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SMALL DISPLACEMENT MEASUREMENTS USING HOLOGRAPHIC INTERFEROMETRY

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704
81

Contents

Abstract	1
Introduction	1
Holographic Interferometry Theory	2
Multiple-Hologram Method	4
Single-Hologram Method	6
Implementation of the Three-Hologram Technique	7
Mapping of a Flat Experimental Surface (Plane)	8
Mapping of a Spherical Surface	9
Magnification Correction	11
Angular Correction	11
HIDI Computer Program	13
Experimental Example	14
Acknowledgment	16

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Abstract

Holographic interferometry produced by coherent laser light can be used to detect three-dimensional displacements as small as $12\text{ }\mu\text{in.}$ on a complex curved surface of a structure deformed by either stresses or temperature gradients and displacements caused by solid-body motion. With this technique, the measuring apparatus is independent of the surface being measured which is an advantage over strain-gauge or photoelasticity techniques. A computer code was developed to give quantitative and qualitative interpretation in the form of displacement contour lines.

Introduction

Three-dimensional displacement measurements as small as $12\text{ }\mu\text{in.}$ can be made using coherent light interferometry produced by He-Ne laser holography. Holographic interferometry can be conveniently used as a noncontacting, non-destructive testing tool to measure displacements of every point on a complex curved surface of a structure deformed by stresses or temperature gradients and displacement caused by solid-body motion. The distinction between surface displacement and body motion is made simply to point out and emphasize that the holographic interferometry technique detects absolute displacement and can, therefore, not distinguish between the two unless special techniques are applied.

The advantage of this technique is that the measuring apparatus is independent of the surface being measured. Unlike a strain gauge which measures strain at a point with a glued-on metallic strip or photoelasticity which requires a grid to be glued on the structure, holographic inter-

ferometry obtains the displacement at every point on the surface only from the laser light reflection from that surface.

Determining the displacement magnitude at a single point is a relatively simple task using holographic interferometry.

However, qualitative interpretation of the interference fringe pattern produced by the spatial motion of a complex surface is very difficult, and this is an obstacle to the general application of this technique as a sensitive nondestructive displacement measurement tool. The virtue of this method is its ability to detect the total field deformation of the complex motion of a three-dimensional surface. This lends itself to the application of the digital computer for multiple-point calculation and the plotting of the result in a manner that clarifies for the user the qualitative interpretation of three-dimensional surface displacement.

Interference fringes are produced when the image of the deformed diffuse surface

is superimposed on the image formed on the hologram of the undeformed surface.

In the method suggested by Aleksandrov and Bonch-Bruevich,¹ the surface motion is described point-by-point on the surface. A phase change due to a path length difference at any point gives rise to interference fringes only at the same observed point on the surface when motion takes place between the initial and the final images. At any other point on the surface, the randomness of the scattered light from a diffuse surface produces no regular pattern of light reinforcement. A fringe line can, therefore, be thought of as a series of points on the surface of equal phase change.

By viewing the surface through a small enough aperture, the object surface and fringes can both be viewed in focus simultaneously. This insures that the observed fringes are a function only of the viewing angle and the spatial displacement, and makes the analysis of every point on the surface independent of every other point.

In practice, two methods may be employed in recording the fringe pattern. One method is the double exposure or

frozen fringes, in which one exposure is made of the object surface in its initial state on a rigidly fixed holographic plate. A second or double exposure is then made after the surface is in its final state. The result is a holographic plate on which can be seen the virtual image of the object and the interference fringes corresponding to a component of the spacial motion of the object surface. Three such holographic plates are made to obtain three displacement components from which the total spacial displacement vector may be determined.

The real-time or live fringe method requires more complex equipment, but is more convenient for counting and recording the fringes. In this method, the first exposure is made as before with the object surface in its initial state. With the holographic plate in place, the formation of the fringes can be observed as the surface is set in motion. With this method it is possible to count the fringes as they pass a certain point on the surface—a capability which is useful for the multiple-hologram method which will be discussed later.

Holographic Interferometry Theory

The essence of coherent light interferometry applied to displacement measurement is its ability to detect the number of fringes (phase changes) which represent the path length difference traversed by

two light rays or vectors. One light path is the traverse from the coherent light source S in Fig. 1 to the point O to the observation point P_1 , or simply SOP_1 . The second path is $SO'P_1$ where O' is the

1. E. B. Aleksandrov and A. M. Bonch-Bruevich, "Investigation of Surface Strains by the Hologram Technique," Soviet Phys. — Tech. Phys. **12**, 258 (1967).

