ANL-7222
Reactor Technology
(TID-4500)
AEC Research and
Development Report

CFSTI PRICES

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

H.C. \$ 3.00; MN 65

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

AN ANALYTIC STUDY OF
THE TRANSIENT BOILING OF SODIUM
IN REACTOR COOLANT CHANNELS

by

Donald R. MacFarlane

Reactor Physics Division

Based on a thesis submitted to the Faculty
of Purdue University
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

June 1966

Operated by The University of Chicago
under
Contract W-31-109-eng-38
with the
U. S. Atomic Energy Commission

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	9
I. INTRODUCTION	10
II. LITERATURE REVIEW AND SURVEY OF PREVIOUS WORK	12
A. Analysis of Fast Reactor Transients	13
B. Transient Two-phase Hydrodynamics	14
C. Experimental Work	17
III. MATHEMATICAL DEVELOPMENT OF PROBLEM	19
A. Objectives of This Work	19
B. Reactor Channel and Flow Model	19
C. Equations	21
D. Boundary Conditions	24
IV. STEADY-STATE SOLUTIONS	25
A. Equations and Method of Solution	25
B. Numerical Results	28
C. Experimental Data for Steady-state Boiling	30
V. TRANSIENT SOLUTION FOR CHANNELS CONTAINING ONLY TWO-PHASE MIXTURE	32
A. Derivation of Equations	32
B. Finite Difference Forms of the Equations	35
C. Numerical Results	39
VI. TRANSIENT SOLUTIONS FOR SINGLE-PHASE, TWO- PHASE CHANNELS WITH HEAT TRANSFER FROM FUEL	52
A. Single-phase Convection Equation	52
B. Fuel Heat Conduction Equations	53
C. The Complete Calculation	57
D. Numerical Results	60

TABLE OF CONTENTS

	<u>Page</u>
VII. SUMMARY OF RESULTS AND CONCLUSIONS	74
ACKNOWLEDGMENTS	76
REFERENCES	77
GENERAL BIBLIOGRAPHY	79
APPENDIXES	
A. Two-phase Flow Correlations	80
B. ALGOL Program for Calculations of Steady-state Two- phase Flow	93
C. ALGOL Program for Transient Solutions with Channels Containing only a Two-phase Mixture	100
D. ALGOL Program for Transient Solutions with Single- phase, Two-phase Channel with Heat Transfer from Fuel .	106
E. Physical Properties of Sodium	116

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	Model of Reactor Channel	20
2.	Annular-flow Model	20
3.	Vapor Void and Quality Profiles for Various Flow Rates-- $\phi_{\max} = 50 \text{ cal/cm}^2\text{-sec.}$	28
4.	Vapor Void and Quality Profiles for Various Flow Rates-- $\phi_{\max} = 100 \text{ cal/cm}^2\text{-sec.}$	29
5.	Pressure Profiles for Various Flow Rates, $P_{\text{ex}} = 6.0 \text{ atm}$.	29
6.	Pressure Profiles for Various Flow Rates, $P_{\text{ex}} = 4.0 \text{ atm}$.	29
7.	Pressure Profiles for Various Flow Rates, $P_{\text{ex}} = 2.0 \text{ atm}$.	29
8.	Total Channel Pressure Drop vs Flow Rate for Various Heat Fluxes.	30
9.	Comparison of Liquid Metal Void Fraction Data.	31
10.	Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.100 \text{ sec}$	41
11.	Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.050 \text{ sec}$	41
12.	Mass Flow Rate vs Channel Position at Various Times for $\omega = -0.100 \text{ sec}$	42
13.	Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.100 \text{ sec.}$	42
14.	Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.050 \text{ sec.}$	43
15.	Vapor Void Fraction vs Channel Position at Various Times for $\omega = -0.100 \text{ sec.}$	43
16.	Vapor Quality vs Channel Position at Various Times for $\omega = 0.100 \text{ sec.}$	44

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
17.	Vapor Quality vs Channel Position at Various Times for $\omega = 0.050$ sec	44
18.	Vapor Quality vs Channel Position at Various Times for $\omega = -0.100$ sec	45
19.	Pressure Profiles at Various Times for $\omega = 0.100$ sec.	46
20.	Pressure Profiles at Various Times for $\omega = 0.050$ sec.	46
21.	Pressure Profiles at Various Times for $\omega = -0.100$ sec.	47
22.	Calculated Values of G_j^i vs Time-step Size.	48
23.	Calculated Values of α_j^i vs Time-step Size.	48
24.	Calculated Values of x_j^i vs Time-step Size.	49
25.	Calculated Values of P_j^i vs Time-step Size.	49
26.	Mesh Layout for Fuel Heat Conduction Equations	54
27.	Interpolation to Locate Single-phase, Two-phase Interface.	57
28.	Variable Space Mesh in Two-phase Region of Channel.	58
29.	Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.10$ sec	62
30.	Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.20$ sec	62
31.	Average Mass Flow Rate vs Time for $\omega = 0.10$ sec	63
32.	Average Mass Flow Rate vs Time for $\omega = 0.20$ sec	63
33.	Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.10$ sec	64
34.	Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.20$ sec	64

