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SIMULATION OF THE LLL BENIOFF SEISMIC SYSTEM

M. D. Denny

May 4, 1973

Prepared for U.S. Atomic Energy Commission under contract No. W-7405-Eng-48

**LAWRENCE
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SIMULATION OF THE LLL BENIOFF SEISMIC SYSTEM

Abstract

For the past decade the Laboratory has conducted a seismic yield measurement program in support of the nuclear testing program at the Nevada Test Site. Initially, two stations were equipped with a short-period vertical-component seismometer and film recorder system designed by Hugo Benioff. Later this system was replaced by a long-period broad-band telemetry system. Since much of the data currently used in the seismic yield determination consists of data recorded on the Benioff system, an equivalent Benioff signal must be filtered from the data recorded on the broad-band system. To determine that signal, the Benioff system was removed from storage and set up for critical laboratory measurements. Unfortunately the damping of the seismometer and of the film recorder galvanometers could not be reconstructed in the lab. Consequently, it was necessary to deduce these values from parameter studies and from routine response tests conducted in the field prior to the recording of a nuclear event. The parameter studies were made with a recursive filter code that simulates the Benioff system. The damping values finally arrived at are compromises as the field tests showed that considerable variation existed from event to event. Nevertheless, the recursive filter code satisfactorily reproduced the Benioff data from the broad-band data when tested against actual Benioff data.

Introduction

For the past decade the Laboratory has conducted a seismic yield measurement program in support of the nuclear testing program at the Nevada Test Site. Initially two stations, one near Mina, Nevada and the other near Kanab, Utah, were equipped with the short period vertical component seismometer and film recorder system designed by Hugo Benioff of Cal Tech. They were built by Geotech Corporation of Garland, Texas. The equipment was operated and maintained in the field on a courtesy basis by personnel of the Geotech Corporation under contract to the Air Force Technical Application Center (AFTAC) as these stations were a part of AFTAC's Long Range Seismic Measurement (LRSM) program. This film recorder system was later replaced by a long period, broad-band, telemetry system which is currently in use at both of the two original stations plus two new ones, one near Landers, California, and one near Elko, Nevada.

Much of the data base currently used in the seismic yield determinations consists of data recorded on the Benioff system. Consequently, an equivalent Benioff signal

must be filtered from the data recorded on the broad-band system. Originally, the filter only approximated the overall system response without regard for the responses of the individual components. Consequently, an empirical correction factor was needed to convert the filtered amplitudes into equivalent Benioff amplitudes. This correction factor was arrived at by a linear regression fit to

$$\log(\text{Benioff amplitude}) = E1 + E2 \log(\text{filtered amplitude})$$

for twenty events recorded on both systems. The standard deviation from this fit was about 7.5%. As this was not a completely satisfactory situation, we felt that a new filter which simulates each part of the system would do a better job.

To fill this need we designed and built an electrical filter, mathematically modeled the system, and then wrote a recursive, digital filter code. All of this requires considerable knowledge of the system and, therefore, it was removed from storage and critical laboratory measurements were made.

The system galvanometers are governed by second order differential equations which can be described satisfactorily with the results of a free period and relative amplitude response curve. However, the seismometer is governed by a third order differential equation and is not as easily characterized. To describe the seismometer we need to know the internal resistance, inductance, and generator constant of the magnetic variable-reluctance circuit, the mass of the moving body, its natural damping, and its natural frequency when the magnetic circuit is open. The opportunity to test the system also allowed us to determine the motor constant of the calibration coil and the effect that channel differences had on event data.

Description of the System

The seismometer, shown in Fig. 1, consists of a large steel cylinder which is free to move along its vertical axis and which is supported by a coil spring located in the center of the cylinder. This cylinder is restrained from any horizontal or torsional motion by six small leaf springs, three at the top and three at the bottom. The bottom of the cylinder is attached to one-half of the variable reluctance transducer. The other half is attached to the frame and is stationary. The two halves are separated by an air gap. When the part attached to the cylinder moves, the gap between the two parts becomes unequal and produces a variable reluctance in the magnetic circuit. This in turn produces a voltage in the coils such that the output of the transducer is proportional to the velocity of the mass of the seismometer.

The damping of the seismometer is controlled by a variable resistor located in a unit called the "Line Termination Module" and connected in series with the transducer. See Fig. 2 for a schematic of the system. The terminal voltage of this unit is amplified by a photo-tube amplifier (PTA) which consists of a light source, a mirror

Fig. 1. Benioff Seismometer.

Fig. 2. Functional wiring diagram of overall Benioff seismometer system.

mounted on a galvanometer and a photo tube deck. The principle of operation is that a voltage on the input causes an angular rotation of the galvanometer which, in turn, causes the light reflected from the mirror to move across the photo deck. The photo deck then produces a voltage which is proportional to the deflection of the light spot. The net effect is to amplify the low frequencies more than the high (above 5 Hz).

Before driving the galvanometer located in the film-recorder, the signal from the photo deck is further filtered by a band pass filter, located in the amplifier power supply. The film recorder operates on much the same principle as the PTA except that, instead of the deflection of the light spot being detected by a photo deck, the light spot exposes 35-mm film which is mounted on a drum. This drum makes one revolution every 6 min and travels horizontally 0.8 mm during each revolution, so there are up to 30 traces on each film strip.

There are two ways to calibrate the system. In the primary method we shake the mass by driving a small electromagnetic coil (calibrated coil, item 6 of Fig. 1) with a current of known amplitude and frequency. The second way is to drop a small known weight onto the mass. This method (known as the ball lift) is of limited value except as a quick check on the damping ratio of the seismometer.

Mathematical Models

SEISMOMETER WITH RESISTIVE LOAD

Because the Benioff seismometer has a higher inductive load than resistive load, it cannot be described by a second order differential equation, as can most seismometers. A second order differential equation predicts a linear relationship between the voltage output and sinusoidal input ground displacement for frequencies above the natural frequency. On the other hand, shake table tests for the Benioff show a constant relationship for these conditions. To obtain agreement between practice and theory a third order differential equation is needed. It is derived by including the inductive load of the magnetic circuit as follows.

The equation of motion is

$$M\ddot{x} + D\dot{x} + Cx + K_g i = -M\ddot{z}, \quad (1)$$

where x is the relative displacement between the mass and frame, z is the absolute displacement of the frame, M is the mass, $D\dot{x}$ is the restoring force resulting from internal viscous damping, Cx is the restoring force of the spring, and $K_g i$ is the electromagnetic restoring force of the current flowing through the transducer coils. The generator constant (K_g) is in N/A and i is the current in the transducer's coils. Now, divide Eq. (1) by M and let $\omega_s^2 = C/M$ and $2h_s \omega_s = D/M$, where ω_s is the undamped

natural circular frequency and h_s is the coefficient of critical damping under no load. This results in the following expression

$$\ddot{x} + 2h_s \omega_s \dot{x} + \omega_s^2 x + \frac{K_s}{M} i = -\ddot{z} \quad (2)$$

The current i is found from the circuit equations for the transducer given below

$$K_s \dot{x} = i(R_s + S) + Li \quad (3)$$

where $K_s \dot{x}$ is the voltage generated by the motion of the mass relative to the frame, $i(R_s + S)$ is the voltage loss in the internal resistance of the coils (R_s), and S and Li (the external damping load) are the voltage losses resulting from coil inductance.

The frequency response of the system can be found directly by taking the Fourier transforms of Eqs. (2) and (3) as follows, where the bar denotes the transformed variable and $j = \sqrt{-1}$. Equation (3) becomes

$$j\omega K_s \bar{x} = \bar{i}(R + j\omega L) \quad (4)$$

where $R = R_s + S$ and (2) becomes

$$\bar{x}(-\omega^2 + 2h_s \omega_s j\omega + \omega_s^2) + \frac{K_s}{M} \bar{i} = \omega^2 \bar{z} \quad (5)$$

The transfer function (the ratio of the relative motion of the mass to the absolute motion of the frame) can now be found by substituting (4) into (5)

$$\text{Transfer Function} \equiv \frac{\bar{x}}{\bar{z}} = \frac{\omega^2}{-\omega^2 + \left(2h_s \omega_s + \frac{K_s^2}{M(R + j\omega L)}\right) j\omega + \omega_s^2} \quad (6)$$

Equation (6) shows that the transfer function is the same for displacement, velocity, and acceleration. This is shown simply by multiplying the top and bottom of the lefthand side by $-i\omega$ and ω^2 respectively. It also shows that the damping term is a complex quantity,

$$2h_s \omega_s + \frac{K_s^2}{M(R + j\omega L)} \quad (7)$$

Seismologists often use magnification and velocity sensitivity to describe instrument response and to reduce response output to physical quantities. They are defined below:

$$\text{Magnification (M)} \equiv \frac{\text{output}}{\text{input ground displacement}} \quad (8)$$

$$\text{Velocity Sensitivity (V. S.)} \equiv \frac{\text{output}}{\text{input ground velocity}} \quad (9)$$

Both are applicable to sinusoidal input ground motion, but velocity sensitivity is more meaningful for general ground motion since the transducer is basically sensitive to velocity. The units of the output are arbitrary and can be measured by displacement on a strip chart or film record or by a voltage or current. If, as in the case of the shake table test to be discussed later, the output is defined as the voltage across the external load, then

$$M = \left| \frac{S_i}{z} \right| \quad \text{and V. S.} = \left| \frac{S_i}{j\omega z} \right|, \quad (10)$$

or using Eq. (4)

$$M = \left| \frac{SK_s j\omega \bar{x}}{(R + j\omega L)\bar{z}} \right| = \left| \frac{SK_s j\omega}{(R + j\omega L)} \right| \quad |\text{Transfer Function}| \quad (11)$$

and

$$\text{V. S.} = \left| \frac{M}{j\omega} \right| = \left| \frac{SK_s}{(R + j\omega L)} \right| \quad |\text{Transfer Function}| \quad (12)$$

Equation (11), after taking the absolute value, or modulus, becomes

$$M = \frac{SK_s}{R} \frac{\omega^3}{\left[\left(\omega_s^2 - \omega^2 - 2h_s \omega_s \omega^2 \frac{L}{R} \right)^2 + \omega^2 \frac{L^2}{R^2} \left(\omega_s^2 - \omega^2 + 2h_s \omega_s \frac{R}{L} + \frac{K_s^2}{LM} \right)^2 \right]^{1/2}} \quad (13)$$

Equation (13) will be used later to reduce the results of the shake table test; it is the most convenient form of the response function for this application, since displacement of the shake table can be measured directly. For small frequencies ($\omega < \omega_s$), it can be seen from Eq. (13) that the magnification is proportional to ω^3 while for high frequencies ($\omega > \omega_s$), it approaches a constant value, SK_s/L .

GALVANOMETER WITH RESISTIVE LOAD

Unlike the seismometer, the galvanometer can be represented satisfactorily by a second order differential equation since its internal inductive load is small

compared to its resistive load. The equation of motion is

$$I\ddot{\theta} + D\dot{\theta} + K\theta = T_g i, \quad (14)$$

where θ is the angular deflection of the galvanometer, I is the moment of inertia, $I\ddot{\theta}$ is the inertial torque, $D\dot{\theta}$ is the internal viscous damping torque, K is the torque rate, $K\theta$ is the restoring spring torque, T_g is the galvanometer sensitivity, $T_g i$ is the driving torque, and i is the driving current.