*"Observation point" is used only as a term to define the points P , the center of the holographic plates. The actual observation is made by the eye or camera behind the holographic plate.

- S = Coherent light source
 P_1 = Observation point
 (Holographic plate 1)
 O = Initial state (surface)
 O' = Final state (surface)
 $\vec{d}$ = Displacement (surface)
 $\vec{R}_1$ = Position vector of P_1
 $\vec{k}_1, \vec{k}_2, \vec{a}_1, \vec{a}_2$ = unit vectors

Fig. 1. Basic light path geometry. Coherent light from source S is propagated along $\vec{k}_1$ and $\vec{a}_1$ and is scattered from surface at O and O' along scatter vectors $\vec{k}_2$ and $\vec{a}_2$ respectively. The path SOP_1 is for the initial surface position. Path $SO'P_1$ is for the final surface position. A similar diagram can be drawn for observation points P_2 and P_3 , omitted from the figure for clarity.

new position of point O . Ultimately the motion OO' , which is the displacement vector $\vec{d}$, can be determined.²

Path length SOP_1 can be written as

$$\delta_1 = \vec{k}_1 \cdot \vec{b}_1 + \vec{k}_2 \cdot (\vec{R}_1 - \vec{b}_1) \quad (1)$$

where the products are vector dot products and $\vec{k}_1$ (source vector) is the unit vector along the light source, $\vec{k}_2$ is the unit vector along which the reflection or scatter is observed, $\vec{b}_1$ is the vector locating point O , $\vec{R}_1$ is the vector locating the observation point P_1 , and $\delta_1 = n_1 \lambda$, where n_1 is the number of wavelengths and λ is the wavelength of light. The path length $SO'P_1$ can be written as

$$\delta_2 = \vec{a}_1 \cdot \vec{b}_2 + \vec{a}_2 \cdot (\vec{R}_1 - \vec{b}_2) \quad (2)$$

where $\vec{a}_1$ is the unit vector along the light source, $\vec{a}_2$ is the unit vector along which the scatter is observed, and $\vec{b}_2$ is the

vector locating the point O' . The length δ_2 is $\delta_2 = n_2 \lambda$, where n_2 is again the number of wavelengths. The path length difference can be written as

$$\delta_1 - \delta_2 = \vec{k}_1 \cdot \vec{b}_1 + \vec{k}_2 \cdot (\vec{R}_1 - \vec{b}_1) - \vec{a}_1 \cdot \vec{b}_2 - \vec{a}_2 \cdot (\vec{R}_1 - \vec{b}_2). \quad (3)$$

The phase change associated with surface interactions are ignored since they cancel out in Eq. (3). However, from Fig. 1

$$\vec{a}_1 = \vec{k}_1 + \vec{\Delta}_1$$

$$\vec{a}_2 = \vec{k}_2 + \vec{\Delta}_2$$

and

$$\delta_1 - \delta_2 = (n_1 - n_2) \lambda = N_1 \lambda$$

which substituted into Eq. (3) gives

$$N_1 \lambda = (\vec{k}_2 - \vec{k}_1) \cdot (\vec{b}_2 - \vec{b}_1) - \vec{\Delta}_1 \cdot \vec{b}_2 - \vec{\Delta}_2 \cdot (\vec{R}_1 - \vec{b}_2), \quad (4)$$

where N_1 is the difference in the number of wavelengths or the phase change which will manifest itself as fringes on an interferogram.

2. J. E. Sollid, "Holographic Interferometry Applied to Measurements of Small Static Displacements of Diffusely Reflecting Surfaces," Appl. Optics 8, 1587 (1969).

By insuring that $\vec{b}_1$ and $\vec{b}_2$ are much larger than $\vec{d}$, which is always the case, the vector $\vec{\Delta}_1$ is nearly perpendicular to $\vec{b}_1$ and $\vec{\Delta}_2$ is nearly perpendicular to $(\vec{R}_1 - \vec{b}_2)$, and their dot product terms,

$$\vec{\Delta}_1 \cdot \vec{b}_2 \approx 0,$$

$$\vec{\Delta}_2 \cdot (\vec{R}_1 - \vec{b}_2) \approx 0,$$

can be neglected in Eq. (4). Since the displacement vector is $\vec{d} = (\vec{b}_2 - \vec{b}_1)$, Eq. (4) can be written

