MASTER

FUNDAMENTAL INVESTIGATION OF LIQUID- METAL LUBRICATED JOURNAL BEARINGS

Quarterly Technical Report No. 4
Contract No. AT (11-1)-1228
Project Agreement No. 8
Modification No. 7

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Department of Aerospace Propulsion Research

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APPROVED:

P. M. Ku, Director
Department of Aerospace
Propulsion Research

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FOREWORD

This report was prepared by Southwest Research Institute under Contract No. AT(11-1)-1228, Project Agreement No. 8, Modification No. 7. The work was administered by the Engineering Development Branch, Division of Reactor Development, U. S. Atomic Energy Commission, Washington 25, D. C. with Mr. Nicholas Grossman as Project Engineer.

This report covers work performed from April 1, 1965 through July 31, 1965.

ABSTRACT

The momentum and continuity equations for a turbulent fluid are integrated across a thin film with arbitrary thickness variation. Velocity profile and friction parameters required for the solution of these equations are calculated for the power-law approximation. The results are applied to calculation of pressures in a short, full journal bearing and a tilted pad without end leakage, showing good agreement with experimental measurements.

Experimental velocity profile data are reported which should help in future improvement of the computations.

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LIST OF SYMBOLS

a	Slider bearing slope
b	Local half-thickness of film
$\bar{b}$	Mean half-thickness of film
C_f	Friction coefficient
C_x	Friction coefficient in tangential direction
c	Radial bearing clearance
D	Journal diameter
e	Eccentricity
H	Dimensionless film thickness
h	Local film thickness
$\bar{h}$	Mean film thickness
h_c	Film thickness at point where only Couette flow prevails
k	Exponent in power-law friction-coefficient equation
L	Bearing length in axial direction
ℓ	Pad bearing length in tangential direction
m	Velocity profile exponent for axial flow
n	Velocity profile exponent for tangential flow
P	Dimensionless pressure, $\frac{p}{\rho(u_s/2)^2}$
p	Gage pressure
p_a	Ambient pressure
R	Reynolds number based on local film thickness, $b u_b / \nu$
$\bar{R}$	Reynolds number based on mean film thickness, $\bar{b} u_s / 2\nu$, $\bar{b} u_{bc} / \nu$

LIST OF SYMBOLS (Cont'd)

r	Journal radius
U	See Equation (69)
U^+	Dimensionless velocity
u	Velocity in tangential direction
V	Velocity vector, $\sqrt{u^2 + w^2}$
v	Velocity across film
W	See Equation (69)
w	Velocity in axial direction
X	See Equations (69) and (118)
x	Coordinate in tangential direction
Y^+	Dimensionless wall distance
y	Coordinate across film
Z	Dimensionless axial coordinate
z	Axial coordinate
β	$\bar{h}/r$
δ^*	Displacement thickness [See Eq. (112)]
ϵ	Eccentricity ratio
ζ	Dimensionless coefficient in integrated turbulent film equation
μ	Absolute viscosity
ν	Kinematic viscosity
ξ	See Equation (120)
ρ	Fluid density
τ	Shear stress
ϕ	Angular coordinate on bearing (See Fig. 5)

LIST OF SYMBOLS (Cont'd)

ψ	Attitude angle (See Fig. 5)
χ	See Equation (121)
$()_b$	Refers to midchannel
$()_{bc}$	Refers to Couette flow at midchannel
$()_{bh}$	Refers to midchannel velocity relative to stationary wall
$()_{bo}$	Refers to midchannel velocity relative to moving wall
$()_c$	Refers to Couette flow
$()_h$	Refers to stationary wall
$()_o$	Refers to moving wall
$()_s$	Refers to surface velocity of journal

INTRODUCTION

This report may be considered to supplement work covered in the Topical Report, SwRI-1228-60, UC-80, Reactor Technology TID-4500 (40th Edition). The principal emphasis is on extension of theoretical and experimental work reported therein. The theoretical effort has been devoted to detailed development of a general approach to turbulent film bearing calculations, and to the development of tables for use with this. Applying this approach, computations have been undertaken for the special cases of (a) full, short journal bearings and (b) tilted pads without end leakage. These are compared with experimental results. Other experimental work is reported where the velocity profile variations at and beyond transition are measured, as well as pressure variations in certain situations.

FORMALIZATION OF THIN BEARING FILM WITH END LEAKAGE

The Fundamental Equations of an Incompressible Turbulent Fluid

The momentum and continuity equations for an incompressible fluid under a steady-state condition are⁽¹⁾:

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} = & - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} \right) \\ & - \left(\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right) \end{aligned} \quad (1)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} = & - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} + \nu \left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2} \right) \\ & - \left(\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'^2}}{\partial y} + \frac{\partial \overline{v'w'}}{\partial z} \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} = & - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \nu \left(\frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{\partial^2 \bar{w}}{\partial z^2} \right) \\ & - \left(\frac{\partial \overline{u'w'}}{\partial x} + \frac{\partial \overline{v'w'}}{\partial y} + \frac{\partial \overline{w'^2}}{\partial z} \right) \end{aligned} \quad (3)$$

and

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \quad (4)$$

In the above equations, the quantities with superscript bar and prime indicate time average and fluctuating components, respectively.

Let the bearing film thickness lie along the y-axis. For very thin film,

the simplifications in view of order of magnitude may be made as follows⁽²⁾:

$$\frac{\partial^2 \bar{u}}{\partial x^2}, \frac{\partial^2 \bar{u}}{\partial z^2}, \frac{\partial^2 \bar{w}}{\partial x^2}, \frac{\partial^2 \bar{w}}{\partial z^2} \ll \frac{\partial^2 \bar{u}}{\partial y^2}, \frac{\partial^2 \bar{w}}{\partial y^2} \quad (5)$$

$$\frac{\partial \overline{u'^2}}{\partial x}, \frac{\partial \overline{u'w'}}{\partial z}, \frac{\partial \overline{u'w'}}{\partial x}, \frac{\partial \overline{w'^2}}{\partial z}, \frac{\partial \overline{u'v'}}{\partial x}, \frac{\partial \overline{v'w'}}{\partial z} \ll \frac{\partial \overline{u'v'}}{\partial y}, \frac{\partial \overline{v'w'}}{\partial y}, \frac{\partial \overline{v'^2}}{\partial y} \quad (6)$$

$$\bar{v} = 0 \quad (7)$$

and

$$\frac{\partial \bar{p}}{\partial y} = 0 \quad (8)$$

By means of Equations (5) to (8) and omitting superscript bar over the quantities, Equations (1) to (4) then reduce to

$$\rho \left[u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right] = - \frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial y} \quad (9)$$

$$\rho \left[u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \right] = - \frac{\partial p}{\partial z} + \frac{\partial \tau_z}{\partial y} \quad (10)$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (11)$$

where τ_x and τ_z are shearing stresses in the x - and z -directions, respectively.

Making use of Equation (11), the following identities may be obtained as

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \equiv \frac{\partial u^2}{\partial x} + \frac{\partial(uw)}{\partial z} \quad (12)$$

and

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \equiv \frac{\partial w^2}{\partial z} + \frac{\partial(uw)}{\partial x} \quad (13)$$

Substituting Equations (12) and (13) into Equations (9) and (10), respectively, and integrating them across the film thickness $h = h(x, z)$, the momentum equations may be simplified to

$$\rho \left[\int_0^h \frac{\partial u^2}{\partial x} dy + \int_0^h \frac{\partial(uw)}{\partial z} dy \right] = - \frac{\partial p}{\partial x} h + \tau_{xh} - \tau_{x0} \quad (14)$$

$$\rho \left[\int_0^h \frac{\partial w^2}{\partial z} dy + \int_0^h \frac{\partial(uw)}{\partial x} dy \right] = - \frac{\partial p}{\partial z} h + \tau_{zh} - \tau_{zo} \quad (15)$$

where

$$\tau_{xh} \equiv \nu \left. \frac{\partial u}{\partial y} \right|_{y=h} - \rho \overline{u'v'} \Big|_{y=h} \quad (16)$$

$$\tau_{xo} \equiv \nu \left. \frac{\partial u}{\partial y} \right|_{y=0} - \rho \overline{u'v'} \Big|_{y=0} \quad (17)$$

$$\tau_{zh} \equiv \nu \left. \frac{\partial w}{\partial y} \right|_{y=h} - \rho \overline{v'w'} \Big|_{y=h} \quad (18)$$

and

$$\tau_{zo} \equiv \nu \left. \frac{\partial w}{\partial y} \right|_{y=0} - \rho \overline{v'w'} \Big|_{y=0} \quad (19)$$

Similarly, integrating Equation (11) across the film thickness, the continuity equation is obtained:

$$\int_0^h \frac{\partial u}{\partial x} dy + \int_0^h \frac{\partial w}{\partial y} dy = 0 \quad (20)$$

Three equations [Eqs. (14), (15) and (20)] with six dependent variables (u, w, p, u', v' and w') are just impossible to deal with. Consequently, further simplification of these three equations is necessary. In terms of three dependent variables ($\Delta u_b, w_b$ and p), three film equations can be handled either numerically or analytically.

Further Simplification of Equations (14) to (20)

Because of the emphasis upon friction coefficient (C_{xc}) and velocity profile parameters (n in x -direction and m in z -direction) in the

experimental program, it has been found desirable to carry through an analytical treatment of thin bearing film, in terms of these three quantities (n , m and C_{xc}). Introduce the definition

$$C_f = \frac{2|\tau|}{\rho u_b^2} \quad (21)$$

such that C_f is defined relative to the wall on which the shear stress is to be measured, and u_b is defined as midchannel velocity relative to that wall.

Let $h = h(x)$ and have continuous derivatives for $x_1 < x < x_2$. In addition, let $u^2(x, z)$, $w^2(x, z)$, and wu be continuous and have continuous derivatives $\frac{\partial u^2}{\partial x}$, $\frac{\partial w^2}{\partial z}$, $\frac{\partial uw}{\partial x}$, $\frac{\partial uw}{\partial z}$ in a domain of the x - z plane. Let the bearing problem be a case of $u_o = u_s$, $u_h = 0$, $w_o = 0$ and $w_h = 0$. Then, by means of Leibnitz's rule, Equations (14), (15) and (20) may be written as

$$\begin{aligned} \rho \left[\left\{ \frac{\partial}{\partial x} \int_0^{h(x)} u^2 dy - u_h^2 \frac{\partial h}{\partial x} \right\} + \left\{ \frac{\partial}{\partial z} \int_0^{h(x)} w u dy - w_h u_h \frac{\partial h}{\partial z} \right\} \right] \\ = - \frac{\partial p}{\partial x} h + \tau_{xh} - \tau_{xo} \end{aligned} \quad (22)$$

$$\begin{aligned} \rho \left[\left\{ \frac{\partial}{\partial z} \int_0^{h(x)} w^2 dy - w_h^2 \frac{\partial h}{\partial z} \right\} + \left\{ \frac{\partial}{\partial x} \int_0^{h(x)} u w dy - u_h w_h \frac{\partial h}{\partial x} \right\} \right] \\ = - \frac{\partial p}{\partial z} h + \tau_{zh} - \tau_{zo} \end{aligned} \quad (23)$$

$$\frac{\partial}{\partial x} \int_0^h u dy - u_h \frac{\partial h}{\partial x} + \frac{\partial}{\partial z} \int_0^h w dy - w_h \frac{\partial h}{\partial z} = 0 \quad (24)$$

Because $h = h(x)$ and $u_h = w_h = 0$, Equations (22) to (24) are reduced to:

$$\rho \left[\frac{\partial}{\partial x} \int_0^h u^2 dy + \frac{\partial}{\partial z} \int_0^h w u dy \right] = - \frac{\partial p}{\partial x} h + \tau_{xh} - \tau_{xo} \quad (25)$$

$$\rho \left[\frac{\partial}{\partial z} \int_0^h w^2 dy + \frac{\partial}{\partial x} \int_0^h u w dy \right] = - \frac{\partial p}{\partial z} h + \tau_{zh} - \tau_{zo} \quad (26)$$

and

$$\frac{\partial}{\partial x} \int_0^h u dy + \frac{\partial}{\partial z} \int_0^h w dy = 0 \quad (27)$$

Introducing dimensionless coefficients, affected by turbulent velocity profile parameters, and friction coefficient, Equations (25) to (27) may be written as

$$\frac{\partial}{\partial x} (u_b^2 h \zeta_1) + \frac{\partial}{\partial z} (u_b w_b h \zeta_2) = - \frac{1}{\rho} \frac{\partial p}{\partial x} h + \frac{1}{\rho} \Delta \tau_x \quad (28)$$

$$\frac{\partial}{\partial z} (w_b^2 h \zeta_3) + \frac{\partial}{\partial x} (w_b u_b h \zeta_4) = - \frac{1}{\rho} \frac{\partial p}{\partial z} h + \frac{1}{\rho} \Delta \tau_z \quad (29)$$

and

$$\frac{\partial}{\partial x} (u_b h \zeta_7) + \frac{\partial}{\partial z} (w_b h \zeta_8) = 0 \quad (30)$$

where

$$|u_{bh}| = |u_{bc} + \Delta u_b| \quad (31)$$

$$|u_{bo}| = |-u_{bc} + \Delta u_b| \quad (32)$$

$$|w_{bh}| = |w_{bo}| \equiv |w_b| \quad (33)$$

$$\begin{aligned} |V_{bh}| &= \left[(u_{bh})^2 + (w_{bh})^2 \right]^{1/2} \\ &= u_{bc} \cdot \left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{1/2} \end{aligned} \quad (34)$$

$$\begin{aligned} |V_{bo}| &= \left[(u_{bo})^2 + (w_{bo})^2 \right]^{1/2} \\ &= u_{bc} \cdot \left[\left(1 - \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{1/2} \end{aligned} \quad (35)$$

$$|\tau_h| = C_{fh\rho} \frac{(V_{bh})^2}{2} \quad (36)$$

$$|\tau_o| = C_{fo\rho} \frac{(V_{bo})^2}{2} \quad (37)$$

$$\begin{aligned} C_{fh} &= C_{xc} \left(\frac{R_{bc}}{R_{bh}} \right)^k \\ &= C_{xc} \left(\frac{u_{bc}}{|V_{bh}|} \right)^k \end{aligned} \quad (38)$$

$$C_{fo} = C_{xc} \left(\frac{u_{bc}}{|V_{bo}|} \right)^k \quad (39)$$

$$|\tau_{xh}| = C_{fh\rho} \cdot \frac{|V_{bh}|^2}{2} \cdot \frac{|u_{bh}|}{|V_{bh}|} \quad (40)$$

$$|\tau_{xo}| = C_{fo\rho} \cdot \frac{|V_{bo}|^2}{2} \cdot \frac{|u_{bo}|}{|V_{bo}|} \quad (41)$$

$$|\tau_{zh}| = C_{fh\rho} \cdot \frac{|V_{bh}|^2}{2} \cdot \frac{|w_{bh}|}{|V_{bh}|} \quad (42)$$

$$|\tau_{zo}| = C_{fo\rho} \cdot \frac{|V_{bo}|^2}{2} \cdot \frac{|w_{bo}|}{|V_{bo}|} \quad (43)$$

$$\zeta_1 = \frac{\int_0^h u^2 dy}{u_b^2 h} \quad (44)$$

$$\zeta_2 \equiv \zeta_4 = \frac{\int_0^h wudy}{w_b u_b h} \quad (45)$$

$$\zeta_3 = \frac{\int_0^h w^2 dy}{w_b^2 h} \quad (46)$$

$$\zeta'_7 = \frac{\int_0^h u dy}{u_b h} \quad (47)$$

and

$$\zeta'_8 = \frac{\int_0^h w dy}{w_b h} \quad (48)$$

The Difference of Wall Shears, $\Delta\tau_x$, in the Tangential Direction for Codirectional Couette and Pressure Flows, with Wall Shear with Same Signs (Couette Flow Dominating)

Refer to Figure 1 where relative velocities u_{bh} and u_{bo} are defined.

By definition, the quantity $\Delta\tau_x$ may be written as

$$\Delta\tau_x = -|\tau_{xh}| - (-1)|\tau_{xo}| \quad (49)$$

Substituting Equations (31), (32), (34), (35), (38) to (41) into (49), $\Delta\tau_x$ is found to be

$$\Delta\tau_x = -2\rho\zeta_5 C_{xc} u_{bc} \Delta u_b \quad (50)$$

where

$$\zeta_5 = \frac{A - B}{4U} \quad (0 \leq |U| \leq 1) \quad (51)$$

$$A \equiv \left| 1 + \frac{\Delta u_b}{u_{bc}} \right| \cdot \left| \left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{\frac{1-k}{2}} \right| \quad (52)$$

and

$$B \equiv \left| -1 + \frac{\Delta u_b}{u_{bc}} \right| \cdot \left| \left[\left(1 - \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{\frac{1-k}{2}} \right| \quad (53)$$

When $U \equiv \frac{\Delta u_b}{u_{bc}} \rightarrow 0$, the limit of ζ_5 in Equation (51) is:

(a)

(b)

FIGURE 1. WALL SHEAR WITH SAME SIGNS
(COUETTE EFFECT DOMINATING)

$$\lim_{U \rightarrow 0} \zeta_5 = \left(1 - \frac{k}{2}\right) \cdot \left| (1 + W^2)^{\frac{1-k}{2}} - \frac{(1-k)W^2}{2} \cdot \left| (1 + W^2)^{\frac{-(1+k)}{2}} \right| \right| \quad (54)$$

The Difference of Wall Shears, $\Delta\tau_x$, in the Tangential Direction for Codirectional Couette and Pressure Flows, with Wall Shear with Opposite Signs (Pressure Flow Dominating)

Lower Surface Positive Shear, Upper Surface Negative Shear

As shown in Figure 2, wall shears appear to be opposite in sign.

The definition of u_{bh} , u_{bo} remains the same as in Figure 1(a). Similarly,

$\Delta\tau_x$ is

$$\begin{aligned} \Delta\tau_x &= -|\tau_{xh}| - |\tau_{xo}| \\ &= -2\rho\zeta_5 C_{xc} u_{bc} \Delta u_b \end{aligned} \quad (55)$$

where

$$\zeta_5 = \frac{A + B}{4U} \quad (1 \leq U) \quad (56)$$

such that A and B in Equation (56) are in the same form as in Equations (52) and (53).

Lower Surface Negative Shear, Upper Surface Positive Shear

Using the definition of $\Delta\tau_x$ and taking the sign of τ_{xh} and τ_{xo} into account, $\Delta\tau_x$ is (see Figure 3)

$$\begin{aligned} \Delta\tau_x &= |\tau_{xh}| - (-1)|\tau_{xo}| \\ &= -2\rho\zeta_5 C_{xc} u_{bc} \Delta u_b \end{aligned} \quad (57)$$

where

$$\zeta_5 = -\frac{(A + B)}{4U} \quad (U \leq -1) \quad (58)$$

FIGURE 2. COUETTE AND PRESSURE FLOW EFFECTS IN THE SAME DIRECTION FOR THE CASE OF U SUCH THAT $1 \leq U$

FIGURE 3. COUETTE AND PRESSURE FLOW EFFECTS IN THE OPPOSITE DIRECTION FOR THE CASE OF U SUCH THAT $U \leq -1$

such that A and B in Equation (58) are in the same form as in Equations (52) and (53).

Remarks

For the above three different cases, ζ_5 is obtained by Equations (51), (56) and (58). Each form for ζ_5 in Equations (51), (56) and (58) will be chosen in a corresponding bearing problem. Although $\Delta\tau_x$ in Equations (50), (55) and (57) is in the same form, the value of ζ_5 in Equations (51), (56) and (58) is different. Consequently, $\Delta\tau_x$ corresponding to each case is different.

The Difference of Wall Shears, $\Delta\tau_z$, in the Axial Direction for Either Positive z-Direction or Negative z-Direction Flow

No matter how the tangential flow varies, the velocity profile of w can always be assumed as a turbulent power law velocity profile symmetrical about $y = b$ as shown in Figure 4.