The circuit equation is

$$e_{in} = i(R_g + R_x) + T_g \dot{\theta}, \quad (15)$$

where e_{in} is the driving voltage, iR_g is the voltage loss resulting from the internal resistance of the galvanometer (R_g), iR_x is the voltage loss resulting from the external resistance used to control the damping (R_x), and $T_g \dot{\theta}$ is the voltage loss resulting from the deflection of the galvanometer. Again the frequency response can be found directly from the Fourier transformation of Eqs. (14) and (15)

$$\bar{\theta}(\omega_g^2 - \omega^2 + 2h_g \omega_g j\omega) = \frac{T_g}{I} \bar{i} \quad (16)$$

$$\bar{e}_{in} = \bar{i}(R_g + R_x) + j\omega \bar{\theta} T_g, \quad (17)$$

where $\omega_g^2 = K/I$, the natural or free angular frequency of the galvanometer under no load, and where $2h_g \omega_g = D/I$, the corresponding damping term. Solving for $\bar{i}$ in Eq. (17) and substituting the result in Eq. (16) yields the galvanometer transfer function

$$\frac{\bar{\theta}}{\bar{e}_{in}} = \frac{(T_g/I)}{\omega_g^2 - \omega^2 + j\omega \left(2h_g \omega_g + \frac{T_g^2/I}{R_g + R_x} \right)}. \quad (18)$$

The damping can be defined as a fraction of critical damping

$$\eta = h_g + \frac{T_g^2/I}{2\omega_g(R_g + R_x)}, \quad (19)$$

where $\eta = 1$ for critical damping. The relative amplitude response is found by taking the absolute value of Eq. (18)

$$\left| \frac{\partial}{\partial \omega} \right|_{\omega_g} \frac{\omega_g^2}{T_g} = \frac{1}{\left[\left(1 - \frac{\omega_g^2}{\omega^2} \right)^2 + 4 \frac{\omega_g^2}{\omega^2} \right]^{1/2}}, \quad (20)$$

which approaches 1 for $\omega < \omega_g$ and approaches ω_g^2/ω^2 for $\omega > \omega_g$ while at $\omega = \omega_g$ it becomes $1/2\eta$. The differential equation represented by Eqs. (18) and (20) is

$$\ddot{\theta} + 2\eta_g \omega_g \dot{\theta} + \omega_g^2 \theta = \frac{T_g}{I_g} e_{in}. \quad (21)$$

SEISMOMETER-GALVANOMETER COMBINATION

Figure 3 shows the electrical schematic of the seismometer-galvanometer combination. In the figure rectangles represent resistors and saw tooth shapes represent the internal resistance of a component. The variable resistor (R_d) is used to control the damping of the seismometer but it also affects the galvanometer. Across the input terminals to the photo tube amplifier (PTA) is an 80Ω bridged "T" attenuator which is supposed to be loaded with 80Ω on both sides. This load achieves the attenuation

Fig. 3. Electrical schematic of seismometer-galvanometer combination. $R_1 = 80/(k-1)$, $R_2 = 80(K-1)$, and $K = 10(\text{dB}/20)$. R_S denotes internal inductance, L is inductance of the coils in the seismometer's transducer, R_g is the internal resistance of the galvanometer, and R_L is the resistance of the cable.

indicated on the dial and maintains the same input or output impedance regardless of dial setting. To balance the input of the attenuator requires a value for R_d of 337.5Ω minus line resistance (R_L) which for Mina is estimated to be 15 to 20Ω . Any other value for this component means the attenuator is potentially unbalanced to some degree. This, in turn, means that the damping load on the galvanometer is not independent of the attenuator setting. For this reason the attenuator has been included in the analysis and the system of coupled equations has been solved. However, as will be seen later, this effect is not significant.

To analyze the system we begin with the equations of motion which are identical to Eqs. (5) and (16) except for the currents

$$\text{seismometer: } \bar{x} (-\omega^2 + 2h_s \omega_s j\omega + \omega_s^2) + \frac{K_s}{M} \bar{i}_1 = \omega^2 \bar{z} \quad (22)$$

$$\text{galvanometer: } \bar{\theta} (-\omega^2 + 2h_g \omega_g j\omega + \omega_g^2) = (T_g/I) \bar{i}_3. \quad (23)$$

Next the circuit equations are required in order to solve for $\bar{i}_1$ and $\bar{i}_3$, the currents out of the seismometer and into the galvanometer, respectively. For the loops defined in Fig. 3, the circuit equations are

$$K_s j\omega \bar{x} = j\omega L \bar{i}_1 + (162.5 + R_d + R_L) \bar{i}_1 - 100 \bar{i}_2 \quad (24)$$

$$0 = \left(180 + \frac{80}{K-1}\right) \bar{i}_2 - 100 \bar{i}_1 - \left(\frac{80}{K-1}\right) \bar{i}_3 - 80 \bar{i}_4 \quad (25)$$

$$0 = \left(163.5 + \frac{80}{K-1}\right) \bar{i}_3 + T_g j\omega - \left(\frac{80}{K-1}\right) \bar{i}_2 - 80 \bar{i}_4 \quad (26)$$

$$0 = (160 + 80(k-1)) \bar{i}_4 - 80 \bar{i}_2 - 80 \bar{i}_3, \quad (27)$$

where, as before, the bar denotes the Fourier transform and K is determined by setting the attenuator in DB, as follows

$$K = 10^{(DB/20)} \quad (28)$$

Straightforward manipulation of the circuit equations yields the following solutions for $\bar{i}_3$ and $\bar{i}_1$

$$\bar{i}_3 = \frac{800 K \bar{i}_1 - T_g j\omega \bar{\theta} (9K^2 - 1)}{1471.5 K^2 - 3.5} \quad (29)$$

and

$$\bar{i}_1 = \left[K_S j\omega \bar{x} - \frac{800 K T_g j\omega \bar{\theta}}{1471.5 K^2 - 3.5} \right] / (j\omega L + R_T), \quad (30)$$

where

$$R_T = 162.5 + R_d + R_L - \frac{500}{(9K^2 - 1)} \left[(K^2 - 1) + \frac{1280 K^2}{(1471.5 K^2 - 3.5)} \right]. \quad (31)$$

The transfer function for the PTA, can now be found by combining Eqs. (29) and (23) so that the ratio of the deflection of the galvanometer to the input current is

$$\frac{\theta}{\bar{i}_1} = \left[\frac{800 K (T_g/I)}{1471.5 K^2 - 3.5} \right] / \left[-\omega^2 + j\omega \left(2h_g \omega_g + \frac{(T_g^2/I) (9K^2 - 1)}{1471.5 K^2 - 3.5} \right) + \omega_g^2 \right]. \quad (32a)$$

The damping term,

$$\eta_g = h_g + \frac{(T_g^2/I) (9K^2 - 1)}{2(1471.5 K^2 - 3.5) \omega_g}. \quad (32b)$$

is seen to be a function of the attenuator setting.

The magnification for the seismometer is found by combining Eqs. (22) and (32) with Eq. (30) and solving for $\bar{i}_1$, the output current.

$$\frac{\bar{i}_1}{z} = \frac{K_S j\omega^3}{(R_T + j\omega L)(\omega_s^2 - \omega^2 + 2h_s \omega_s j\omega) + j\omega \left[\frac{K_s^2}{M} + C_0^2 \left(\frac{T_g^2}{I} \right) \left(\frac{\omega_s^2 - \omega^2 + 2h_s \omega_s j\omega}{\omega_g^2 - \omega^2 + 2\eta_g \omega_g j\omega} \right) \right]}, \quad (33)$$

where

$$C_0 = \frac{800 K}{1471.5 K^2 - 3.5}. \quad (34)$$

If the inductance L could be neglected the damping of the system would be governed by the coefficient of $j\omega$, namely,

$$\frac{K_s^2}{M} + C_0^2 \left(\frac{T_g^2}{I} \right) \left[\frac{\omega_s^2 - \omega^2 + 2h_s \omega_s j\omega}{\omega_g^2 - \omega^2 + 2\eta_g \omega_g j\omega} \right], \quad (35)$$

which is a function of not only the load on the seismometer resulting from the magnetic field and coils but also of the load due to the galvanometer magnetic field. The extent to which this is important is determined by the attenuator. It turns out, as can be shown from Eq. (35), that this is a noticeable effect for an attenuator setting of less than about 18 dB.

The overall magnification for the seismometer-galvanometer combination is the product of Eqs. (32) and (33).

$$\frac{\bar{\theta}}{z} = C_0 (T_g/I) K_s j\omega^3 / (R_T + j\omega L) (\omega_s^2 - \omega^2 + 2h_s \omega_s j\omega) (\omega_g^2 - \omega^2 + j\omega 2\eta_g \omega_g) + j\omega \left[\frac{K_s^2}{M} (\omega_g^2 - \omega^2 + j\omega 2\eta_g \omega_g) + C_0^2 \left(\frac{T_g^2}{I} \right) (\omega_s^2 - \omega^2 + 2h_s \omega_s j\omega) \right]. \quad (36)$$

The fifth order differential equation represented by Eq. (36) can be found by carrying out the term-by-term and cross multiplications as indicated by the equation and by reversing the transformation. That is, $j\omega \bar{\theta}$ becomes (as it was originally) simply $d\theta/dt$ and $-\omega^2 \bar{\theta}$ becomes $d^2\theta/dt^2$ and so on. The result is

$$L \frac{d^5\theta}{dt^5} + A_4 \frac{d^4\theta}{dt^4} + A_3 \frac{d^3\theta}{dt^3} + A_2 \frac{d^2\theta}{dt^2} + A_1 \frac{d\theta}{dt} + \omega_s^2 \omega_g^2 \theta = -C_0 \left(\frac{T_g}{I} \right) K_s \frac{d^3z}{dt^3}, \quad (37)$$

where

$$A_4 = R_T + L(2h_s \omega_s + 2\eta_g \omega_g),$$

$$A_3 = R_T(2h_s \omega_s + 2\eta_g \omega_g) + L(\omega_s^2 + 4h_s \eta_g \omega_s \omega_g + \omega_g^2) + \frac{K_s^2}{M} + C_0^2 (T_g^2/I),$$

$$A_2 = R_T(\omega_s^2 + 4h_s \eta_g \omega_s \omega_g + \omega_g^2) + L(2\eta_g \omega_g \omega_s^2 + 2h_s \omega_s \omega_g^2) + \frac{K_s^2}{M} 2\eta_g \omega_g + C_0^2 \left(\frac{T_g^2}{I} \right) 2h_s \omega_s,$$

and

$$A_1 = L \omega_s^2 \omega_g^2 + R_T(2\eta_g \omega_g \omega_s^2 + 2h_s \omega_s \omega_g^2) + \frac{K_s^2}{M} \omega_g^2 + C_0^2 \left(\frac{T_g^2}{I} \right) \omega_s^2.$$

PTA FILTER

The filter in the PTA is described in the Geotech Operation and Maintenance Manual as a band pass filter with 6 dB/oct attenuation on the low frequency side and 12 dB/oct on the high side. A model for this filter can be obtained by combining

a first order differential equation, which simulates the high pass side, with a second order differential equation, which simulates the low pass side. This is achieved with the following equations

$$\text{First order: } \dot{e}_1 + \omega_1 e_1 = \dot{e}_0 \quad (38)$$

$$\text{Second order: } \ddot{e}_2 + 2\eta\omega_2\dot{e}_2 + \omega_2^2 e_2 = \omega_2^2 e_1, \quad (39)$$

where e_0 is the input voltage to the high pass portion, e_1 is the corresponding output and also the input to the low pass section, ω_1 and ω_2 are the circular frequencies at which the amplitude responses are down 3 dB, when $\eta = \sqrt{2}/2$.

