investigate the effects of the waves in the metal, the velocity of light (V_0) may be assumed infinitely large. One result of this assumption is to simplify considerably the reflection and transmission factors. Another assumption is that the metal is non-ferrous so that $\mu_1 = \mu_0$. With the aid of these assumptions, from Eqs. 3, 22, and 24,

$$K_{z0} = \sqrt{\lambda^2 + \mu_0 \epsilon_0 s^2} = \sqrt{\lambda^2 + \left(\frac{s}{V_0}\right)^2} \approx \lambda, \quad (29)$$

$$R_{01} = \frac{\lambda - K_{z1}}{\lambda + K_{z1}} = -1 + \frac{2\lambda}{\lambda + K_{z1}} = -1 + T_{01}, \quad (30)$$

and

$$T_{01} = \frac{2\lambda}{\lambda + K_{z1}} = R_{01} + 1. \quad (31)$$

Then, from Eqs. 25, 26, 28, 29, and 30,

$$L(e_c) = \frac{2H_0 NA\mu_0 s}{\pi} \int_0^\infty \frac{\lambda}{\lambda^2 + \frac{1}{x_0^2}} \left[e^{\lambda(z-z_0)} \sin(\lambda x - \theta) + R_{01} e^{-\lambda(z+z_0)} \sin(\lambda x + \theta) \right] d\lambda. \quad (32)$$

The first term in the integrand of Eq. 32 is a second-order impulse in time, which represents the sum of all the waves moving in the negative z direction and coming from the plane of the source. The second term is the sum of all the waves reflected from the metal and moving in the positive z direction. If the metal were a perfect conductor, $\sigma_1 \rightarrow \infty$, $K_{z1} \rightarrow \infty$, $T_{01} \rightarrow 0$, and $R_{01} \rightarrow -1$. For a perfect conductor, then, the second term is also a second-order impulse in time, but the waves travel in the positive z direction.

The two second-order impulses may be made to cancel one another in the following manner: From Eq. 32,

$$\int_0^\infty \frac{\lambda}{\lambda^2 + \frac{1}{x_0^2}} \left[e^{\lambda(z-z_0)} \sin(\lambda x - \theta) - e^{-\lambda(z+z_0)} \sin(\lambda x + \theta) \right] d\lambda = 0 \quad (33)$$

or

$$M_1 \cos \theta - M_2 \sin \theta = 0, \quad (34)$$

where

$$\begin{aligned} M_1 &= \int_0^\infty \frac{\lambda e^{-\lambda z_0}}{\lambda^2 + \frac{1}{x_0^2}} (\sinh \lambda z) (\sin \lambda x) d\lambda \\ &= \int_0^\infty \frac{\xi e^{-\xi w_0}}{\xi^2 + 1} (\sin \xi w) (\sin \xi u) d\xi, \end{aligned}$$

$$u = \frac{x}{x_0}, \quad w = \frac{z}{x_0}, \quad w_0 = \frac{z_0}{x_0}, \quad \xi = \lambda x_0,$$

and

$$\begin{aligned} M_2 &= \int_0^\infty \frac{\lambda e^{-\lambda z_0}}{\lambda^2 + \frac{1}{x_0^2}} (\cosh \lambda z) (\cos \lambda x) d\lambda \\ &= \int_0^\infty \frac{\xi e^{-\xi w_0}}{\xi^2 + 1} (\cosh \xi w) (\cos \xi u) d\xi. \end{aligned}$$

A method of computing M_1 and M_2 is given in Appendix A. From Eq. 34,

$$\theta = \tan^{-1} \frac{M_1}{M_2}. \quad (35)$$

One result from Appendix A is that if $u = 1.0$, $w_0 = 2.5$, and $w = 0.5$, $\theta = 14.81^\circ$.

If Eqs. 31 and 33 are substituted in Eq. 32, then

$$\begin{aligned} L(e_c) &= \frac{2H_0NA\mu_0s}{\pi} \int_0^\infty \frac{\lambda e^{-\lambda(z+z_0)}}{\lambda^2 + \frac{1}{x_0^2}} T_{01} \sin(\lambda x + \theta) d\lambda \\ &= \frac{4\mu_0H_0NAs}{\pi} \int_0^\infty \frac{\lambda^2 e^{-\lambda(z+z_0)} \sin(\lambda x + \theta)}{\left(\lambda^2 + \frac{1}{x_0^2}\right) \left(\sqrt{\sigma_1\mu_0s + \lambda^2} + \lambda\right)} d\lambda. \end{aligned} \quad (36)$$

From Eq. 11, p. 88 of Ref. 7,

$$L^{-1} \frac{1}{\sqrt{\sigma_1\mu_0s + \lambda^2} + \lambda} = \frac{\lambda}{\sigma_1\mu_0} \left[\frac{\exp\left(-\frac{\lambda^2 t}{\sigma_1\mu_0}\right)}{\sqrt{\frac{\pi\lambda^2 t}{\sigma_1\mu_0}}} - \text{Erfc} \sqrt{\frac{\lambda^2 t}{\sigma_1\mu_0}} \right], \quad (37)$$

and from Eq. 37,

$$L^{-1} \frac{s}{\sqrt{\sigma_1\mu_0s + \lambda^2} + \lambda} = - \frac{\exp\left(-\frac{\lambda^2 t}{\sigma_1\mu_0}\right)}{2\sqrt{\pi\sigma_1\mu_0} t^{3/2}}. \quad (38)$$

Now if the inverse Laplace transform of Eq. 36 is taken and the order of the integrals is reversed, with the aid of Eq. 38,

$$e_c = - \frac{2\sqrt{\frac{\mu_0}{\sigma_1}} H_0NA}{(\pi t)^{3/2}} \int_0^\infty \frac{\lambda^2 e^{-\lambda(z+z_0)}}{\lambda^2 + \frac{1}{x_0^2}} \exp\left(-\frac{\lambda^2 t}{\sigma_1\mu_0}\right) \sin(\lambda x + \theta) d\lambda. \quad (39)$$

Now use the dimensionless variables of Eq. 34 in Eq. 39, and substitute

$$\tau = \frac{t}{\sigma_1\mu_0x_0^2}:$$

$$e_c = - \frac{2 \mu_0 H_0 N A}{\sigma_1^2 \mu_0^2 x_0^4 (\pi \tau)^{3/2}} \int_0^\infty \frac{\xi^2 e^{-\xi(w_0+w)}}{\xi^2 + 1} e^{-\xi^2 \tau} \sin(\xi u + \theta) d\xi. \quad (40)$$

In Eq. 40, notice that under the integral sign the constants of the metal appear only in the variable τ . The pickup voltage in Eq. 40 has an infinite value at the origin, which is caused by the $\tau^{-3/2}$ term and which results from the use of impulses as the magnetic-field driving function. If the driving function were a finite pulse, the response could be obtained from Eq. 40 by using a convolution integral; consequently, a pulse with an initial step as the driving function would produce a response varying as $\tau^{-1/2}$, which would still have an infinite value at the origin. However, if the driving pulse started as either a ramp or a parabolic function of time, the response would start from zero at the origin. The ramp or the parabolic function would be much closer to the actual pulses used in experimental work.