$$N_1 \lambda = (\vec{k}_2 - \vec{k}_1) \cdot \vec{d}. \quad (5)$$

From this relationship we can determine $\vec{d}_1$, the displacement component of vector $\vec{d}$ along the vector $(\vec{k}_2 - \vec{k}_1)$. The difference vector $(\vec{k}_2 - \vec{k}_1)$ bisects the angle ψ between $\vec{k}_1$ and $-\vec{k}_2$ as shown in Fig. 2a, where the notation $\vec{F}_1$ is substituted for $(\vec{k}_2 - \vec{k}_1)$. In general, the angles between the unit scatter vectors $\vec{k}_2$, $\vec{k}_3$, and $\vec{k}_4$ and the source vector $\vec{k}_1$ are ψ_1 , ψ_2 and ψ_3 respectively. These angles as well as the difference vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ in the global x, y, z coordinate system are shown in Fig. 2b.

To determine the total vector $\vec{d}$, two more components $\vec{d}_1$ and $\vec{d}_2$ are needed. If two additional observation points P_2 and P_3 are established, then two additional equations may be written:

$$N_2 \lambda = (\vec{k}_3 - \vec{k}_1) \cdot \vec{d}, \quad (6)$$

$$N_3 \lambda = (\vec{k}_4 - \vec{k}_1) \cdot \vec{d}, \quad (7)$$

where the unit vectors $\vec{k}_2$ and $\vec{k}_3$ correspond to $\vec{k}_1$, and N_2 and N_3 correspond to N_1 .

Fig. 2. The scatter and difference vectors in the global reference system. (a) The component $\vec{d}_1$ of the displacement vector $\vec{d}$ lying along the difference vector $(\vec{k}_2 - \vec{k}_1)$ makes an angle $\psi_1/2$ with $\vec{k}_1$ and $\vec{k}_2$. (b) The relationship of the difference vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ with the source vector $\vec{k}_1$ and scatter vectors $\vec{k}_2$, $\vec{k}_3$, and $\vec{k}_4$ in the global x, y, z coordinate system.

MULTIPLE-HOLOGRAM METHOD

In the multiple-hologram method, a holographic plate is placed at each observation point and perpendicular to the unit vectors $\vec{k}_2$, $\vec{k}_3$, and $\vec{k}_4$. Substituting the notation $\vec{F}_i$ for the difference vectors (see Fig. 2) in Eqs. (5-7) gives

$$\vec{N}_1 \lambda = \vec{F}_1 \cdot \vec{d} = |\vec{F}_1| |\vec{d}| \cos \theta_1, \quad (8)$$

$$\vec{N}_2 \lambda = \vec{F}_2 \cdot \vec{d} = |\vec{F}_2| |\vec{d}| \cos \theta_2, \quad (9)$$

$$\vec{N}_3 \lambda = \vec{F}_3 \cdot \vec{d} = |\vec{F}_3| |\vec{d}| \cos \theta_3, \quad (10)$$

where θ_1 , θ_2 , and θ_3 are determined from the geometry, and are defined in Fig. 3. The above equations can be written as

$$d_1 = \frac{N_1 \lambda}{|\vec{F}_1|} = |\vec{d}| \cos \theta_1, \quad (11)$$

$$d_2 = \frac{N_2 \lambda}{|\vec{F}_2|} = |\vec{d}| \cos \theta_2, \quad (12)$$

$$d_3 = \frac{N_3 \lambda}{|\vec{F}_3|} = |\vec{d}| \cos \theta_3, \quad (13)$$

which give the magnitude of the components $\vec{d}_1$, $\vec{d}_2$, and $\vec{d}_3$ of $\vec{d}$ along the non-orthogonal coordinate system $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ respectively. The relationship of the component to the total displacement vector $\vec{d}$ is shown in Fig. 3. As long as the difference vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ are not coplanar, the displacement vector $\vec{d}$ is completely defined since the unit vectors $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_3$ and $\vec{k}_4$ are known and the fringe counts N_1 , N_2 , and N_3 are obtained from the interferograms. It should be noted that the fringe count is with

respect to the zeroth-order fringe which is a point on the surface that experiences no motion in the component direction corresponding to the hologram in question. It can be detected by the fact that it has no parallax. A more convenient way to get the fringe count is to count the fringes as they pass a point on the surface in the real-time method discussed earlier. This method also is capable of letting the initial state for the first exposure be at some pre-set strain (displacement). The fringe count for the pre-set strain condition must, of course, be known to determine the total absolute displacement.