That Eqs. (38) and (39) have the desired frequency response can be shown by taking the Fourier transforms of each of them and solving for the ratio of the output to the input. Carrying out these operations results in the following expression for the first and second order equations, respectively.

$$\frac{\bar{e}_1}{\bar{e}_0} = \frac{j\omega}{j\omega + \omega_1} \quad \text{or} \quad \left| \frac{\bar{e}_1}{\bar{e}_0} \right| = \frac{\omega}{[\omega^2 + \omega_1^2]^{1/2}} \quad (40)$$

$$\frac{\bar{e}_2}{\bar{e}_1} = \frac{\omega_2^2}{\omega_2^2 - \omega^2 + 2\eta\omega_2 j\omega} \quad \text{or} \quad \left| \frac{\bar{e}_2}{\bar{e}_1} \right| = \frac{\omega_2^2}{[(\omega_2^2 - \omega^2)^2 + 4\eta^2\omega_2^2\omega^2]^{1/2}}. \quad (41)$$

Equation (40) shows that the response approaches unity for $\omega > \omega_1$ while the response decreases linearly with decreasing frequency for $\omega < \omega_1$. Similarly, Eq. (41) shows that the response approaches unity for $\omega < \omega_2$ and that it decreases with increasing frequency to the second power for $\omega > \omega_2$. Both of these are desired frequency responses and it remains only to specify ω_1 and ω_2 .

The complete model for the filter is obtained by combining Eqs. (40) and (41) to get a single expression for the output in terms of the input

$$\bar{e}_2 = \frac{j\omega\omega_2^2\bar{e}_0}{(j\omega + \omega_1)(\omega_2^2 - \omega^2 + 2\eta\omega_2 j\omega)}. \quad (42)$$

If the denominator of Eq. (42) is removed by cross multiplication and all other multiplications carried out as well, it can be rewritten as a third order differential equation

$$\ddot{e}_2 + (2\eta\omega_2 + \omega_1)\dot{e}_2 + (\omega_2^2 + 2\eta\omega_2\omega_1)\dot{e}_2 + \omega_1\omega_2^2 e_2 = \omega_2^2 \dot{e}_0. \quad (43)$$

FINITE DIFFERENCE EQUATIONS FOR THE ENTIRE SYSTEM

Of the four frequency sensitive items (seismometer, PTA galvanometer, PTA band-pass filter, and film recorder galvanometer), only the seismometer and PTA galvanometer interact with each other and this is only appreciable when the attenuator is set at a low value. The PTA galvanometer is isolated from the PTA filter by the light source and photo-cell arrangement while the PTA filter is in turn decoupled from the film recorder galvanometer by a signal isolation transformer. Consequently, the deflection of the PTA galvanometer is scaled by a constant to convert angular deflection into volts. This deflection becomes the input to the PTA filter, whose output, in turn, becomes the input to the film recorder galvanometer. The ultimate outcome is, of course, a trace on 35-mm film with an amplitude proportional to the angular deflection, θ_f , of the film recorder galvanometer. To summarize, the entire system can then be described by the following differential equations.

(a) Equation (37)

$$L \frac{d^5 \theta_g}{dt^5} + A_4 \frac{d^4 \theta_g}{dt^4} + A_3 \frac{d^3 \theta_g}{dt^3} + A_2 \frac{d^2 \theta_g}{dt^2} + A_1 \frac{d \theta_g}{dt} + \omega_s^2 \omega_g^2 \theta_g = C_0 \left(\frac{T_g}{I_g} \right) K_s \frac{d^2 v}{dt^2}, \quad (44)$$

where ground velocity (v) has replaced ground displacement (z).

(b) Equation (43)

$$\frac{d^3 e}{dt^3} + (2\eta \omega_2 + \omega_1) \frac{d^2 e}{dt^2} + (\omega_2^2 + 2\eta \omega_2 \omega_1) \frac{de}{dt} + \omega_1 \omega_2^2 e = \omega_2^2 C_1 \frac{d \theta_g}{dt}, \quad (45)$$

where e is the voltage output of the filter, and C_1 is a constant which converts the angular displacement of the PTA galvanometer into volts at the input to the PTA filter.

(c) Equation (21)

$$\frac{d^2 \theta_f}{dt^2} + 2\eta_f \omega_f \frac{d \theta_f}{dt} + \omega_f^2 \theta_f = \frac{T_f}{I_f} C_2 e, \quad (46)$$

where θ_f is the angular deflection of the film recorder galvanometer, and C_2 is a constant to scale the amplitude of the PTA filter output to the input of the film recorder galvanometer.

The three above equations have been solved numerically by approximating the derivatives with backward differences such as

$$\frac{d\theta_g}{dt} \approx \frac{\theta_g(n) - \theta_g(n-1)}{\Delta t}, \quad (47)$$

where n replaces the continuous variable time, t , but is related to it by

$$t = n\Delta t$$

where

$$n = 0, 1, 2, \dots, \quad (48)$$

and Δt is the time increment between samples. The results, after collecting terms and solving for the present values, are

$$\begin{aligned} a) \quad \theta_g(n) = & \left\{ -(\Delta t)^3 C_0 \frac{T}{I} K_s [V(n) - 2V(n-1) + V(n-2)] + B_1 \theta_g(n-1) \right. \\ & \left. - B_2 \theta_g(n-2) + B_3 \theta_g(n-3) - B_4 \theta_g(n-4) + L \theta_g(n-5) \right\} / B_0, \end{aligned} \quad (49)$$

where

$$B_0 = L + A_4(\Delta t) + A_3(\Delta t)^2 + A_2(\Delta t)^3 + A_1(\Delta t)^4 + \omega_s^2 \omega_g^2 (\Delta t)^5$$

$$B_1 = 5L + 4A_4(\Delta t) + 3A_3(\Delta t)^2 + 2A_2(\Delta t)^3 + A_1(\Delta t)^4$$

$$B_2 = 10L + 6A_4(\Delta t) + 3A_3(\Delta t)^2 + A_2(\Delta t)^3$$

$$B_3 = 10L + 4A_4(\Delta t) + A_3(\Delta t)^2$$

$$B_4 = 5L + A_4(\Delta t)$$

and

$$\theta_g(-1) = \theta_g(-2) = \theta_g(-3) = \theta_g(-4) = \theta_g(-5) = V(-1) = V(-1) = V(-2) = 0.$$

$$b) \quad e(n) = \left\{ \omega_2^2 (\Delta t)^2 C_1 [\theta_g(n) - \theta_g(n-1)] + D_1 e(n-1) - D_2 e(n-2) + e(n-2) \right\} / D_0, \quad (50)$$

where

$$D_0 = 1 + (2\eta \omega_2 + \omega_1)(\Delta t) + (\omega_2^2 + 2\eta \omega_2 \omega_1)(\Delta t)^2 + \omega_1 \omega_2^2 (\Delta t)^3$$

$$D_1 = 3 + 2(2\eta \omega_2 + \omega_1)(\Delta t) + (\omega_2^2 + 2\eta \omega_2 \omega_1)(\Delta t)^2$$

$$D_2 = 3 + (2\eta \omega_2 + \omega_1)(\Delta t)$$

and

$$e(-1) = e(-2) = 3(-3) = 0.$$

$$c) \quad \theta_f(n) = (T_f/I) C_2 \left\{ e(n) + E_1 \theta_f(n-1) - \theta_f(n-2) \right\} / E_0, \quad (51)$$

where

$$E_0 = 1 + 2\eta_f \omega_f(\Delta t) + \omega_f^2(\Delta t)^2$$

$$E_1 = 2(1 + \eta_f \omega_f(\Delta t))$$

and

$$\theta_f(-1) = \theta_f(-2) = 0.$$

Equations (49), (50), and (51) are known as recursive because past values of the solution are used to calculate the present value. The input is thought to be ground velocity because the transducer is basically sensitive to velocity and because the system which replaced the Benioff system has a flat response to velocity over the entire passband of the Benioff system.

A code has been written which makes the above calculations; numerical results are discussed in another section of this document. A listing of the code is given in the Appendix.

Measurement of Parameters

SEISMOMETER

Transducer Internal Resistance and Inductance

To measure the internal resistance (R_s) of the transducer, we bolted the mass of the seismometer in the center position to eliminate earth noise-generated currents in the transducer. Thus, we eliminated the contribution of the inductance to the impedance measurement. The value read at the seismometer terminals on a five place digit voltmeter was

$$R_s = 63.3\Omega.$$

We also measured the inductance while the mass was locked into the center position. This measurement was made using a function generator to drive the transducer in series with a known resistor and at a known frequency. The voltages across the transducer terminals and across the known resistor were recorded on a high impedance strip chart recorder. The ratio of the peak-to-peak amplitudes were then used to solve for the inductance. Eight frequencies from 1 to 20 Hz were used with a sample mean and standard deviation of

$$L = 6.97 \pm 0.06 \text{ H.}$$

Natural Damping and Free Period

Natural damping is the amount of damping present and free period is the length of time during which the instrument will vibrate when perturbed under either zero or negligible electrical load. Natural frequency, the reciprocal of free period, can be created by switching in the 200-k Ω resistor in the line termination module (Fig. 2) and then applying a momentary force with either the ball lift or electro-magnetic calibrators. The normal resistive load (R) is on the order of 160 Ω . Thus, Eq. (6) shows that increasing R to over 200 k Ω clearly reduces the inductive term to insignificance.

Since this test was routinely made by the Geotech personnel following every NTS event, abundant information is available to evaluate this parameter. We selected one of these tests and microscopically examined the peak-to-peak amplitudes. The same test was performed in the laboratory after the system had been reassembled. Figure 4 shows the results of both tests; note that the rate of decay had not changed.

The following equation was used to compute the natural damping

$$\log_e \frac{A_i}{A_{i+1}} = \frac{2\pi h}{\sqrt{1 - h^2}}, \quad (52)$$

where A_i is the peak-to-peak amplitude. An average value of the natural log of the ratio of A_i to A_{i+1} was found from the slope of the data of Fig. 3 and this value was used in Eq. (52) to calculate

$$h = 0.0096.$$

However, the free period was not the same as in the field. As will be shown later, considerable variation existed in the free period from one test to the next.

Fig. 4. Benioff natural damping test.

Static Test

The static test is simpler and consists of placing a small known weight on the mass of the seismometer and then applying a constant current to the transducer to drive the mass back to its original position. At this point the force acting in the transducer should equal the force of the weight so that the generator constant can be computed from

$$K_B = \frac{9.8 m}{I}, \quad (54)$$

where 9.8 is the gravitational constant, m is the mass in kg of the test weight and I is the constant current.

The chief difficulty in performing this test is the problem of determining when the mass has been returned to its original position. If the test weight is 250 g and the effective spring constant is 3.71 n/mm,* then the mass must be returned to within 0.033 mm of its initial position for the results to be accurate within 5%. To accomplish

* Computed from $\omega_s = 5.875$ 1/sec and $M = 107.5$ kg.

The free period averaged over 60 cycles in the lab and during the shake table test. This yielded an undamped period of

$$T' = 1.0693 \text{ sec},$$

where T is the undamped period and the prime is the observed period. The two periods are related by

$$T = \sqrt{1 - h^2} T' \quad (53)$$

However, since h is so small, the observed and true periods are nearly identical.

Transducer Generator Constant

To determine the generator constant we performed static and dynamic tests and found that the results agreed to within 4%.

this a non-contacting displacement gage, made by the Bentley Corporation of Nevada,* was borrowed from the transducer lab. It was mounted to the frame of the seismometer so that its output was proportional to the relative displacement between the frame and the mass. The sensitivity of the gage was 20.11 V/mm so that the 5% figure could be easily attained, since a displacement of 0.033 mm would produce a voltage of 0.664 V which is easily recorded.

Figure 5 shows the results of the tests. The testing procedure was to first record the output of the Bentley gage and then to apply weight. Next, a constant current was applied to the transducer and we recorded current readings at several levels and the corresponding Bentley output. Finally, after removing the current and the test weight, the output of the Bentley gage was noted. The zero load and zero current reading of the Bentley before and after the test differed by 40 mV or approximately 0.002 mm. Thus, the force resulting from spring rate amounts to approximately 0.3% of the test weight. However, the corresponding uncertainty in the current is approximately 1% (Fig. 5), and this results in a generator constant in N/A of

$$365 \leq K_s \leq 369.$$

Fig. 5. Weight lift test.

*Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Atomic Energy Commission to the exclusion of others that may be suitable.