To evaluate Eq. 40, in particular for small values of time, $e^{-\xi^2 \tau}$ can be expanded in a power series of τ , and then by interchanging the summation and the integration,

$$e_c = - \frac{2 \mu_0 H_0 N A}{\sigma_1^2 \mu_0^2 x_0^4 (\pi \tau)^{3/2}} \sum_{n=0}^{\infty} \frac{(-1)^n \tau^n}{n!} \left[S_{2(n+1)}(w_0+w, u) \cos \theta + C_{2(n+1)}(w_0+w, u) \sin \theta \right] \quad (41)$$

where

$$S_n(x, y) = \int_0^\infty \frac{t^n e^{-xt} (\sin yt) dt}{t^2 + 1}$$

and

$$C_n(x, y) = \int_0^\infty \frac{t^n e^{-xt} (\cos yt) dt}{t^2 + 1}.$$

Now

$$S_n(x, y) = \int_0^\infty t^{(n-2)} e^{-xt} (\sin yt) dt - S_{n-2}(x, y) \quad (42)$$

and

$$C_n(x, y) = \int_0^\infty t^{(n-2)} e^{-xt} (\cos yt) dt - C_{n-2}(x, y). \quad (43)$$

From Eq. 42,

$$S_2(x, y) = \frac{y}{x^2 + y^2} - S_0(x, y);$$

$$S_4(x, y) = \frac{2y(3x^2 - y^2)}{(x^2 + y^2)^3} - S_2(x, y),$$

$$S_6(x, y) = \frac{24y(5x^4 - 10x^2y^2 + y^4)}{(x^2 + y^2)^5} - S_4(x, y),$$

$$S_8(x, y) = \frac{720y(7x^6 - 35x^4y^2 + 21x^2y^4 - y^6)}{(x^2 + y^2)^7} - S_6(x, y),$$

and

$$S_{10}(x, y) = \frac{40320y(9x^8 - 84x^6y^2 + 126x^4y^4 - 36x^2y^6 + y^8)}{(x^2 + y^2)^9} - S_8(x, y).$$

From Eq. 43,

$$C_2(x, y) = \frac{x}{x^2 + y^2} - C_0(x, y),$$

$$C_4(x, y) = \frac{2x(x^2 - 3y^2)}{(x^2 + y^2)^3} - C_2(x, y),$$

$$C_6(x, y) = \frac{24x(x^4 - 10x^2y^2 + 5y^4)}{(x^2 + y^2)^5} - C_4(x, y),$$

$$C_8(x, y) = \frac{720x(x^6 - 21x^4y^2 + 35x^2y^4 - 7y^6)}{(x^2 + y^2)^7} - C_6(x, y),$$

and

$$C_{10}(x, y) = \frac{40320x(x^8 - 36x^6y^2 + 126x^4y^4 - 84x^2y^6 + 9y^8)}{(x^2 + y^2)^9} - C_8(x, y).$$

As an example, take $w_0 + w = 2.0$ and $u = 2.0$; then w may be taken anywhere in the range from zero to unity, as shown in Table I. The corresponding values of w_0 and θ computed from Eq. 35 are also presented. The particular value of $w = 0.55$ has been selected so that the coefficient of the first term ($n = 0$) of the series in Eq. 41 is zero; i.e.,

$$S_2(w_0 + w, u)(\cos \theta) + C_2(w_0 + w, u)(\sin \theta) = 0. \quad (44)$$

TABLE I. Values of w , w_0 , and θ

w	w_0	θ
0.00	2.0	0.0°
0.5	1.5	68.67°
0.55	1.45	75.93°
1.0	1.0	135.22°

If any other value of w and the corresponding θ are used, the coefficient for the $n = 0$ term will not be zero. Notice that the relationship given in Eq. 44 does not depend on the constants of the metal. When the above values of the parameters u , w , w_0 , and θ , along with Eqs. 42-44, are substituted in Eq. 41, then

$$e_c = -K\tau^{-3/2}(-0.0467\tau + 0.02327\tau^2 + 0.0504\tau^3 - 0.1118\tau^4 + \dots), \quad (45)$$

where

$$K = \frac{2\mu_0 H_0 N A}{(\sigma_1 \mu_0 x_0^2)^2 \pi^{3/2}}.$$

The first term in Eq. 45 varies as $\tau^{-1/2}$ and approaches infinity as $\tau \rightarrow 0$. The rest of the terms start from zero and become larger as τ increases. This variation is shown in Fig. 4, where the first term is most important initially, but the remainder of the terms begin to be important for larger values of τ . The effect of using a finite pulse, as mentioned previously, would be to decrease the importance of the first term considerably and to increase correspondingly the importance of the remainder of the terms.

To show the effect of the different conductivities of various metals, consider $\tau = t/(\sigma_1 \mu_0 x_0^2)$; the time scale would be much larger for metals of good conductivity such as copper, compared with metals of relatively poor conductivity such as Type 304 stainless steel. Also, for the amplitude factor,

$$K = \frac{2\mu_0 H_0 N A}{(\sigma_1 \mu_0 x_0^2)^2 \pi^{3/2}},$$

thus, the amplitude for Type 304 stainless steel would be much larger than that for copper. This is illustrated in Fig. 5, which shows the voltage of the pickup coil for the three metals (copper, tin, and Type 304 stainless steel) chosen to represent good, fair, and relatively poor conductivity. Notice that the response to the stainless steel is much higher than for the copper;

Fig. 4. The Voltage e_c of the Pickup Coil

Fig. 5. The Voltage e_c of the Pickup Coil for Various Metals

might be to obtain a series, probably asymptotic, in $1/\tau$ for the large values of τ . Neither of these approaches have been tried.

however, stainless steel has a much shorter time scale than copper, with tin somewhere in between.