In the positive z-direction, $\Delta\tau_z$

$$\Delta\tau_z = -\left|\tau_{zh}\right| - \left|\tau_{zo}\right| \quad (59)$$

Substituting Equations (31) to (35), (38), (39), (42) and (43) into Equation (59), $\Delta\tau_z$ is obtained as

$$\Delta\tau_z = -\frac{C_{xc}\zeta_6}{2} \rho u_{bc} w_b \quad (60)$$

where

$$\zeta_6 = E + F \quad (61)$$

$$E = \left| \left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{\frac{1-k}{2}} \right| \quad (62)$$

FIGURE 4. TURBULENT VELOCITY PROFILE IN AXIAL DIRECTION

and

$$F = \left| \left[\left(1 - \frac{\Delta u_b}{u_{bc}} \right)^2 + \left(\frac{w_b}{u_{bc}} \right)^2 \right]^{\frac{1-k}{2}} \right| \quad (63)$$

In the negative z-direction, $\Delta\tau_z$ is

$$\begin{aligned} \Delta\tau_z &= \left| \tau_{zh} \right| - (-1) \left| \tau_{zo} \right| \\ &= - \frac{C_{xc} \zeta_6}{2} \rho u_{bc} w_b \end{aligned} \quad (64)$$

where

$$\zeta_6 = E + F \quad (65)$$

Comparing Equations (60) and (61) with (64) and (65), ζ_6 remains the same in either the positive or negative z-direction.

The Simplified Fundamental Thin Film Equations of Incompressible Turbulent Fluid

Considering Equations (50), (55), (57) and (60), and neglecting

small quantities of $\frac{\partial \zeta_1}{\partial x}$, $\frac{\partial \zeta_2}{\partial z}$, $\frac{\partial \zeta_3}{\partial z}$, $\frac{\partial \zeta_4}{\partial x}$, $\frac{\partial(\zeta'_7)}{\partial x}$, and $\frac{\partial(\zeta'_8)}{\partial z}$, Equations (28) to

(30) may be simplified to

$$\begin{aligned} \frac{\zeta_1}{\zeta_2} \left[2(1+U) \frac{\partial U}{\partial X} + \frac{(dH/dX)}{H} (1+U)^2 \right] + W \frac{\partial U}{\partial Z} + (1+U) \frac{\partial W}{\partial Z} \\ = - \frac{1}{\zeta_2} \frac{\partial P}{\partial X} - \frac{2\zeta_5}{H\zeta_2} C_{xc} U \end{aligned} \quad (66)$$

$$\begin{aligned} \frac{2\zeta_3}{\zeta_2} W \frac{\partial W}{\partial Z} + (1+U) \frac{\partial W}{\partial X} + W \frac{\partial U}{\partial X} + \frac{(dH/dX)}{H} W (1+U) \\ = - \frac{1}{\zeta_2} \frac{\partial P}{\partial Z} - \zeta_6 C_{xc} \frac{W}{H} \end{aligned} \quad (67)$$

$$\frac{(dH/dX)}{H} (1+U) + \frac{\partial U}{\partial X} + \zeta_7 \frac{\partial W}{\partial Z} = 0 \quad (68)$$

where

$$\frac{p}{\rho u_{bc}^2} = P, \quad \frac{\Delta u_b}{u_{bc}} = U, \quad \frac{w_b}{u_{bc}} = W, \quad \frac{h}{\bar{h}} = H, \quad \frac{x}{\bar{h}} = X, \quad \frac{z}{\bar{h}} = Z \text{ and}$$

$$\bar{h} = \text{the mean film thickness} \quad (69)$$

and

$$\zeta_7 = \frac{\zeta'_8}{\zeta'_7} \quad (70)$$

Given sets of $\zeta_1, \zeta_2, \zeta_3, \zeta_5, \zeta_6, \zeta_7$ and C_{xc} , Equations (66) to (68) constitute the starting point for the mathematical treatment of thin turbulent bearing fluid film subjected to the given boundary values.

EVALUATION OF THE DIMENSIONLESS COEFFICIENTS

The dimensionless coefficients (ζ_1 , ζ_2 , ζ_3 and ζ_7), are evaluated to account for the variations of U , n and m . Irrespective of n and m , ζ_5 and ζ_6 are obtained to take the quantities U , k and W into account. It is of interest to determine the general magnitude of all the dimensionless coefficients and their sensitivity to the quantities U , W , n , m and k .

Velocity Profiles

The velocity profiles for u as shown in Figures 1 to 3 are represented by the turbulent power law as follows:

u - profile:

$$u = 2u_{bc} - (u_{bc} - \Delta u_b) \left(\frac{y_o}{b} \right)^n \quad (0 \leq \frac{y_o}{b} \leq 1) \quad (71)$$

$$u = (u_{bc} + \Delta u_b) \left(\frac{y_h}{b} \right)^n \quad (0 \leq \frac{y_h}{b} \leq 1) \quad (72)$$

The velocity profile as shown in Figure 4 is the w - profile:

$$w = w_b \left(\frac{y_o}{b} \right)^m \quad (0 \leq \frac{y_o}{b} \leq 1) \quad (73)$$

$$w = w_b \left(\frac{y_h}{b} \right)^m \quad (0 \leq \frac{y_h}{b} \leq 1) \quad (74)$$

Let y_o and y_h denote the reference position across channel with respect to lower and upper surface of the wall, respectively. The values of n and m refer to the velocity profiles in the tangential and axial directions, respectively. In the axial direction, the w - profile is assumed to be symmetrical about $y = b$.

Evaluation of ζ_1 , ζ_2 , ζ_3 , ζ_6 and ζ_7

Substituting Equations (71) to (74) into Equations (44) to (48) and (70), the general magnitude of the dimensionless coefficients and their sensitivity to the shape of the turbulent velocity profile in the tangential and axial directions, respectively, are obtained as

$$\begin{aligned}\zeta_1 &= \frac{b \left[\int_0^1 (u_{bc} + \Delta u_b)^2 \left(\frac{y_h}{b} \right)^{2n} d \left(\frac{y_h}{b} \right) + \int_0^1 \left\{ 2u_{bc} - (u_{bc} - \Delta u_b) \left(\frac{y_o}{b} \right)^n \right\}^2 d \left(\frac{y_o}{b} \right) \right]}{2b(u_{bc} + \Delta u_b)^2} \\ &= \frac{(1 + U^2)/(1 + 2n) + 2(n + U)/(1 + n)}{(1 + U)^2}\end{aligned}\quad (75)$$

$$\begin{aligned}\zeta_2 \equiv \zeta_4 &= \frac{b \left[\int_0^1 w_b(u_{bc} + \Delta u_b) \left(\frac{y_h}{b} \right)^{n+m} d \left(\frac{y_h}{b} \right) + \int_0^1 \left\{ 2u_{bc} - (u_{bc} - \Delta u_b) \left(\frac{y_o}{b} \right)^n \right\} w_b \left(\frac{y_o}{b} \right)^m d \left(\frac{y_o}{b} \right) \right]}{2bw_b u_b} \\ &= \frac{\frac{1}{1+m} + \frac{U}{1+n+m}}{1+U}\end{aligned}\quad (76)$$

$$\lim_{U \rightarrow -1} \frac{\zeta_1}{\zeta_2} = \frac{2(1+m)(1+n+m)}{(1+n)(1+2n)} \quad (77)$$

$$\begin{aligned}\zeta_3 &= \frac{b \left[\int_0^1 w_b^2 \left(\frac{y_o}{b} \right)^{2m} d \left(\frac{y_o}{b} \right) + \int_0^1 w_b^2 \left(\frac{y_h}{b} \right)^{2m} d \left(\frac{y_h}{b} \right) \right]}{2b(w_b)^2} \\ &= \frac{1}{1+2m}\end{aligned}\quad (78)$$

$$\begin{aligned}\zeta_7' &= \frac{b \left[\int_0^1 (u_{bc} + \Delta u_b) \left(\frac{y_h}{b} \right)^n d \left(\frac{y_h}{b} \right) + \int_0^1 \left\{ 2u_{bc} - (u_{bc} - \Delta u_b) \left(\frac{y_o}{b} \right)^n \right\} d \left(\frac{y_o}{b} \right) \right]}{2bu_b} \\ &= \frac{1 + U/(1+n)}{1+U}\end{aligned}\quad (79)$$

$$\begin{aligned}\zeta_8' &= \frac{b \left[\int_0^1 w_b \left(\frac{y_h}{b} \right)^m d \left(\frac{y_h}{b} \right) + \int_0^1 w_b \left(\frac{y_o}{b} \right)^m d \left(\frac{y_o}{b} \right) \right]}{2bw_b} \\ &= \frac{1}{1+m}\end{aligned}\quad (80)$$

and

$$\zeta_7 \equiv \zeta'_8 / \zeta'_7$$

$$= \frac{1 + U}{(1 + m) \left(1 + \frac{U}{1 + n}\right)} \quad (81)$$

The value of ζ_6 in Equation (61), used for evaluating its general magnitude, is rewritten as

$$\zeta_6 = \frac{\left| [(1 + U)^2 + W^2]^{\frac{1-k}{2}} \right|}{2} + \frac{\left| [(-1 + U)^2 + W^2]^{\frac{1-k}{2}} \right|}{2} \quad (82)$$

For both cases of Couette and pressure flows effect dominating, the equations of ζ_1 , ζ_2 , ζ_3 , ζ_6 and ζ_7 remain unchanged because of identical equations for velocity profiles in the x and z directions, respectively.

However, the equation of ζ_5 will be different for each case.

Evaluation of ζ_5

Wall Shear with Same Sign (Couette Flow Dominating). Equation (51)

of ζ_5 will be used

$$\zeta_5 = \frac{A - B}{4U} \quad (0 \leq |U| \leq 1) \quad (83)$$

where A and B are the same as Equation (52) and Equation (53).

Wall Shear with Opposite Sign (Pressure Flow Dominating)

Lower Surface Positive Shear, Upper Surface Negative Shear.

The value of ζ_5 in Equation (56) will be used

$$\zeta_5 = \frac{A + B}{4U} \quad (1 \leq U) \quad (84)$$

Lower Surface Negative Shear, Upper Surface Positive Shear.

The value of ζ_5 in Equation (58) will be used

$$\zeta_5 = - \frac{(A + B)}{4U} \quad (U \leq -1) \quad (85)$$

Results

The corresponding symbols in the analysis and computer programs are listed as follows:

TABLE I. CORRESPONDING SYMBOLS IN ANALYSIS
AND COMPUTER PROGRAMS

<u>Symbols</u>	
<u>In Analysis</u>	<u>In Computer Programs</u>
U	U
W	W
n	N
m	M
k	K
ζ_1	ZETA 1
ζ_2	ZETA 2
ζ_3	ZETA 3
ζ_5	ZETA 5
ζ_6	ZETA 6
ζ_7	ZETA 7

All the dimensionless coefficients expressed by Equations (75) to (85) are calculated for the range of U such that $-6 \leq U \leq 6$ in the following Tables II to XX. For example, in Table II corresponding to $U = -6$, the general magnitudes of ζ_1 , ζ_2 , ζ_3 and ζ_7 , and of ζ_5 and ζ_6 are listed in Table II(a) and (b), respectively. Correspondingly, Tables III to XX refer to $U = -5$ to 6 , respectively. Analyzing Tables II to XX, the effect of U on the extreme values of ζ_1 , ζ_2 , ζ_3 and ζ_7 for $n \left(\frac{1}{10} \leq n \leq \frac{1}{3} \right)$ and $m \left(\frac{1}{10} \leq m \leq \frac{1}{3} \right)$ are shown in Table XXI. Similarly, Table XXII shows the effect of U on the extreme values of ζ_5 and ζ_6 (independent of n and m) for W ($0 \leq W \leq 2$) and $k \left(\frac{1}{4} \leq k \leq \frac{1}{2} \right)$.

TABLE II. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -6$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq n, m \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.54800	0.57000	0.60000	1.07143				
	0.25000	0.54800	0.59789	0.66667	1.14286				
	0.20000	0.54800	0.61594	0.71429	1.19048				
	0.16667	0.54800	0.62857	0.75000	1.22449				
	0.14286	0.54800	0.63790	0.77778	1.25000				
0.25000	0.12500	0.54800	0.64508	0.80000	1.26984				
	0.11111	0.54800	0.65077	0.81818	1.28571				
	0.10000	0.54800	0.65539	0.83333	1.29870				
	0.33333	0.61867	0.60789	0.60000	0.98684				
	0.25000	0.61867	0.64000	0.66667	1.05263				
0.20000	0.20000	0.61867	0.66092	0.71429	1.09649				
	0.16667	0.61867	0.67563	0.75000	1.12782				
	0.14286	0.61867	0.68654	0.77778	1.15132				
	0.12500	0.61867	0.69495	0.80000	1.16959				
	0.11111	0.61867	0.70163	0.81818	1.18421				
0.16667	0.10000	0.61867	0.70707	0.83333	1.19617				
	0.33333	0.67048	0.63261	0.60000	0.93750				
	0.25000	0.67048	0.66759	0.66667	1.00000				
	0.20000	0.67048	0.69048	0.71429	1.04167				
	0.16667	0.67048	0.70662	0.75000	1.07143				
0.14286	0.14286	0.67048	0.71862	0.77778	1.09375				
	0.12500	0.67048	0.72788	0.80000	1.11111				
	0.11111	0.67048	0.73525	0.81818	1.12500				
	0.10000	0.67048	0.74126	0.83333	1.13636				
	0.33333	0.71000	0.65000	0.60000	0.90517				
0.12500	0.25000	0.71000	0.68706	0.66667	0.96552				
	0.20000	0.71000	0.71138	0.71429	1.00575				
	0.16667	0.71000	0.72857	0.75000	1.03448				
	0.14286	0.71000	0.74136	0.77778	1.05603				
	0.12500	0.71000	0.75125	0.80000	1.07280				
0.11111	0.11111	0.71000	0.75913	0.81818	1.08621				
	0.10000	0.71000	0.76555	0.83333	1.09718				
	0.33333	0.74111	0.66290	0.60000	0.88235				
	0.25000	0.74111	0.70154	0.66667	0.94118				
	0.20000	0.74111	0.72695	0.71429	0.98039				
0.10000	0.16667	0.74111	0.74494	0.75000	1.00840				
	0.14286	0.74111	0.75833	0.77778	1.02941				
	0.12500	0.74111	0.76870	0.80000	1.04575				
	0.11111	0.74111	0.77696	0.81818	1.05882				
	0.10000	0.74111	0.78370	0.83333	1.06952				
0.08333	0.33333	0.76622	0.67286	0.60000	0.86538				
	0.25000	0.76622	0.71273	0.66667	0.92308				
	0.20000	0.76622	0.73899	0.71429	0.96154				
	0.16667	0.76622	0.75760	0.75000	0.98901				
	0.14286	0.76622	0.77148	0.77778	1.00962				
0.07692	0.12500	0.76622	0.78222	0.80000	1.02564				
	0.11111	0.76622	0.79079	0.81818	1.03846				
	0.10000	0.76622	0.79777	0.83333	1.04895				
	0.33333	0.78691	0.68077	0.60000	0.85227				
	0.25000	0.78691	0.72163	0.66667	0.90909				
0.07059	0.20000	0.78691	0.74859	0.71429	0.94697				
	0.16667	0.78691	0.76770	0.75000	0.97403				
	0.14286	0.78691	0.78196	0.77778	0.99432				
	0.12500	0.78691	0.79301	0.80000	1.01010				
	0.11111	0.78691	0.80182	0.81818	1.02273				
0.06494	0.10000	0.78691	0.80901	0.83333	1.03306				
	0.33333	0.80424	0.68721	0.60000	0.84184				
	0.25000	0.80424	0.72889	0.66667	0.89796				
	0.20000	0.80424	0.75641	0.71429	0.93537				
	0.16667	0.80424	0.77594	0.75000	0.96210				
0.05944	0.14286	0.80424	0.79052	0.77778	0.98214				
	0.12500	0.80424	0.80181	0.80000	0.99773				
	0.11111	0.80424	0.81083	0.81818	1.01020				
	0.10000	0.80424	0.81818	0.83333	1.02041				

TABLE III. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -5$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq n, m \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.53333	0.53333	0.53750	0.56250	0.60000	1.09091				
	0.25000	0.53750	0.58947	0.66667	1.16364				
	0.20000	0.53750	0.60688	0.71429	1.21212				
	0.16667	0.53750	0.61905	0.75000	1.24675				
	0.14286	0.53750	0.62802	0.77778	1.27273				
	0.12500	0.53750	0.63492	0.80000	1.29293				
0.25000	0.11111	0.53750	0.64038	0.81818	1.30909	0.50000	0.00000	1.13485	2.22474
	0.10000	0.53750	0.64482	0.83333	1.32231		0.25000	1.13556	2.22625
	0.33333	0.60833	0.60197	0.60000	1.00000		0.66667	1.13985	2.23538
	0.25000	0.60833	0.63333	0.66667	1.06667		1.00000	1.14601	2.24843
	0.20000	0.60833	0.65374	0.71429	1.11111		1.25000	1.15212	2.26139
	0.16667	0.60833	0.66807	0.75000	1.14286		1.33333	1.15443	2.26628
0.20000	0.14286	0.60833	0.67869	0.77778	1.16667		1.66667	1.16496	2.28855
	0.12500	0.60833	0.68687	0.80000	1.18519		2.00000	1.17741	2.31480
	0.11111	0.60833	0.69337	0.81818	1.20000	0.33333	0.00000	1.49455	2.91088
	0.10000	0.60833	0.69865	0.83333	1.21212		0.25000	1.49577	2.91348
	0.33333	0.66071	0.62772	0.60000	0.94737		0.66667	1.50323	2.92921
	0.25000	0.66071	0.66207	0.66667	1.01053		1.00000	1.51392	2.95175
0.16667	0.20000	0.66071	0.68452	0.71429	1.05263		1.25000	1.52457	2.97418
	0.16667	0.66071	0.70035	0.75000	1.08271		1.33333	1.52861	2.98266
	0.14286	0.66071	0.71210	0.77778	1.10526		1.66667	1.54702	3.02136
	0.12500	0.66071	0.72117	0.80000	1.12281	0.25000	2.00000	1.56887	3.06718
	0.11111	0.66071	0.72839	0.81818	1.13684		0.00000	1.71578	3.33104
	0.10000	0.66071	0.73427	0.83333	1.14833		0.25000	1.71736	3.33436
0.14286	0.33333	0.70089	0.64583	0.60000	0.91304		0.66667	1.72693	3.35449
	0.25000	0.70089	0.68235	0.66667	0.97391		1.00000	1.74067	3.38336
	0.20000	0.70089	0.70630	0.71429	1.01449		1.25000	1.75437	3.41212
	0.16667	0.70089	0.72321	0.75000	1.04348		1.33333	1.75956	3.42300
	0.14286	0.70089	0.73580	0.77778	1.06522		1.66667	1.78329	3.47272
	0.12500	0.70089	0.74552	0.80000	1.08213		2.00000	1.81151	3.53173
0.12500	0.11111	0.70089	0.75326	0.81818	1.09565				
	0.10000	0.70089	0.75957	0.83333	1.10672				
	0.33333	0.73264	0.65927	0.60000	0.88889				
	0.25000	0.73264	0.69744	0.66667	0.94815				
	0.20000	0.73264	0.72252	0.71429	0.98765				
	0.16667	0.73264	0.74026	0.75000	1.01587				
0.11111	0.14286	0.73264	0.75347	0.77778	1.03704				
	0.12500	0.73264	0.76369	0.80000	1.05350				
	0.11111	0.73264	0.77184	0.81818	1.06667				
	0.10000	0.73264	0.77847	0.83333	1.07744				
	0.33333	0.75833	0.66964	0.60000	0.87097				
	0.25000	0.75833	0.70909	0.66667	0.92903				
0.10000	0.20000	0.75833	0.73506	0.71429	0.96774				
	0.16667	0.75833	0.75346	0.75000	0.99539				
	0.14286	0.75833	0.76717	0.77778	1.01613				
	0.12500	0.75833	0.77778	0.80000	1.03226				
	0.11111	0.75833	0.78624	0.81818	1.04516				
	0.10000	0.75833	0.79314	0.83333	1.05572				
0.08333	0.33333	0.77955	0.67788	0.60000	0.85714				
	0.25000	0.77955	0.71837	0.66667	0.91429				
	0.20000	0.77955	0.74506	0.71429	0.95238				
	0.16667	0.77955	0.76398	0.75000	0.97959				
	0.14286	0.77955	0.77809	0.77778	1.00000				
	0.12500	0.77955	0.78901	0.80000	1.01587				
0.06667	0.11111	0.77955	0.79773	0.81818	1.02857				
	0.10000	0.77955	0.80484	0.83333	1.03896				
	0.33333	0.79735	0.68459	0.60000	0.84615				
	0.25000	0.79735	0.72593	0.66667	0.90256				
	0.20000	0.79735	0.75321	0.71429	0.94017				
	0.16667	0.79735	0.77256	0.75000	0.96703				
0.05000	0.14286	0.79735	0.78700	0.77778	0.98718				
	0.12500	0.79735	0.79819	0.80000	1.00285				
	0.11111	0.79735	0.80711	0.81818	1.01538				
	0.10000	0.79735	0.81439	0.83333	1.02564				