Dynamic Test

The purpose of this test was to obtain an experimental magnification plot from which, with the aid of Eq. (13), the generator constant (K_g) and also the motor constant of the calibration coil can be determined. To meet this objective the test was run at 14 frequencies ranging from 0.1 to 30 Hz and at four levels of displacement. The displacement levels were nominally 0.254, 0.508, 1.016, and 1.27 mm peak-to-peak. Useful data were obtained on 49 of the 64 runs. The displacement of the seismometer base was monitored by a linear voltage displacement transformer (LVDT). The outputs of the LVDT and the seismometer's transducer were recorded on magnetic tape. The output of the transducer was terminated into a 79.7- Ω resistor.

Using the LVDT, we measured the ratio of transducer voltage output to displacement. This ratio is called experimental magnification, and is in units of mV/mm. Many records contained enough noise to make measurement of the peak-to-peak amplitudes unreliable. Consequently, data from the LVDT and the transducer were digitized over identical segments of time and transformed into the frequency domain through the use of the fast Fourier transform algorithm. The resultant transform, in general, showed a large peak value at the frequency of the test. The ratio of these peak values was used to obtain the magnification values given in Table 1.

Table 1 also gives an indication of the linearity of the seismometer. Note that 3 readings of the magnification at 0.2 Hz have a deviation of approximately 1/2% but at 10 Hz three readings have a deviation of 7.5%, while the mean deviation for 11 frequencies is 3.4%. These data do not show any nonlinearity because there is no apparent trend, that is, data from the highest test level are neither consistently high nor low.

The generator constant values given in Table 1 were found by evaluating Eq. (13) and by plotting the magnification against the generator constant for each of the test frequencies. The generator constant for each experimental value of magnification was then read directly from these plots.* The sample mean and standard deviation for the thirteen values in Table 1 are

$$K_g = 355.2 \pm 11.8 \text{ n/A.}$$

To evaluate Eq. (13) we wrote code using the following constants

$$\omega_s = 2\pi (0.9355) = 5.878 \text{ r/sec}^\dagger$$

$$h_s = 0.0096$$

$$L = 6.97 \text{ H}$$

$$R = 143 \text{ } \Omega$$

$$M = 107.5 \text{ kg}$$

$$S = 79.7 \text{ } \Omega$$

* Not included in this document.

† The natural frequency as it existed at the time of the test.

Table 1. Magnification and deduced generator constant magnification $\frac{mv}{mm}$.

Frequency	Run				Mean	Generator constant K_g (n/A)
	0.254 mm	0.508 mm	1.016 mm	1.27 mm		
0.2	—	11.057	11.146	11.000	11.068 ± 0.54%	353
0.5	—	156.24	159.22	159.59	158.02 ± 0.8%	366
0.8	554.26	553.78	545.98	577.29	559.2 ± 2.11%	354
1.0	952.84	—	914.08	967.37	944.7 ± 2.38%	361
1.2	1502.7	1439.9	1355.7	—	1432.8 ± 4.2%	355
1.5	2177.1	2142.1	1946.4	2019.1	2071.2 ± 4.49%	386
2.0	3219.1	3100.7	2863.5	2978.2	3040.4 ± 4.37%	349
2.5	3753.7	3480.1	3301.1	—	3511.6 ± 5.3%	351
3.0	4052.3	3756.81	3827.4	—	3878.8 ± 3.25%	362
5.0	3956.6	3962.9	3762.5	3920.9	3900.7 ± 2.1%	348
10.0	—	3918.3	3678.9	4400.0	3999.0 ± 7.5%	352
20.0	3872.8	4085.3	—	—	3979.0	348
30.0	3767.0	3839.0	—	—	3803.0	333

The following confidence intervals result from using the Student "t" distribution:

Confidence coefficient	Interval on mean generator constant
90%	349 - 361
95%	347.8 - 362.6
99%	346.1 - 364.3

The estimated population standard deviations using the Chi-Squared distribution are

Confidence coefficient	Interval on standard deviation
90%	8.9 - 17.9
96%	8.3 - 20.0
98%	8.0 - 21.8

Note that the above results of the weight lift test and the shake table test are in agreement within experimental error.

Figure 6 shows the experimental magnification values and the theoretical curve for $K_g = 355$ n/A.

Calibration Coil Motor Constant

Static and dynamic tests were made to determine the motor constant (G) of the calibration coil. The results are in agreement to within 5%.

Fig. 6. Magnification determined from shake table data.

Static Test

In this case, the accuracy of the weight lift test suffered because of both the low strength of the coil itself and the limitations on the maximum current which could be applied. Figure 7 shows the results of the weight lift test. The error bar is intended

Fig. 7. Weight lift test to determine calibrated coil motor constant.

to show the uncertainty involved in returning the mass to its initial unloaded position. The mean value and standard deviation for the test sample are

$$G = 2.58 \pm 0.16 \text{ n/A.}$$

The largest calibration ever used in the field to calibrate the system was a 14 mA peak-to-peak sine wave, equivalent in power to approximately 5 mA dc. Because of the cultural noise level in the lab, the power levels used (Fig. 7) all exceeded the largest value used in the field. An acceptable current based on the size of the wire used in the calibration coil was calculated to be approximately 55 mA dc. This fact raised speculation about the linearity of the device at such high levels, and indeed, Fig. 6 does appear to show a nonlinear trend. To resolve this question, we ran a test to obtain the output of the data coils as a function of the input to the calibration coil. For this test a frequency of 1.4 Hz was used as this is the approximate value of input at which the output is maximum. The results are plotted in Fig. 8 and show that the calibration coil is linear at least up to 115 mA peak-to-peak ac, with a mean slope of

$$E/I = 0.558 \pm 0.015 \text{ mV/mA.}$$

Consequently, the apparent nonlinear trend in the weight lift test data is not credible, at least up to 81 mA dc. If the two values above this level are eliminated from the sample (as possibly beyond the linear range) and if the two lowest ones are also eliminated, simply because of the uncertainty resulting from the size of the test weight involved, then the sample mean and standard deviation are

$$G = 2.47 \pm 0.11 \text{ n/A.}$$

Dynamic Test

The motor constant can be found from the dynamic test by equating the magnification from the shake table data with that found by using the calibration coil. The magnification is determined using the calibration coil by replacing the inertial force on the mass with an equivalent force produced by the calibration coil, driven with current, I , that is,

Fig. 8. Calibrated coil linearity.

$$- M\ddot{z} = GI, \quad (55)$$

which when transformed is

$$\bar{z} = GI/M\omega^2. \quad (56)$$

By substituting Eq. (56) into Eq. (10), the magnification determined by the calibration coil is then

$$M = \left| \frac{\bar{S}_i}{\bar{z}} \right| = \frac{M\omega^2}{G} \left| \frac{\bar{E}}{\bar{I}} \right|, \quad (57)$$

where $\bar{E} = \bar{S}_i$ is the voltage across the external damping resistance. The motor constant is now found by equating Eq. (57) to the experimental shake table data M_{exp} , that is

$$G = \frac{M\omega^2}{M_{exp}} \left| \frac{\bar{E}}{\bar{I}} \right|. \quad (58)$$

Table 2 lists the results of a test in which we measured current in the calibration coil and the corresponding voltage across the external damping resistance (same as the one used during the shake table test). (See Fig. 9.) These values were then used in Eq. (58) to calculate the motor constant, G.

The mean value and standard deviation of the values of G given in Table 2 are

$$G = 2.42 \pm 0.18 \text{ N/A.}$$

Table 3 lists the confidence intervals for the three cases discussed. Note that although the dynamic test has the largest standard deviation it has the smallest confidence interval for a given confidence coefficient because of the larger sample size. For this reason the results of the dynamic test are preferred over the weight lift.

Ball Lift Matching Test

A third kind of test was employed by the Geotech personnel to determine the calibration coil motor constant. This test consisted of dropping a known weight onto the mass of the seismometer and recording the peak output value. The weight was then removed and a step in current was applied to the calibration coil. This procedure was then repeated until the resulting output amplitude was identical to that produced by the weight. Since the dropping of the weight and the step in current are both a step in force on the seismometer, the seismometer responds to both in the same

manner. The motor constant is then found by equating the force of the weight to the force in the calibration coil as in Eq. (54). The value reported by the Geotech personnel was

2.44 N/A.

Fig. 9. Response of seismometer to sinusoidal current input in calibrated coil.

Table 2. Calibration coil motor constant.

Frequency	I(ma)	E(mv)	$E/I(\frac{v}{a})$	$M(\frac{v}{m})$	$G(\frac{nt}{amp})$
0.2	110.6	15.25	0.138	11.07	2.117
0.5	110.6	35.25	0.319	158.02	2.142
0.8	110.6	49.75	0.450	559.2	2.186
1.0	111.3	56.25	0.506	944.7	2.274
1.2	110.6	59.25	0.536	1432.8	2.286
1.5	110.6	61.49	0.553	2071.2	2.548
2.0	110.6	51.5	0.466	3040.0	2.602
2.5	110.6	38.50	0.348	3511.6	2.630
3.0	110.6	28.5	0.258	3878.8	2.54
5.0	110.6	10.25	0.0927	3900.7	2.52
10.0	108.75	2.5	0.0230	3999.0	2.44
20.0	103.75	0.586	0.00565	3979.0	2.560
30.0	98.125	0.25	0.00255	3803.0	2.561

Table 3. Confidence intervals.

	Degrees of freedom	Coefficient	Interval of true mean (n/A)
Weight lift test	9	90%	2.48-2.68
		95%	2.46-2.70
		98%	2.43-2.73
Weight lift test	5	90%	2.37-2.57
		95%	2.34-2.60
		98%	2.30-2.64
Dynamic test	12	90%	2.38-2.56
		95%	2.36-2.58
		98%	2.33-2.61

DAMPING AND LINE RESISTANCES

Figure 3 shows the damping and line resistances, R_d and R_L . They are not well known because the line had been removed before this investigation. The damping resistor was used to adjust the overshoot ratio obtained on the ball lift test. Geotech personnel checked this ratio daily and corrected it by adjusting the damping resistance if it was greater than 20/1 or less than 12/1. Consequently, it was changed from time to time. No records of such changes were kept. The only available information is

that on the day the seismometer, line termination module, and photo tube amplifier were returned to Livermore, the damping resistance, R_d , was set at 32Ω . This value will be assumed to be representative of the damping resistance for lack of better information.

While the actual cable from the seismometer to the line termination module had been removed by the Geotech personnel on their departure, it is known that the cable run was approximately one-third of a mile of Spiral Four cable whose resistance is approximately $20\Omega/\text{mile}$.^{*} Thus, the cable had a resistance of about 7Ω . In addition to this there were four lightning arrestors in the line, each with a resistance of approximately $2\frac{1}{4}\Omega$.[†] These two items result in a total line resistance of approximately 17Ω :

Cable	$\sim 1/3$ mile at $20\Omega/\text{mile} = \sim 7\Omega$
Lightning arrestors	4 at $\sim 2\frac{1}{4}\Omega = \sim 9\Omega$
Total	$R_L = \sim 17\Omega$

PHOTOTUBE AMPLIFIER

Attenuator

We used an 80Ω -bridged T attenuator on the input to the PTA galvanometer. This means that unless the input is loaded with 80Ω , the combined resistances of the attenuator and load will vary with attenuator setting. This could mean that the damping of the galvanometer is a function of attenuator setting and therefore the response of the galvanometer would be affected every time the attenuator setting is changed, as shown by Eq. (32). To determine whether this is a problem, the seismometer was connected to the line termination module which was in turn connected to the PTA (Fig. 3). The output side of the attenuator was then disconnected from the galvanometer and the impedance [R_x of Eq. (18)] was measured as a function of damping resistance (R_d) in the line termination module. Table 4 lists the results.