An important point should be noted from Eqs. (11), (12) and (13). Since the difference vector $\vec{F}_1$ in Eq. (11), for example, lies along the bisector of the angle between $\vec{k}_2$ and $-\vec{k}_1$, the component along $\vec{F}_1$ will be zero and no fringes will form for a displacement $\vec{d}$ which is perpendicular to the bisector $\vec{F}_1$, since the cosine becomes zero. The implication here is that as both the illumination vector $\vec{k}_1$ and observation vector $\vec{k}_2$ become nearly normal to the direction of the displacement vector $\vec{d}$, the ability to detect that displacement decreases.

The vector $\vec{d}$ is known in terms of components along the general non-orthogonal coordinate system $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$. To determine the components of $\vec{d}$ in the orthogonal x, y, z global coordinate system shown in Fig. 3, the transformation matrix must first be determined.

The vector $\vec{d}$ can be defined by the intersection of the three planes normal to the vectors $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ and passing through the respective points defined by the components $\vec{d}_1$, $\vec{d}_2$ and $\vec{d}_3$. The three planes can be written as

$$(\vec{d} - \vec{d}_1) \cdot \vec{d}_1 = \vec{d} \cdot \vec{d}_1 - \vec{d}_1 \cdot \vec{d}_1 = 0, \quad (14)$$

Fig. 3. The difference vectors in the global system. The relationship of the cosine angles θ_1 , θ_2 , and θ_3 and the displacement vector $\vec{d}$ to the difference vectors $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ along which the components d_1 , d_2 , and d_3 lie is shown in some global coordinate system x, y, z .

$$(\vec{d} - \vec{d}_2) \cdot \vec{d}_2 = \vec{d} \cdot \vec{d}_2 - \vec{d}_2 \cdot \vec{d}_2 = 0, \quad (15)$$

$$(\vec{d} - \vec{d}_3) \cdot \vec{d}_3 = \vec{d} \cdot \vec{d}_3 - \vec{d}_3 \cdot \vec{d}_3 = 0. \quad (16)$$

Expanding these equations in terms of the x, y, z components gives

$$x_d d_{1i} + y_d d_{1j} + z_d d_{1k} = d_1^2, \quad (17)$$

$$x_d d_{2i} + y_d d_{2j} + z_d d_{2k} = d_2^2, \quad (18)$$

$$x_d d_{3i} + y_d d_{3j} + z_d d_{3k} = d_3^2, \quad (19)$$

where x_d , y_d and z_d are the magnitude of the components of $\vec{d}$ in the global x, y, z

Fig. 4. The displacement vector $\vec{d}'$ of point 0 defined by spherical coordinates ϕ and θ in the global x, y, z coordinate system.

coordinate system; d_{1i} , d_{1j} , d_{1k} , ..., d_{3k} are the components in the global system of the vectors $\vec{d}_1$, $\vec{d}_2$, and $\vec{d}_3$, and d_1 , d_2 , and d_3 are defined in Eqs. (11-13). By solving for x_d , y_d , and z_d the solution for the magnitude of $\vec{d}$ follows, and the displacement component in any other direction in the x, y, z system can easily be determined.

The magnitude of the displacement vector $\vec{d}'$ in any direction defined by ϕ , a rotation in the x, y plane, and θ , a rotation in a plane parallel to the z axis and passing through the origin shown in Fig. 4, can be found by taking the dot product

$$d' = \vec{d} \cdot \vec{u} \quad (20)$$

where $\vec{u}$ is the unit vector along the displacement $\vec{d}'$ and is defined by

$$\vec{u} = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k} \quad (21)$$

where u_1 , u_2 , and u_3 are the direction cosines. In terms of spherical coordinates

ϕ and θ , the direction cosines can be written as

$$\begin{aligned}u_1 &= \frac{\sin \theta \cos \phi}{D}, \\u_2 &= \frac{\sin \theta \sin \phi}{D}, \\u_3 &= \frac{\cos \theta}{D},\end{aligned}\quad (22)$$

where

$$D = \sqrt{\sin^2 \theta \cos^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \theta}.$$

SINGLE-HOLOGRAM METHOD

In the single-hologram method of measuring displacement the three observation or recording points P_1 , P_2 , and P_3 are obtained from three different points on the same holographic plate, eliminating the need of making two additional holograms. Any reference point chosen is inherently the same for every point on the hologram so that only the difference in fringe count that is observed in moving from one observation point to another is needed. Any combination of observation point unit

vectors from Eqs. (5-7) and a reference point, say P_5 with unit vector $\vec{k}_5$, is valid. Therefore, Eqs. (5-7) can be rewritten for the single-hologram method as

$$N_4 \lambda = (\vec{k}_2 - \vec{k}_5) \cdot \vec{d}, \quad (23)$$

$$N_5 \lambda = (\vec{k}_3 - \vec{k}_5) \cdot \vec{d}, \quad (24)$$

$$N_6 \lambda = (\vec{k}_4 - \vec{k}_5) \cdot \vec{d}, \quad (25)$$

where N_4 , N_5 , and N_6 are the fringe count difference as one moves from viewing along one unit vector to another. From this point on, the analysis to determine the displacement $\vec{d}$ is identical to that used in the multiple-hologram method.