Unfortunately, the series of Eqs. 41 and 45 does not appear to converge for large values of time, which is the reason the curves of Figs. 4 and 5 were not extended for large values of τ and t . An evaluation of Eq. 40 by means of a computer would be necessary to determine large values of τ . Since Eq. 40 approaches zero as $\tau \rightarrow \infty$, another possibility

III. ANALYSIS--METAL OF FINITE THICKNESS

Preparatory to obtaining the response for defects in metals, an analysis of the response of a finite thickness of metal would be helpful, particularly for a defect with finite dimensions. Figure 6 shows the two-dimensional model for a first metal of finite thickness h_1 positioned in air, or above a second metal of infinite thickness. As in the previous analysis, the field components of Eqs. 13-15 will be used in region 0. In region I, the field components are

$$\hat{H}_z = \int_0^\infty \left[C_1(\lambda) e^{K_{z1}z} + D_1(\lambda) e^{-K_{z1}z} \right] (\sin \lambda x) d\lambda, \quad (46)$$

$$\hat{H}_x = \int_0^\infty \frac{K_{z1}}{\lambda} \left[C_1(\lambda) e^{K_{z1}z} - D_1(\lambda) e^{-K_{z1}z} \right] (\cos \lambda x) d\lambda, \quad (47)$$

and

$$\hat{E}_y = \mu_1 s \int_0^\infty \frac{1}{\lambda} \left[C_1(\lambda) e^{K_{z1}z} + D_1(\lambda) e^{-K_{z1}z} \right] (\cos \lambda x) d\lambda. \quad (48)$$

Similarly, in region II,

Fig. 6. Two-dimensional Model for a Metal Sheet Positioned in Air or above a Base Metal of Infinite Depth

$$\hat{H}_z = \int_0^\infty C_2(\lambda) e^{K_{z2}z} (\sin \lambda x) d\lambda, \quad (49)$$

where

$$K_{z2} = \sqrt{\lambda^2 + (\sigma_2 + \epsilon_2 s) \mu_2 s},$$

$$\hat{H}_x = \int_0^\infty \frac{K_{z2}}{\lambda} C_2(\lambda) e^{K_{z2}z} (\cos \lambda x) d\lambda, \quad (50)$$

and

$$\hat{E}_y = \mu_2 s \int_0^\infty \frac{1}{\lambda} C_2(\lambda) e^{K_{z2}z} (\cos \lambda x) d\lambda. \quad (51)$$

If more layers of metals are to be considered in applying the boundary conditions, the development of an impedance concept would only likely be preferable, or the use of matrices to systematize the algebraic manipulations. The boundary conditions at $z = 0$ give

$$K_{z0} D_0(\lambda) e^{-K_{z0} z_0} + K_{z1} C_1(\lambda) - K_{z1} D_1(\lambda) = \frac{2H_0 K_{z0} \lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} e^{-K_{z0} z_0} \quad (52)$$

and

$$-\mu_0 D_0(\lambda) e^{-K_{z0} z_0} + \mu_1 C_1(\lambda) + \mu_1 D_1(\lambda) = \frac{2H_0 \mu_0 \lambda}{\pi \left(\lambda^2 + \frac{1}{x_0^2} \right)} e^{-K_{z0} z_0} \quad (53)$$

Similarly, at $z = -h_1$,

$$K_{z1} C_1(\lambda) e^{-K_{z1} h_1} - K_{z1} D_1(\lambda) e^{K_{z1} h_1} - K_{z2} C_2(\lambda) e^{-K_{z2} h_1} = 0 \quad (54)$$

and

$$\mu_1 C_1(\lambda) e^{-K_{z1} h_1} + \mu_1 D_1(\lambda) e^{K_{z1} h_1} - \mu_2 C_2(\lambda) e^{-K_{z2} h_1} = 0. \quad (55)$$

From Eqs. 54 and 55,

$$\begin{aligned} C_2(\lambda) e^{-K_{z2} h_1} &= \frac{K_{z1}}{K_{z2}} \left[C_1(\lambda) e^{-K_{z1} h_1} - D_1(\lambda) e^{K_{z1} h_1} \right] \\ &= \frac{\mu_1}{\mu_2} \left[C_1(\lambda) e^{-K_{z1} h_1} + D_1(\lambda) e^{K_{z1} h_1} \right] \end{aligned} \quad (56)$$

From Eq. 56,

$$\frac{D_1(\lambda)}{C_1(\lambda)} e^{2K_{Z1}h_1} = \frac{\mu_2 K_{Z1} - \mu_1 K_{Z2}}{\mu_2 K_{Z1} + \mu_1 K_{Z2}} = R_{12}, \quad (57)$$

where R_{12} is the reflection factor at the I-II interface. Now substitute Eq. 57 in Eqs. 52 and 53 and solve for $D_0(\lambda)$:

$$D_0(\lambda) = \frac{2H_0\lambda}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} R_{01}, \quad (58)$$

where

$$R_{01} = \frac{\mu_1 K_{Z0} \left(1 + R_{12} e^{-2K_{Z1}h_1}\right) - \mu_0 K_{Z1} \left(1 - R_{12} e^{-2K_{Z1}h_1}\right)}{\mu_1 K_{Z0} \left(1 + R_{12} e^{-2K_{Z1}h_1}\right) + \mu_0 K_{Z1} \left(1 - R_{12} e^{-2K_{Z1}h_1}\right)} \quad (59)$$

is the reflection factor for the 0-I (air-to-metal) interface and includes the effects of the finite thickness h_1 of the metal of region I and the constants for all the metals and the air. Also notice that when $h_1 \rightarrow \infty$ the R_{01} of Eq. 59 becomes the same as the R_{01} of Eq. 22.

In a similar manner to that used to obtain $D_0(\lambda)$ in Eq. 58,

$$C_1(\lambda) = \frac{\mu_0}{\mu_1} \frac{2H_0\lambda T_{01}}{\pi\left(\lambda^2 + \frac{1}{x_0^2}\right)} e^{-K_{Z0}z_0}, \quad (60)$$

where

$$T_{01} = \frac{2\mu_1 K_{Z0}}{K_{Z0}\mu_1 \left(1 + R_{12} e^{-2K_{Z1}h_1}\right) + K_{Z1}\mu_0 \left(1 - R_{12} e^{-2K_{Z1}h_1}\right)} \quad (61)$$

is the transmission coefficient for the 0-I (air-to-metal) interface. Also notice that when $h_1 \rightarrow \infty$, the T_{01} of Eq. 61 becomes the same as the T_{01} of Eq. 24.