If the load on the attenuator is 80Ω and if $R_g = 20.5\Omega$ then the total damping load on the galvanometer is the sum of R_x and R_g . Table 4 shows that the damping load is only affected significantly at the zero and 6-dB settings. In actual usage at Mina, Nevada, the least attenuation used was 24 dB and a typical value was 36 dB.

Galvanometer Internal Resistance

Using a digital multimeter, we measured the internal damping resistance. After the galvanometer had come to rest, R_g equaled 20.5Ω . This value is in agreement with the nominal design value of the Geotech specifications.

^{*} Personal communication with station operator.

[†] Measured on a similar component at the station near Kanab, Utah.

Table 4. External damping resistance as a function of attenuator setting.

Attenuator setting (dB)	R_x (ohms)			$(R_x + R_g)/163.5$		
	$R_d = 0 \Omega$	$R_d = 32 \Omega$	$R_d = 500 \Omega$	$R_d = 0 \Omega$	$R_d = 32 \Omega$	$R_d = 500 \Omega$
0	102.2	112.3	148.8	0.75	0.812	1.035
6	130.3	134.0	144.4	0.922	0.945	1.009
12	139.7	140.7	143.4	0.980	0.986	1.002
18	142.2	142.5	143.2	0.995	0.997	1.001
24	142.8	142.9	143.1	0.999	0.999	1.000
30	143.0	143.0	143.1	1.0	1.0	1.000
36	143.0	143.0	143.1	1.0	1.0	1.000
42	143.0	143.0	143.0	1.0	1.0	1.000

Galvanometer Natural Damping and Free Period

We put the PTA galvanometer into free period vibration by setting the attenuator switch to zero, removing the input cable, and then applying a small transient voltage to the input terminals and removing it. We recorded the result on a strip chart and then plotted the peak-to-peak amplitudes one period apart (Fig. 10). Again, Eq. (52) was used to compute the natural damping from an average value of the natural log of the ratio of A_i to A_{i+1} found from the slope of the data of Fig. 9. From this data the natural damping of the PTA galvanometer was calculated to be

$$h_g = 0.004045.$$

The free period averaged over 48 cycles from the same test is

$$T' = 1/4.977 \text{ sec.}$$

Galvanometer Sensitivity and Moment of Inertia

The ratio of the square of the sensitivity to the moment of inertia (T_g^2/I) mathematically characterizes the galvanometer. This ratio together with the resistive load ($R_g + R_x$) governs the damping and, therefore, the response of the system. The values T_g and I aid in designing the system since they control the amplitude of motion. However, in modeling the system, we have emphasized relative amplitude response because the final product of this study, the digital code, will be normalized to unit magnification at 1 Hz.

The quantity T_g^2/I can be found by experimentally evaluating Eq. (20). We used a function generator to drive an 80- Ω matching network on the input to the PTA attenuator which was set at 42 dB. The output of the function generator and the galvanometer (actually the photocells) were recorded on a Brush strip chart. The

Fig. 10. PTA galvanometer free period damping test.

a mean value of $\eta = 0.675$.

The quantity $T_g^2/1$ can now be determined using Eq. (19) and the previously found values of h_g , ω_g and R_g . We have also determined that R_x is 79.9Ω .

Fig. 11. PTA galvanometer frequency response.

results are listed in Table 5 and plotted in Fig. 11. We ran the zero frequency with a square wave whose period was ten times that of the galvanometer. Thus, the transient response of the galvanometer died out before the next cycle of the square wave.

Table 5 also lists the values of η deduced from Eq. (20) and the experimental data. Note that the values of η tend to increase on both ends. This may be explained by noisy data at the high frequency end and by the difficulty in dividing a poorly known small number by another small number at the low frequency end. Thus, the values of η for frequencies 2 through 6 have been selected arbitrarily as the best data from which to calculate

Table 5. Frequency response of the PTA galvanometer.

f(Hz)	Relative amplitude response	η	Eq. (20) $\eta_{\text{mean}} = 0.675$
0	1.	—	1.
0.01	0.995	—	1.
0.05	1.	—	1.
0.1	0.996	—	1.
0.25	0.996	—	1.
0.5	0.991	0.9765	1.002
1.0	0.996	0.7351	1.037
2.0	1.	0.6782	1.001
3.0	0.973	0.6701	0.969
4.0	0.881	0.6728	0.879
5	0.746	0.6702	0.741
6	0.589	0.6832	0.596
7	0.454	0.708	0.472
8	0.353	0.7579	0.375
9	0.286	0.7458	0.303
10	0.229	0.7933	0.248
12.5	0.152	0.7930	
15	0.097	1.08	
20	0.048	—	
30	0.019	—	

$$T_g^2/I = 6861 \Omega/\text{sec.}$$

The value of R_x which will result in critical damping, that is, $\eta = 1$, is sometimes called CDRX for external critical damping resistance and is used by Geotech in their specifications. Applying h_g , ω_g , and R_g , we find that CDRX is 89.6Ω .

PTA Filter

We used the bandpass filter to eliminate 60-Hz power line noise as well as long period drifts in the output voltage. Laboratory tests established that the -3-dB points are at 0.01 Hz and 10.0 Hz, which are some of the available options specified in the Geotech manual. In other words, ω_1 and ω_2 of Eqs. (38) and (39), respectively, are

$$\omega_1 = 0.02 \pi$$

$$\omega_2 = 20 \pi$$

and

$$\eta = 0.707.$$

FILM RECORDER

Galvanometer Internal Resistance

Using a digital multimeter, we measured the internal resistances of the four galvanometers in the film recorder. Table 6 lists these measurements.

Table 6. Galvanometer internal resistance.

Cable channel	Galvanometer internal resistance (R_g) (Ω)
A	20.5
B	18.8
C	20.9
D	19.1

Galvanometer Natural Damping and Free Period

The film recorder has a switch setting that puts the galvanometer into free period. Free period tests were run on all four channels. However, the results were not measured with the same precision as on the free period

test of the galvanometer in the PTA. Instead, the film from these tests was compared visually with the film from the PTA galvanometer test by overlaying one on the other. In each case the results were virtually identical. Thus, the natural damping for all four galvanometers is negligible (approximately 0.004) and the natural frequency is 5 Hz.

Galvanometer Termination

Three of the four galvanometers have front panel controls labelled "E-P Channel" while the fourth, Channel D, has a set labelled "AMP Channel". The chief difference is that the E-P panel has a damping control while the AMP panel does not. With this damping control the external damping resistance (Table 7) can be varied as follow:

Table 7. Range of external damping resistance.

Channel	R_x (Ω)
A	2 - 135
B	4 - 144
C	5 - 145

On Channel D (the AMP panel) the external damping resistance is fixed at 16.9 Ω and it cannot be changed.

Galvanometer Sensitivity and Moment of Inertia

Both the PTA galvanometer, and the ratio T_g^2/I were found by experimentally evaluating Eq. (20). However, the input to the E-P channels required a

60- Ω matching network while the AMP channel required a 22-k Ω matching network. The damping adjustments were not touched on the assumption that the settings were still the same as in the field. The external damping resistances at these settings were measured and the values recorded in Table 8. We ran the frequency response

Table 8. External damping resistance used in frequency response test.

Channel	R_x (Ω)
A	88.8
B	78
C	97

test with the Table 8 settings and the results were recorded in Table 9. As before, the zero frequency datum was found from a step input, and η was calculated from the normalized amplitude, Eq. (20), and the values of ω_g and h_g given above.

As before, the computed value of η begins to increase rapidly when the normalized amplitude is either close to unity or very small. Consequently, not

all computed values of η were used to compute a mean value. The selection was arbitrary but, in general, only high values from either end were omitted (as indicated in the table). We used the mean values from each channel to calculate T_g^2/I and CDRX (Table 10).

Table 9. Frequency response of film recorder galvanometers.

Frequency	Channel A		Channel B		Channel C		Channel D	
	Normalized amplitude	η	Normalized amplitude	η	Normalized amplitude	η	Normalized amplitude	η
0	1.	—	1	—	1.	—	1.	—
0.1	0.981	—	0.977	—	0.98	—	0.967	—
0.3	0.971	2.17	0.955	2.68	0.958	2.5926	0.869	4.797
1	0.914	1.312	0.796	2.0258 ^a	0.823	1.8621	0.498	4.4092 ^a
2	0.784	1.1998 ^a	0.566	1.9427 ^a	0.6018	1.7922 ^a	0.277	4.3888 ^a
3	0.648	1.170 ^a	0.412	1.9511	0.4469	1.7868 ^a	0.19	4.3534
4	0.521	1.178 ^a	0.318	1.8495 ^a	0.346	1.7923 ^a	0.141	4.4269
5	0.426	1.1748 ^a	0.255	1.9608 ^a	0.279	1.7921 ^a	0.116	4.3103
7	0.287	1.4037	0.179	2.0245 ^a	0.195	1.8633 ^a	0.086	4.167 ^a
16	0.172	1.6356	1157	2.2872	0.128	2.092	0.06	4.2336 ^a
20	0.0495	—	0.0385	—	0.047	—	0.025	—
30	0.0208	—	0.0188	—	0.0239	—	0.015	—

^aUsed to compute mean value of η .

Table 10. Film recorder galvanometer parameters.

Channel	R_x (Ω)	η mean	R_g (Ω)	T_g^2/I (Ω/sec)	CDRX (Ω)	η min
A	88.8	1.18	20.5	8081	109	0.831
B	78.	1.96	18.8	11880	171	1.155
C	97.	1.82	20.9	13420	194	1.291
D	16.9	4.33	19.1	9778	137	4.33

Table 10 shows that there is a wide variation between galvanometers, so much so, in fact, that only Channel A can be adjusted to critical damping, as indicated by the tabulated values of CDRX and η min.

Recursive Code Calculations

As pointed out in the introduction one objective of this study is to develop a digital filter to simulate the Benioff system; the system parameters were systematically measured for this purpose. However, the seismometer and film recorder galvanometer damping were subject to change by the operators and, consequently, system response was not constant from event to event nor even from channel to channel of the film recorder. Therefore, the problem is not to numerically duplicate one set of conditions, but rather to find a set of parameters which best reproduces the average or typical system response.

CODE VERIFICATION AND PARAMETER MODIFICATIONS

As presently set up, the code is intended to filter broad-band data transmitted over the seismic telemetry system, F.M. recorded on magnetic tape, demodulated and digitized and stored on the computer photo-store facilities. A digitizing rate of 300 pt/sec has been used because this value allows 15 min of data from ten channels to be recorded on one digital tape. It also allows us to evaluate the discrete Fourier transform of portions of the signal up to 150 Hz, which is approximately 15 to 30 times the pass band of the Benioff seismic system. The digitizing rate affects the response obtained from the finite difference approximation, that is, the lower the digitizing rate the greater the attenuation at a given frequency. Thus, some of the parameters measured in the previous section must be modified slightly to obtain the desired response at specified points in the passband. This is more important for the low pass than the high pass elements of the system.