One severe limitation in the single-hologram method is the inability to detect motion perpendicular to the hologram normal, as was pointed out earlier. For a holographic plate 4 in. square and a typical plate-to-object distance of 30 in., the included half-angle between unit vectors is 4 deg or a cosine of 0.105. This is the greatest sensitivity that can be achieved for motion parallel to the holographic plate.

Implementation of the Three-Hologram Technique

The multiple-hologram technique requires that the three holographic plates be rigidly mounted perpendicular to the position vectors for the first exposure and remain in the same position for real-time observation of the fringe formation as the deformation of the experimental object takes place. In this way it is possible to count the fringes formed from the initial state to the final state. Finally, a photograph is taken of the fringe pattern formed in the final deformed state.

The distance from the surface to the hologram need not be known, except to describe the unit vectors in the global x, y, z reference frame, but, as discussed earlier, the condition that $b_1 \gg \vec{d}$ must be met. This means that some reference dimension must be established in each hologram since each photograph taken of the fringes might be of different size and orientation. To correct for orientation and dimension, a short straight reference line of known orientation and length is

drawn on the surface being measured which will appear in each photograph.

The fringes and reference line in each interferograph (photograph of the interference fringes) are digitized in terms of x'' , y'' points in some local x'' , y'' coordinate system arbitrarily chosen within the plane of each interferogram, as shown in Fig. 5. Each fringe line is, therefore, defined in terms of a series of points along that line.

It is necessary that the digitized interference fringes for each of the three interferograms be superimposed on top of each other to establish the fringe count from each interferograph at any point on the deformed surface. To accomplish this, a common reference frame must be established into which every point defined in terms of the local x_1'' , y_1'' coordinate system on each interferograph must be mapped. A common reference frame chosen is the global x , y , z coordinate system.

MAPPING OF A FLAT EXPERIMENTAL SURFACE (PLANE)

For the case in which the experimental surface is flat and the reference line is drawn along the global x -axis, the first step in the mapping operation is to rotate the x_1'' , y_1'' axis an angle δ_1 so that the reference line $\vec{R}_f$ in Fig. 5b lies along the new local axis x_1' , y_1' and is given by

$$x' = x_1'' \cos \delta_1 + y_1'' \sin \delta_1, \quad (26)$$

$$y' = -x_1'' \sin \delta_1 + y_1'' \cos \delta_1, \quad (27)$$

or

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \begin{bmatrix} \cos \delta_1 & \sin \delta_1 \\ -\sin \delta_1 & \cos \delta_1 \end{bmatrix} \begin{bmatrix} x_1'' \\ y_1'' \end{bmatrix}, \quad (27a)$$

(b)

(a)

Fig. 5. The holographic plate at P_1 in the global x , y , z system. (a) The holographic plate is normal to the scatter vector $\vec{k}_1$. (b) Plan view of the holographic plate. The x_1' , y_1' coordinate system is rotated by an angle δ within the arbitrarily chosen local x_1'' , y_1'' system. Fringe digitization is made in the x_1'' , y_1'' system. The reference line R lies along the x_1' axis.

where δ_1 is the angle between the reference line as it appears on the interferograph and the arbitrarily chosen local x_1'' axis used for the digitization operation. The x_1'' , y_1'' plane and x_1' , y_1' plane both lie in the plane of the interferogram which is normal to $\vec{k}_1$.