The vertical and horizontal components of the magnetic field in the air ($0 \leq z \leq z_0$) are given by Eqs. 25 and 26 with the R_{01} of Eq. 59 replacing that of Eq. 22. Also, the pickup coil at an angle θ may be used so that Eq. 28 is valid. Two special cases may now be considered. The first is that region II would be a metal, so $\sigma_2 \gg |\epsilon_2 s|$. Also, if the metals are not ferromagnetic, $\mu_2 = \mu_1 = \mu_0$, and then

$$K_{Z2} = \sqrt{\lambda^2 + \sigma_2 \mu_0 s}.$$

The second case is that region II is air, and the metal of region I is not ferromagnetic; then $\sigma_2 = 0$, and $\mu_2 = \mu_1 = \mu_0$. Also, the velocity in the air is assumed to be very large so that $K_{z2} = \lambda$. The second case will be the one considered.

From Eq. 57,

$$R_{12} = \frac{\sqrt{\lambda^2 + \sigma_1 \mu_0 s} - \lambda}{\sqrt{\lambda^2 + \sigma_1 \mu_0 s} + \lambda} \quad (62)$$

Again, assume $K_{z0} = \lambda$ and use Eqs. 59 and 62 to obtain

$$R_{01} = -1 + M, \quad (63)$$

where

$$M = \frac{2\lambda}{N_1} \left[\sqrt{\lambda^2 + \sigma_1 \mu_0 s} + \lambda + \left(\sqrt{\lambda^2 + \sigma_1 \mu_1 s} - \lambda \right) \exp \left(-2h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) \right]$$

and

$$N_1 = \lambda \left[\sqrt{\lambda^2 + \sigma_1 \mu_0 s} + \lambda + \left(\sqrt{\lambda^2 + \sigma_1 \mu_0 s} - \lambda \right) \exp \left(-2h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) \right] \\ + \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \left[\sqrt{\lambda^2 + \sigma_1 \mu_0 s} + \lambda - \left(\sqrt{\lambda^2 + \sigma_1 \mu_0 s} - \lambda \right) \exp \left(-2h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) \right].$$

Equation 63 may also be written in terms of hyperbolic functions, or

$$M = \frac{2\lambda}{N_2} \left[\sqrt{\lambda^2 + \sigma_1 \mu_0 s} \cosh \left(h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) + \lambda \sinh \left(h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) \right], \quad (64)$$

where

$$N_2 = 2\lambda \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \cosh \left(h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right) + (2\lambda^2 + \sigma_1 \mu_0 s) \sinh \left(h_1 \sqrt{\lambda^2 + \sigma_1 \mu_0 s} \right).$$

The -1 term of Eq. 63 may be canceled as outlined previously in Eqs. 33-35, and in Appendix A. Then,

$$L(e_c) = \frac{2H_0 N_A \mu_0}{\pi} \int_0^\infty \frac{\lambda}{\lambda^2 + \frac{1}{x_0^2}} (sM) e^{-\lambda(z+z_0)} \sin(\lambda x + \theta) d\lambda. \quad (65)$$

The inverse Laplace transform of sM is obtained in Appendix B.

From Eqs. B.9 and 65,

$$e_c = - \frac{8\mu_0 H_0 N A}{\pi(\sigma_1 \mu_0 h_1^2)^2} \int_0^\infty \frac{\lambda a}{\lambda^2 + \frac{1}{x_0^2}} \left\{ \sum_{n=1}^\infty \frac{\beta_n^2(\beta_n^2 + a^2) \exp\left(-\frac{\beta_n^2 + a^2}{\sigma_1 \mu_0 h_1^2} t\right)}{\beta_n^2 + a^2 + 2a} \right\} e^{-\lambda(z+z_0)} \sin(\lambda x + \theta) d\lambda. \quad (66)$$

Now substitute the dimensionless quantities $u = x/x_0$, $w = z/x_0$, $w_0 = z_0/x_0$, $h = h_1/x_0$, $\tau = t/(\sigma_1 \mu_0 x_0^2)$, $\xi = \lambda x_0$, $\beta_n = b_n h$, and $a = \xi h$ in Eq. 66 to produce

$$e_c = - \frac{8\mu_0 H_0 N A}{\pi(\sigma_1 \mu_0 x_0^2)^2} \int_0^\infty \frac{\xi^2}{\xi^2 + 1} \left\{ \sum_{n=1}^\infty \frac{b_n^2(b_n^2 + \xi^2) e^{-(b_n^2 + \xi^2)\tau}}{(b_n^2 + \xi^2) h + 2\xi} \right\} e^{-\xi(w_0+w)} \sin(\xi u + \theta) d\xi. \quad (67)$$

The infinite series in the integrand of Eq. 67 appears to converge for all values of t greater than zero, but no attempt has been made to evaluate the integral. The constants of the metal, including the thickness, are all contained within the infinite series, except for the σ_1^{-2} in front of the integral. A similar expression could be derived for a pickup coil in region II of Fig. 6.

IV. A SURVEY OF DEFECTS

The literature was searched briefly for discussions of artificial defects, for comparisons with real defects, and for analyses of defects. These references were almost all concerned with eddy-current testing, although some touched on other methods of nondestructive testing.

The earliest reference found⁸ showed the response of an eddy-current instrument to milled slots in, and to holes drilled through, the wall of a stainless steel seamless tube. The second reference⁹ involved the use of models to simulate defects in metallic objects. Some work was done that showed the difficulty of producing bonding defects in Zircaloy.¹⁰ Mercury models¹¹⁻¹⁴ have been used to simulate cracks and other defects, and milled slots and drilled holes were used to simulate subsurface defects.¹⁵ Fuel elements were fabricated¹⁶ to simulate inclusions, seams, scratches, pits, strings, and unsized areas. The suggestion^{17,18} was made that holes be used as standard defects, and that an electromagnetic circuit or tape recording could also be used to simulate a standard defect.

The response of a through-transmission type of pulsed system to holes of various sizes in a metal plate was reported.¹⁹ Further work with

drilled holes as standards in eddy-current testing was reported.^{20,21} A tube containing a dimensional change, a slot, and a small piece of lead on the inside diameter was used to determine the best frequency, and a mercury tube with various threads and wires inside was used to simulate artificial defects.²² Spark machining was advocated to make standard defects,²³ and milled flaws in the surface of a tube were used to determine the sensitivity of an eddy-current system.²⁴ Drilled holes were advocated as a proposed standard defect,²⁵ and a mercury model²⁶ with plastic threads was used to simulate defects.

A two-dimensional oscilloscope presentation²⁷ was used to indicate the difference between probe wobble, a dent, notches, holes, and corrosion. The response was presented.^{28,29} A method³⁰ of making standard defects by the use of holes and notches was given, and defects in a mercury model were investigated;³¹ the defects were made from plastic sheets and balsa wood, and agreement was obtained with the earlier work.^{11,12} A mathematical theory of defects³² was presented that used the idea of a dipole inside the metal. Various possible artificial defects were discussed,³³ and a secondary standard³⁴ in the form of drilled holes in aluminum alloy tubing was suggested.