TABLE IV THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -4$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq n, m \leq \frac{1}{3}$ on the values of ζ_1, ζ_2
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.52222	0.55000	0.60000	1.12500				
	0.25000	0.52222	0.57544	0.66667	1.20000				
	0.20000	0.52222	0.59179	0.71429	1.25000				
0.25000	0.16667	0.52222	0.60317	0.75000	1.28571				
	0.14286	0.52222	0.61156	0.77778	1.31250				
	0.12500	0.52222	0.61799	0.80000	1.33333				
0.20000	0.11111	0.52222	0.62308	0.81818	1.35000	0.50000	0.00000	1.02353	1.98406
	0.10000	0.52222	0.62720	0.83333	1.36364		0.25000	1.02453	1.98626
	0.33333	0.59259	0.59211	0.60000	1.02273		0.66667	1.03055	1.99949
0.16667	0.25000	0.59259	0.62222	0.66667	1.09091		1.00000	1.03908	2.01819
	0.20000	0.59259	0.64176	0.71429	1.13636		1.25000	1.04746	2.03650
	0.16667	0.59259	0.65546	0.75000	1.16883		1.33333	1.05061	2.04335
0.12500	0.14286	0.59259	0.66560	0.77778	1.19318		1.66667	1.06477	2.07414
	0.12500	0.59259	0.67340	0.80000	1.21212		2.00000	1.08122	2.10971
	0.11111	0.59259	0.67959	0.81818	1.22727	0.33333	0.00000	1.30377	2.50205
0.10000	0.10000	0.59259	0.68462	0.83333	1.23967		0.25000	1.30543	2.50567
	0.33333	0.64550	0.61957	0.60000	0.96429		0.66667	1.31547	2.52751
	0.25000	0.64550	0.65287	0.66667	1.02857		1.00000	1.32974	2.55847
0.08333	0.20000	0.64550	0.67460	0.71429	1.07143		1.25000	1.34380	2.58890
	0.16667	0.64550	0.68990	0.75000	1.10204		1.33333	1.34909	2.60032
	0.14286	0.64550	0.70124	0.77778	1.12500		1.66667	1.37302	2.65187
0.07500	0.12500	0.64550	0.70999	0.80000	1.14286		2.00000	1.40097	2.71183
	0.11111	0.64550	0.71695	0.81818	1.15714	0.25000	0.00000	1.47231	2.81160
	0.10000	0.64550	0.72261	0.83333	1.16883		0.25000	1.47440	2.81613
0.06667	0.33333	0.68651	0.63889	0.60000	0.92647		0.66667	1.48704	2.84348
	0.25000	0.68651	0.67451	0.66667	0.98824		1.00000	1.50502	2.88231
	0.20000	0.68651	0.69783	0.71429	1.02941		1.25000	1.52279	2.92057
0.05833	0.16667	0.68651	0.71429	0.75000	1.05882		1.33333	1.52947	2.93494
	0.14286	0.68651	0.72652	0.77778	1.08088		1.66667	1.55979	2.99995
	0.12500	0.68651	0.73596	0.80000	1.09804		2.00000	1.59532	3.07581
0.05000	0.11111	0.68651	0.74348	0.81818	1.11176				
	0.10000	0.68651	0.74960	0.83333	1.12299				
	0.33333	0.71914	0.65323	0.60000	0.90000				
0.04500	0.25000	0.71914	0.69060	0.66667	0.96000				
	0.20000	0.71914	0.71513	0.71429	1.00000				
	0.16667	0.71914	0.73247	0.75000	1.02857				
0.04000	0.14286	0.71914	0.74537	0.77778	1.05000				
	0.12500	0.71914	0.75535	0.80000	1.06667				
	0.11111	0.71914	0.76329	0.81818	1.08000				
0.03750	0.10000	0.71914	0.76977	0.83333	1.09091				
	0.33333	0.74568	0.66429	0.60000	0.88043				
	0.25000	0.74568	0.70303	0.66667	0.93913				
0.03500	0.20000	0.74568	0.72851	0.71429	0.97826				
	0.16667	0.74568	0.74654	0.75000	1.00621				
	0.14286	0.74568	0.75998	0.77778	1.02717				
0.03250	0.12500	0.74568	0.77037	0.80000	1.04348				
	0.11111	0.74568	0.77865	0.81818	1.05652				
	0.10000	0.74568	0.78541	0.83333	1.06719				
0.03000	0.33333	0.76768	0.67308	0.60000	0.86538				
	0.25000	0.76768	0.71293	0.66667	0.92308				
	0.20000	0.76768	0.73917	0.71429	0.96154				
0.02750	0.16667	0.76768	0.75776	0.75000	0.98901				
	0.14286	0.76768	0.77162	0.77778	1.00962				
	0.12500	0.76768	0.78236	0.80000	1.02564				
0.02500	0.11111	0.76768	0.79091	0.81818	1.03846				
	0.10000	0.76768	0.79789	0.83333	1.04895				
	0.33333	0.78620	0.68023	0.60000	0.85345				
0.02250	0.25000	0.78620	0.72099	0.66667	0.91034				
	0.20000	0.78620	0.74786	0.71429	0.94828				
	0.16667	0.78620	0.76692	0.75000	0.97537				
0.02000	0.14286	0.78620	0.78113	0.77778	0.99569				
	0.12500	0.78620	0.79214	0.80000	1.01149				
	0.11111	0.78620	0.80092	0.81818	1.02414				
	0.10000	0.78620	0.80808	0.83333	1.03448				

TABLE V. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -3$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \\ \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \\ \text{on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.43333	0.33333	0.50000	0.52500	0.60000	1.20000	0.50000	0.00000	0.90237	1.70711
	0.25000	0.50000	0.54737	0.66667	1.28000		0.25000	0.90393	1.71083
	0.20000	0.50000	0.56159	0.71429	1.33333		0.66667	0.91324	1.73285
	0.16667	0.50000	0.57143	0.75000	1.37143		1.00000	0.92607	1.76295
0.25000	0.14286	0.50000	0.57863	0.77778	1.40000	0.33333	1.25000	0.93834	1.79144
	0.12500	0.50000	0.58413	0.80000	1.42222		1.33333	0.94286	1.80188
	0.11111	0.50000	0.58846	0.81818	1.44000		1.66667	0.96281	1.84759
	0.10000	0.50000	0.59197	0.83333	1.45455		2.00000	0.98521	1.89827
0.20000	0.33333	0.56667	0.57237	0.60000	1.07143	0.25000	0.00000	1.10451	2.05362
	0.25000	0.56667	0.60000	0.66667	1.14286		0.25000	1.10698	2.05937
	0.20000	0.56667	0.61782	0.71429	1.19048		0.66667	1.12168	2.09355
	0.16667	0.56667	0.63025	0.75000	1.22449		1.00000	1.14209	2.14063
0.16667	0.14286	0.56667	0.63942	0.77778	1.25000	0.25000	1.25000	1.16175	2.18558
	0.12500	0.56667	0.64646	0.80000	1.26984		1.33333	1.16904	2.20216
	0.11111	0.56667	0.65204	0.81818	1.28571		1.66667	1.20140	2.27521
	0.10000	0.56667	0.65657	0.83333	1.29870		2.00000	1.23814	2.35721
0.20000	0.33333	0.61905	0.60326	0.60000	1.00000	0.25000	0.00000	1.22311	2.25511
	0.25000	0.61905	0.63448	0.66667	1.06667		0.25000	1.22612	2.26208
	0.20000	0.61905	0.65476	0.71429	1.11111		0.66667	1.24414	2.30360
	0.16667	0.61905	0.66899	0.75000	1.14286		1.00000	1.26925	2.36102
0.16667	0.14286	0.61905	0.67952	0.77778	1.16667	0.25000	1.25000	1.29353	2.41608
	0.12500	0.61905	0.68763	0.80000	1.18519		1.33333	1.30255	2.43644
	0.11111	0.61905	0.69407	0.81818	1.20000		1.66667	1.34273	2.52648
	0.10000	0.61905	0.69930	0.83333	1.21212		2.00000	1.38860	2.62815
0.16667	0.33333	0.66071	0.62500	0.60000	0.95455	0.25000	0.00000	1.24414	2.30360
	0.25000	0.66071	0.65882	0.66667	1.01818		0.25000	1.26925	2.36102
	0.20000	0.66071	0.68089	0.71429	1.06061		0.66667	1.29353	2.41608
	0.16667	0.66071	0.69643	0.75000	1.09091		1.00000	1.30255	2.43644
0.14286	0.14286	0.66071	0.70795	0.77778	1.11364	0.25000	1.25000	1.34273	2.52648
	0.12500	0.66071	0.71685	0.80000	1.13131		1.33333	1.38860	2.62815
	0.11111	0.66071	0.72391	0.81818	1.14545		1.66667	1.30255	2.43644
	0.10000	0.66071	0.72967	0.83333	1.15702		2.00000	1.34273	2.52648
0.14286	0.33333	0.69444	0.64113	0.60000	0.92308	0.25000	0.00000	1.24414	2.30360
	0.25000	0.69444	0.67692	0.66667	0.98462		0.25000	1.26925	2.36102
	0.20000	0.69444	0.70035	0.71429	1.02564		0.66667	1.29353	2.41608
	0.16667	0.69444	0.71688	0.75000	1.05494		1.00000	1.30255	2.43644
0.12500	0.14286	0.69444	0.72917	0.77778	1.07692	0.25000	1.25000	1.34273	2.52648
	0.12500	0.69444	0.73865	0.80000	1.09402		1.33333	1.38860	2.62815
	0.11111	0.69444	0.74620	0.81818	1.10769		1.66667	1.30255	2.43644
	0.10000	0.69444	0.75235	0.83333	1.11888		2.00000	1.34273	2.52648
0.12500	0.33333	0.72222	0.65357	0.60000	0.90000	0.25000	0.00000	1.24414	2.30360
	0.25000	0.72222	0.69091	0.66667	0.96000		0.25000	1.26925	2.36102
	0.20000	0.72222	0.71541	0.71429	1.00000		0.66667	1.29353	2.41608
	0.16667	0.72222	0.73272	0.75000	1.02857		1.00000	1.30255	2.43644
0.11111	0.14286	0.72222	0.74560	0.77778	1.05000	0.25000	1.25000	1.34273	2.52648
	0.12500	0.72222	0.75556	0.80000	1.06667		1.33333	1.38860	2.62815
	0.11111	0.72222	0.76348	0.81818	1.08000		1.66667	1.30255	2.43644
	0.10000	0.72222	0.76994	0.83333	1.09091		2.00000	1.34273	2.52648
0.11111	0.33333	0.74545	0.66346	0.60000	0.88235	0.25000	0.00000	1.24414	2.30360
	0.25000	0.74545	0.70204	0.66667	0.94118		0.25000	1.26925	2.36102
	0.20000	0.74545	0.72740	0.71429	0.98039		0.66667	1.29353	2.41608
	0.16667	0.74545	0.74534	0.75000	1.00840		1.00000	1.30255	2.43644
0.10000	0.14286	0.74545	0.75870	0.77778	1.02941	0.25000	1.25000	1.34273	2.52648
	0.12500	0.74545	0.76904	0.80000	1.04575		1.33333	1.38860	2.62815
	0.11111	0.74545	0.77727	0.81818	1.05882		1.66667	1.30255	2.43644
	0.10000	0.74545	0.78399	0.83333	1.06952		2.00000	1.34273	2.52648
0.10000	0.33333	0.76515	0.67151	0.60000	0.86842	0.25000	0.00000	1.24414	2.30360
	0.25000	0.76515	0.71111	0.66667	0.92632		0.25000	1.26925	2.36102
	0.20000	0.76515	0.73718	0.71429	0.96491		0.66667	1.29353	2.41608
	0.16667	0.76515	0.75564	0.75000	0.99248		1.00000	1.30255	2.43644
0.10000	0.14286	0.76515	0.76940	0.77778	1.01316	0.25000	1.25000	1.34273	2.52648
	0.12500	0.76515	0.78005	0.80000	1.02924		1.33333	1.38860	2.62815
	0.11111	0.76515	0.78853	0.81818	1.04211		1.66667	1.30255	2.43644
	0.10000	0.76515	0.79545	0.83333	1.05263		2.00000	1.34273	2.52648

TABLE VI. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -2.5$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.48889	0.50000	0.60000	1.28571				
	0.25000	0.48889	0.51930	0.66667	1.37143				
	0.20000	0.48889	0.53140	0.71429	1.42857				
	0.16667	0.48889	0.53968	0.75000	1.46939				
	0.14286	0.48889	0.54570	0.77778	1.50000				
	0.12500	0.48889	0.55026	0.80000	1.52381				
0.25000	0.33333	0.54815	0.55263	0.60000	1.12500	0.50000	0.00000	0.83850	1.54779
	0.25000	0.54815	0.57778	0.66667	1.20000		0.25000	0.84060	1.55319
	0.20000	0.54815	0.59387	0.71429	1.25000		0.66667	0.85283	1.58439
	0.16667	0.54815	0.60504	0.75000	1.28571		1.00000	0.86916	1.62528
	0.14286	0.54815	0.61325	0.77778	1.31250		1.25000	0.88434	1.66258
	0.12500	0.54815	0.61953	0.80000	1.33333		1.33333	0.88985	1.67598
0.20000	0.33333	0.54815	0.62449	0.81818	1.35000		1.66667	0.91373	1.73316
	0.25000	0.54815	0.62851	0.83333	1.36364	0.33333	2.00000	0.93989	1.79445
	0.20000	0.59788	0.58696	0.60000	1.03846		0.00000	1.00338	1.80779
	0.16667	0.59788	0.61609	0.66667	1.10769		0.25000	1.00656	1.81576
	0.14286	0.59788	0.63492	0.71429	1.15385		0.66667	1.02520	1.86214
	0.12500	0.59788	0.64808	0.75000	1.18681		1.00000	1.05040	1.92378
0.16667	0.33333	0.63889	0.61111	0.60000	0.98438		1.25000	1.07410	1.98077
	0.25000	0.63889	0.64314	0.66667	1.05000		1.33333	1.08277	2.00140
	0.20000	0.63889	0.66396	0.71429	1.09375		1.66667	1.12068	2.09044
	0.16667	0.63889	0.67857	0.75000	1.12500	0.25000	2.00000	1.16282	2.18746
	0.14286	0.63889	0.68939	0.77778	1.14844		0.00000	1.09892	1.95714
	0.12500	0.63889	0.69773	0.80000	1.16667		0.25000	1.10273	1.96659
0.14286	0.33333	0.67284	0.62903	0.60000	0.94737		0.66667	1.12519	2.02175
	0.25000	0.67284	0.66325	0.66667	1.01053		1.00000	1.15573	2.09556
	0.20000	0.67284	0.68558	0.71429	1.05263		1.25000	1.18462	2.16425
	0.16667	0.67284	0.70130	0.75000	1.08271		1.33333	1.19522	2.18922
	0.14286	0.67284	0.71296	0.77778	1.10526		1.66667	1.24181	2.29755
	0.12500	0.67284	0.72196	0.80000	1.12281		2.00000	1.29395	2.41655
0.12500	0.33333	0.70123	0.64286	0.60000	0.92045				
	0.25000	0.70123	0.67879	0.66667	0.98182				
	0.20000	0.70123	0.70231	0.71429	1.02273				
	0.16667	0.70123	0.71889	0.75000	1.05195				
	0.14286	0.70123	0.73122	0.77778	1.07386				
	0.12500	0.70123	0.74074	0.80000	1.09091				
0.11111	0.33333	0.72525	0.65385	0.60000	0.90000				
	0.25000	0.72525	0.69116	0.66667	0.96000				
	0.20000	0.72525	0.71563	0.71429	1.00000				
	0.16667	0.72525	0.73292	0.75000	1.02857				
	0.14286	0.72525	0.74578	0.77778	1.05000				
	0.12500	0.72525	0.75572	0.80000	1.06667				
0.10000	0.33333	0.74579	0.66279	0.60000	0.88393				
	0.25000	0.74579	0.70123	0.66667	0.94286				
	0.20000	0.74579	0.72650	0.71429	0.98214				
	0.16667	0.74579	0.74436	0.75000	1.01020				
	0.14286	0.74579	0.75766	0.77778	1.03125				
	0.12500	0.74579	0.76795	0.80000	1.04762				
	0.11111	0.74579	0.77615	0.81818	1.06071				
	0.10000	0.74579	0.78283	0.83333	1.07143				

TABLE VII. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -2$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7				
0.33333	0.33333	0.50000	0.45000	0.60000	1.50000				
	0.25000	0.50000	0.46316	0.66667	1.60000				
	0.20000	0.50000	0.47101	0.71429	1.66667				
	0.16667	0.50000	0.47619	0.75000	1.71429				
	0.14286	0.50000	0.47984	0.77778	1.75000				
0.25000	0.12500	0.50000	0.48254	0.80000	1.77778				
	0.11111	0.50000	0.48462	0.81818	1.80000				
	0.10000	0.50000	0.48626	0.83333	1.81818				
	0.33333	0.53333	0.51316	0.60000	1.25000				
	0.25000	0.53333	0.53333	0.66667	1.33333				
0.20000	0.20000	0.53333	0.54598	0.71429	1.38889				
	0.16667	0.53333	0.55462	0.75000	1.42857				
	0.14286	0.53333	0.56090	0.77778	1.45833				
	0.12500	0.53333	0.56566	0.80000	1.48148				
	0.11111	0.53333	0.56939	0.81818	1.50000				
0.16667	0.10000	0.53333	0.57239	0.83333	1.51515				
	0.33333	0.57143	0.55435	0.60000	1.12500				
	0.25000	0.57143	0.57931	0.66667	1.20000				
	0.20000	0.57143	0.59524	0.71429	1.25000				
	0.16667	0.57143	0.60627	0.75000	1.28571				
0.14286	0.14286	0.57143	0.61436	0.77778	1.31250				
	0.12500	0.57143	0.62055	0.80000	1.33333				
	0.11111	0.57143	0.62542	0.81818	1.35000				
	0.10000	0.57143	0.62937	0.83333	1.36364				
	0.33333	0.60714	0.58333	0.60000	1.05000				
0.12500	0.25000	0.60714	0.61176	0.66667	1.12000				
	0.20000	0.60714	0.63008	0.71429	1.16667				
	0.16667	0.60714	0.64286	0.75000	1.20000				
	0.14286	0.60714	0.65227	0.77778	1.22500				
	0.12500	0.60714	0.65950	0.80000	1.24444				
0.11111	0.11111	0.60714	0.66522	0.81818	1.26000				
	0.10000	0.60714	0.66986	0.83333	1.27273				
	0.33333	0.63889	0.60484	0.60000	1.00000				
	0.25000	0.63889	0.63590	0.66667	1.06667				
	0.20000	0.63889	0.65603	0.71429	1.11111				
0.10000	0.16667	0.63889	0.67013	0.75000	1.14286				
	0.14286	0.63889	0.68056	0.77778	1.16667				
	0.12500	0.63889	0.68858	0.80000	1.18519				
	0.11111	0.63889	0.69494	0.81818	1.20000				
	0.10000	0.63889	0.70010	0.83333	1.21212				
0.08333	0.33333	0.66667	0.62143	0.60000	0.96429				
	0.25000	0.66667	0.65455	0.66667	1.02857				
	0.20000	0.66667	0.67610	0.71429	1.07143				
	0.16667	0.66667	0.69124	0.75000	1.10204				
	0.14286	0.66667	0.70246	0.77778	1.12500				
0.07143	0.12500	0.66667	0.71111	0.80000	1.14286				
	0.11111	0.66667	0.71798	0.81818	1.15714				
	0.10000	0.66667	0.72356	0.83333	1.16883				
	0.33333	0.69091	0.63462	0.60000	0.93750				
	0.25000	0.69091	0.66939	0.66667	1.00000				
0.06250	0.20000	0.69091	0.69209	0.71429	1.04167				
	0.16667	0.69091	0.70807	0.75000	1.07143				
	0.14286	0.69091	0.71994	0.77778	1.09375				
	0.12500	0.69091	0.72909	0.80000	1.11111				
	0.11111	0.69091	0.73636	0.81818	1.12500				
0.05556	0.10000	0.69091	0.74229	0.83333	1.13636				
	0.33333	0.71212	0.64535	0.60000	0.91667				
	0.25000	0.71212	0.68148	0.66667	0.97778				
	0.20000	0.71212	0.70513	0.71429	1.01852				
	0.16667	0.71212	0.72180	0.75000	1.04762				
0.05000	0.14286	0.71212	0.73420	0.77778	1.06944				
	0.12500	0.71212	0.74376	0.80000	1.08642				
	0.11111	0.71212	0.75138	0.81818	1.10000				
	0.10000	0.71212	0.75758	0.83333	1.11111				