The x_1' , y_1' coordinates' transformation into the x , y system is given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \end{bmatrix} \quad (28)$$

where

$$a_1 = \frac{1}{\cos \alpha_1}, \quad (29)$$

$$b_1 = \frac{1}{\cos \beta_1}, \quad (30)$$

and where α_1 is the angle between $\vec{k}_1$ and the yz plane and β_1 is the angle between $\vec{k}_1$ and the xz plane. The angles α_1 and β can be found by taking the dot product

$$\cos \alpha = \frac{\vec{k}_1 \cdot \vec{T}_{yz1}}{|\vec{k}_1| |\vec{T}_{yz1}|}, \quad (31)$$

$$\cos \beta = \frac{\vec{k}_1 \cdot \vec{T}_{xz1}}{|\vec{k}_1| |\vec{T}_{xz1}|}, \quad (32)$$

$$\alpha_1 = \cos^{-1} \left[\frac{\vec{k}_1 \cdot \vec{T}_{yz1}}{|\vec{k}_1| |\vec{T}_{yz1}|} \right], \quad (33)$$

$$\beta_1 = \cos^{-1} \left[\frac{\vec{k}_1 \cdot \vec{T}_{xz1}}{|\vec{k}_1| |\vec{T}_{xz1}|} \right], \quad (34)$$

where the vectors $\vec{T}_{yz}$ and $\vec{T}_{xz}$ lie in the y, z and x, z planes respectively and can be written in terms of $\vec{k}_1$ as

$$\vec{T}_{yz1} = k_{1y}\vec{i} + k_{1z}\vec{k} \quad (35)$$

$$\vec{T}_{xz1} = k_{1x}\vec{i} + k_{1z}\vec{k} \quad (36)$$

where k_{1x} , k_{1y} , and k_{1z} are the x , y , and z components of $\vec{k}_1$ in the global coordinate system.

Substituting Eq. (27) into (28) gives

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \begin{bmatrix} \cos \delta_1 & \sin \delta_1 \\ -\sin \delta_1 & \cos \delta_1 \end{bmatrix} \begin{bmatrix} x_1'' \\ y_1'' \end{bmatrix} \quad (37)$$

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 \cos \delta_1 & a_1 \sin \delta_1 \\ -b_1 \sin \delta_1 & b_1 \cos \delta_1 \end{bmatrix} \begin{bmatrix} x_1'' \\ y_1'' \end{bmatrix}. \quad (38)$$

Equation (38) gives the transformation matrix to map the points digitized from the interferograph taken from observation point P_1 on a plane experimental surface into a common x, y coordinate system. In general, the common coordinate system can be defined as $x_m y_m$, which does not necessarily correspond to the global x, y coordinate system. A similar equation can be written for the scatter vectors $\vec{k}_2$ and $\vec{k}_3$ by permuting the subscripts.

MAPPING OF A SPHERICAL SURFACE

The transformation matrix will have to be modified, of course, for complex surfaces. If the common coordinate system $x_m y_m$ is defined as the plane tangent to the experimental surface at a point O in the center of the experimental surface, then the fringes are correctly defined by Eq. (38) for an experimental plane surface.

However, if the experimental surface deviates from a plane, the fringes are actually the projections of the experimental surface onto the $x_m y_m$ plane along the scatter vector. The degree of surface distortion is, therefore, a function of deviation of the experimental surface from the $x_m y_m$ plane. For the case of a spherical experimental surface, the mapping onto the $x_m y_m$ plane is well defined. The projection of the spherical surface onto the $x_m y_m$ plane along a line normal to the $x_m y_m$ plane is chosen as the common basis. This is established by determining the new x_m', y_m' coordinates

Fig. 6. The projection of the spherical surface onto the plane. The vector $O_1 P_1$ passes through the $x_m y_m$ plane at O_1 and intersects the spherical surface at O , establishing a projection of the spherical surface on the $x_m y_m$ plane along the z_m axis.

formed when the scatter vector passes through the $x_m y_m$ coordinates at O_1 and intersects the spherical surface at O , as shown in Fig. 6. The equation for a sphere is

$$x_m^2 + y_m^2 + (z_m - z_{m0})^2 = r^2 \quad (39)$$

where r is the spherical radius and the center of the sphere is at $(0, 0, z_{m0})$.

The equation of the scatter vector passing through O_1 is

$$\frac{(x_m - x_{m1})}{\ell} = \frac{(y_m - y_{m1})}{m} = \frac{z_m}{n}, \quad (40)$$

or solving for z_m and y_m in terms of x_m gives

$$z_m = \frac{n}{\ell} (x_m - x_{m1}), \quad (41)$$

$$y_m = \frac{m}{\ell} (x_m - x_{m1}) + y_{m1}, \quad (42)$$

where ℓ , m , and n are the direction cosines and x_{m1} and y_{m1} are the coordinates of point O_1 . Substituting Eqs. (41) and (42) into (39) gives