Further information³⁵ of the two-dimensional oscilloscope presentation²⁷ was given, and the detection of various electromachined notches outside and inside Zircaloy tubing³⁶ was described. Conformal mapping with an approximate solution of an integral³⁷ was used to determine the effect of a small radial crack in a circular metal cylinder. Electrodischarge machining was investigated³⁸ for preparing artificially notched longitudinal and circumferential defects in bars and tubes. Possible standard flaws and some experimental results with a sampling system were discussed.³⁹ The use of artificial and natural flaws in inspection work⁴⁰ was described. The ideas of a "point" defect and of matching filters to the defect⁴¹ were presented. The effect of an artificial crack on the response of various sizes of probe coils was investigated,⁴² and a tube with standard defects and its response⁴³ were described. The use of simulated defects along with the action of sampling and filtering in a pulsed eddy-current system⁴⁴ was presented. Some calculations have been made of the effect of defects⁴⁵ on sinusoidal eddy currents.

V. CONCLUSIONS

The analysis presented here is useful in determining the response to a pulsed magnetic field applied above (1) an infinitely deep metal and (2) a metal sheet of finite thickness. The results in turn will give some indication of what response a defect might produce.

Additional computations by an electronic computer are needed so that the complete pulse waveforms may be obtained. The next studies should be (1) the effect of a pulsed dipole³² in the metal and (2) the effect of a finite-size defect, say rectangular, in the metal. For this task, the work on the metal sheet of finite thickness should be helpful.

APPENDIX A
Computation of the Angle θ

By the use of Eqs. 5.2.12, 5.2.5, and 5.2.6 of Ref. 5, we get

$$f(z) = \int_0^{\infty} \frac{e^{-zt}}{t^2 + 1} dt = \text{Ci}(z)(\sin z) + \left[\frac{\pi}{2} - \text{Si}(z)\right](\cos z). \quad (\text{A.1})$$

Now use Eqs. 5.2.21 and 5.2.23 of Ref. 5 in Eq. A.1 to obtain

$$\begin{aligned} f(z) &= -\frac{\sin z}{2} [E_1(jz) + E_1(-jz)] - \frac{\cos z}{2j} [E_1(jz) - E_1(-jz)] \\ &= \frac{j}{2} [e^{jz} E_1(jz) - e^{-jz} E_1(-jz)]. \end{aligned} \quad (\text{A.2})$$

Also, if $z = x - jy$ in Eq. A.1, then

$$\begin{aligned} f(z) &= \int_0^{\infty} \frac{e^{-xt}(\cos yt)}{t^2 + 1} dt + j \int_0^{\infty} \frac{e^{-xt}(\sin yt)}{t^2 + 1} dt \\ &= C_0(x, y) + jS_0(x, y), \end{aligned} \quad (\text{A.3})$$

where

$$C_0(x, y) = \int_0^{\infty} \frac{e^{-xt}(\cos yt)}{t^2 + 1} dt,$$

and

$$S_0(x, y) = \int_0^{\infty} \frac{e^{-xt}(\sin yt)}{t^2 + 1} dt.$$

Then from Eqs. A.2 and A.3,

$$\begin{aligned} C_0(x, y) &= \int_0^{\infty} \frac{e^{-xt}(\cos yt)}{t^2 + 1} dt \\ &= \text{Re} \left\{ \frac{j}{2} [e^{jz} E_1(jz) - e^{-jz} E_1(-jz)] \right\}, \end{aligned} \quad (\text{A.4})$$

and

$$\begin{aligned} S_0(x, y) &= \int_0^{\infty} \frac{e^{-xt}(\sin yt)}{t^2 + 1} dt \\ &= \text{Im} \left\{ \frac{j}{2} [e^{jz} E_1(jz) - e^{-jz} E_1(-jz)] \right\}. \end{aligned} \quad (\text{A.5})$$

Similarly, from Eqs. 5.2.13, 5.2.5, and 5.2.7 of Ref. 5,

$$g(z) = \int_0^{\infty} \frac{te^{-zt}}{t^2 + 1} dt = -\text{Ci}(z)(\cos z) + \left[\frac{\pi}{2} - \text{Si}(z) \right] (\sin z). \quad (\text{A.6})$$

Also, substitution of Eqs. 5.2.24 and 5.2.23 of Ref. 5 in Eq. A.6 gives

$$\begin{aligned} g(z) &= \frac{\cos z}{2} [E_1(jz) + E_1(-jz)] - \frac{\sin z}{2j} [E_1(jz) - E_1(-jz)] \\ &= \frac{1}{2} [e^{jz} E_1(jz) + e^{-jz} E_1(-jz)]. \end{aligned} \quad (\text{A.7})$$

If $z = x - jy$ in Eq. A.6, then

$$\begin{aligned} g(z) &= \int_0^{\infty} \frac{te^{-xt}(\cos yt)}{t^2 + 1} dt + j \int_0^{\infty} \frac{te^{-xt}(\sin yt)}{t^2 + 1} dt \\ &= C_1(x, y) + jS_1(x, y), \end{aligned} \quad (\text{A.8})$$

where

$$C_1(x, y) = \int_0^{\infty} \frac{te^{-xt}(\cos yt)}{t^2 + 1} dt,$$

and

$$S_1(x, y) = \int_0^{\infty} \frac{te^{-xt}(\sin yt)}{t^2 + 1} dt.$$

Then from Eqs. A.7 and A.8,

$$C_1(x, y) = \int_0^{\infty} \frac{te^{-xt}(\cos yt)}{t^2 + 1} dt = \text{Re} \left\{ \frac{1}{2} [e^{jz} E_1(jz) + e^{-jz} E_1(-jz)] \right\}, \quad (\text{A.9})$$

and

$$\begin{aligned}
 S_1(x, y) &= \int_0^{\infty} \frac{te^{-xt}(\sin yt) dt}{t^2 + 1} \\
 &= \text{Im} \left\{ \frac{1}{2} [e^{jz} E_1(jz) + e^{-jz} E_1(-jz)] \right\}. \quad (\text{A.10})
 \end{aligned}$$

Tables of $E_1(z)$ and $e^z E_1(z)$ were available⁶ and were used in the computations. As an example, take $x = 2.0$ and $y = 1.0$, so that $z = 2.0 - j1.0$ and $jz = 1.0 + j2.0$.