TABLE VIII THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -1.5$

(a) Effect of the Values of n and m Such That
 $\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k
 $\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7				
0.33333	0.33333	0.80000	0.30000	0.60000	3.00000				
	0.25000	0.80000	0.29474	0.66667	3.20000				
	0.20000	0.80000	0.28986	0.71429	3.33333				
	0.16667	0.80000	0.28571	0.75000	3.42857				
	0.14286	0.80000	0.28226	0.77778	3.50000				
	0.12500	0.80000	0.27937	0.80000	3.55556				
	0.11111	0.80000	0.27692	0.81818	3.60000				
	0.10000	0.80000	0.27484	0.83333	3.63636				
0.25000	0.33333	0.66667	0.39474	0.60000	1.87500				
	0.25000	0.66667	0.40000	0.66667	2.00000				
	0.20000	0.66667	0.40230	0.71429	2.08333				
	0.16667	0.66667	0.40336	0.75000	2.14286				
	0.14286	0.66667	0.40385	0.77778	2.18750				
	0.12500	0.66667	0.40404	0.80000	2.22222				
	0.11111	0.66667	0.40408	0.81818	2.25000				
	0.10000	0.66667	0.40404	0.83333	2.27273				
0.20000	0.33333	0.61905	0.45652	0.60000	1.50000				
	0.25000	0.61905	0.46897	0.66667	1.60000				
	0.20000	0.61905	0.47619	0.71429	1.66667				
	0.16667	0.61905	0.48084	0.75000	1.71429				
	0.14286	0.61905	0.48404	0.77778	1.75000				
	0.12500	0.61905	0.48637	0.80000	1.77778				
	0.11111	0.61905	0.48814	0.81818	1.80000				
	0.10000	0.61905	0.48951	0.83333	1.81818				
0.16667	0.33333	0.60714	0.50000	0.60000	1.31250				
	0.25000	0.60714	0.51765	0.66667	1.40000				
	0.20000	0.60714	0.52846	0.71429	1.45833				
	0.16667	0.60714	0.53571	0.75000	1.50000				
	0.14286	0.60714	0.54091	0.77778	1.53125				
	0.12500	0.60714	0.54480	0.80000	1.55556				
	0.11111	0.60714	0.54783	0.81818	1.57500				
	0.10000	0.60714	0.55024	0.83333	1.59091				
0.14286	0.33333	0.61111	0.53226	0.60000	1.20000				
	0.25000	0.61111	0.55385	0.66667	1.28000				
	0.20000	0.61111	0.56738	0.71429	1.33333				
	0.16667	0.61111	0.57662	0.75000	1.37143				
	0.14286	0.61111	0.58333	0.77778	1.40000				
	0.12500	0.61111	0.58842	0.80000	1.42222				
	0.11111	0.61111	0.59241	0.81818	1.44000				
	0.10000	0.61111	0.59561	0.83333	1.45455				
0.12500	0.33333	0.62222	0.55714	0.60000	1.12500				
	0.25000	0.62222	0.58182	0.66667	1.20000				
	0.20000	0.62222	0.59748	0.71429	1.25000				
	0.16667	0.62222	0.60829	0.75000	1.28571				
	0.14286	0.62222	0.61620	0.77778	1.31250				
	0.12500	0.62222	0.62222	0.80000	1.33333				
	0.11111	0.62222	0.62697	0.81818	1.35000				
	0.10000	0.62222	0.63080	0.83333	1.36364				
0.11111	0.33333	0.63636	0.57692	0.60000	1.07143				
	0.25000	0.63636	0.60408	0.66667	1.14286				
	0.20000	0.63636	0.62147	0.71429	1.19048				
	0.16667	0.63636	0.63354	0.75000	1.22449				
	0.14286	0.63636	0.64241	0.77778	1.25000				
	0.12500	0.63636	0.64919	0.80000	1.26984				
	0.11111	0.63636	0.65455	0.81818	1.28571				
	0.10000	0.63636	0.65888	0.83333	1.29870				
0.10000	0.33333	0.65152	0.59302	0.60000	1.03125				
	0.25000	0.65152	0.62222	0.66667	1.10000				
	0.20000	0.65152	0.64103	0.71429	1.14583				
	0.16667	0.65152	0.65414	0.75000	1.17857				
	0.14286	0.65152	0.66379	0.77778	1.20313				
	0.12500	0.65152	0.67120	0.80000	1.22222				
	0.11111	0.65152	0.67706	0.81818	1.23750				
	0.10000	0.65152	0.68182	0.83333	1.25000				

TABLE IX. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -1$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	ZETA1/ZETA2				
.33333	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.00000				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	1.78125				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	1.65600				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	1.57500				
	0.14286	INFINITE	INFINITE	0.77778	0.00000	1.51837				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	1.47656				
0.25000	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.44444				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.41900				
	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.25185				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.00000				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	1.85600				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	1.76296				
0.20000	0.14286	INFINITE	INFINITE	0.77778	0.00000	1.69796				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	1.65000				
	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.61317				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.58400				
	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.43386				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.15774				
0.16667	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.00000				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	1.89815				
	0.14286	INFINITE	INFINITE	0.77778	0.00000	1.82702				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	1.77455				
	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.73427				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.70238				
0.14286	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.57143				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.27679				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.10857				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	2.00000				
	0.14286	INFINITE	INFINITE	0.77778	0.00000	1.92420				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	1.86830				
0.12500	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.82540				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.79143				
	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.67901				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.36979				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.19333				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	2.07948				
0.11111	0.14286	INFINITE	INFINITE	0.77778	0.00000	2.00000				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	1.94141				
	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.89643				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.86083				
	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.76543				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.44444				
0.10000	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.26133				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	2.14321				
	0.14286	INFINITE	INFINITE	0.77778	0.00000	2.06077				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	2.00000				
	0.11111	INFINITE	INFINITE	0.81818	0.00000	1.95336				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.91644				
0.08333	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.83636				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.50568				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.31709				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	2.19545				
	0.14286	INFINITE	INFINITE	0.77778	0.00000	2.11057				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	2.04801				
0.06667	0.11111	INFINITE	INFINITE	0.81818	0.00000	2.00000				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	1.96200				
	0.33333	INFINITE	INFINITE	0.60000	0.00000	2.89562				
	0.25000	INFINITE	INFINITE	0.66667	0.00000	2.55682				
	0.20000	INFINITE	INFINITE	0.71429	0.00000	2.36364				
	0.16667	INFINITE	INFINITE	0.75000	0.00000	2.23906				
0.05000	0.14286	INFINITE	INFINITE	0.77778	0.00000	2.15213				
	0.12500	INFINITE	INFINITE	0.80000	0.00000	2.08807				
	0.11111	INFINITE	INFINITE	0.81818	0.00000	2.03891				
	0.10000	INFINITE	INFINITE	0.83333	0.00000	2.00000				
	0.08333	INFINITE	INFINITE	0.85714	0.00000	1.96200				
	0.06667	INFINITE	INFINITE	0.88889	0.00000	1.91644				

TABLE X. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = -0.5$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \text{ on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	2.00000	0.90000	0.60000	0.60000	0.50000	0.00000	0.74178	0.96593
	0.25000	2.00000	0.96842	0.66667	0.64000		0.25000	0.73795	0.99042
	0.20000	2.00000	1.01449	0.71429	0.66667		0.66667	0.73268	1.09704
	0.16667	2.00000	1.04762	0.75000	0.68571		1.00000	0.74266	1.20002
	0.14286	2.00000	1.07258	0.77778	0.70000		1.25000	0.75793	1.27882
	0.12500	2.00000	1.09206	0.80000	0.71111		1.33333	0.76417	1.30499
	0.11111	2.00000	1.10769	0.81818	0.72000		1.66667	0.79329	1.40827
0.25000	0.10000	2.00000	1.12051	0.83333	0.72727	0.33333	2.00000	0.82690	1.50847
	0.33333	1.73333	0.86842	0.60000	0.62500		0.00000	0.82529	0.97017
	0.25000	1.73333	0.93333	0.66667	0.66667		0.25000	0.82214	1.00050
	0.20000	1.73333	0.97701	0.71429	0.69444		0.66667	0.82225	1.13853
	0.16667	1.73333	1.00840	0.75000	0.71429		1.00000	0.84163	1.27923
	0.14286	1.73333	1.03205	0.77778	0.72917		1.25000	0.86684	1.39072
	0.12500	1.73333	1.05051	0.80000	0.74074		1.33333	0.87687	1.42840
0.20000	0.11111	1.73333	1.06531	0.81818	0.75000	0.25000	1.66667	0.92318	1.57991
	0.10000	1.73333	1.07744	0.83333	0.75758		2.00000	0.97656	1.73091
	0.33333	1.57143	0.84783	0.60000	0.64286		0.00000	0.86790	0.97500
	0.25000	1.57143	0.91034	0.66667	0.68571		0.25000	0.86543	1.00795
	0.20000	1.57143	0.95238	0.71429	0.71429		0.66667	0.86960	1.16119
	0.16667	1.57143	0.98258	0.75000	0.73469		1.00000	0.89504	1.32154
	0.14286	1.57143	1.00532	0.77778	0.75000		1.25000	0.92638	1.45081
0.16667	0.12500	1.57143	1.02306	0.80000	0.76190	0.16667	1.33333	0.93873	1.49486
	0.11111	1.57143	1.03729	0.81818	0.77143		1.66667	0.99553	1.67371
	0.10000	1.57143	1.04895	0.83333	0.77922		2.00000	1.06102	1.85432
	0.33333	1.46429	0.83333	0.60000	0.65625				
	0.25000	1.46429	0.89412	0.66667	0.70000				
	0.20000	1.46429	0.93496	0.71429	0.72917				
	0.16667	1.46429	0.96429	0.75000	0.75000				
0.14286	0.14286	1.46429	0.98636	0.77778	0.76563	0.14286			
	0.12500	1.46429	1.00358	0.80000	0.77778				
	0.11111	1.46429	1.01739	0.81818	0.78750				
	0.10000	1.46429	1.02871	0.83333	0.79545				
	0.33333	1.38889	0.82258	0.60000	0.66667				
	0.25000	1.38889	0.88205	0.66667	0.71111				
	0.20000	1.38889	0.92199	0.71429	0.74074				
0.12500	0.16667	1.38889	0.95065	0.75000	0.76190	0.12500			
	0.14286	1.38889	0.97222	0.77778	0.77778				
	0.12500	1.38889	0.98905	0.80000	0.79012				
	0.11111	1.38889	1.00253	0.81818	0.80000				
	0.10000	1.38889	1.01358	0.83333	0.80808				
	0.33333	1.33333	0.81429	0.60000	0.67500				
	0.25000	1.33333	0.87273	0.66667	0.72000				
0.11111	0.20000	1.33333	0.91195	0.71429	0.75000	0.11111			
	0.16667	1.33333	0.94009	0.75000	0.77143				
	0.14286	1.33333	0.96127	0.77778	0.78750				
	0.12500	1.33333	0.97778	0.80000	0.80000				
	0.11111	1.33333	0.99101	0.81818	0.81000				
	0.10000	1.33333	1.00186	0.83333	0.81818				
	0.33333	1.29091	0.80769	0.60000	0.68182				
0.10000	0.25000	1.29091	0.86531	0.66667	0.72727	0.10000			
	0.20000	1.29091	0.90395	0.71429	0.75758				
	0.16667	1.29091	0.93168	0.75000	0.77922				
	0.14286	1.29091	0.95253	0.77778	0.79545				
	0.12500	1.29091	0.96879	0.80000	0.80808				
	0.11111	1.29091	0.98182	0.81818	0.81818				
	0.10000	1.29091	0.99249	0.83333	0.82645				
0.10000	0.33333	1.25758	0.80233	0.60000	0.68750	0.10000			
	0.25000	1.25758	0.85926	0.66667	0.73333				
	0.20000	1.25758	0.89744	0.71429	0.76389				
	0.16667	1.25758	0.92481	0.75000	0.78571				
	0.14286	1.25758	0.94540	0.77778	0.80208				
	0.12500	1.25758	0.96145	0.80000	0.81481				
	0.11111	1.25758	0.97431	0.81818	0.82500				
0.10000	0.10000	1.25758	0.98485	0.83333	0.83333	0.10000			

TABLE XI. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 0$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7				
0.33333	0.33333	1.10000	0.75000	0.60000	0.75000				
	0.25000	1.10000	0.80000	0.66667	0.80000				
	0.20000	1.10000	0.83333	0.71429	0.83333				
	0.16667	1.10000	0.85714	0.75000	0.85714				
	0.14286	1.10000	0.87500	0.77778	0.87500				
0.25000	0.12500	1.10000	0.88889	0.80000	0.88889				
	0.11111	1.10000	0.90000	0.81818	0.90000				
	0.10000	1.10000	0.90909	0.83333	0.90909				
	0.33333	1.06667	0.75000	0.60000	0.75000				
	0.25000	1.06667	0.80000	0.66667	0.80000				
0.20000	0.20000	1.06667	0.83333	0.71429	0.83333				
	0.16667	1.06667	0.85714	0.75000	0.85714				
	0.14286	1.06667	0.87500	0.77778	0.87500				
	0.12500	1.06667	0.88889	0.80000	0.88889				
	0.11111	1.06667	0.90000	0.81818	0.90000				
0.16667	0.10000	1.06667	0.90909	0.83333	0.90909				
	0.33333	1.04762	0.75000	0.60000	0.75000				
	0.25000	1.04762	0.80000	0.66667	0.80000				
	0.20000	1.04762	0.83333	0.71429	0.83333				
	0.16667	1.04762	0.85714	0.75000	0.85714				
0.14286	0.14286	1.04762	0.87500	0.77778	0.87500				
	0.12500	1.04762	0.88889	0.80000	0.88889				
	0.11111	1.04762	0.90000	0.81818	0.90000				
	0.10000	1.04762	0.90909	0.83333	0.90909				
	0.33333	1.03571	0.75000	0.60000	0.75000				
0.12500	0.25000	1.03571	0.80000	0.66667	0.80000				
	0.20000	1.03571	0.83333	0.71429	0.83333				
	0.16667	1.03571	0.85714	0.75000	0.85714				
	0.14286	1.03571	0.87500	0.77778	0.87500				
	0.12500	1.03571	0.88889	0.80000	0.88889				
0.11111	0.11111	1.03571	0.90000	0.81818	0.90000				
	0.10000	1.03571	0.90909	0.83333	0.90909				
	0.33333	1.02778	0.75000	0.60000	0.75000				
	0.25000	1.02778	0.80000	0.66667	0.80000				
	0.20000	1.02778	0.83333	0.71429	0.83333				
0.10000	0.16667	1.02778	0.85714	0.75000	0.85714				
	0.14286	1.02778	0.87500	0.77778	0.87500				
	0.12500	1.02778	0.88889	0.80000	0.88889				
	0.11111	1.02778	0.90000	0.81818	0.90000				
	0.10000	1.02778	0.90909	0.83333	0.90909				
0.08333	0.33333	1.02222	0.75000	0.60000	0.75000				
	0.25000	1.02222	0.80000	0.66667	0.80000				
	0.20000	1.02222	0.83333	0.71429	0.83333				
	0.16667	1.02222	0.85714	0.75000	0.85714				
	0.14286	1.02222	0.87500	0.77778	0.87500				
0.07143	0.12500	1.02222	0.88889	0.80000	0.88889				
	0.11111	1.02222	0.90000	0.81818	0.90000				
	0.10000	1.02222	0.90909	0.83333	0.90909				
	0.33333	1.01818	0.75000	0.60000	0.75000				
	0.25000	1.01818	0.80000	0.66667	0.80000				
0.06250	0.20000	1.01818	0.83333	0.71429	0.83333				
	0.16667	1.01818	0.85714	0.75000	0.85714				
	0.14286	1.01818	0.87500	0.77778	0.87500				
	0.12500	1.01818	0.88889	0.80000	0.88889				
	0.11111	1.01818	0.90000	0.81818	0.90000				
0.05556	0.10000	1.01818	0.90909	0.83333	0.90909				
	0.33333	1.01515	0.75000	0.60000	0.75000				
	0.25000	1.01515	0.80000	0.66667	0.80000				
	0.20000	1.01515	0.83333	0.71429	0.83333				
	0.16667	1.01515	0.85714	0.75000	0.85714				
0.05000	0.14286	1.01515	0.87500	0.77778	0.87500				
	0.12500	1.01515	0.88889	0.80000	0.88889				
	0.11111	1.01515	0.90000	0.81818	0.90000				
	0.10000	1.01515	0.90909	0.83333	0.90909				
	0.33333	1.01212	0.75000	0.60000	0.75000				
0.04545	0.25000	1.01212	0.80000	0.66667	0.80000				
	0.20000	1.01212	0.83333	0.71429	0.83333				
	0.16667	1.01212	0.85714	0.75000	0.85714				
	0.14286	1.01212	0.87500	0.77778	0.87500				
	0.12500	1.01212	0.88889	0.80000	0.88889				
0.04167	0.11111	1.01212	0.90000	0.81818	0.90000				
	0.10000	1.01212	0.90909	0.83333	0.90909				
	0.33333	1.00909	0.75000	0.60000	0.75000				
	0.25000	1.00909	0.80000	0.66667	0.80000				
	0.20000	1.00909	0.83333	0.71429	0.83333				
0.03846	0.16667	1.00909	0.85714	0.75000	0.85714				
	0.14286	1.00909	0.87500	0.77778	0.87500				
	0.12500	1.00909	0.88889	0.80000	0.88889				
	0.11111	1.00909	0.90000	0.81818	0.90000				
	0.10000	1.00909	0.90909	0.83333	0.90909				
0.03571	0.33333	1.00606	0.75000	0.60000	0.75000				
	0.25000	1.00606	0.80000	0.66667	0.80000				
	0.20000	1.00606	0.83333	0.71429	0.83333				
	0.16667	1.00606	0.85714	0.75000	0.85714				
	0.14286	1.00606	0.87500	0.77778	0.87500				
0.03333	0.12500	1.00606	0.88889	0.80000	0.88889				
	0.11111	1.00606	0.90000	0.81818	0.90000				
	0.10000	1.00606	0.90909	0.83333	0.90909				
	0.33333	1.00303	0.75000	0.60000	0.75000				
	0.25000	1.00303	0.80000	0.66667	0.80000				
0.03125	0.20000	1.00303	0.83333	0.71429	0.83333				
	0.16667	1.00303	0.85714	0.75000	0.85714				
	0.14286	1.00303	0.87500	0.77778	0.87500				
	0.12500	1.00303	0.88889	0.80000	0.88889				
	0.11111	1.00303	0.90000	0.81818	0.90000				
0.02941	0.10000	1.00303	0.90909	0.83333	0.90909				
	0.33333	1.00000	0.75000	0.60000	0.75000				
	0.25000	1.00000	0.80000	0.66667	0.80000				
	0.20000	1.00000	0.83333	0.71429	0.83333				
	0.16667	1.00000	0.85714	0.75000	0.85714				
0.02778	0.14286	1.00000	0.87500	0.77778	0.87500				
	0.12500	1.00000	0.88889	0.80000	0.88889				
	0.11111	1.00000	0.90000	0.81818	0.90000				
	0.10000	1.00000	0.90909	0.83333	0.90909				
	0.33333	0.99702	0.75000	0.60000	0.75000				
0.02632	0.25000	0.99702	0.80000	0.66667	0.80000				
	0.20000	0.99702	0.83333	0.71429	0.83333				
	0.16667	0.99702	0.85714	0.75000	0.85714				
	0.14286	0.99702	0.87500	0.77778	0.87500				
	0.12500	0.99702	0.88889	0.80000	0.88889				
0.02500	0.11111	0.99702	0.90000	0.81818	0.90000				
	0.10000	0.99702	0.90909	0.83333	0.90909				
	0.33333	0.99404	0.75000	0.60000	0.75000				
	0.25000	0.99404	0.80000	0.66667	0.80000				
	0.20000	0.99404	0.83333	0.71429	0.83333				
0.02381	0.16667	0.99404	0.85714	0.75000	0.85714				
	0.14286	0.99404	0.87500	0.77778	0.87500				
	0.12500	0.99404	0.88889	0.80000	0.88889				
	0.11111	0.99404	0.90000	0.81818	0.90000				
	0.10000	0.99404	0.90909	0.83333	0.90909				
0.02273	0.33333	0.99106	0.75000	0.60000	0.75000				
	0.25000	0.99106	0.80000	0.66667	0.80000				
	0.20000	0.99106	0.83333	0.71429	0.83333				
	0.16667	0.99106	0.85714	0.75000	0.85714				
	0.14286	0.99106	0.87500	0.77778	0.87500				
0.02176	0.12500	0.99106	0.88889	0.80000	0.88889				
	0.11111	0.99106	0.90000	0.81818	0.90000				
	0.10000	0.99106	0.90909	0.83333	0.90909				
	0.33333	0.98808	0.75000	0.60000	0.75000				
	0.25000	0.98808	0.80000	0.66667	0.80000				
0.02083	0.20000	0.98808	0.83333	0.71429	0.83333				
	0.16667	0.98808	0.85714	0.75000	0.85714				
	0.14286	0.98808	0.87500	0.77778	0.87500				
	0.12500	0.98808	0.88889	0.80000	0.88889				
	0.11111	0.98808	0.90000	0.81818	0.90000				
0.02000									