Seismometer

Equation (13) shows that the inductance can be neglected for $\omega \leq \omega_s$ and that the inductance is the important term for $\omega \geq \omega_s$. Therefore, the seismometer and its inductance will be considered separately when determining modified parameters, that is, we will assume that the inductive term can be factored out of the expression for the response of the seismometer. The differential equation to be considered is derived from Eqs. (2) and (3) by assuming zero inductance and replacing ground displacement with ground velocity. The result is

$$\frac{di^2}{dt^2} + 2\eta_s \omega_s \frac{di}{dt} + \omega_s^2 i = \frac{-k_s}{R} \frac{d^2V}{dt^2}, \quad (59)$$

where

$$\eta_s = h_s + \frac{k_s^2}{2\omega_s R M}.$$

The relative amplitude response is

$$\left| \frac{i}{v} \right| \frac{R}{k_s} = \frac{1}{\left[\left(\left(\frac{\omega}{\omega_s} \right)^2 - 1 \right)^2 + \left(2\eta_s \frac{\omega}{\omega_s} \right)^2 \right]^{1/2}} \quad (60)$$

The relative amplitude response to velocity of the recursive solution is derived from the Z transform,¹ evaluated for

$$Z = e^{-\omega \Delta t} \quad (61)$$

The derivation yields

$$\left| \frac{i(\omega)}{v(\omega)} \right| \frac{R}{k_s} = \frac{4 \sin^2 \frac{\omega \Delta t}{2}}{\left[\left(\omega_s^2 \Delta t^2 + 4B \sin^2 \left(\frac{\omega \Delta t}{2} \right) - 2 \sin^2 \omega \Delta t \right)^2 + (2B \sin \omega \Delta t - \sin 2\omega \Delta t)^2 \right]^{1/2}} \quad (62)$$

where

$$B = 1 + \eta_s \omega_s \Delta t,$$

and

Δt = the time increment between points (the reciprocal of the digitizing rate).

Both Eqs. (60) and (62) approach $(\omega/\omega_s)^2$ for $\omega < \omega_s$. However, Eq. (60) approaches unity for $\omega > \omega_s$ whereas Eq. (62) becomes

$$\frac{1}{1 + \eta_s \omega_s \Delta t + \left(\frac{\omega_s \Delta t}{2} \right)^2} \quad (63)$$

Equation (62) can only be made to approach unity by increasing the digitizing rate, which has already been fixed by other considerations. The alternative is to require that Eq. (62) equal Eq. (60) at a specified point in the pass band of the system.

Arbitrarily, then, Eqs. (60) and (62) are evaluated at $\omega = \omega_s$. Assuming small angles, the result is

1. B. Gold and C. Rader, Digital Processing of Signals (McGraw-Hill, New York, 1969), Ch. 2.

$$2\eta_s = 2 \left(\eta'_s + \frac{\omega_s \Delta t}{2} \right) \sqrt{1 + \left(\frac{\omega_s \Delta t}{2} \right)^2} \quad (64)$$

where the prime denotes a modified value to be used in the recursive code. Evaluating Eq. (64) yields

$$\eta'_s = \eta_s - 0.01047,$$

which implies that no modification should be required to obtain the desired response at $\omega = \omega_s$.

This is not the case for the inductive term, however. The relative amplitude response of Eq. (3) is

$$\left| \frac{\tilde{i}(\omega)}{\tilde{v}(\omega)} \right| = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R} \right)^2}} \quad (65)$$

while the relative amplitude response of the recursive solution is

$$\left| \frac{\tilde{i}(\omega)}{\tilde{v}(\omega)} \right| = \frac{1}{\left[1 + \frac{4L}{R\Delta t} \left(1 + \frac{L}{R\Delta t} \right) \sin^2 \left(\frac{\omega \Delta t}{2} \right) \right]^{1/2}} \quad (66)$$

Both Eqs. (65) and (66) are unity at $\omega = 0$, but Eq. (66) is less than Eq. (65) for $0 < \omega < \omega_1$. At $\omega = \omega_1$, Eq. (66) is again equal to Eq. (65), where ω_1 is determined from

$$\frac{\sin \left(\frac{\omega_1 \Delta t}{2} \right)}{\frac{\omega_1 \Delta t}{2}} = \frac{L}{\sqrt{R\Delta t} + L} \quad (67)$$

A modified inductance (L') can be found for small angles, that is $\frac{\omega \Delta t}{2} < 6^\circ$ (frequencies up to 10 Hz), by equating the approximate form of (66) to (65)

$$\frac{L'}{R} (R\Delta t + L')^2 = \left(\frac{L}{R} \right)^2 \quad (68)$$

which, when solved, for L' yields

$$L' = -\frac{R\Delta t}{2} + \sqrt{L^2 + \left(\frac{R\Delta t}{2}\right)^2}. \quad (69)$$

Assuming $R = 140 \, \Omega$ and $L = 6.971 \, \Omega \cdot \text{sec}$, the modified inductance for $t = 1/300 \, \text{sec}$ is

$$L' = 6.754 \, \Omega \cdot \text{sec}.$$

Equation (69) is included in the code.

Figure 12a shows the results of the recursive solution and the Fourier transform of the exact solution [Eq. (12)]. The latter has been evaluated for the following parameters

$$\begin{array}{lll} \omega_s = 2\pi \, 1/\text{sec} & M = 107.5 \, \text{kg} & K_s = 355 \, \text{Vsec/m} \\ h_s = 0.0096 & R = 139.9 \, \Omega & L = 6.97 \, \Omega \cdot \text{sec} \end{array}$$

To make the comparison in Fig. 11a, we gave the recursive code the above parameters and calculated the current output of the seismometer for a unit impulse in velocity. This calculation was made using a finite difference approximation to Eq. (32). It was not made a permanent feature of the final code listed in the Appendix because it is incidental to the final result; it is only useful to verify the coding of Eq. (49). The finite Fourier transform (FFT) of the current was then computed. Note that the recursive solution matches the exact solution [Eq. (11)].

PTA Galvanometer and Filter

As both the galvanometer and the PTA filter are low pass filters, both can be described in general terms by the relative amplitude response of Eq. (39)

$$\left| \frac{\bar{e}_2}{\bar{e}_1} \right| = \frac{\omega_2^2}{\left[(\omega_2^2 - \omega^2)^2 + (2\eta\omega_2\omega)^2 \right]^{1/2}}, \quad (70)$$

which is unity at $\omega = 0$ and which approaches $(\omega_2/\omega)^2$ for $\omega > \omega_2$. The relative amplitude response of the recursive solution to Eq. (39) is

$$\left| \frac{e_2(\omega)}{e_1(\omega)} \right| = \frac{\omega_2^2 \Delta t^2}{\left[(\omega_2^2 \Delta t^2 + 4B \sin^2(\frac{\omega \Delta t}{2}) - 2 \sin^2 \omega \Delta t)^2 + (2B \sin \omega \Delta t - \sin 2\omega \Delta t)^2 \right]^{1/2}}, \quad (71)$$

(a) Seismometer

Fig. 12. Response of finite difference approximations to the Benioff system.

where $B = 1 + \eta \omega_2 \Delta t$. Equation (71) is unity at $\omega = 0$, but it does not approach $(\omega_2/\omega)^2$ when $\omega > \omega_2$, as does Eq. (70), but rather is less than it up to a point, beyond which it is larger than Eq. (70). To ensure that Eq. (70) and (71) have the same attenuation at ω_2 we used a modified value (η') in Eq. (70). Both Eqs. (70) and (71) are evaluated at $\omega_2 = 10\pi$, and then equated. The resultant is then solved for η'

$$\eta' = 1.00044 \eta - 0.05235, \quad (72)$$

where η' is the modified value to be used in Eq. (71). In the previous section η was found to be 0.675. Thus,

$$\eta' = 0.622,$$

(b) PTA galvanometer and filter.

Fig. 12 (cont.)

which in turn implies

$$T'_g = 79.42.$$

The PTA filter is treated similarly. Previously, the PTA filter was down 3 dB at $\omega_2 = 20\pi$, i. e., at $\omega_2 = 20\pi$ the relative amplitude response is 0.707. Following the procedure outlined above, the modified value is

$$\eta' = 0.604.$$

The high pass section of the PTA is represented by Eq. (38), whose relative amplitude response is

$$\left| \frac{\bar{e}_1}{\bar{e}_0} \right| = \frac{\omega}{\sqrt{\omega^2 + \omega_1^2}}, \quad (73)$$

which approaches unity for $\omega > \omega_1$ and approaches (ω/ω_1) for $\omega < \omega_1$. The relative amplitude response of the recursive solution is

$$\left| \frac{\bar{e}_1(\omega)}{\bar{e}_0(\omega)} \right| = \frac{2\sin\left(\frac{\omega\Delta t}{2}\right)}{\sqrt{(1 - \cos\omega\Delta t + \omega_1^2\Delta t^2) + \sin^2\omega\Delta t}}, \quad (74)$$

which is nearly identical to Eq. (73). Therefore, no modification is required on the high pass section.

Figure 12b shows the responses of the recursive solutions which simulate the PTA galvanometer and filter. Note that the desired response has been achieved. Again, as in the case of the current from the seismometer, we have calculated the response curves using the FFT of the impulse response of the recursive equations.

Film Recorder Galvanometers

The damping terms in the recursive solutions for the film recorder and in the PTA galvanometer require the same modification [given by Eq. (72)]. However, as the correct damping is not known, a range of values must be considered. Table 8 lists damping terms and minimum possible damping. For each of the damping terms a corresponding modified value has been calculated using Eq. (72) (Table 11).

Table 11. Modified film recorder damping values.

Channel	η_{test}	η_{test}^1	η_{min}	η_{min}^1
A	1.18	1.128	0.831	0.779
B	1.96	1.909	1.155	1.103
C	1.82	1.768	1.291	1.239

SEISMOMETER AND FILM RECORDER DAMPING: CRITERIA AND PARAMETER STUDY

During the period of overlap of the two systems, 18 of the events recorded by the broad band system were digitized. These events form the basis for evaluating the

digital filter in the same way as the electronic filter was evaluated. Following each of these events, the operators ran a routine free period, ball lift, and an eight point frequency response test, all of which vary from event to event. The ball lift test will be used to choose the seismometer damping while the frequency response test will be used to choose the film recorder galvanometer damping.

Free Period Tests

To determine the free period for the 18 events we measured the time required for a minimum of six complete cycles. The values range from 0.99 sec to 1.099 sec, with a mean and standard deviation of 1.02 ± 0.03 sec. No explanation for this variation is available. However, the speculation is that the mass drifted about its center position and that the period is a function of the mass position. This cannot be confirmed because all efforts to measure the free period at various positions were unsuccessful.

Ball Lift Tests

The ball lift test consists of alternately lifting and dropping a small weight on the mass of the seismometer and then lifting it off. Essentially, the response of the system results from seismometer damping and the high pass section of the PTA filter. The latter, however, is fixed and known. Consequently, any changes can be attributed to changes in the damping potentiometer. Furthermore, the effect of the film recorder response is so small over the range of its most likely damping values that the ball lift test provides a good criterion for choosing the seismometer damping.

Figure 13 shows the system response to one such test, which has been traced from a 12.05X enlargement of the 35-mm film from Channel B. The ratio of the amplitudes X_1 and X_2 , shown in Fig. 13, is called the damping over-shoot ratio. The

Fig. 13. Channel B response to a ball lift test.

long tail on the response results from the high pass section of the PTA filter. That tail makes measurement of X_2 difficult and somewhat uncertain because the base line must be extrapolated from the trace ahead of the test. The operators usually applied about a half dozen tests only 10 sec apart, which is too short a time for the PTA filter to return to zero. This makes the determination of the base line even more difficult and the measurement more uncertain. For this reason only the first of the ball lift tests was measured for any given event. For the 18 events under consideration the range was from 8.8 to 11.2 with a mean and a standard deviation of 9.71 ± 0.62 .

We made a parameter study with the recursive code to determine a damping resistance which would yield the mean damping overshoot ratio. The calculation included the PTA filter but excluded the film recorder, that is, the results of Eq. (50) were printed for an input step in force. The damping resistance shown in Fig. 14 includes the contribution of the line resistance. The input to the code was

$$\Delta t = 1/300 \text{ sec}, h_s = 0.0096, f_s = 1.0 \text{ Hz}, K_s = 355 \text{ Vsec/m}, M = 107.5 \text{ kg}$$

$$L = 6.97 \text{ } \Omega \text{ sec}, T'_g = 79.42 \text{ Vsec}, I = 1 \text{ nt msec}^2, h_g = 0.004, f_g = 5.0 \text{ Hz}.$$

In Fig. 14 the damping resistance that corresponds to a damping overshoot ratio of 9.7 is $61.5 \text{ } \Omega$. We made four calculations with $R_d = 61.5 \text{ } \Omega$. Film recorder damping factors are 0.7, 0.9, 1.1, and 1.9. The output of the film recorder [Eq. (51)] was printed out. The damping overshoot ratios were 9.69, 9.70, 9.9, and 12.5, respectively. As it turns out the last of these is of no interest.