From pages 608 and 609 of Ref. 6,

$$e^{jz} E_1(jz) = e^{(1+j2)} E_1(1+j2) = 0.230131 - j0.273692.$$

From pages 618 and 619 of Ref. 6,

$$e^{(-1+j2)} E_1(-1+j2) = -0.027749 - j0.434335.$$

From Eq. 5.1.13 of Ref. 5,

$$E_1(z^*) = E_1^*(z),$$

where the asterisk means the complex conjugate. Then,

$$e^{-jz} E_1(-jz) = e^{(-1-j2)} E_1(-1-j2) = -0.027749 + j0.434335.$$

From Eq. A.4,

$$\begin{aligned}
 C_0(2.0, 1.0) &= \text{Re} \left\{ \frac{j}{2} [e^{(1+j2)} E_1(1+j2) - e^{(-1-j2)} E_1(-1-j2)] \right\} \\
 &= \text{Re} \left\{ \frac{j}{2} [(0.230131 - j0.273692) - (-0.027749 + j0.434335)] \right\} \\
 &= 0.354014.
 \end{aligned}$$

Similarly, from Eq. A.5,

$$\begin{aligned}
 S_0(2.0, 1.0) &= \text{Im} \left\{ \frac{j}{2} [e^{(1+j2)} E_1(1+j2) - e^{(-1+j2)} E_1(-1-j2)] \right\} \\
 &= \text{Im} \left\{ \frac{j}{2} [(0.230131 - j0.273692) - (-0.027749 + j0.434335)] \right\} \\
 &= 0.128940.
 \end{aligned}$$

From Eq. A.9,

$$\begin{aligned}
 C_1(2.0, 1.0) &= \operatorname{Re} \left\{ \frac{1}{2} [e^{(1+j2)} E_1(1+j2) + e^{(-1-j2)} E_1(-1-j2)] \right\} \\
 &= \operatorname{Re} \left\{ \frac{1}{2} [(0.230131 - j0.273692) + (-0.027749 + j0.434335)] \right\} \\
 &= 0.101191.
 \end{aligned}$$

From Eq. A.10,

$$\begin{aligned}
 S_1(2.0, 1.0) &= \operatorname{Im} \left\{ \frac{1}{2} [e^{(1+j2)} E_1(1+j2) + e^{(-1-j2)} E_1(-1-j2)] \right\} \\
 &= \operatorname{Im} \left\{ \frac{1}{2} [(0.230131 - j0.273692) + (-0.027749 + j0.434335)] \right\} \\
 &= 0.080322.
 \end{aligned}$$

To determine θ , M_1 and M_2 must be computed, and this may be done by using C_1 from Eq. A.9 and S_1 from Eq. A.10. From Eq. 34,

$$\begin{aligned}
 M_1 &= \int_0^\infty \frac{\xi e^{-\delta w_0}}{\xi^2 + 1} (\sinh \xi w) (\sin \xi u) d\xi \\
 &= \frac{1}{2} \int_0^\infty \frac{\xi e^{-\delta(w_0-w)}}{\xi^2 + 1} (\sin \xi u) d\xi - \frac{1}{2} \int_0^\infty \frac{\xi e^{-\xi(w_0+w)}}{\xi^2 + 1} (\sin \xi u) d\xi \\
 &= \frac{1}{2} S_1(w_0 - w, u) - \frac{1}{2} S_1(w_0 + w, u), \tag{A.11}
 \end{aligned}$$

and

$$\begin{aligned}
 M_2 &= \int_0^\infty \frac{\xi e^{-\xi w_0}}{\xi^2 + 1} (\cosh \xi w) (\cos \xi u) d\xi \\
 &= \frac{1}{2} \int_0^\infty \frac{\xi e^{-\xi(w_0-w)}}{\xi^2 + 1} (\cos \xi u) d\xi + \frac{1}{2} \int_0^\infty \frac{\xi e^{-\xi(w_0+w)}}{\xi^2 + 1} (\sin \xi u) d\xi \\
 &= \frac{1}{2} C_1(w_0 - w, u) + \frac{1}{2} C_1(w_0 + w, u). \tag{A.12}
 \end{aligned}$$

As an example, take $u = 1.0$, $w_0 = 2.5$, and $w = 0.5$. Then $S_1(w_0 - w, u) = S_1(2.0, 1.0)$ and $C_1(w_0 - w, u) = C_1(2.0, 1.0)$ were obtained in the preceding example. In a similar manner, $S_1(w_0 + w, u) = S_1(3.0, 1.0) = 0.036429$ and $C_1(w_0 + w, u) = C_1(3.0, 1.0) = 0.064880$. Then, from Eq. A.11,

$$M_1 = \frac{1}{2}(0.080322) - \frac{1}{2}(0.036429) = 0.021947,$$

and from Eq. A.12,

$$M_2 = \frac{1}{2}(0.101191) + \frac{1}{2}(0.064880) = 0.083036.$$

From Eq. 35, $\theta = \tan^{-1} \frac{0.021947}{0.083036} = 14.81^\circ$.

A computer program to obtain M_1 , M_2 , and θ has been developed, and the program is given in Table II. A number of values were computed with this program and are given in Table III. For comparison, the values for $w_0 = 10.0$, $u = 10.0$, and $w = 5.0$ were computed by using the method involving the tables for the exponential integral; computed values shown in Table IV seem to agree, to four significant figures, with the values obtained from the computer. The tables for the complex exponential integral are probably not dependable beyond four significant figures in this range.

TABLE II. FORTRAN Computer Program for the Calculation of M_1 , M_2 , and θ

```

COMPILER OPTIONS - NAME= MAIN,OPT=00,LINECNT=57,SOURCE,EBCDIC,LIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
C      PROGRAM
ISN 0002      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
ISN 0003      DIMENSION A(6,6),B(6,2),C(6,6),E(2),EX(2),X(2),P(6,2,2),PU(6),T(2,2,1)
ISN 0004      N=3
C-----
ISN 0005      DO 5 L1=1,N
ISN 0006      C(L1,1)=1.
ISN 0007      DO 4 IR=2,L1
ISN 0008      4 C(L1,IR)=C(L1,IR-1)*(L1-IR+1)
ISN 0009      5 CONTINUE
ISN 0010      DO 8 IR=1,6
ISN 0011      DO 8 IA=1,2
ISN 0012      DO 8 IB=1,2
ISN 0013      8 P(IR,IA,IB)=0.D0
C-----
ISN 0014      WRITE(6,1000)
ISN 0015      1000 FORMAT(1H1,45X,26HF(X)=X/(X**2+1)*EXP(-X*V0),/,5X,2HV0,6X,1HU,7X,1
2HV,9X,29HN=SF(X)*SINH(X*V)*SIN(X*V)*DX,9X,29HD=SF(X)*COSH(X*V)*COS
3(X*U)*DX,9X,23HTHETA=ARCTAN(N/D), DEG.)
ISN 0016      1001 FORMAT(1X,3F8.3,10X,F19.9,8X,F19.9,14X,F19.9)
ISN 0017      N1=N-1
ISN 0018      V0=.01
ISN 0019      DO 200 I=1,3
ISN 0020      V0=V0*10.
ISN 0021      U=.01
ISN 0022      DO 200 J=1,3
ISN 0023      U=U*10.
C-----
ISN 0024      PU(1)=1./U
ISN 0025      DO 10 JK=2,N
ISN 0026      10 PU(JK)=PU(JK-1)/U
C-----
ISN 0027      V=0.0000      DO
ISN 0028      DO 200 K=1,3
ISN 0029      V=V+.25*V0