TABLE XII. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 0.5$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \text{ on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.88889	0.70000	0.60000	0.81818				
	0.25000	0.88889	0.74386	0.66667	0.87273				
	0.20000	0.88889	0.77295	0.71429	0.90909				
	0.16667	0.88889	0.79365	0.75000	0.93506				
	0.14286	0.88889	0.80914	0.77778	0.95455				
	0.12500	0.88889	0.82116	0.80000	0.96970				
	0.11111	0.88889	0.83077	0.81818	0.98182				
	0.10000	0.88889	0.83862	0.83333	0.99174				
0.25000	0.33333	0.90370	0.71053	0.60000	0.80357				
	0.25000	0.90370	0.75556	0.66667	0.85714				
	0.20000	0.90370	0.78544	0.71429	0.89286				
	0.16667	0.90370	0.80672	0.75000	0.91837				
	0.14286	0.90370	0.82265	0.77778	0.93750				
	0.12500	0.90370	0.83502	0.80000	0.95238				
	0.11111	0.90370	0.84490	0.81818	0.96429				
	0.10000	0.90370	0.85297	0.83333	0.97403				
0.20000	0.33333	0.91534	0.71739	0.60000	0.79412				
	0.25000	0.91534	0.76322	0.66667	0.84706				
	0.20000	0.91534	0.79365	0.71429	0.88235				
	0.16667	0.91534	0.81533	0.75000	0.90756				
	0.14286	0.91534	0.83156	0.77778	0.92647				
	0.12500	0.91534	0.84416	0.80000	0.94118				
	0.11111	0.91534	0.85424	0.81818	0.95294				
	0.10000	0.91534	0.86247	0.83333	0.96257				
0.16667	0.33333	0.92460	0.72222	0.60000	0.78750				
	0.25000	0.92460	0.76863	0.66667	0.84000				
	0.20000	0.92460	0.79946	0.71429	0.87500				
	0.16667	0.92460	0.82143	0.75000	0.90000				
	0.14286	0.92460	0.83788	0.77778	0.91875				
	0.12500	0.92460	0.85066	0.80000	0.93333				
	0.11111	0.92460	0.86087	0.81818	0.94500				
	0.10000	0.92460	0.86922	0.83333	0.95455				
0.14286	0.33333	0.93210	0.72581	0.60000	0.78261				
	0.25000	0.93210	0.77265	0.66667	0.83478				
	0.20000	0.93210	0.80378	0.71429	0.86957				
	0.16667	0.93210	0.82597	0.75000	0.89441				
	0.14286	0.93210	0.84259	0.77778	0.91304				
	0.12500	0.93210	0.85550	0.80000	0.92754				
	0.11111	0.93210	0.86582	0.81818	0.93913				
	0.10000	0.93210	0.87426	0.83333	0.94862				
0.12500	0.33333	0.93827	0.72857	0.60000	0.77885				
	0.25000	0.93827	0.77576	0.66667	0.83077				
	0.20000	0.93827	0.80713	0.71429	0.86538				
	0.16667	0.93827	0.82949	0.75000	0.89011				
	0.14286	0.93827	0.84624	0.77778	0.90865				
	0.12500	0.93827	0.85926	0.80000	0.92308				
	0.11111	0.93827	0.86966	0.81818	0.93462				
	0.10000	0.93827	0.87817	0.83333	0.94406				
0.11111	0.33333	0.94343	0.73077	0.60000	0.77586				
	0.25000	0.94343	0.77823	0.66667	0.82759				
	0.20000	0.94343	0.80979	0.71429	0.86207				
	0.16667	0.94343	0.83230	0.75000	0.88670				
	0.14286	0.94343	0.84916	0.77778	0.90517				
	0.12500	0.94343	0.86226	0.80000	0.91954				
	0.11111	0.94343	0.87273	0.81818	0.93103				
	0.10000	0.94343	0.88129	0.83333	0.94044				
0.10000	0.33333	0.94781	0.73256	0.60000	0.77344				
	0.25000	0.94781	0.78025	0.66667	0.82500				
	0.20000	0.94781	0.81197	0.71429	0.85938				
	0.16667	0.94781	0.83459	0.75000	0.88393				
	0.14286	0.94781	0.85153	0.77778	0.90234				
	0.12500	0.94781	0.86470	0.80000	0.91667				
	0.11111	0.94781	0.87523	0.81818	0.92813				
	0.10000	0.94781	0.88384	0.83333	0.93750				

TABLE XIII. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 1$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \\ \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \\ \text{on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.80000	0.67500	0.60000	0.85714				
	0.25000	0.80000	0.71579	0.66667	0.91429				
	0.20000	0.80000	0.74215	0.71429	0.95238				
	0.16667	0.80000	0.76190	0.75000	0.97959				
	0.14286	0.80000	0.77621	0.77778	1.00000				
	0.12500	0.80000	0.78730	0.80000	1.01587				
0.25000	0.11111	0.80000	0.79615	0.81818	1.02857	0.50000	0.00000	0.70711	0.70711
	0.10000	0.80000	0.80338	0.83333	1.03896		0.25000	0.70985	0.95985
	0.33333	0.83333	0.69079	0.60000	0.83333		0.66667	0.72598	1.13423
	0.25000	0.83333	0.73333	0.66667	0.88889		1.00000	0.74767	1.24767
	0.20000	0.83333	0.76149	0.71429	0.92593		1.25000	0.76787	1.32689
	0.16667	0.83333	0.78151	0.75000	0.95238		1.33333	0.77519	1.35254
0.20000	0.14286	0.83333	0.79647	0.77778	0.97222		1.66667	0.80676	1.45225
	0.12500	0.83333	0.80808	0.80000	0.98765		2.00000	0.84090	1.54800
	0.11111	0.83333	0.81735	0.81818	1.00000	0.33333	0.00000	0.79370	0.79370
	0.10000	0.83333	0.82492	0.83333	1.01010		0.25000	0.79781	0.99624
	0.33333	0.85714	0.70109	0.60000	0.81818		0.66667	0.82207	1.20364
	0.25000	0.85714	0.74483	0.66667	0.87273		1.00000	0.85499	1.35499
0.16667	0.20000	0.85714	0.77381	0.71429	0.90909		1.25000	0.88592	1.46612
	0.16667	0.85714	0.79443	0.75000	0.93506		1.33333	0.89720	1.50291
	0.14286	0.85714	0.80984	0.77778	0.95455		1.66667	0.94624	1.64910
	0.12500	0.85714	0.82180	0.80000	0.96970		2.00000	1.00000	1.79370
	0.11111	0.85714	0.83136	0.81818	0.98182	0.25000	0.00000	0.84090	0.84090
	0.10000	0.85714	0.83916	0.83333	0.99174		0.25000	0.84580	1.02258
0.14286	0.33333	0.87500	0.70833	0.60000	0.80769		0.66667	0.87479	1.24368
	0.25000	0.87500	0.75294	0.66667	0.86154		1.00000	0.91429	1.41429
	0.20000	0.87500	0.78252	0.71429	0.89744		1.25000	0.95158	1.54267
	0.16667	0.87500	0.80357	0.75000	0.92308		1.33333	0.96523	1.58563
	0.14286	0.87500	0.81932	0.77778	0.94231		1.66667	1.02477	1.75820
	0.12500	0.87500	0.83154	0.80000	0.95726		2.00000	1.09051	1.93140
0.12500	0.11111	0.87500	0.84130	0.81818	0.96923				
	0.10000	0.87500	0.84928	0.83333	0.97902				
	0.33333	0.88889	0.71371	0.60000	0.80000				
	0.25000	0.88889	0.75897	0.66667	0.85333				
	0.20000	0.88889	0.78901	0.71429	0.88889				
	0.16667	0.88889	0.81039	0.75000	0.91429				
0.11111	0.14286	0.88889	0.82639	0.77778	0.93333				
	0.12500	0.88889	0.83881	0.80000	0.94815				
	0.11111	0.88889	0.84873	0.81818	0.96000				
	0.10000	0.88889	0.85684	0.83333	0.96970				
	0.33333	0.90000	0.71786	0.60000	0.79412				
	0.25000	0.90000	0.76364	0.66667	0.84706				
0.10000	0.20000	0.90000	0.79403	0.71429	0.88235				
	0.16667	0.90000	0.81567	0.75000	0.90756				
	0.14286	0.90000	0.83187	0.77778	0.92647				
	0.12500	0.90000	0.84444	0.80000	0.94118				
	0.11111	0.90000	0.85449	0.81818	0.95294				
	0.10000	0.90000	0.86271	0.83333	0.96257				
0.09090	0.33333	0.90909	0.72115	0.60000	0.78947				
	0.25000	0.90909	0.76735	0.66667	0.84211				
	0.20000	0.90909	0.79802	0.71429	0.87719				
	0.16667	0.90909	0.81988	0.75000	0.90226				
	0.14286	0.90909	0.83623	0.77778	0.92105				
	0.12500	0.90909	0.84894	0.80000	0.93567				
0.08333	0.11111	0.90909	0.85909	0.81818	0.94737				
	0.10000	0.90909	0.86739	0.83333	0.95694				
	0.33333	0.91667	0.72384	0.60000	0.78571				
	0.25000	0.91667	0.77037	0.66667	0.83810				
	0.20000	0.91667	0.80128	0.71429	0.87302				
	0.16667	0.91667	0.82331	0.75000	0.89796				
0.07692	0.14286	0.91667	0.83980	0.77778	0.91667				
	0.12500	0.91667	0.85261	0.80000	0.93122				
	0.11111	0.91667	0.86284	0.81818	0.94286				
	0.10000	0.91667	0.87121	0.83333	0.95238				

TABLE XIV. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 1.5$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \text{ on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.75200	0.66000	0.60000	0.88235				
	0.25000	0.75200	0.69895	0.66667	0.94118				
	0.20000	0.75200	0.72464	0.71429	0.98039				
	0.16667	0.75200	0.74286	0.75000	1.00840				
	0.14286	0.75200	0.75645	0.77778	1.02941				
	0.12500	0.75200	0.76698	0.80000	1.04575				
	0.11111	0.75200	0.77538	0.81818	1.05882				
	0.10000	0.75200	0.78224	0.83333	1.06952				
	0.33333	0.79467	0.67895	0.60000	0.85227				
	0.25000	0.79467	0.72000	0.66667	0.90909				
0.25000	0.20000	0.79467	0.74713	0.71429	0.94697				
	0.16667	0.79467	0.76639	0.75000	0.97403				
	0.14286	0.79467	0.78077	0.77778	0.99432				
	0.12500	0.79467	0.79192	0.80000	1.01010				
	0.11111	0.79467	0.80082	0.81818	1.02273				
	0.10000	0.79467	0.80808	0.83333	1.03306				
	0.33333	0.82476	0.69130	0.60000	0.83333				
	0.25000	0.82476	0.73379	0.66667	0.88889				
	0.20000	0.82476	0.76190	0.71429	0.92593				
	0.16667	0.82476	0.78188	0.75000	0.95238				
0.20000	0.14286	0.82476	0.79681	0.77778	0.97222				
	0.12500	0.82476	0.80839	0.80000	0.98765				
	0.11111	0.82476	0.81763	0.81818	1.00000				
	0.10000	0.82476	0.82517	0.83333	1.01010				
	0.33333	0.84714	0.70000	0.60000	0.82031				
	0.25000	0.84714	0.74353	0.66667	0.87500				
	0.20000	0.84714	0.77236	0.71429	0.91146				
	0.16667	0.84714	0.79286	0.75000	0.93750				
	0.14286	0.84714	0.80818	0.77778	0.95703				
	0.12500	0.84714	0.82007	0.80000	0.97222				
0.16667	0.11111	0.84714	0.82957	0.81818	0.98438				
	0.10000	0.84714	0.83732	0.83333	0.99432				
	0.33333	0.86444	0.70645	0.60000	0.81081				
	0.25000	0.86444	0.75077	0.66667	0.86486				
	0.20000	0.86444	0.78014	0.71429	0.90090				
	0.16667	0.86444	0.80104	0.75000	0.92664				
	0.14286	0.86444	0.81667	0.77778	0.94595				
	0.12500	0.86444	0.82880	0.80000	0.96096				
	0.11111	0.86444	0.83848	0.81818	0.97297				
	0.10000	0.86444	0.84640	0.83333	0.98280				
0.14286	0.33333	0.87822	0.71143	0.60000	0.80357				
	0.25000	0.87822	0.75636	0.66667	0.85714				
	0.20000	0.87822	0.78616	0.71429	0.89286				
	0.16667	0.87822	0.80737	0.75000	0.91837				
	0.14286	0.87822	0.82324	0.77778	0.93750				
	0.12500	0.87822	0.83556	0.80000	0.95238				
	0.11111	0.87822	0.84539	0.81818	0.96429				
	0.10000	0.87822	0.85343	0.83333	0.97403				
	0.33333	0.88945	0.71538	0.60000	0.79787				
	0.25000	0.88945	0.76082	0.66667	0.85106				
0.12500	0.20000	0.88945	0.79096	0.71429	0.88652				
	0.16667	0.88945	0.81242	0.75000	0.91185				
	0.14286	0.88945	0.82848	0.77778	0.93085				
	0.12500	0.88945	0.84095	0.80000	0.94563				
	0.11111	0.88945	0.85091	0.81818	0.95745				
	0.10000	0.88945	0.85905	0.83333	0.96712				
	0.33333	0.89879	0.71860	0.60000	0.79327				
	0.25000	0.89879	0.76444	0.66667	0.84615				
	0.20000	0.89879	0.79487	0.71429	0.88141				
	0.16667	0.89879	0.81654	0.75000	0.90659				
0.11111	0.14286	0.89879	0.83276	0.77778	0.92548				
	0.12500	0.89879	0.84535	0.80000	0.94017				
	0.11111	0.89879	0.85541	0.81818	0.95192				
	0.10000	0.89879	0.86364	0.83333	0.96154				

TABLE XV. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 2$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \text{ on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7				
0.33333	0.33333	0.72222	0.65000	0.60000	0.90000				
	0.25000	0.72222	0.68772	0.66667	0.96000				
	0.20000	0.72222	0.71256	0.71429	1.00000				
	0.16667	0.72222	0.73016	0.75000	1.02857				
	0.14286	0.72222	0.74328	0.77778	1.05000				
	0.12500	0.72222	0.75344	0.80000	1.06667				
	0.11111	0.72222	0.76154	0.81818	1.08000				
	0.10000	0.72222	0.76815	0.83333	1.09091				
	0.33333	0.77037	0.67105	0.60000	0.86538				
0.25000	0.25000	0.77037	0.71111	0.66667	0.92308				
	0.20000	0.77037	0.73755	0.71429	0.96154				
	0.16667	0.77037	0.75630	0.75000	0.98901				
	0.14286	0.77037	0.77030	0.77778	1.00962				
	0.12500	0.77037	0.78114	0.80000	1.02564				
	0.11111	0.77037	0.78980	0.81818	1.03846				
	0.10000	0.77037	0.79686	0.83333	1.04895				
	0.33333	0.80423	0.68478	0.60000	0.84375				
	0.25000	0.80423	0.72644	0.66667	0.90000				
	0.20000	0.80423	0.75397	0.71429	0.93750				
	0.16667	0.80423	0.77352	0.75000	0.96429				
	0.14286	0.80423	0.78812	0.77778	0.98438				
	0.12500	0.80423	0.79944	0.80000	1.00000				
	0.11111	0.80423	0.80847	0.81818	1.01250				
	0.10000	0.80423	0.81585	0.83333	1.02273				
0.16667	0.33333	0.82937	0.69444	0.60000	0.82895				
	0.25000	0.82937	0.73725	0.66667	0.88421				
	0.20000	0.82937	0.76558	0.71429	0.92105				
	0.16667	0.82937	0.78571	0.75000	0.94737				
	0.14286	0.82937	0.80076	0.77778	0.96711				
	0.12500	0.82937	0.81243	0.80000	0.98246				
	0.11111	0.82937	0.82174	0.81818	0.99474				
	0.10000	0.82937	0.82935	0.83333	1.00478				
0.14286	0.33333	0.84877	0.70161	0.60000	0.81818				
	0.25000	0.84877	0.74530	0.66667	0.87273				
	0.20000	0.84877	0.77423	0.71429	0.90909				
	0.16667	0.84877	0.79481	0.75000	0.93506				
	0.14286	0.84877	0.81019	0.77778	0.95455				
	0.12500	0.84877	0.82212	0.80000	0.96970				
	0.11111	0.84877	0.83165	0.81818	0.98182				
	0.10000	0.84877	0.83943	0.83333	0.99174				
0.12500	0.33333	0.86420	0.70714	0.60000	0.81000				
	0.25000	0.86420	0.75152	0.66667	0.86400				
	0.20000	0.86420	0.78092	0.71429	0.90000				
	0.16667	0.86420	0.80184	0.75000	0.92571				
	0.14286	0.86420	0.81749	0.77778	0.94500				
	0.12500	0.86420	0.82963	0.80000	0.96000				
	0.11111	0.86420	0.83933	0.81818	0.97200				
	0.10000	0.86420	0.84725	0.83333	0.98182				
0.11111	0.33333	0.87677	0.71154	0.60000	0.80357				
	0.25000	0.87677	0.75646	0.66667	0.85714				
	0.20000	0.87677	0.78625	0.71429	0.89286				
	0.16667	0.87677	0.80745	0.75000	0.91837				
	0.14286	0.87677	0.82331	0.77778	0.93750				
	0.12500	0.87677	0.83562	0.80000	0.95238				
	0.11111	0.87677	0.84545	0.81818	0.96429				
	0.10000	0.87677	0.85349	0.83333	0.97403				
0.10000	0.33333	0.88721	0.71512	0.60000	0.79839				
	0.25000	0.88721	0.76049	0.66667	0.85161				
	0.20000	0.88721	0.79060	0.71429	0.88710				
	0.16667	0.88721	0.81203	0.75000	0.91244				
	0.14286	0.88721	0.82807	0.77778	0.93145				
	0.12500	0.88721	0.84051	0.80000	0.94624				
	0.11111	0.88721	0.85046	0.81818	0.95806				
	0.10000	0.88721	0.85859	0.83333	0.96774				

TABLE XVI. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 2.5$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2, \zeta_3 \text{ and } \zeta_7$$

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2) \text{ on the Values of } \zeta_5 \text{ and } \zeta_6$$