$$ax_m^2 + bx_m + c = 0 \quad (43)$$

where

$$a = 1 + \frac{n^2}{\ell^2} + \frac{m^2}{\ell^2},$$

$$b = \frac{2}{\ell} \left[my_{m1} - nz_{m0} - \frac{x_{m1}}{\ell} (m^2 + n^2) \right],$$

$$c = z_{m0}^2 + y_{m1}^2 + \frac{x_{m1}^2}{\ell^2} (m^2 + n^2)$$

$$+ \frac{2z_{m0}n}{\ell} - \frac{2my_{m1}x_{m1}}{\ell} - r^2,$$

and where z_{m0} refers to point O on the spherical surface. Solving for x_m which is now designated x'_m gives

$$x'_{m1} = x_m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (44)$$

Substituting Eq. (44) into (42) gives the value of y'_m . The coordinates $x'_m y'_m$

now establish a common system so that the fringe count for any point on the experimental surface can be determined by simply referring to the same x'_m, y'_m point on each interferograph.

MAGNIFICATION CORRECTION

After the mapping is completed for each of the three interferographs, the reference lines may not all be of equal length since the interferographs taken at the observation points are not necessarily at equal distance from the surface. This means that the picture of the object and fringes, and therefore the reference lines, will be of different sizes. Furthermore, the location of the reference line with respect to the arbitrary digitizing axes x'', y'' is not the same in each interferograph. It is necessary to enlarge or decrease as well as to translate the digitized information so that it corresponds with one of the interferograms. In this case, the interferograms at observation points 2 and 3 are made to correspond to the interferogram at point 1.

ANGULAR CORRECTION

The scatter vector must be defined for some point O on the experimental surface, as shown in Fig. 2b. For a different point O on the surface, the scatter vector as well as the incident vector change and must, therefore, be recalculated for every point on the surface being tested. In some cases, however, using constant scatter and incident vectors, as defined by some known point O_0 at the center of the surface, produces very little error as the point O is moved over the surface. It is of interest, however, to first look at a varying k.

The calculation of the variation in the scatter and incident vectors with point O can easily be made, but requires some knowledge of the surface being tested. For a spherical surface, for example, the change in viewing angle $\Delta\gamma$ in moving from point O_0 to point O in the i, j plane, shown in Fig. 7 is directly related to the rotation of the spherical radius r through an angle ω .

The position vector $\vec{k}$ defined here as the vector $R_1 - b_1$ shown in Fig. 1 becomes $\vec{k}'$ at O and can be defined as

$$\vec{k} = k_x \vec{i} + k_y \vec{j} \quad (45)$$

$$\vec{k}' = \vec{k} + \vec{v}_1 + \vec{v}_2 \quad (46)$$

where k_x and k_y are the vector components of the unit vectors $\vec{i}$ and $\vec{j}$ respectively in some rectangular coordinate system, and

$$\vec{v}_1 = (r - r \cos \omega) \vec{j} \quad (47)$$

$$\vec{v}_2 = -(r \sin \omega) \vec{i}. \quad (48)$$

Fig. 7. The scatter vector relationship to the angle $\Delta\gamma$. The scatter vector becomes $\vec{k}'$ as $\vec{k}$ traverses the angle $\Delta\gamma$ from point O_0 to point O.

Substituting Eqs. (45), (47), and (48) into (46) gives

$$\vec{k}^i = (k_x - r \sin \omega) \vec{i} + (k_y + r - r \cos \omega) \vec{j}. \quad (49)$$

Taking the dot product gives

$$\vec{k} \cdot \vec{k}^i = |\vec{k}| |\vec{k}^i| \cos \Delta\gamma, \quad (50)$$

and finally, solving for the inverse cosine the angle is

$$\Delta\gamma = \cos^{-1} \left(\frac{\vec{k} \cdot \vec{k}^i}{|\vec{k}| |\vec{k}^i|} \right). \quad (51)$$

The relationship of ω and the i, j plane to the x, y, z global coordinate system is obtained from the mapping of a spherical surface onto a flat plane. However, for

Fig. 8. The displacement profile of a sphere displaced in the negative x direction. The outline is the common area among the three interferograms.

Fig. 9. Fringe matrix which describes each fringe in the P_1 interferogram as a number in a grid of 60×60 numbers.

an angle of $\gamma = 60$ deg and a ratio $|\vec{k}|/r = 9$, which is the ratio of the distance between point O_0 and P to spherical radius r , the angular change $\Delta\gamma$ is less than 3 deg and results in an error of not more than 3% in Eqs. (11), (12), and (13). There is justification, therefore, for using a

constant value for γ which is calculated for point O_0 , the center point of the field of interest on the experimental object.