```

TABLE II. (Contd.)

```

ISN 0030      DX=.1D0
ISN 0031      LA=2
ISN 0032      ASSIGN 120 TO M
ISN 0033      ASSIGN 130 TO M1
ISN 0034      ASSIGN 25 TO M2
ISN 0035      15 X(2)=0.D0
ISN 0036      EN=0.D0
ISN 0037      ED=0.D0
ISN 0038      X2=1.D0
ISN 0039      E(2)=1.D0
ISN 0040      E(1)=1.D0
ISN 0041      20 EX(1)=DEXP(-DX*(V0-V))
ISN 0042      GOTOM2,(25,30)
ISN 0043      25 EX(2)=DEXP(-DX*(V0+V))
ISN 0044      30 X(1)=X(2)
ISN 0045      DO 60 L1=1,N
ISN 0046      Q=X(2)/X2
ISN 0047      DO 40 , IA=1,LA
ISN 0048      E(IA)=E(IA)*EX(IA)
ISN 0049      40 B(L1,IA)=E(IA)*Q
ISN 0050      A(L1,1)=1.
ISN 0051      DO 50 J1=2,N
ISN 0052      50 A(L1,J1)=A(I1,J1-1)*X(2)
ISN 0053      X(2)=X(2)+DX
ISN 0054      60 X2=X(2)*X(2)+1.D0
ISN 0055      GOTO M,(120,62)
ISN 0056      62 GOTO M1,(64,130)
ISN 0057      64 IF(B(1,1).LT.1.D-8)GOTO150
ISN 0059      65 CALL MATINV(A,N,B,LA,1.,6)
ISN 0060      DO 90 IA=1,LA
ISN 0061      DO 90 IB=1,2
ISN 0062      DO 80 IR=1,N1
ISN 0063      P(IR,IA,IB)=B(N,IA)*C(N,IR)
ISN 0064      DO 70 I1=IR,N1
ISN 0065      L2=N1-I1+IR
ISN 0066      70 P(IR,IA,IB)=P(IR,IA,IB)*X(IB)+B(L2,IA)*C(L2,IR)
ISN 0067      P(N,IA,IB)=B(N,IA)*C(N,IR)
ISN 0068      80 P(IR,IA,IB)=P(IR,IA,IB)*X(IB)+B(L2,IA)*C(L2,IR)
ISN 0069      P(1,IA,IB)=P(1,IA,IB)-P(3,IA,IB)+P(5,IA,IB)
ISN 0070      90 P(2,IA,IB)=P(2,IA,IB)-P(4,IA,IB)+P(6,IA,IB)
ISN 0071      T(1,2)=DSIN(U*X(2))
ISN 0072      T(2,1)=DCOS(U*X(2))
ISN 0073      T(1,1)=-DSIN(U*X(1))
ISN 0074      T(2,1)=-DCOS(U*X(1))
ISN 0075      IA=1
ISN 0076      95 DO 100 IB=1,2
ISN 0077      DO 100 IR=1,2
ISN 0078      ED=P(IR,IA,IB)*T(IR,IB)+ED
ISN 0079      100 EN=P(IR,IA,IB)*T(3-IR,IB)-EN
ISN 0080      IF(LA.EQ.1)GOTO110
ISN 0082      EN=-EN
ISN 0083      IA=IA+1
ISN 0084      IF(IA.EQ.2)GOTO95
ISN 0086      110 DX=DX*2.0000000
ISN 0087      GOTO 20
ISN 0088      120 IF(B(N1,2).LT.B(N,2))GOTO125
ISN 0090      DX=DX*.20000000
ISN 0091      GOTO15
ISN 0092      125 ASSIGN 62 TO M
ISN 0093      GOTO 65
ISN 0094      130 IF(B(1,2).GT.1.D-8)GOTO64
ISN 0096      LA=1
ISN 0097      ASSIGN 64 TO M1
ISN 0098      ASSIGN 30 TO M2
ISN 0099      GOTO64
ISN 0100      150 Z=EN/ED
ISN 0101      TH=DATAN(Z)*90.000000/1.5709630.
ISN 0102      IF(EN)160,170,170
ISN 0103      160 TH=180.D0+TH
ISN 0104      GOTO180
ISN 0105      170 IF(ED)171,180,180
ISN 0106      171 TH=360.D0+TH
ISN 0107      180 EN=EN/2.D0
ISN 0108      ED=ED/2.D0
ISN 0109      WRITE(6,1001)V0,U,V,EN,ED,TH
ISN 0110      200 CONTINUE
ISN 0111      STOP
ISN 0112      END