Frequency Response Tests

The Geotech personnel ran the routine frequency response tests by applying a sine in current to the calibration coil. One current level was used on each channel,

Fig. 14. Ratio X_1/X_2 vs damping resistance.

each of which was returned to its routine attenuator settings following an event, assuming additional attenuation was required for the event. One difficulty with this is that the lowest and the highest frequencies run (0.3 to 0.5 Hz) were often very noisy because the seismometer response (Fig. 9) at these frequencies was small compared with the center frequency of about 1.5 Hz. Consequently, the measurement of response amplitude on film is more uncertain on either end of the spectrum than in the middle.

Note that when tested in the lab, Channel A's normalized frequency response was higher at the high frequency

end of the spectrum than were Channels B and C, both of which were nearly the same. Since five events were recorded on Channel A, ten on Channel B, and three on Channel C, systematic differences can be expected between events recorded on Channel A and those recorded on Channels B or C. Thus, all the amplitudes of the frequency response tests were read for Channels A and B. The mean and standard deviation of the response for each frequency over the set of 18 events were calculated. The values listed in Table 12 have been normalized to the response at 1 Hz. These values are shown in Fig. 15 a. Note that the overall system response through Channel A is broader than

Table 12. Mean and standard deviation of system response following 18 events.

Frequency (Hz)	Channel A	Channel B	Event channel
0.3	0.1058 ± 0.0029	0.1114 ± 0.0025	0.1099 ± 0.0044
0.5	0.2918 ± 0.0073	0.304 ± 0.004	0.3011 ± 0.0074
0.7	0.548 ± 0.0065	0.5651 ± 0.0038	0.560 ± 0.01
1.5	1.532 ± 0.041	1.458 ± 0.029	1.483 ± 0.045
2.0	1.558 ± 0.08	1.414 ± 0.066	1.455 ± 0.078
3.0	1.120 ± 0.10	0.919 ± 0.070	0.977 ± 0.104
5.0	0.403 ± 0.075	0.3044 ± 0.0375	0.334 ± 0.057

(a) Mean and standard deviation channels A and B on 18 occasions.

(b) Mean and standard deviation of event channel response on 18 events.

Fig. 15. Mean and standard deviation of Benioff systems response.

through B or C. If we assume that an event was large enough to be read on Channel B but not large enough to clip (or go off the film on Channel A) then the reduced or scaled amplitude of the event, as read from Channel A, would be larger than Channel B's amplitude.

Nevertheless, we still have to find one set of parameters for the digital filter which will satisfactorily reproduce the film data from the digitized data. Therefore, we calculated the mean and standard deviation of the overall system response based on the test results from the same channel on which the event was recorded. (See Table 12 and Fig. 15b.)

Table 11 gives a range of 0.78 to 1.96 for modified film recorder damping values. Therefore, four sets of code calculations were made based on the parameters of the previous section and on $\eta'_f = 0.7, 0.9, 1.1, \text{ and } 1.9$, respectively. In each set we calculated the response to 0.3, 0.5, 0.7, 1.0, 1.5, 2.0, 3.0, and 5.0 Hz sine waves in ground velocity. The results were then normalized to 1 Hz and plotted in Figs. 15a and b. Figure 15a seems to show that Channel A would be best represented by a film recorder damping factor of about seven tenths of critical, while Channel T would be best represented by a film recorder damping factor of about critical. A factor of 1.9 (Table 12) seems out of the question. Figure 15b shows that the 0.7 case has the best fit for the center of the spectrum but not the ends, the 0.9 case fits well on the ends but not in the center, the 1.1 case fits well only on the low end of the spectrum, and the 1.9 case is so far off it hasn't been shown.

No one case is clearly superior, and the final choice will have to be based on how well the filtered data for the three cases just discussed reproduce the film data.

COMPARISON OF DIGITALLY-FILTERED DATA WITH FILM DATA

In this section we compare three sets of filtered amplitudes with the same amplitudes recorded on film. The film data have been scaled to displacement in millimicrons by dividing the amplitude on the film trace by the system magnification

1 Hz. Consequently, the recursively filtered data have also been scaled to displacement at 1 Hz.

Scaling of Filtered Data

In the code the constants C_0 , C_1 , and C_2 of Eqs. (44), (45), and (46), respectively, have been lumped together into one constant called SCALE, which can be used to scale the filtered data in whatever units are desired. In this case the desired result is displacement at 1 Hz, i.e., set SCALE equal to the reciprocal of the magnification at 1 Hz

$$\text{SCALE} = \frac{1}{M_1} = \frac{1}{2\pi V.S.1}, \quad (75)$$

where $V.S._1$ is the velocity sensitivity at 1 Hz. The velocity sensitivity can be found directly because the code operates on ground velocity [see Eq. (44)]. Thus, SCALE is initially set equal to unity and the code is simply given a sine wave whose amplitude is taken to be 1 mm/sec (the broad band data are in units of velocity). The velocity sensitivity is then

$$V.S._1 = \left(\frac{\text{Code Output}}{\text{Ground Velocity}} \right)_{1 \text{ Hz}} \text{ sec/mm}, \quad (76)$$

where the code output is taken to be dimensionless. The velocity sensitivity, magnification, and SCALE at 1 Hz for the three filters are given in Table 13.

Table 13. Velocity sensitivity, magnification, and SCALE at 1 Hz for three filters.^a

Filter	η_f	$V.S._1$ (sec/mm)	M_1 (1/mm)	SCALE (mm)
1	0.7	8.0669×10^{-4}	5.069×10^{-3}	197.294
2	0.9	7.8628×10^{-4}	4.94×10^{-3}	202.415
3	1.1	7.6293×10^{-4}	4.794×10^{-3}	208.610

^aThe following parameters are common to all three filters:
 $\Delta t = 1/300 \text{ sec}$, $h_s = 0.0096$, $f_s = 1.0 \text{ Hz}$, $K_s = 355 \text{ Vsec/m}$
 $M = 107.5 \text{ kg}$, $L = 6.97 \Omega \cdot \text{sec}$, $T_g = 79.42 \text{ V} \cdot \text{sec}$, $I = 1 \text{ n msec}^2$
 $h_g = 0.004$, $f_g = 5.0 \text{ Hz}$

Filtered Data Compared with Film Data

The 18 events mentioned previously have been filtered using three sets of parameters. The only difference between the three filters is the film recorder damping factors and the corresponding SCALE values. Table 14 lists the amplitudes found by the recursive filters and subsequently recorded on film. In this table amplitude a is the first positive peak, b' is the first negative peak, and c' is the second positive peak. All three amplitudes are measured from base line to peak. These amplitudes were then used to obtain a linear regression fit to the following equation

$$\log(\text{film amplitude}) = E1 + E2 \log(\text{filter amplitude}). \quad (77)$$

The results obtained and the standard deviation from the fit are listed in Table 15. For comparison the same results for the electronic filter currently in use are also shown. Note that all three recursive filters out-perform the analog filter in terms of standard deviation and nearness to unit slope. This is particularly true on c'. However, the choice of the best filter is not immediately obvious from the table. Ideally, the one nearest to unit slope and with the smallest standard deviation would be the best choice. No one filter has both of these properties, but filter 2 comes close. It has nearly the same mean slope as filter 3 and it has a better mean deviation.

Table 14. Event amplitudes from film and from recursive filters.

Channel	Benioff film			Filter 1 $\eta_f = 0.7$			Filter 2 $\eta_f = 0.3$			Filter 3 $\eta_f = 1.1$		
	a	b'	c'	a	b'	c'	a	b'	c'	a	b'	c'
A	11.6	31.1	42.7	10.95	30.9	39.1	10.3	28.3	36.7	9.9	26.2	34.4
A	14.4	39.2	43.8	13.89	36.3	41.9	13.4	33.2	39.5	13.05	30.7	37.6
A	27.1	95.9	82.1	25.3	91.9	77.4	24.0	85.7	76.5	22.9	80.8	75.1
B	32.2	17.8	98.2	34.5	128.4	102.8	32.8	120.1	92.4	31.4	114.9	83.9
B	41.5	151.5	130.5	41.4	162.5	152.2	39.7	151.0	132.4	38.3	141.9	116.6
A	41.7	133.3	146.7	40.2	127.2	133.3	38.1	117	120	36.5	109	108
A	53.8	134.2	187.8	52.5	131.2	190.9	49.9	125.2	185.5	47.9	120.4	180.6
H	54.3	170.7	255.3	48.4	187.2	253.9	45.8	175.7	232.7	43.7	166.5	214.9
B	162	557	259	170.4	584.5	305.3	163.1	547.8	264.7	157.4	517.8	234.2
B	493	1,347	813	500	1,350	1,222	1,311	1,141	459	1,270	1,062	
C	506	1,494	1,796	523	1,587	1,955	496.4	1,513	1,798	475.8	1,450	1,663
B	584	1,196	179	569	1,338	232	553	1,211	157	532	1,112	109
C	1,085	2,365	1,665	1,086	2,577	1,817	1,041	2,437	1,699	1,006	2,361	1,577
B	3,150	8,670	5,190	3,016	9,150	5,460	2,882	8,541	4,973	2,776	8,055	4,540
B	3,230	6,000	2,700	3,128	6,221	2,685	2,994	5,897	2,443	2,881	5,633	2,206
B	7,110	12,390	5,610	6,939	12,787	5,718	6,610	12,144	4,908	6,352	11,616	4,187
B	8,400	19,600	9,800	8,787	22,044	11,519	8,433	20,606	10,172	8,154	19,452	9,007
C	29,500	60,200	6,700	28,792	63,800	13,914	27,761	59,869	10,028	26,956	56,609	7,205

Table 15. Results of linear regression fit.

Filter	Amplitude	E1	E2	σ (%)
1	a	0.016047	0.995926	4.69
	b'	0.022190	0.986919	5.12
	c'	0.108315	0.944798	19.52
	mean	0.049	0.976	9.78
2	a	0.040161	0.994516	4.87
	b'	0.058293	0.984074	5.16
	c'	0.113011	0.960573	14.9
	mean	0.070	0.980	8.31
3	a	0.059996	0.993244	5.05
	b'	0.088467	0.981919	5.75
	c'	0.132992	0.971769	16.49
	mean	0.094	0.982	9.1
electronic	a	-0.045827	0.994696	6.73
	b'	-0.020299	0.983359	7.52
	c'	0.112427	0.914014	25.82
	mean	0.015	0.964	13.36

Therefore, number 2 would seem to be the best choice to simulate the mean operating conditions of the old Benioff system.

To measure amplitudes b' and c' we depart from the traditional practice of measuring peak-to-peak amplitudes. That is, we traditionally measure from the first positive peak to the first negative peak for b and from the first negative peak to the second positive peak for c . The following relationships then hold:

$$b = a + b' \quad (78)$$

and

$$c = b' + c' \quad (79)$$

With this scheme of measuring, we obtained a smaller standard deviation in c than in c' because b' is, in general, as large as or larger than c' . Similarly, a smaller deviation is found for b than for b' . The results of a regression fit to Eq. (77) for b and c are given in Table 16.

Table 16. Results of regression fit for filter 2 using normal method.

Amplitude	E1	E2	σ (%)
b	0.055687	0.987263	4.14
c	0.076569	0.979419	6.12

However, this method of measurement is deceptive because the second positive peak appears to be better known than it really is. This effect is clearly seen in Fig. 16 where the recursive filter output for the last event listed in Table 14 is scaled to displacement in microns at 1 Hz for the three filters. Also shown is the film data for the last event listed in Table 14. The film data were read manually with a microscope equipped with X-Y readout on a card punch machine. Therefore, the data are not even spaced. Note that filter 1 is closest on amplitude a , filter 2 on amplitude b' , and filter 3 is nearest on amplitude c' . Clearly, the first method of measurement gives the best indication of how well the filter is reproducing the film data.