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.70204	0.64286	0.60000	0.91304				
	0.25000	0.70204	0.67970	0.66667	0.97391				
	0.20000	0.70204	0.70393	0.71429	1.01449				
	0.16667	0.70204	0.72109	0.75000	1.04348				
	0.14286	0.70204	0.73387	0.77778	1.06522				
0.25000	0.12500	0.70204	0.74376	0.80000	1.08213				
	0.11111	0.70204	0.75165	0.81818	1.09565	0.50000	0.00000	0.83850	1.54779
	0.10000	0.70204	0.75808	0.83333	1.10672		0.25000	0.84060	1.55319
	0.33333	0.75374	0.66541	0.60000	0.87500		0.66667	0.85283	1.58439
	0.25000	0.75374	0.70476	0.66667	0.93333		1.00000	0.86916	1.62528
0.20000	0.20000	0.75374	0.73071	0.71429	0.97222		1.25000	0.88434	1.66258
	0.16667	0.75374	0.74910	0.75000	1.00000		1.33333	0.88985	1.67598
	0.14286	0.75374	0.76282	0.77778	1.02083		1.66667	0.91373	1.73316
	0.12500	0.75374	0.77345	0.80000	1.03704		2.00000	0.93989	1.79445
	0.11111	0.75374	0.78192	0.81818	1.05000	0.33333	0.00000	1.00338	1.80779
0.16667	0.10000	0.75374	0.78884	0.83333	1.06061		0.25000	1.00656	1.81576
	0.33333	0.79009	0.68012	0.60000	0.85135		0.66667	1.02520	1.86214
	0.25000	0.79009	0.72118	0.66667	0.90811		1.00000	1.05040	1.92378
	0.20000	0.79009	0.74830	0.71429	0.94595		1.25000	1.07410	1.98077
	0.16667	0.79009	0.76755	0.75000	0.97297		1.33333	1.08277	2.00140
0.14286	0.14286	0.79009	0.78191	0.77778	0.99324		1.66667	1.12068	2.09044
	0.12500	0.79009	0.79305	0.80000	1.00901		2.00000	1.16282	2.18746
	0.11111	0.79009	0.80194	0.81818	1.02162	0.25000	0.00000	1.09892	1.95714
	0.10000	0.79009	0.80919	0.83333	1.03194		0.25000	1.10273	1.96659
	0.33333	0.81706	0.69048	0.60000	0.83523		0.66667	1.12519	2.02175
0.12500	0.25000	0.81706	0.73277	0.66667	0.89091		1.00000	1.15573	2.09556
	0.20000	0.81706	0.76074	0.71429	0.92803		1.25000	1.18462	2.16425
	0.16667	0.81706	0.78061	0.75000	0.95455		1.33333	1.19522	2.18922
	0.14286	0.81706	0.79545	0.77778	0.97443		1.66667	1.24181	2.29755
	0.12500	0.81706	0.80696	0.80000	0.98990		2.00000	1.29395	2.41655
0.11111	0.11111	0.81706	0.81615	0.81818	1.00227				
	0.10000	0.81706	0.82365	0.83333	1.01240				
	0.33333	0.83787	0.69816	0.60000	0.82353				
	0.25000	0.83787	0.74139	0.66667	0.87843				
	0.20000	0.83787	0.77001	0.71429	0.91503				
0.10000	0.16667	0.83787	0.79035	0.75000	0.94118				
	0.14286	0.83787	0.80556	0.77778	0.96078				
	0.12500	0.83787	0.81735	0.80000	0.97603				
	0.11111	0.83787	0.82676	0.81818	0.98824				
	0.10000	0.83787	0.83445	0.83333	0.99822				
0.08333	0.33333	0.85442	0.70408	0.60000	0.81466				
	0.25000	0.85442	0.74805	0.66667	0.86897				
	0.20000	0.85442	0.77718	0.71429	0.90517				
	0.16667	0.85442	0.79789	0.75000	0.93103				
	0.14286	0.85442	0.81338	0.77778	0.95043				
0.07692	0.12500	0.85442	0.82540	0.80000	0.96552				
	0.11111	0.85442	0.83499	0.81818	0.97759				
	0.10000	0.85442	0.84283	0.83333	0.98746				
	0.33333	0.86790	0.70879	0.60000	0.80769				
	0.25000	0.86790	0.75335	0.66667	0.86154				
0.07059	0.20000	0.86790	0.78289	0.71429	0.89744				
	0.16667	0.86790	0.80390	0.75000	0.92308				
	0.14286	0.86790	0.81962	0.77778	0.94231				
	0.12500	0.86790	0.83182	0.80000	0.95726				
	0.11111	0.86790	0.84156	0.81818	0.96923				
0.06522	0.10000	0.86790	0.84952	0.83333	0.97902				
	0.33333	0.87910	0.71262	0.60000	0.80208				
	0.25000	0.87910	0.75767	0.66667	0.85556				
	0.20000	0.87910	0.78755	0.71429	0.89120				
	0.16667	0.87910	0.80881	0.75000	0.91667				
0.06090	0.14286	0.87910	0.82471	0.77778	0.93576				
	0.12500	0.87910	0.83706	0.80000	0.95062				
	0.11111	0.87910	0.84692	0.81818	0.96250				
	0.10000	0.87910	0.85498	0.83333	0.97222				

TABLE XVII. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 3$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.68750	0.63750	0.60000	0.92308				
	0.25000	0.68750	0.67368	0.66667	0.98462				
	0.20000	0.68750	0.69746	0.71429	1.02564				
	0.16667	0.68750	0.71429	0.75000	1.05495				
	0.14286	0.68750	0.72681	0.77778	1.07692				
	0.12500	0.68750	0.73651	0.80000	1.09402				
	0.11111	0.68750	0.74423	0.81818	1.10769				
	0.10000	0.68750	0.75053	0.83333	1.11888				
0.25000	0.33333	0.74167	0.66118	0.60000	0.88235				
	0.25000	0.74167	0.70000	0.66667	0.94118				
	0.20000	0.74167	0.72557	0.71429	0.98039				
	0.16667	0.74167	0.74370	0.75000	1.00840				
	0.14286	0.74167	0.75721	0.77778	1.02941				
	0.12500	0.74167	0.76768	0.80000	1.04575				
	0.11111	0.74167	0.77602	0.81818	1.05882				
	0.10000	0.74167	0.78283	0.83333	1.06952				
0.20000	0.33333	0.77976	0.67663	0.60000	0.85714				
	0.25000	0.77976	0.71724	0.66667	0.91429				
	0.20000	0.77976	0.74405	0.71429	0.95238				
	0.16667	0.77976	0.76307	0.75000	0.97959				
	0.14286	0.77976	0.77726	0.77778	1.00000				
	0.12500	0.77976	0.78826	0.80000	1.01587				
	0.11111	0.77976	0.79703	0.81818	1.02857				
	0.10000	0.77976	0.80420	0.83333	1.03896				
0.16667	0.33333	0.80804	0.68750	0.60000	0.84000				
	0.25000	0.80804	0.72941	0.66667	0.89600				
	0.20000	0.80804	0.75711	0.71429	0.93333				
	0.16667	0.80804	0.77679	0.75000	0.96000				
	0.14286	0.80804	0.79148	0.77778	0.98000				
	0.12500	0.80804	0.80287	0.80000	0.99556				
	0.11111	0.80804	0.81196	0.81818	1.00800				
	0.10000	0.80804	0.81938	0.83333	1.01818				
0.14286	0.33333	0.82986	0.69556	0.60000	0.82759				
	0.25000	0.82986	0.73846	0.66667	0.88276				
	0.20000	0.82986	0.76684	0.71429	0.91954				
	0.16667	0.82986	0.78701	0.75000	0.94581				
	0.14286	0.82986	0.80208	0.77778	0.96552				
	0.12500	0.82986	0.81377	0.80000	0.98084				
	0.11111	0.82986	0.82310	0.81818	0.99310				
	0.10000	0.82986	0.83072	0.83333	1.00313				
0.12500	0.33333	0.84722	0.70179	0.60000	0.81818				
	0.25000	0.84722	0.74545	0.66667	0.87273				
	0.20000	0.84722	0.77437	0.71429	0.90909				
	0.16667	0.84722	0.79493	0.75000	0.93506				
	0.14286	0.84722	0.81030	0.77778	0.95455				
	0.12500	0.84722	0.82222	0.80000	0.96970				
	0.11111	0.84722	0.83174	0.81818	0.98182				
	0.10000	0.84722	0.83952	0.83333	0.99174				
0.11111	0.33333	0.86136	0.70673	0.60000	0.81081				
	0.25000	0.86136	0.75102	0.66667	0.86486				
	0.20000	0.86136	0.78037	0.71429	0.90090				
	0.16667	0.86136	0.80124	0.75000	0.92664				
	0.14286	0.86136	0.81685	0.77778	0.94595				
	0.12500	0.86136	0.82896	0.80000	0.96096				
	0.11111	0.86136	0.83864	0.81818	0.97297				
	0.10000	0.86136	0.84654	0.83333	0.98280				
0.10000	0.33333	0.87311	0.71076	0.60000	0.80488				
	0.25000	0.87311	0.75556	0.66667	0.85854				
	0.20000	0.87311	0.78526	0.71429	0.89431				
	0.16667	0.87311	0.80639	0.75000	0.91986				
	0.14286	0.87311	0.82220	0.77778	0.93902				
	0.12500	0.87311	0.83447	0.80000	0.95393				
	0.11111	0.87311	0.84427	0.81818	0.96585				
	0.10000	0.87311	0.85227	0.83333	0.97561				

TABLE XVIII. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 4$

(a) Effect of the Values of n and m Such That

$\frac{1}{10} \leq [n, m] \leq \frac{1}{3}$ on the Values of $\zeta_1, \zeta_2,$
 ζ_3 and ζ_7

(b) Effect of the Values of k

$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ and $W(0 \leq W \leq 2)$
on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.66800	0.63000	0.60000	0.93750				
	0.25000	0.66800	0.66526	0.66667	1.00000				
	0.20000	0.66800	0.68841	0.71429	1.04167				
	0.16667	0.66800	0.70476	0.75000	1.07143				
	0.14286	0.66800	0.71694	0.77778	1.09375				
0.25000	0.12500	0.66800	0.72635	0.80000	1.11111				
	0.11111	0.66800	0.73385	0.81818	1.12500	0.50000	0.00000	1.02353	1.98406
	0.10000	0.66800	0.73996	0.83333	1.13636		0.25000	1.02453	1.98626
	0.33333	0.72533	0.65526	0.60000	0.89286		0.66667	1.03055	1.99949
	0.25000	0.72533	0.69333	0.66667	0.95238		1.00000	1.03908	2.01819
0.20000	0.20000	0.72533	0.71839	0.71429	0.99206		1.25000	1.04746	2.03650
	0.16667	0.72533	0.73613	0.75000	1.02041		1.33333	1.05061	2.04335
	0.14286	0.72533	0.74936	0.77778	1.04167		1.66667	1.06477	2.07414
	0.12500	0.72533	0.75960	0.80000	1.05820		2.00000	1.08122	2.10971
	0.11111	0.72533	0.76776	0.81818	1.07143	0.33333	0.00000	1.30377	2.50205
0.20000	0.10000	0.72533	0.77441	0.83333	1.08225		0.25000	1.30543	2.50567
	0.33333	0.76571	0.67174	0.60000	0.86538		0.66667	1.31547	2.52751
	0.25000	0.76571	0.71172	0.66667	0.92308		1.00000	1.32974	2.55847
	0.20000	0.76571	0.73810	0.71429	0.96154		1.25000	1.34380	2.58890
	0.16667	0.76571	0.75679	0.75000	0.98901		1.33333	1.34909	2.60032
0.16667	0.14286	0.76571	0.77074	0.77778	1.00962		1.66667	1.37302	2.65187
	0.12500	0.76571	0.78155	0.80000	1.02564		2.00000	1.40097	2.71183
	0.11111	0.76571	0.79017	0.81818	1.03846	0.25000	0.00000	1.47231	2.81160
	0.10000	0.76571	0.79720	0.83333	1.04895		0.25000	1.47440	2.81613
	0.33333	0.79571	0.68333	0.60000	0.84677		0.66667	1.48704	2.84348
0.14286	0.25000	0.79571	0.72471	0.66667	0.90323		1.00000	1.50502	2.88231
	0.20000	0.79571	0.75203	0.71429	0.94086		1.25000	1.52279	2.92057
	0.16667	0.79571	0.77143	0.75000	0.96774		1.33333	1.52947	2.93494
	0.14286	0.79571	0.78591	0.77778	0.98790		1.66667	1.55979	2.99995
	0.12500	0.79571	0.79713	0.80000	1.00358		2.00000	1.59532	3.07581
0.12500	0.11111	0.79571	0.80609	0.81818	1.01613				
	0.10000	0.79571	0.81340	0.83333	1.02639				
	0.33333	0.81889	0.69194	0.60000	0.83333				
	0.25000	0.81889	0.73436	0.66667	0.88889				
	0.20000	0.81889	0.76241	0.71429	0.92593				
0.11111	0.16667	0.81889	0.78234	0.75000	0.95238				
	0.14286	0.81889	0.79722	0.77778	0.97222				
	0.12500	0.81889	0.80876	0.80000	0.98765				
	0.11111	0.81889	0.81797	0.81818	1.00000				
	0.10000	0.81889	0.82550	0.83333	1.01010				
0.10000	0.33333	0.83733	0.69857	0.60000	0.82317				
	0.25000	0.83733	0.74182	0.66667	0.87805				
	0.20000	0.83733	0.77044	0.71429	0.91463				
	0.16667	0.83733	0.79078	0.75000	0.94077				
	0.14286	0.83733	0.80599	0.77778	0.96037				
0.10000	0.12500	0.83733	0.81778	0.80000	0.97561				
	0.11111	0.83733	0.82719	0.81818	0.98780				
	0.10000	0.83733	0.83488	0.83333	0.99778				
	0.33333	0.85236	0.70385	0.60000	0.81522				
	0.25000	0.85236	0.74776	0.66667	0.86957				
0.10000	0.20000	0.85236	0.77684	0.71429	0.90580				
	0.16667	0.85236	0.79752	0.75000	0.93168				
	0.14286	0.85236	0.81297	0.77778	0.95109				
	0.12500	0.85236	0.82497	0.80000	0.96618				
	0.11111	0.85236	0.83455	0.81818	0.97826				
0.10000	0.10000	0.85236	0.84237	0.83333	0.98814				
	0.33333	0.86485	0.70814	0.60000	0.80882				
	0.25000	0.86485	0.75259	0.66667	0.86275				
	0.20000	0.86485	0.78205	0.71429	0.89869				
	0.16667	0.86485	0.80301	0.75000	0.92437				
0.10000	0.14286	0.86485	0.81868	0.77778	0.94363				
	0.12500	0.86485	0.83084	0.80000	0.95861				
	0.11111	0.86485	0.84055	0.81818	0.97059				
	0.10000	0.86485	0.84848	0.83333	0.98039				

TABLE XIX. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 5$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2,$$

ζ_3 and ζ_7

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2)$$

on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7				
0.33333	0.33333	0.65556	0.62500	0.60000	0.94737				
	0.25000	0.65556	0.65965	0.66667	1.01053				
	0.20000	0.65556	0.68237	0.71429	1.05263				
	0.16667	0.65556	0.69841	0.75000	1.08271				
	0.14286	0.65556	0.71035	0.77778	1.10526				
0.25000	0.12500	0.65556	0.71958	0.80000	1.12281				
	0.11111	0.65556	0.72692	0.81818	1.13684				
	0.10000	0.65556	0.73291	0.83333	1.14833				
	0.33333	0.71481	0.65132	0.60000	0.90000				
	0.25000	0.71481	0.68889	0.66667	0.96000				
0.20000	0.20000	0.71481	0.71360	0.71429	1.00000				
	0.16667	0.71481	0.73109	0.75000	1.02857				
	0.14286	0.71481	0.74412	0.77778	1.05000				
	0.12500	0.71481	0.75421	0.80000	1.06667				
	0.11111	0.71481	0.76224	0.81818	1.08000				
0.16667	0.10000	0.71481	0.76880	0.83333	1.09091				
	0.33333	0.75661	0.66848	0.60000	0.87097				
	0.25000	0.75661	0.70805	0.66667	0.92903				
	0.20000	0.75661	0.73413	0.71429	0.96774				
	0.16667	0.75661	0.75261	0.75000	0.99539				
0.14286	0.14286	0.75661	0.76640	0.77778	1.01613				
	0.12500	0.75661	0.77708	0.80000	1.03226				
	0.11111	0.75661	0.78559	0.81818	1.04516				
	0.10000	0.75661	0.79254	0.83333	1.05572				
	0.33333	0.78770	0.68056	0.60000	0.85135				
0.12500	0.25000	0.78770	0.72157	0.66667	0.90811				
	0.20000	0.78770	0.74864	0.71429	0.94595				
	0.16667	0.78770	0.76786	0.75000	0.97297				
	0.14286	0.78770	0.78220	0.77778	0.99324				
	0.12500	0.78770	0.79331	0.80000	1.00901				
0.11111	0.11111	0.78770	0.80217	0.81818	1.02162				
	0.10000	0.78770	0.80941	0.83333	1.03194				
	0.33333	0.81173	0.68952	0.60000	0.83721				
	0.25000	0.81173	0.73162	0.66667	0.89302				
	0.20000	0.81173	0.75946	0.71429	0.93023				
0.10000	0.16667	0.81173	0.77922	0.75000	0.95681				
	0.14286	0.81173	0.79398	0.77778	0.97674				
	0.12500	0.81173	0.80543	0.80000	0.99225				
	0.11111	0.81173	0.81456	0.81818	1.00465				
	0.10000	0.81173	0.82201	0.83333	1.01480				
0.08333	0.33333	0.83086	0.69643	0.60000	0.82653				
	0.25000	0.83086	0.73939	0.66667	0.88163				
	0.20000	0.83086	0.76782	0.71429	0.91837				
	0.16667	0.83086	0.78802	0.75000	0.94461				
	0.14286	0.83086	0.80311	0.77778	0.96429				
0.07692	0.12500	0.83086	0.81481	0.80000	0.97959				
	0.11111	0.83086	0.82416	0.81818	0.99184				
	0.10000	0.83086	0.83179	0.83333	1.00186				
	0.33333	0.84646	0.70192	0.60000	0.81818				
	0.25000	0.84646	0.74558	0.66667	0.87273				
0.07059	0.20000	0.84646	0.77448	0.71429	0.90909				
	0.16667	0.84646	0.79503	0.75000	0.93506				
	0.14286	0.84646	0.81039	0.77778	0.95455				
	0.12500	0.84646	0.82231	0.80000	0.96970				
	0.11111	0.84646	0.83182	0.81818	0.98182				
0.06522	0.10000	0.84646	0.83959	0.83333	0.99174				
	0.33333	0.85943	0.70640	0.60000	0.81148				
	0.25000	0.85943	0.75062	0.66667	0.86557				
	0.20000	0.85943	0.77991	0.71429	0.90164				
	0.16667	0.85943	0.80075	0.75000	0.92740				
0.06090	0.14286	0.85943	0.81633	0.77778	0.94672				
	0.12500	0.85943	0.82842	0.80000	0.96175				
	0.11111	0.85943	0.83807	0.81818	0.97377				
	0.10000	0.85943	0.84596	0.83333	0.98361				

TABLE XX. THE GENERAL MAGNITUDES OF THE
DIMENSIONLESS COEFFICIENTS FOR $U = 6$

(a) Effect of the Values of n and m Such That

$$\frac{1}{10} \leq [n, m] \leq \frac{1}{3} \text{ on the Values of } \zeta_1, \zeta_2,$$