```

TABLE III. Computed Values of M_1 , M_2 , and θ

w_0	u	w	M_1	M_2	θ , deg
10.0	0.10	2.50	0.000154215	0.011279847	0.783200297
10.0	0.10	5.00	0.000563908	0.019186940	1.683272629
10.0	0.10	7.50	0.003100203	0.053914352	3.290670333
10.0	1.00	2.50	0.001484995	0.010828328	7.807993937
10.0	1.00	5.00	0.005317695	0.017632477	16.780755334
10.0	1.00	7.50	0.026274924	0.041924560	32.072756043
10.0	10.0	2.50	0.001243316	-0.000316442	284.287478199
10.0	10.0	5.00	0.001985653	-0.001702606	310.616811420
10.0	10.0	7.50	0.001452668	-0.003648873	338.294106340
1.0	0.10	0.250	0.017630537	0.360628194	2.798573891
1.0	0.10	0.500	0.046796089	0.433749330	6.157022166
1.0	0.10	0.750	0.130043561	0.615065445	11.936986701
1.0	1.0	0.250	0.059662043	0.141798078	22.816675401
1.0	1.0	0.500	0.122596001	0.124808996	44.482792710
1.0	1.0	0.750	0.188317889	0.088594996	64.798275053
1.0	10.0	0.250	-0.000529957	-0.010323860	182.938283241
1.0	10.0	0.500	-0.001529957	-0.010264201	185.874404999
1.0	10.0	0.750	-0.001574975	-0.010165400	188.806156281
0.10	0.10	0.0250	0.121921662	1.527765241	4.562274158
0.10	0.10	0.0500	0.250785556	1.522700980	9.351538125
0.10	0.10	0.0750	0.389958928	1.505161501	14.523349869
0.10	1.0	0.0250	0.009975848	0.002253228	77.264074656
0.10	1.0	0.0500	0.019916094	0.001342551	86.134365592
0.10	1.0	0.0750	0.029785349	-0.000171100	270.338641940
0.10	10.0	0.0250	0.000058656	-0.010780520	180.311706646
0.10	10.0	0.0500	-0.000117307	-0.010779785	180.623407702
0.10	10.0	0.075	-0.000175947	-0.010778561	180.935101560

TABLE IV. Comparison of the Values of M_1 , M_2 , and θ for $w_0 = 10.0$, $u = 10.0$, and $w = 5.0$

Parameter	Values from Tables ⁶	Values from Computer
M_1	0.001986	0.001985653
M_2	-0.001702	-0.001702606
θ , deg	310.596	310.616811420

APPENDIX B

Determination of Inverse Laplace Transform

The inverse transform of sM , as given in Eq. 63 or 64, for use in Eq. 65 can be found by noting that

$$L^{-1}(sM) = \frac{d}{dt} L^{-1}(M). \quad (B.1)$$

Now use the theorem concerning change of scale in $L^{-1}(M)$, and replace $\sigma_1 \mu_0 s$ by s' . Then in M each square root becomes

$$\sqrt{\lambda^2 + \sigma_1 \mu_0 s} = \sqrt{\lambda^2 + s'}. \quad (B.2)$$

Next use the shifting theorem, and replace $s' + \lambda^2$ by s'' . Again in M each square root becomes

$$\sqrt{\lambda^2 + s'} = \sqrt{s''}. \quad (B.3)$$

Now drop the primes for the time being so that

$$L^{-1}(M) = L^{-1} \left\{ \frac{2\lambda[s^{1/2} \cosh(h_1 s^{1/2}) + \lambda \sinh(h_1 s^{1/2})]}{2\lambda s^{1/2} \cosh(h_1 s^{1/2}) + (\lambda^2 + s) \sinh(h_1 s^{1/2})} \right\}. \quad (B.4)$$

Now Eq. B.4 has no branch points, but does have a series of poles, which are found by putting the denominator of Eq. B.4 equal to zero and substituting $jv = h_1 s^{1/2}$ and $a = \lambda h_1$. The result is

$$\tan v = \frac{2av}{v^2 - a^2}. \quad (B.5)$$

Equation B.5 has as roots a series of positive and negative numbers, but only the positive roots need be used. Let the roots be β_n , where $n =$

1, 2, 3, ..., and $0 < \beta_1 < \beta_2 < \beta_3 < \dots < \beta_n < \dots$. The left-hand side (lhs) and the right-hand side (rhs) of Eq. B.5 are sketched in Fig. 7 for $0 < a < \pi/2$, and the crossing points of the two curves are shown as $\beta_1, \beta_2, \beta_3$, etc. When $a = 0$, the β 's become the roots of $\tan v = 0$; at $a = \pi/2$, $\beta_1 = \pi/2$.

Fig. 7. Sketch of the Crossing Points for $0 < a < \pi/2$

If a is given, Eq. B.5 may be solved either graphically or numerically for v . However, reversing the process proved a much simpler solution. Thus, for a given v , a may be obtained from

$$a = v \frac{\pm 1 - \cos v}{\sin v}. \quad (\text{B.6})$$

For the $\pm$ sign in Eq. B.6, use the sign that will give a positive value of a . A calculated curve of β_1 for various values of a is shown in Fig. 8, where the inverse-tangent scale was used for the abscissa so that the entire range of a from zero to infinity could be included. A corresponding curve for β_2 is shown in Fig. 9, and similar curves may be made for the other β 's. When used in the equations, the β 's should be expressed in radians.

Fig. 8. Variation of β_1 with a

Fig. 9. Variation of β_2 with a

By the use of Eqs. B.4 and B.5,

$$L^{-1}(M) = \frac{4\lambda}{h_1} \sum_{n=1}^{\infty} \frac{\beta_n^2 \exp\left(-\frac{\beta_n^2 t}{h_1^2}\right)}{\beta_n^2 + a^2 + 2a}. \quad (\text{B.7})$$

Now by using Eq. B.7 and undoing the work of the shifting theorem, we obtain

$$L^{-1}(M) = e^{-\lambda^2 t} \frac{4\lambda}{h_1} \sum_{n=1}^{\infty} \frac{\beta_n^2 \exp\left(-\frac{\beta_n^2 t}{h_1^2}\right)}{\beta_n^2 + a^2 + 2a}.$$

Similarly, for the change-of-scale theorem,

$$L^{-1}(M) = \frac{4\lambda}{\sigma_1 \mu_0 h_1} e^{-\frac{\lambda^2 t}{\sigma_1 \mu_0}} \sum_{n=1}^{\infty} \frac{\beta_n^2 \exp\left(-\frac{\beta_n^2 t}{\sigma_1 \mu_0 h_1^2}\right)}{\beta_n^2 + a^2 + 2a} \quad (\text{B.8})$$

(Contd.)

or

$$L^{-1}(M) = \frac{4a}{\sigma_1 \mu_0 h_1^2} \sum_{n=1}^{\infty} \frac{\beta_n^2 \exp\left(-\frac{\beta_n^2 + a^2}{\sigma_1 \mu_0 h_1^2} t\right)}{\beta_n^2 + a^2 + 2a} \quad \begin{array}{l} \text{(Contd.)} \\ \text{(B.8)} \end{array}$$

From Eqs. B.1 and B.8,

$$L^{-1}(sM) = - \frac{4a}{(\sigma_1 \mu_0 h_1^2)^2} \sum_{n=1}^{\infty} \frac{\beta_n^2 (\beta_n^2 + a^2) \exp\left(-\frac{\beta_n^2 + a^2}{\sigma_1 \mu_0 h_1^2} t\right)}{\beta_n^2 + a^2 + 2a} \quad \text{(B.9)}$$

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