Conclusions

We have written a recursive code which satisfactorily reproduces the Benioff film data. Considerable effort was required to measure the physical parameters which describe the system. Unfortunately, seismometer and film galvanometer damping could not be reconstructed in the lab and we had to deduce these values from the routine ball lift and frequency response tests run by the Geotech personnel and from parameter studies made with the code.

The values finally arrived at are compromises because routine frequency response tests showed that considerable variation existed from event to event and between channels. This fact could account for some of the scatter that exists in the seismic data. For example, assume that the damping factors for Channels A and B are 0.7 and 0.9; then there would be a 3.7% difference in a , 6.6% in b' , and 39% in c'

Fig. 16. Comparison for one event of the recursively filtered data with the film data. Amplitudes are scaled to displacement in microns from system magnification at 1 Hz.

for the largest event in Table 14. This suggests that the present practice of measuring peak-to-peak should be discontinued because a poorly reproducible amplitude, c' , is receiving more weight than it deserves.

The recursion technique used in the code has the advantage that it can be used in an on-line mode of operation — should the funds for on-line digital processing of the data ever become available. The code is also better than an analog filter because once its parameters are specified, its frequency response does not change from time to time, as the analog filters have been known to do.

We have shown that the seismometer is effectively decoupled from the PTA galvanometer for the PTA attenuator values actually used to record events. Also, the damping of the galvanometer is unaffected at these high values of attenuation (24 dB and greater). However, this is not the case for small values of attenuation.

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Appendix Benioff Recursive Filter

```

USE COMRLK
DIMENSION SAVEUF(2),SAVEF(2),STV(5),SFO(3),ABC(3)
EQUIVALENCE (WORK,SAVEUF),(WORK(3),SAVEF),(WORK(5),STV),
*          (WORK(10),SFO),(WORK(13),ABC)
REAL MASS
DATA (MS=.0096),(FQS=1.),(GENC=355.),(MASS=107.5),(DUCT=6.97),
*      (RD=61.5),(ATT=41.),(TG=79.42),(ERT=1.),(HG=.004),(FQS=5.),
*      (ETAF=.9),(SCALE=202.415)

IN1 FORMAT(12F5.0,F10.0)
OUT1 FORMAT(///2X,36HPARAMETER VALUES FOR BENIOFF FILTER:5X,5HTPPC=,
1          F10.8,5X,5HGENC=,F10.2,5X,3HRD=,F10.2,5X,4HATT=,F10.2,5X,
2          2HMS=,F10.5,5X,6HSCALE=,F10.3/6X,4HFQS=,F10.4,5X,5HDUCT=,
3          F10.4,5X,3HTG=,F10.2,4X,5HETAF=,F10.2//)
OUT2 FORMAT(3X,26HPEAK FILTERED AMPLITUDES -.4X,7HCHANNEL,RX,1HA,14X,
*      1HB,14X,1HC,12X,6HOFFSET,10X,3HRMS)
OUT3 FORMAT(35X,12,3X,5E15.5)

```

```

N1=INSTR(ICNT+1)
ICNT=ICNT+2
N2=INSTR(ICNT)

```

```

C      TPP=TIME IN SECONDS BETWEEN POINTS IN A CHANNEL
C      MS=MECHANICAL DAMPING FACTOR
C      FQS=NATURAL FREQUENCY OF SEISMOMETER IN HZ
C      GENC=GENERATOR CONSTANT IN NEWTONS PER AMP
C      MASS=MASS OF SEISMOMETER IN KG
C      DUCT=INDUCTANCE OF DATA COILS IN OHM*SEC.
C      RD=DAMPING RESISTANCE IN OHMS
C      ATT=PTA ATTENUATOR SETTING IN DB
C      TG=GALVANOMETER SENSITIVITY IN NT*4/AMP
C      ERT=INERTIA OF GALVANOMETER IN KG*M**2
C      HG=MECHANICAL DAMPING FACTOR
C      FQG=NATURAL FREQUENCY OF GALVANOMETER
C      ETAF=DAMPING FACTOR FOR FILM RECORDER, DECIMAL
C      SCALE=NORMALIZING FACTOR
C      TV=PTA GALVANOMETER DEFLECTION OR VOLTAGE OUTPUT
C      CURV=CURRENT OUT OF SEISMOMETER
C      FOD=PTA FILTER OUTPUT
C      FR=FILM RECORDER DEFLECTION

```

```

C      COMPUTE FILTER CONSTANTS

1PP=TPP*NCHANS
OMS=2.*(3.1415927)*FQS

```

```

ATK=10.** (ATT/20.)
BTK=ATK**2.
CON1=1471.5*BTK-3.5
CON2=9.*BTK-1.
RT=167.3*RD-500.*(BTK-1.+(1280*BTK/CON1))/CON2
S=RT-43.3
PI=3.141592654
TEMP=(20.*PI*DUCT)/(RT*SINF(10.*PI*TPP))
DUCT=(RT*TPP/2.)*(-1.+SQRT(1.+TEMP*TEMP))
OMG=2.*(3.1415927)*FQG
ETAG=HG*(TG**2*CON2)/(CON1*2.*OMG*ERT)
AS=1.2.*HS*OMS*TPP*(OMS*TPP)**2.
BS=(1.*HS*OMS*TPP)*2.
AG=1.*2.*ETAG*OMG*TPP*(OMG*TPP)**2.
BG=2.*(1.*ETAG*OMG*TPP)
C=((800.*ATK*TG*TPP)/CON1)**2/ERT
D=DUCT*RT*TPP
E=(GENC*TPP)**2./MASS
BI=(AS*D+E)/C
BII=(HS*D*DUCT*AS+E)/C
BIII=(DUCT*BS*D)/C
AI=AG*BI+AS
AII=-(AG*BII+BG*BI)-AS-BI
AIII=AG*BIII+BG*BII+BI+1.*BS
AIV=-(AG*DUCT/C)-BG*BIII-BII-1.
AV=(BG*DUCT/C)*BIII
AVI=-DUCT/C
F=SCALE*((GENC*TPP*CON1)/(800.*ATK*TG))
G=CON1*ERT/(800*ATK*TG*TPP**2)
F1=3947.86*TPP*TPP
F2=3.*177.842*TPP+3953.44*TPP*TPP
F3=3.*88.921*TPP
F4=1.*TPP*(88.921+3953.44*TPP+248.052*TPP*TPP)
FR1=(OMG*TPP)**2.
FR2=2.*(1.*ETAF*OMG*TPP)
FR3=FR1+FR2-1.
WOT 3,OUT1,TPP,GENC,RD,ATT,S,SCALE,FQS,DUCT,TG,ETAF

```

C APPLY FILTER TO SPECIFIED CHANNELS

```

DO 200 NOCH=N1,42
IF (ICOR(NOCH)) *200.
N3=0
SAVEUF(1)=SAVEUF(2)=0.
SAVEF(1)=SAVEF(2)=0.
DO 10 I=1.5
10 STV(I)=0.
DO 15 I=1.3
15 SFO(I)=0.
IF (IDADSK-2) 30,30
NX=NXW=0 $ NMBR=NMBRW=NBRPTS $ NT=NTW=NDADSK
25 CALL ADDSKRD(21,DATA,NX,NMBR,NT,IFLAG)

```



```

30 DO 100 I=NOCH,NMBR,NCHANS
    TV=-(F*(DATA(I)-2.*SAVEUF(1)+SAVEUF(2))+4II*STV(1)+4III*STV(2)+
    * AIV*STV(3)+AV*STV(4)+AVI*STV(5))/AI
    FO=(F1*(TV-STV(1))+F2*SFO(1)-F3*SFO(2)+SFO(3))/F4
    FR=(FR1*FO+FR2*SAVEF(1)-SAVEF(2))/FR3
    SAVEUF(2)=SAVEUF(1)
    SAVEUF(1)=DATA(I)
    SAVEF(2)=SAVEF(1)
    SAVEF(1)=FR
    DO 50 J=5,2,-1
50 STV(J)=STV(J-1)
    STV(1)=TV
    SFO(3)=SFO(2)
    SFO(2)=SFO(1)
    SFO(1)=FO
100 DATA(I)=-FR
    CALL AMIMX(DATA(NOCH),NMBR,XMIN,XMAX,NCHANS)
    IF(N3) 120,120
    AMIN(NOCH)=XMIN
    AMAX(NOCH)=XMAX
    N3=1
    GO TO 150
120 AMIN(NOCH)=MIN1F(AMIN(NOCH),XMIN)
    AMAX(NOCH)=MAX1F(AMAX(NOCH),XMAX)
150 IF(IDADSK-2) 200,200
    CALL ADDSKWR(2),DATA,NX,NMBR,NTW,JFLAG)
    IF(IFLAG) 25.
200 CONTINUE
    TPP=TPP/NCHANS

```

C FIND FILTERED PEAK AMPLITUDES

```

WOT 3.OUT?
NSR=1./TPP
NCNT=2*NSR/NCHANS

```

```

DO 500 NOCH=N1,N2
    IF(ICOR(NOCH)) 500.
    ARC(1)=ABC(2)=ABC(3)=0.
    XRAR=STD=SVAR=0.
    NPS=NSR
    L=1
    IF(IDADSK-2) 225,225
    NX=0 * NMBR=NBRPTS * NT=NDADSK
210 CALL ADDSKRD(2),DATA,NX,NMBR,NT,IFLAG)

```

```

225 GO TO (250,300,350,400,450,400),L
250 NPS=NPS-NMBR
    IF(NPS) 490,490
    NSTART=NMBR+NPS+NOCH
    J=0

```

```

L=3
GO TO 310

300 NSTART=NOCH
310 DO 330 I=NSTART,NMBR,NCHANS
    J=J+1
    IF (J=NCNT) **340
    XBAR=XBAR+(DATA(I)-XBAR)/J
330 SVAR=SVAR+(DATA(I)-XBAR)*(DATA(I)-XBAR)
    GO TO 490
340 STD=SQRT(SVAR/(NCNT-1))
    DIFF=4.*STD
    THRESH=XBAR+DIFF
    NSTART=I
    L=3
    GO TO 360

350 NSTART=NOCH
360 DO 380 I=NSTART,NMBR,NCHANS
    IF (DATA(I)-THRESH) *390*390
380 CONTINUE
    GO TO 490
390 NSTART=I+NCHANS
    DLAST=DATA(I)
    L=4
    GO TO 410

400 NSTART=NOCH
410 DO 430 I=NSTART,NMBR,NCHANS
    IF (DATA(I)-DLAST) *430*425
    IF (DATA(I)-DLAST+DIFF) 440*430*430
425 DLAST=DATA(I)
430 CONTINUE
    GO TO 490
440 ABC(L-3)=DLAST
    IF (L-6) *495*
    NSTART=I
    L=5
    GO TO 460

450 NSTART=NOCH
460 DO 480 I=NSTART,NMBR,NCHANS
    IF (DATA(I)-DLAST) 475*480*
    IF (DATA(I)-DLAST+DIFF) 480*480*485
475 DLAST=DATA(I)
480 CONTINUE
    GO TO 490
485 ABC(2)=DLAST
    NSTART=I
    L=6
    GO TO 410

490 IF (IDASK-2) 495,*495
    IF (IFLAG) ,210,
495 ABC(3)=ABC(3)-ABC(2)
    ABC(2)=ABC(1)-ABC(2)
    ABC(1)=ABC(1)-XBAR
    WOT 3,OUT,NOCH,(ABC(1)+I*3)*XBAR,STD
500 CONTINUE
    RETURN
END

```