ζ_3 and ζ_7

(b) Effect of the Values of k

$$\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right) \text{ and } W(0 \leq W \leq 2)$$

on the Values of ζ_5 and ζ_6

N	M	ZETA 1	ZETA 2	ZETA 3	ZETA 7	K	W	ZETA 5	ZETA 6
0.33333	0.33333	0.64694	0.62143	0.60000	0.95455				
	0.25000	0.64694	0.65564	0.66667	1.01818				
	0.20000	0.64694	0.67805	0.71429	1.06061				
	0.16667	0.64694	0.69388	0.75000	1.09091				
	0.14286	0.64694	0.70565	0.77778	1.11364				
0.25000	0.12500	0.64694	0.71474	0.80000	1.13131				
	0.11111	0.64694	0.72198	0.81818	1.14545				
	0.10000	0.64694	0.72788	0.83333	1.15702				
	0.33333	0.70748	0.64850	0.60000	0.90517				
	0.25000	0.70748	0.68571	0.66667	0.96552				
0.20000	0.20000	0.70748	0.71018	0.71429	1.00575				
	0.16667	0.70748	0.72749	0.75000	1.03448				
	0.14286	0.70748	0.74038	0.77778	1.05603				
	0.12500	0.70748	0.75036	0.80000	1.07280				
	0.11111	0.70748	0.75831	0.81818	1.08621				
0.16667	0.10000	0.70748	0.76479	0.83333	1.09718				
	0.33333	0.75024	0.66615	0.60000	0.87500				
	0.25000	0.75024	0.70542	0.66667	0.93333				
	0.20000	0.75024	0.73129	0.71429	0.97222				
	0.16667	0.75024	0.74963	0.75000	1.00000				
0.14286	0.14286	0.75024	0.76330	0.77778	1.02083				
	0.12500	0.75024	0.77388	0.80000	1.03704				
	0.11111	0.75024	0.78232	0.81818	1.05000				
	0.10000	0.75024	0.78921	0.83333	1.06061				
	0.33333	0.78207	0.67857	0.60000	0.85465				
0.12500	0.25000	0.78207	0.71933	0.66667	0.91163				
	0.20000	0.78207	0.74623	0.71429	0.94961				
	0.16667	0.78207	0.76531	0.75000	0.97674				
	0.14286	0.78207	0.77955	0.77778	0.99709				
	0.12500	0.78207	0.79058	0.80000	1.01292				
0.11111	0.11111	0.78207	0.79938	0.81818	1.02558				
	0.10000	0.78207	0.80656	0.83333	1.03594				
	0.33333	0.80669	0.68779	0.60000	0.84000				
	0.25000	0.80669	0.72967	0.66667	0.89600				
	0.20000	0.80669	0.75735	0.71429	0.93333				
0.10000	0.16667	0.80669	0.77699	0.75000	0.96000				
	0.14286	0.80669	0.79167	0.77778	0.98000				
	0.12500	0.80669	0.80304	0.80000	0.99556				
	0.11111	0.80669	0.81212	0.81818	1.00800				
	0.10000	0.80669	0.81953	0.83333	1.01818				
0.08333	0.33333	0.82630	0.69490	0.60000	0.82895				
	0.25000	0.82630	0.73766	0.66667	0.88421				
	0.20000	0.82630	0.76595	0.71429	0.92105				
	0.16667	0.82630	0.78604	0.75000	0.94737				
	0.14286	0.82630	0.80106	0.77778	0.96711				
0.07692	0.12500	0.82630	0.81270	0.80000	0.98246				
	0.11111	0.82630	0.82199	0.81818	0.99474				
	0.10000	0.82630	0.82958	0.83333	1.00478				
	0.33333	0.84230	0.70055	0.60000	0.82031				
	0.25000	0.84230	0.74402	0.66667	0.87500				
0.07059	0.20000	0.84230	0.77280	0.71429	0.91146				
	0.16667	0.84230	0.79326	0.75000	0.93750				
	0.14286	0.84230	0.80854	0.77778	0.95703				
	0.12500	0.84230	0.82040	0.80000	0.97222				
	0.11111	0.84230	0.82987	0.81818	0.98437				
0.06516	0.10000	0.84230	0.83760	0.83333	0.99432				
	0.33333	0.85560	0.70515	0.60000	0.81338				
	0.25000	0.85560	0.74921	0.66667	0.86761				
	0.20000	0.85560	0.77839	0.71429	0.90376				
	0.16667	0.85560	0.79914	0.75000	0.92958				
0.06073	0.14286	0.85560	0.81466	0.77778	0.94894				
	0.12500	0.85560	0.82669	0.80000	0.96401				
	0.11111	0.85560	0.83630	0.81818	0.97606				
	0.10000	0.85560	0.84416	0.83333	0.98592				

TABLE XXI. EFFECT OF U ON THE EXTREME VALUES OF ζ_1 , ζ_2 , ζ_3 , AND ζ_7 ,
 FOR $n \left(\frac{1}{10} \leq n \leq \frac{1}{3} \right)$ AND $m \left(\frac{1}{10} \leq m \leq \frac{1}{3} \right)$

U	ζ_1		ζ_2		ζ_3		ζ_7		$\lim_{U \rightarrow -1} \frac{\zeta_1}{\zeta_2}$	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
$-6 \leq U < -1$	0.80424	0.48889	0.81818	0.27484	$\uparrow$	$\uparrow$	3.63636	0.84615	2.89562	1.419
$U = -1$	∞	∞	∞	∞			0	0		
$U = -\frac{1}{2}$	2	1.25758	1.12051	0.80769			0.83333	0.6		
$U = 0$	1.1	1.01515	0.90909	0.75			0.90909	0.75		
$U = \frac{1}{2}$	0.94781	0.88889	0.88384	0.7			0.99174	0.81818		
$U = 1$	0.91667	0.8	0.87121	0.675	$\downarrow$	$\downarrow$	1.03896	0.8	1.15702	0.79327
$1 < U \leq 6$	0.89879	0.64694	0.86364	0.62143						

TABLE XXII. EFFECT OF U ON THE EXTREME VALUES OF
 ζ_5 AND ζ_6 FOR $W(0 \leq W \leq 2)$, $k\left(\frac{1}{4} \leq k \leq \frac{1}{2}\right)$ AND ANY
 VALUE OF n AND m SUCH THAT $\frac{1}{10} \leq (n, m) \leq \frac{1}{3}$

U	W	k	ζ_5		ζ_6	
			Max	Min	Max	Min
$-6 \leq U < -1$		1/2	1.27042	0.71773	2.50938	1.14412
		1/3	1.73565	0.82	3.4143	1.23599
		1/4	2.02915	0.87796	3.98356	1.29139
U = -1		1/2	0.8409	0.7	1.548	0.70711
		1/3	1	0.7937	1.7937	0.7937
		1/4	1.09051	0.8409	1.9314	0.8409
U = - $\frac{1}{2}$		1/2	0.8269	0.73268	1.50847	0.96593
		1/3	0.97656	0.82214	1.73091	0.97017
		1/4	1.06102	0.86543	1.85432	0.975
U = 0		1/2	0.82244	0.73789	1.49535	1
		1/3	0.96899	0.82606	1.70998	1
		1/4	1.05143	0.87193	1.82858	1
U = $\frac{1}{2}$		1/2	0.8269	0.73268	1.50847	0.96593
		1/3	0.97656	0.82214	1.73091	0.97017
		1/4	1.06102	0.86543	1.85432	0.975
U = 1		1/2	0.8409	0.70711	1.548	0.70711
		1/3	1	0.7937	1.7937	0.7937
		1/4	1.09051	0.8409	1.9314	0.8409
$1 < U \leq 6$		1/2	1.27042	0.71773	2.50938	1.14412
		1/3	1.73565	0.82	3.4143	1.23599
		1/4	2.02915	0.87796	3.98356	1.29139

PRESSURE SOLUTION OF SHORT FULL JOURNAL BEARING

Because $\frac{L}{D} \doteq 0.3$, the experimental apparatus represents the "short-bearing" regime of film behavior. Consequently, it is pertinent to set up equations for turbulent film behavior, applicable exactly to the limiting case of $\frac{L}{D} \rightarrow 0$ (i.e., infinitely narrow bearing), and presumably applicable with diminishing accuracy as $\frac{L}{D}$ becomes larger.

Short-Bearing Equations

The short-bearing limiting case may be characterized by a tangential Couette flow and an axial pressure flow. This combined flow may be obtained from the flow in the finite bearing in Equations (66) to (68), when the tangential pressure flow component of velocity U approaches zero. When $U = 0$, Equations (66) to (68) are, thus, reduced to

$$\frac{2\zeta_3}{\zeta_2} W \frac{\partial W}{\partial Z} + \frac{\partial W}{\partial X} + \frac{1}{H} \frac{dH}{dX} W = - \frac{1}{\zeta_2} \frac{\partial P}{\partial Z} - \frac{\zeta_6}{\zeta_2} C_{xc} \frac{W}{H} \quad (86)$$

and

$$\frac{1}{H} \frac{dH}{dX} + \zeta_7 \frac{\partial W}{\partial Z} = 0 \quad (87)$$

By using the condition $W = 0$ at $Z = 0$, Equation (87) can be integrated as

$$W = - \frac{1}{\zeta_7} \frac{\frac{dH}{dX}}{H} Z \quad (88)$$

Differentiating W with respect X and Z , respectively,

$$\frac{\partial W}{\partial X} = - \frac{H \frac{d^2 H}{dX^2} - \left(\frac{dH}{dX} \right)^2}{\zeta_7 H^2} \cdot Z \quad (89)$$

and

$$\frac{\partial W}{\partial Z} = - \frac{1}{\zeta_7} \frac{\frac{dH}{dX}}{H} \quad (90)$$

Substituting Equations (88) to (90) into Equation (80), $\frac{\partial P}{\partial Z}$ is

$$\frac{\partial P}{\partial Z} = 2 A(X) Z \quad (91)$$

where

$$\begin{aligned} 2A(X) = & \frac{\zeta_6 C_{xc}}{\zeta_7} \cdot \frac{\frac{dH}{dX}}{H^2} - \frac{2\zeta_3}{(\zeta_7)^2} \left(\frac{\frac{dH}{dX}}{H} \right)^2 \\ & + \frac{\zeta_4}{\zeta_7} \cdot \frac{\left\{ H \frac{d^2 H}{dX^2} - \left(\frac{dH}{dX} \right)^2 \right\}}{H^2} + \frac{\zeta_4}{\zeta_7} \left(\frac{\frac{dH}{dX}}{H} \right)^2 \end{aligned} \quad (92)$$

Integrating $\frac{\partial P}{\partial Z}$ in Equation (91) with respect to Z , and using the boundary condition $P = 0$ at $Z = Z_{\max} = \frac{L/2}{2r} = \frac{L/2}{D}$, $P(X, Z)$ is

$$P(X, Z) = \left\{ A(X) \right\} \cdot \left\{ Z^2 - Z_{\max}^2 \right\} \quad (93)$$

The pressure distribution in Equation (93) is parabolic in Z .

Short-Bearing Equations For Full Journal Bearing

Using the notation shown in Figure 5, the dimensionless film equation, H , is

$$H = 1 + \epsilon \cos \phi = 1 + \epsilon \cos \beta X \quad (94)$$

FIGURE 5. NOTATION FOR SHORT FULL JOURNAL BEARING

Substituting Equation (94) into Equation (93), P is

$$P(\phi, z) = \left\{ 1 - \left(\frac{2z}{L} \right)^2 \right\} \cdot \{ P_{\text{friction}} + P_{\text{inertia}} \} \quad (95)$$

where

$$\begin{aligned} P_f &= P_{\text{friction}} \\ &= \frac{\zeta_6 C_{xc} \epsilon \left(\frac{L}{D} \right)}{2 \left(\frac{h}{L} \right)} \cdot \frac{\sin \phi}{(1 + \epsilon \cos \phi)^2} \end{aligned} \quad (96)$$

and

$$\begin{aligned} P_i &= P_{\text{inertia}} \\ &= \zeta_4 \frac{\epsilon \left(\frac{L}{D} \right)^2 \cos \phi}{(1 + \epsilon \cos \phi)} + 2 \frac{\zeta_3}{\zeta_7} \frac{\left(\epsilon \frac{L}{D} \right)^2 \sin^2 \phi}{(1 + \epsilon \cos \phi)^2} \end{aligned} \quad (97)$$

In the right hand side of Equation (95), the first and second terms denote the pressure due to friction and inertia, respectively. If the inertia effect upon the pressure is neglected, $P(\phi, z)$ is reduced to

$$P(\phi, z) = P_f(\phi, z) \quad (98)$$

where $P_f(\phi, z)$ is in the same form as in Equation (96).

The pressure distribution in Equations (95) and (98) is parabolic in z . For the case of $P = P_f$ in Equation (98), $P(\phi, z)$ is antisymmetrical about $\phi = \pi$ and $P = 0$; that is, the region of negative pressures is identical to that of positive pressures.

At the central line, P is obtained by putting $z = 0$ in Equation (95)

$$P(\phi, 0) = P_f + P_i \quad (99)$$

where P_f and P_i are in the same form as in Equations (96) and (97), respectively.

Numerical Example for Comparison Between Theoretical and Experimental

Results

Experimental Results. In Figure 29 of Reference (3), the given values of the bearing-model experiments are such that $\epsilon = 0.5$, $\bar{h} = .045$ ft, $r \doteq 3$ ft, $D = 2r$, $L = \frac{10}{6}$ ft and $\beta = \frac{\bar{h}}{r} = .015$. In Figure 6, P , obtained from the previous experiment⁽³⁾, is replotted for three runs ($\bar{R} = 2400, 2100$ and 1800).

Theoretical Results. Taking the mean value of $\bar{R}$ for three runs, $\bar{R} = 2100$, and using Figure 9(a), for the case of $\frac{h}{\bar{h}} = 1 \left(\frac{\delta^*}{b} \right)$ for $\epsilon = 0.5$ may be approximated by that for $\epsilon = 0$, $\frac{\delta^*}{b}$ is found to be 0.11. Since $\frac{\delta^*}{b} = \frac{n}{1+n}$ and $n \doteq m$, m may be calculated to be $1/8$. For $U = 0$, Table XI shows the values of all coefficients. For $m = 1/8$ and any value of n in Table XI (a), ζ_2 , ζ_3 and ζ_7 can be found as .88889, 0.8, and 0.88889. Because of very strong Couette flow in the short bearing, we may assume $W \ll 1$ (i. e. $W \doteq 0$). In Table XI(b), no matter which value of k is chosen, $\zeta_6 = 1$ for $W \doteq 0$. As for the evaluation of C_{xc} , we may use $\bar{R} = 2100$ and Figure 13⁽³⁾ to obtain C_{xc} equal to the value of 0.0075.

Substituting the numerical values of the dimensionless coefficients ($\zeta_2 = 0.8889$, $\zeta_3 = 0.8$, $\zeta_6 = 1$, $\zeta_7 = 0.8889$ and $C_{xc} = 0.0075$), and geometrical parameters ($\epsilon = 0.5$, $\bar{h} = .045$ ft, $D = 6$ ft, $L = \frac{10}{6}$ ft and $\beta = 0.015$) into Equation (99), $P(\phi, 0)$ is calculated and plotted in Figure 6.

Remarks

$P(\phi, 0)$, predicted by theory and measured by experiment, is seen to correspond well in regions of positive pressure. This fact supports

FIGURE 6. COMPARISON OF P PREDICTED BY THEORY WITH THAT OBTAINED FROM EXPERIMENT FOR SHORT FULL JOURNAL BEARING WITH $\epsilon = 1/2$

the arguments that the very strong tangential Couette flow is significant in this short bearing model $\left(\frac{L}{D} \doteq \frac{1}{3}\right)$, in such regions. In the regions of negative pressure this discrepancy may result from the components of momentum of the fluid being drawn from the ambient into the film.

RECENT WORK ON THE UNIVERSAL VELOCITY PROFILE

Recently Rasmussen and Karamcheti (4) have derived a single formula for the "law of the wall" or the "universal velocity profile" by postulating an expression for the eddy viscosity valid near the wall and using this to derive a single wall-law equation. Details of their method will not be repeated here, but it is informative to compare their results with those obtained using the single wall-law equation suggested during the present work.

Figure 7 shows the wall-law function relating $U^+ \equiv u\sqrt{\rho/\tau_w}$ and $Y^+ \equiv y\sqrt{\tau_w/\rho}/\nu$ as proposed by Reference (4), the equation being

$$Y^+ = U^+ + \frac{2}{\beta} \left(\cosh KU^+ - \frac{(KU^+)^2}{2} - 1 \right) \quad (100)$$

where K and β are experimentally determined constants. The wall-law equation proposed in the present work

$$Y^+ = U^+ + \left(\frac{U^+}{8.74} \right)^7 \quad (101)$$

is also shown in the figure. It is seen that both expressions fit the experimental data of Laufer (5) very well in this range. Also, for comparison, some points are included from an empirical expression by Deissler (6).

Comparisons can also be made in terms of shear stress by using the relationship

$$\tau = \rho (\nu + \epsilon) \frac{\partial u}{\partial y} \quad (102)$$

where τ is the total shear stress, composed of laminar and turbulent

FIGURE 7. UNIVERSAL VELOCITY PROFILE SHOWING PROPOSED APPROXIMATIONS AS COMPARED WITH LAUFER'S DATA

shear, ρ is the density, ν the kinematic viscosity, ϵ the virtual kinematic viscosity, u is the time-mean velocity in the x -direction (direction of flow), and y is measured normal to the flow. The assumption is made that in a region near the wall

$$\tau \doteq \tau_w \quad (103)$$

where τ_w is the wall shear stress. Then using Equation (102)

$$\tau_w = \rho(\nu + \epsilon) \frac{\partial u}{\partial y} \quad (104)$$

The turbulent component τ' of the total shear stress is

$$\tau' = \rho \epsilon \frac{\partial u}{\partial y} \quad (105)$$

then

$$\tau'/\tau_w = (\epsilon/\nu) / (1 + \epsilon/\nu) \quad (106)$$

To put Equation (106) in terms of U^+ and Y^+ , Equation (104) may be rearranged as

$$(\tau_w/\rho\nu) \frac{\partial u}{\partial y} = 1 + \epsilon/\nu \quad (107)$$

Introducing the friction velocity $u^* = \sqrt{\tau_w/\rho}$ into the numerator and denominator of the left-hand side of Equation (107)

$$\frac{\tau_w}{\rho} \frac{\partial(yu^*/\nu)}{\partial(yu^*)} = 1 + \epsilon/\nu \quad (108)$$

Using the definition of friction velocity $\tau_w = \rho u^{*2}$ then

$$u^{*2} \frac{\partial(yu^*/\nu)}{\partial(yu^*)} = 1 + \epsilon/\nu \quad (109)$$

In terms of U^+ and Y^+ Equation (109) becomes

$$\frac{\partial Y^+}{\partial U^+} = 1 + \epsilon/\nu \quad (110)$$

Then Equation (106) may be written as

$$\tau'/\tau_w = (\epsilon/\nu) / (1 + \epsilon/\nu) = \frac{\frac{\partial Y^+}{\partial U^+} - 1}{\frac{\partial Y^+}{\partial U^+}} \quad (111)$$

With the use of Equation (111) and the proposed wall-law expression Equation (101), a plot of the dimensionless turbulent shear stress τ'/τ_w may be obtained as shown in Figure 8. The relationship from Reference (4) is shown on the figure as well as the data of Schubauer (7). Again the agreement of the two proposed approximations with the experimental data is seen to be good, the curve from Reference (4) fitting the data slightly better than Equation (101) for $Y^+ < 50$.

FIGURE 8. TURBULENT SHEARING STRESS NEAR A WALL SHOWING PROPOSED APPROXIMATIONS AS COMPARED WITH SCHUBAUER'S DATA

ADDITIONAL DATA ON DISPLACEMENT THICKNESS FOR THE FULL JOURNAL BEARING

In Report No. SwRI-1228-60, Reference (3), experimental data were presented on the variation of the dimensionless displacement thickness δ^*/b with local film thickness and Reynolds numbers R and $\bar{R}$ based respectively on local and mean film thickness (see table of symbols). Additional data have been taken, mostly at lower Reynolds numbers, in an attempt to better determine the displacement-thickness/Reynolds-number relationship for various local film thicknesses in the low Reynolds number range. The results are in general agreement with the previous data, and it is clear that additional measurements will not improve them greatly, since experimental-scatter limits the resolution possible.

The earlier data have been replotted, along with the more recent results, in Figures 9, 10, and 11, where now the results for each local film thickness are shown on a separate plot and compared with the data obtained with the journal operating concentrically.