HIDI COMPUTER PROGRAM

The HIDI (Holographic Interferometry Displacement Interpretation) computer

program* solves the three-hologram and the single-hologram problem. To use the single-hologram method, some minor changes must be made within the computer program.

The HIDI computer program is written in Fortran IV and compiles on the Chat compiler only. The input is the digitized

*For further information concerning the program or to obtain a copy of the program, contact the author.

information from tape or cards generated by the Eyeball machine or some digitizer. The output is both a quantitative value of displacement as well as a qualitative interpretation of the displacement field in the form of iso-displacement lines or contour lines, as shown in Fig. 8 for a solid-body motion. The program runs very fast, using only a few seconds for a 100×100 grid of the type shown in Fig. 9.

Experimental Example

The multiple hologram method was used to experimentally determine a predetermined solid-body displacement of a spherical ball. The experimental setup, shown in Fig. 10, was mounted on a rigid table well isolated to eliminate any external vibration motion. A He-Ne laser with a $24.91\text{-}\mu\text{m}$. wavelength was used as the coherent light source for the reference beams $\vec{L}_1$, $\vec{L}_2$, and $\vec{L}_3$ and the illumination source beam k_1 . Diverging lenses were used to obtain spherical wave fronts since large-diameter light collimators were not available.

The holograms were mounted normal to the hologram position vectors $\vec{R}_1$, $\vec{R}_2$, and $\vec{R}_3$ shown in Fig. 11 where $\vec{R}_1$, $\vec{R}_2$, and the source vector $\vec{S}$ are in a plane parallel to the table and passes through the center of the ball. The vector $\vec{R}_3$ is rotated 11° upward from $\vec{R}_1$. The vectors $\vec{R}_1$ and $\vec{R}_2$ are rotated 30° from $\vec{S}$. The ball, coated with a material to give a diffuse surface, is mounted on an air bearing which allows a predetermined motion of 0.001 ± 0.0001 in. in the negative x direction only. The xy plane, shown

Fig. 10. Plan view of the experimental setup of the three-hologram method.

in Fig. 11, coincides with the $\vec{R}_1$ $\vec{R}_2$ plane, and the z axis is normal and directed upward from this plane.

Fig. 11. Schematic of the hologram position vector geometry in the global reference frame for an experiment in which a ball is moved 0.0001 in. in the negative x direction. The z axis extends upward from the center of the ball. The center of the ball, P_1 , S , and P_2 are in the x-y plane.

A photograph of the interference fringes was taken of the holographic plate at each of the observation points P_1 , P_2 , and P_3 , and are shown in Fig. 12. The fringe count was established by counting the fringes as they formed when the ball was moved 0.001 in. in the negative x direction. The origins of the local $x'' y''$ coordinate system used to digitize the fringes were arbitrarily set at the corners of the photograph shown in Fig. 12. The reference lines in the figure were drawn in only for clarity. Actually all that is necessary is to mark the ends of the lines with an x.

The digitized fringes are converted in the computer program into a fringe matrix

Fig. 12. The interferograms as seen from three observation points of a ball displaced 0.001 in. in the negative x direction.

of numbers shown in Fig. 9 for P_1 , which is a grid of 60×60 numbers. Increasing the grid density will round the corners of the lines, but will not increase the number of lines for increased displacement resolution since that is controlled by the total number of fringes actually

present and the separation between fringes.

The resulting displacement profile shown in Fig. 8 is a single line with a magnitude of 0.000969 in., indicating that every point on the body experienced the same displacement. The outline circumscribes an area common to the fringe matrix of each interferograph. The zeros in Fig. 9 indicate areas where no fringes exist.

This result varies from the known displacement by 0.001 - 0.000969 = 0.00031 in., or 3%. Considering that the displacement is known to a tolerance of $\pm 10\%$, the holographic interferometry displacement measuring technique as calculated using the HIDI computer pro-

gram gives results with accuracy which are higher than other displacement measurement techniques for displacements up to several mills. Obtaining this kind of accuracy in displacement ranges below 0.001 in. is what makes this technique useful.

It is no problem to apply holographic interferometry to displacement measurement of surfaces which are given solid-body motion as well as local surface deformation since the fringes caused by solid-body motion are straight lines which can be neglected in the fringe count. The fringes due to local deformation are easily identified as perturbations in the field of straight lines.

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NOTICE

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