In Figure 9 the data for δ^*/b have been plotted against $\bar{R}$, the mean Reynolds number. Figure 9(a) shows the data for $h/\bar{h} = 1.00$ (concentric journal) along with the curve predicted by using the universal velocity profile to compute δ^*/b . For the Reynolds number range shown, the universal velocity profile is seen to overestimate the experimental value of δ^*/b at any given $\bar{R}$, but it appears that at higher Reynolds numbers the two curves may tend to coincide. The dotted curve is drawn as the best representation of the experimental data. Note that at $\bar{R} = 100$, $\delta^*/b = 0.5$

FIGURE 9. EFFECT OF LOCAL FILM THICKNESS ON THE RELATIONSHIP BETWEEN NOMINAL REYNOLDS NUMBER $\bar{R}$ AND DISPLACEMENT THICKNESS FOR ECCENTRIC JOURNAL PLACEMENT (a) $h/\bar{h} = 1.00$, (b) $h/\bar{h} = 1.67$, (c) $h/\bar{h} = 0.46$, (d) $h/\bar{h} = 0.18$

FIGURE 10. EFFECT OF LOCAL FILM THICKNESS ON THE RELATIONSHIP BETWEEN LOCAL REYNOLDS NUMBER R AND DISPLACEMENT THICKNESS FOR ECCENTRIC JOURNAL PLACEMENT (a) $h/\bar{h} = 1.00$, (b) $h/\bar{h} = 1.67$, (c) $h/\bar{h} = 0.46$, (d) $h/\bar{h} = 0.18$

(a)

(b)

(c)

(d)

FIGURE 11. EFFECT OF LOCAL FILM THICKNESS ON THE RELATIONSHIP BETWEEN TAYLOR NUMBER AND DISPLACEMENT THICKNESS FOR ECCENTRIC JOURNAL PLACEMENT (a) $h/\bar{h} = 1.00$, (b) $h/\bar{h} = 1.67$, (c) $h/\bar{h} = 0.46$, (d) $h/\bar{h} = 0.18$

indicating that the flow is laminar, thus for $\bar{R} < 100$ the value of δ^*/b will also be 0.5. Figure 9(b), (c), and (d) show the experimental values of δ^*/b for three different local film thicknesses compared with the experimentally obtained curve for the concentric journal. The data for the thick film shown in Figure 9(b) fit the curve for the concentric journal well, but for the two thinner films, shown in Figure 9(c) and (d), the curve for the concentric journal would underestimate the experimental values for δ^*/b .

Figure 10 shows the data for δ^*/b plotted against the local Reynolds number R . In Figure 10(a) the dotted curve has been drawn as the best representation of the experimental data for the journal operating concentrically. Figure 10(b), (c), and (d) show the values for three different local film thicknesses, again compared with the experimentally obtained curve for the concentric journal. The agreement is seen to be good except for the thinner film of $h/\bar{h} = 0.18$, where below $R = 100$ the experimental values are considerably below those for the concentrically operating journal.

Figure 11 shows plots similar to Figures 9 and 10 where δ^*/b has been plotted against local Taylor number ($T \equiv 4R\sqrt{h/r}$). Here the experimental values for $h/\bar{h} = 0.18$ show poor agreement with the curve obtained for the journal operating concentrically.

If, for purposes of simplifying the analysis, a single functional relationship between δ^*/b and Reynolds number for all film thicknesses was sought, then the δ^*/b vs $\bar{R}$ curve for Couette flow ($h/\bar{h} = 1.00$) might be chosen as a reasonable estimate for the range of local film thicknesses

investigated. Any δ^*/b value obtained from this curve can be converted to the velocity profile exponent n by using the defining equation for displacement thickness in the half channel,

$$\delta^* = \int_0^b (1 - u/u_b) dy \quad (112)$$

Then using the equation

$$u/u_b = (y/b)^n \quad (113)$$

Equation (112) becomes

$$\delta^* = \int_0^b [1 - (y/b)^n] dy \quad (114)$$

Then

$$\delta^*/b = \frac{n}{n+1} \quad (115)$$

or

$$n = \frac{\delta^*/b}{1 - \delta^*/b} \quad (116)$$

In Figure 12, Equation (116) has been used to compute the velocity profile exponent n from the δ^*/b values of Figure 9(a). Also included in the figure are some data of Reichardt (8) and Robertson (9) for plane-Couette flow. The n vs $\bar{R}$ relationship as predicted by the universal velocity profile is also plotted and shows fair agreement with the data of Reichardt and Robertson. The data from the present work depart considerably from both the universal velocity profile and from the data of Reichardt and Robertson.

FIGURE 12. VARIATION OF VELOCITY PROFILE EXPONENT n WITH NOMINAL REYNOLDS NUMBER $\bar{R}$ FOR CONCENTRIC JOURNAL PLACEMENT

The experimental values of the velocity profile exponent n shown in Figure 12 are used in the computation of the dimensionless coefficients in the film equation in the analysis section of this report. As seen previously in this report these dimensionless quantities are relatively insensitive to variation in the velocity profile exponent n , so that no large errors are introduced in the film equations if n is only a reasonable approximation.

EXPERIMENTAL RESULTS ON A PAD BEARING

Test Apparatus

To simulate a pad bearing, panels were removed from the 360° bearing model, isolating a single panel to be used as a test pad. This pad can be placed concentric with the journal, resulting in a constant film thickness of 0.54 in., or pivoted to various angles. The length, l , of the pad measured along its arc is 33.75 in., and the width is 20 in. From these dimensions, it is evident that the pad represents the case of the "short" bearing with end leakage, where "short" by usual convention, refers to the axial dimension. To approximate the infinitely "long" bearing without end leakage, shields were attached to the edges of the pad on both sides which extended across the film to the journal and along the complete length of the pad, thus blocking the flow in the axial direction.

Experimental Results on a Concentric Pad with End Leakage

The pad was first set concentric with no restriction on the flow in the axial direction and pressures were measured at the axial midspan of the pad along its length for two mean Reynolds numbers $\bar{R}$. These are shown in Figure 13 where the gage pressure in atmospheres is plotted against dimensionless position measured from the leading edge, x/l . It is seen that the pressures are all negative, except for one point, and are extremely small. Consequently, there is considerable experimental scatter.

FIGURE 13. VARIATION OF PRESSURE WITH DISTANCE FROM LEADING EDGE ON A PAD-TYPE BEARING OPERATING CONCENTRICALLY WITH END LEAKAGE (EXAGGERATED PRESSURE SCALE)

Figure 14(a) shows these same pressures in terms of the dimensionless pressure $P \equiv \frac{p}{\rho(u_s/2)^2}$, and it is seen that this nondimensionalization tends to group the data slightly better than is shown in Figure 13. The data show negative peak pressures at both the entry and the exit portions of the pad. The presence of this peak at the entry has been previously observed and reported and was attributed to a separation of the flow at the entry. The peak at the pad exit is not understood at present; however, it is possible that a slight distortion in curvature exists in the pad at the exit causing a slightly lower pressure. All of the pressures measured in this connection are small compared with strong pressures when operated with some amount of tilt. Consequently these small variations do not need to be explained fully in order to proceed toward an understanding of performance during realistic operating conditions.

Velocity profiles for three values of mean Reynolds number, $\bar{R}$, for the concentric pad with end leakage are shown in Figure 14(b), (c), and (d). At each mean Reynolds number, velocity profiles were obtained at three positions: 0.5 in. behind the leading edge, 0.5 in. ahead of the trailing edge, and at a point between the leading and trailing edge. These positions are expressed as a fraction of the pad length by x/ℓ . Figure 14(b) shows the profiles for $\bar{R} = 2400$ and thus corresponds to the pressure profile in Figure 14(a). There is not much change in the profile between the leading edge, $x/\ell = 0.015$, and the trailing edge, $x/\ell = 0.985$. The only change occurs between $x/\ell = 0.734$ and 0.985 , where it is seen that the

FIGURE 14. PRESSURE AND VELOCITY PROFILES ON A PAD-TYPE BEARING OPERATING CONCENTRICALLY WITH END LEAKAGE (a) PRESSURE VARIATION, (b) VELOCITY PROFILES FOR $\bar{R} = 2400$, (c) VELOCITY PROFILES FOR $\bar{R} = 1200$, VELOCITY PROFILES FOR $\bar{R} = 250$

profile is incremented in the positive direction. This agrees with the small favorable pressure gradient between $x/\ell = 0.734$ and 0.985 , as shown in Figure 14(a). Figure 14(c) and (d) show the profiles for $\bar{R} = 1200$ and 250 respectively. As the Reynolds number is decreased below 1200 , the pressures become immeasurably small, thus none were obtained at $\bar{R} = 250$.

Experimental Results on a Concentric Pad without End Leakage

As was mentioned previously, to approximate the infinitely long bearing, shields were attached to the pad to restrict flow in the axial direction. This was first done with the pad concentric, and pressure and velocity profiles were obtained for several values of mean Reynolds number. The results of the pressure measurements are shown in Figure 15, where the gage pressure in atmospheres at the axial midspan of the pad is plotted against dimensionless distance from the leading edge. As seen in the figure the pressures are negative over the complete length of the pad and the gradient is adverse between $x/\ell = 0.015$ and 0.985 .

Figure 16(a) shows these same pressures in terms of the dimensionless pressure P . Figure 16(b) shows the velocity profiles at the leading and trailing edges of the pad for $\bar{R} = 2400$, and it is seen that the profiles are essentially the same at the leading and trailing edges. This is to be expected since it is predicted by continuity for the case of no end leakage. The decrement in velocity at the center of the film at $y/h = 0.5$ agrees with the adverse pressure gradient which prevails between $x/\ell = 0.015$ and 0.985 . Velocity profiles are shown in Figure 16(c) and (d) for

FIGURE 15. VARIATION OF PRESSURE WITH DISTANCE FROM LEADING EDGE OF A PAD-TYPE BEARING OPERATING CONCENTRICALLY WITHOUT END LEAKAGE (EXAGGERATED PRESSURE SCALE)

FIGURE 16. PRESSURE AND VELOCITY PROFILES ON A PAD-TYPE BEARING OPERATING CONCENTRICALLY WITHOUT END LEAKAGE
 (a) PRESSURE VARIATION, (b) VELOCITY PROFILES FOR $\bar{R} = 2400$, (c) VELOCITY PROFILES FOR $\bar{R} = 1200$,
 (d) VELOCITY PROFILES FOR $\bar{R} = 250$

$\bar{R} = 1200$ and 250 respectively. Both sets are again in agreement with the continuity equation.

Experimental Results on a Tilted Pad with End Leakage

To obtain velocity and pressure data on a tilted pad, the pad was pivoted about the point $x/\ell = 0.734$ where the film thickness was maintained at the mean value $\bar{h} \equiv 0.54$ in. The film thickness at the leading edge was 1.0 in. and 0.26 in. at the trailing edge. As before, pressures were measured at the axial midspan of the pad along its length for two values of mean Reynolds number, $\bar{R} = 2400$ and 1200 and are shown in Figure 17(a), where the gage pressure in atmospheres is plotted against the dimensionless position from the leading edge x/ℓ . The pressure is positive at all points and appears to peak at or near the pivot point. The curves are shown as dashed lines past $x/\ell = 0.734$ since no measurements were made between $x/\ell = 0.734$ and 0.985. The reason for the almost zero pressure gradient between $x/\ell = 0.015$ and 0.30 for both $\bar{R} = 2400$ and 1200 is not understood at present. Velocity profiles for $\bar{R} = 2400$ and 1200 are shown in Figure 17(b) and (c) respectively. The profiles for $\bar{R} = 250$ are shown in Figure 17(d).

Experimental Results on a Tilted Pad without End Leakage

Data were obtained on a tilted pad without end leakage by attaching the shields to restrict the axial flow. The geometry of the pad was identical with that used for the tilted pad with end leakage.

The pressures measured at the axial midspan of the pad for two values of mean Reynolds number $\bar{R}$ are shown in Figure 18(a). Note that

(a)

(b)

(c)

(d)

FIGURE 17. PRESSURE AND VELOCITY PROFILES ON A TILTED-PAD BEARING WITH END LEAKAGE (a) PRESSURE VARIATION, (b) VELOCITY PROFILES FOR $\bar{R} = 2400$, (c) VELOCITY PROFILES FOR $\bar{R} = 1200$, (d) VELOCITY PROFILES FOR $\bar{R} = 250$

FIGURE 18. PRESSURE AND VELOCITY PROFILES ON A TILTED-PAD BEARING WITHOUT END LEAKAGE (a) PRESSURE VARIATION, (b) VELOCITY PROFILES FOR $\bar{R} = 2400$, (c) VELOCITY PROFILES FOR $\bar{R} = 1200$, (d) VELOCITY PROFILES FOR $\bar{R} = 250$

the peak pressure for both values of $\bar{R}$ is about twice that obtained with the pad with end leakage shown in Figure 17(a). The velocity profiles at the axial midspan of the pad for $\bar{R} = 2400$ and 1200 are shown in Figure 18(b) and (c), respectively. Note that while the shapes of the two pressure profiles show certain similarities, the profiles for the two values of $\bar{R}$ are different. An interesting point is that for $\bar{R} = 1200$ the pressure profile is nearly flat between $x/\ell = 0.015$ and 0.734 indicating essentially pure shear flow; yet the corresponding velocity profiles show a pressure flow in this region. The velocity profiles for $\bar{R} = 250$ are shown in Figure 18(d). Additional experimental checks are to be made to provide a critical test of the profile and pressure gradient relationships.

For purposes of comparing the results with and without end leakage, the pressures shown in Figure 17(a) and Figure 18(a) have been plotted in Figure 19 where the pressures are in terms of the dimensionless pressure P .

Analytic Solution Compared with Experimental Results on a Tilted Pad without End Leakage

Approximating a tilted pad, an inclined flat slider on a flat surface is used for calculating the theoretical pressure distribution compared with the experimental one. From Equation (75) in Reference (3), the solution for the dimensionless pressure for an inclined flat slider on a flat surface, used for approximating dimensionless pressure in a tilted pad without end leakage, is:

FIGURE 19. VARIATION OF PRESSURE WITH DISTANCE FROM LEADING EDGE ON A TILTED-PAD BEARING WITH AND WITHOUT END LEAKAGE

$$-P = \frac{\zeta}{2\chi^2} \cdot \frac{1}{H^2} + \frac{2}{a} \frac{\xi}{\chi} C_{xc} \left(\frac{1}{H} + \ell \ln H \right) + \frac{\zeta(\chi - 1)}{\chi^2} \cdot \frac{1}{H} + \text{constant} \quad (117)$$

where

$$H = 1 - aX, \quad \frac{x}{h_c} = X, \quad \frac{\Delta u_b}{u_{bc}} = U, \quad \frac{p}{\rho u_{bc}^2} = P \quad (118)$$

$$\zeta = \frac{(1 + U^2)/(1 + 2n) + 2(n + U)/(1 + n)}{(1 + U)^2} \quad (119)$$

$$\xi = \frac{(1 + U)^{2-k} - (1 - U)^{2-k}}{4U} \quad (120)$$

and

$$\chi = \frac{1}{1 + n} \quad (121)$$

Equations (119) to (121) are identical with Equations (40), (35) and (46) in Reference (3). The general magnitudes of ζ , ξ and χ in Equations (119) to (121) are valid for any kind of bearing films without end leakage.

The experimental curve for $\bar{R} = 2400$ without end leakage is compared in Figure 19 with that dimensionless pressure distribution predicted by Equation (117). Given the values of U , n and k from the experiment, the values of ζ , ξ and χ can be evaluated by means of Equations (119) to (121), respectively. Taking the values of U at the different locations expressed by x/ℓ in Figure 18(b) and plotting curves of U vs x/ℓ , the mean value of U , equal to the total area under the curve divided by the total length of x/ℓ (i.e., $x/\ell \equiv 1$), is evaluated to be -0.31. As for C_{xc} , we may use $\bar{R} = 2400$ and Figure 13(3) to obtain C_{xc} which is equal to the value of 0.0075. In view of $C_{xc} = 0.0075$ and the previous Figure 13, in Reference (3), k may be approximated to be 0.25.

From the present experimental work on a tilted pad without end leakage, the values of δ^*/b on the journal and bearing sides of the film are listed in Table XXIII.

TABLE XXIII. VALUES OF δ^*/b FOR THE JOURNAL AND BEARING SIDES OF THE FILM FOR $\bar{R} = 2400$

$\bar{R}$	Side of Film	x/ℓ	δ^*/b
2400	Journal Side	0	0.101
		0.7	0.139
		1	0.186
	Bearing Side	0	0.085
		0.7	0.074
		1	0.101

Two curves of δ^*/b vs x/ℓ can be plotted for the journal and bearing sides, respectively. The mean value of δ^*/b for each side [i.e. $(\delta^*/b)_{J.S.}$ and $(\delta^*/b)_{B.S.}$], is equal to the total area under the curve divided by the total length of x/ℓ (i.e. $x/\ell \equiv 1$). Then, the mean value of $(\delta^*/b)_{J.S.}$ and $(\delta^*/b)_{B.S.}$ across the film is $[(\delta^*/b)_{J.S.} + (\delta^*/b)_{B.S.}] / 2$ where J.S. and B.S. refer to the journal side and bearing side, respectively. Values of $(\delta^*/b)_{J.S.}$ and $(\delta^*/b)_{B.S.}$ are 0.1327 and 0.0822, respectively. The mean value of δ^*/b and n at $\bar{R} = 2400$ across the film is calculated to be $\delta^*/b = 0.1075$ and $n \doteq 0.125$.

Corresponding to the experimental model, a is evaluated to be $a = 0.0187$. All the required quantities in Equation (117) are listed in Table XXIV, where $P_{L.E.}$ is the dimensionless pressure at the leading edge of the pad.

TABLE XXIV . REQUIRED QUANTITIES IN EQUATION (117)

<u>$\bar{R}$</u>	<u>U</u>	<u>k</u>	<u>n</u>	<u>ζ</u>	<u>ξ</u>	<u>χ</u>	<u>C_{xc}</u>	<u>a</u>	<u>$P_{L.E.}$</u>
2400	-0.31	0.25	0.125	1.155	0.621	0.889	0.0075	0.0187	0.172

Equation (117) was considered to be sufficiently simple for numerical integration with a digital computer using the quantities in Table XXIV , and P predicted by theory, and measured by experiment, is seen to correspond well as shown in Figure 19.

PRESSURE MEASUREMENTS ON A PAD BEARING WITH BLOCKED FLOW

Pressure measurements were previously reported(3) showing the pressure distribution at entry to a pad-type bearing operating concentrically with an air film thickness of 0.54 in. and mean Reynolds number $\bar{R} = 2400$. These results showed the existence of negative pressures at the leading edge of the pad, these remaining negative for at least 90° behind the leading edge.

To study the effect of a strong tangential pressure gradient on the pressure distribution at entry to the concentric pad, a barrier in the form of a 0.5×0.5 in. blockage, was placed in the 0.54 in. thick film which blocked the flow in the tangential direction and forced the air to divert and pass out the ends of the bearing clearance. The pressure measuring instrument was connected to a static pressure tap at a fixed position behind the leading edge of the pad, the barrier was set to various positions downstream of the point of pressure measurement and the pressure was obtained for each position of the barrier. This was done for 4 different placements of the pressure measuring device and at a mean Reynolds number $\bar{R} = 2400$. The results are shown in Figure 20, where the gage pressure p is nondimensionalized with the atmospheric pressure p_a and plotted against the position of the barrier relative to the point of pressure measurement. All pressures are measured at the axial midspan of the bearing. Also shown on the figure are the negative pressures measured previously at the 1° , 18° , 60° , and 90° positions behind the leading edge without the barrier

FIGURE 20. VARIATION OF PRESSURE WITH POSITION OF BARRIER IN BLOCKED FLOW (EXAGGERATED PRESSURE SCALE)

in place. It is seen that the presence of the barrier has a significant effect on the pressure distribution and is still creating positive pressures when placed as far as 50° downstream of the static pressure tap. It can also be seen that when the barrier is placed about 15° or more downstream of the point of pressure measurement, the value of the pressure is not dependent upon the location of the pressure tap with respect to the leading edge. When the barrier is placed very close to the static pressure tap (within 1°) there is some variation in the pressure with the location of the tap with respect to the leading edge of the pad. Figure 21 shows these same pressures in terms of the dimensionless pressure P plotted against the dimensionless barrier position relative to the static pressure tap, $x/\bar{h}$.

FIGURE 21. VARIATION OF PRESSURE WITH POSITION OF BARRIER IN BLOCKED FLOW

CONCLUSIONS

With regard to the general analysis we conclude that it does, indeed, provide a manageable approach to the generation of bearing data. Insofar as it could be applied to check experimental measurements the agreement appears to be satisfactory.

In the experiments, evidence of momentum or inertial effects has been provided. These effects appear as a leading-edge pressure rise in pad-type bearings. In the short, full journal bearing inertial effects are shown to account for approximately half of the theoretical maximum pressure. Since this pressure agrees closely with the measured value one must conclude that the inertial contribution is indeed significant. To a large extent inertial effects are amplified by the relatively large film thickness which has been used. On the other hand, since the analysis is shown to deal with these well when they are predominant, there should be considerable confidence in going to applications where the effects of inertia are secondary.

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