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
35.	Vapor Quality vs Channel Position at Various Times for $\omega = 0.10$ sec	65
36.	Vapor Quality vs Channel Position at Various Times for $\omega = 0.20$ sec	65
37.	Pressure Profiles at Various Times for $\omega = 0.10$ sec	67
38.	Pressure Profiles at Various Times for $\omega = 0.20$ sec	67
39.	Fuel Center Temperature Profiles at Various Times for $\omega = 0.10$ sec	68
40.	Fuel Center Temperature Profiles at Various Times for $\omega = 0.20$ sec	68
41.	Coolant Temperature Profiles at Various Times for $\omega = 0.10$ sec	69
42.	Coolant Temperature Profiles at Various Times for $\omega = 0.20$ sec	69
43.	Mass Flow Rate vs Time for Flow Decay Transient	71
44.	Fuel Center Temperature Profiles at Various Times for Flow Decay Transient	71
45.	Coolant Temperature Profiles at Various Times for Flow Decay Transient.	72
46.	Block Diagram of Steady-state Two-phase ALGOL Program.	96
47.	Block Diagram of Transient Two-phase ALGOL Program . .	102
48.	Block Diagram of Transient Single-phase, Two-phase ALGOL Program	108

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I.	Parameters for Two-phase Transient Problems	39
II.	Summary of Results of Stability Studies with Two-phase Difference Equations.	50
III.	Input Parameters for Single-phase, Two-phase Transient Problems	60
IV.	Linear Velocities at Channel Exit during Coolant Expulsion.	61
V.	Heat Fluxes and Fuel Power Generation Rates during Transients	70
VI.	Calculated Slip Ratios Implicit in the Martinelli Void Fraction Correlation.	83
VII.	Calculated Slip Ratios Implicit in the Modified Balzhiser Void Fraction Correlation	88
VIII.	Listing of Steady-state Two-phase ALGOL Program.	97
IX.	Listing of Transient Two-phase ALGOL Program	103
X.	Listing of Transient Single-phase, Two-phase ALGOL Program.	109
XI.	Values for x , α , and G during Inner Iterations with Two- phase Hydrodynamics Equations	114

AN ANALYTIC STUDY OF THE TRANSIENT BOILING OF SODIUM IN REACTOR COOLANT CHANNELS

by

Donald R. MacFarlane

ABSTRACT

The purposes of this study were to develop an analytic model and to use it to perform calculations aimed at predicting the time-dependent behavior of boiling sodium coolant in the channels of a nuclear reactor. Specifically, the information sought is a detailed description of the sodium coolant density (vapor void fraction) in space and time during abnormal reactor operation which results in overheating and coolant boiling. Such information is necessary accurately to calculate feedback reactivity effects in transient analyses of large (1000 MWe) fast breeder reactors.

The approach taken in this analysis was to seek solutions for the coupled, nonlinear, partial differential equations of fluid dynamics and heat conduction written for a single reactor coolant channel and fuel rod. Two basic types of transients were studied: 1) Mild overpower transients in which the driving function is an exponentially increasing heat generation rate in the fuel and, 2) coastdown of the coolant flow resulting from an exponentially decreasing available pressure head at the coolant channel inlet. These two cases correspond to the inadvertent insertion of reactivity and to coolant-pump failure, respectively. Numerical solutions were obtained for the nonlinear forms of the coupled partial differential equations by means of implicit finite difference equations and an iterative method in an ALGOL computer program.

In a typical boiling transient the coolant entered the core hundreds of degrees below its boiling point, was heated to the boiling point in the channel, and exited from the channel with a high vapor void fraction. Results of calculations for a typical fast breeder configuration with UO_2 fuel rods and 50 v/o sodium coolant led to the following general conclusions: 1) the rate of coolant expulsion is extremely rapid once boiling starts and is sensitive to the coolant temperature profile in the liquid portion of the channel; 2) the inlet flow decays rapidly to zero, typically by the time 20% of the channel is boiling; 3) pressure buildup in the channel is not excessively large, typically less than 15 atm.

In this analysis sonic effects and critical flow phenomena were not treated; however, the two-phase coolant density is pressure dependent, and a temporal and spatial varying liquid/vapor slip velocity is included.

I. INTRODUCTION

The work described herein is part of a continuing program to obtain a better understanding of coolant voiding effects in large fast power breeder reactors. It has been pointed out that an accurate description of the kinetic behavior of a reactor during abnormal operation which leads to coolant boiling depends on more detailed information about the voiding of sodium in such an accident.¹ In addition, the boiling of sodium, in most cases, is a prelude to fuel meltdown and could generate pressures large enough to influence the motion of molten fuel or even cause core damage prior to melting.

The safety aspects of fast breeder reactors differ from those of thermal reactors mainly because of the much shorter times of prompt-neutron generation typical of fast reactors; this is important should the reactivity become greater than one dollar. Below this value, the low-power kinetics of fast breeder reactors are similar to thermal reactors. However, once the reactor exceeds prompt critical, the resulting rapid exponential increase of power is almost certain to damage the reactor core. Under the worst possible circumstances meltdown of the core and subsequent reassembly of molten fuel into a critical mass could lead to a nuclear explosion. This is possible because of the relatively large inventory of fissionable material contained in a fast reactor core. The rapid sequence of events in a superprompt critical excursion is a direct consequence of the short neutron lifetime, which leads to power increases too rapid to be checked by control-rod movement. Thus, the analysis of fast reactor transients falls into two general areas: 1) analysis of response to perturbations which produce initial reactivity insertions of less than one dollar to see if subsequent events lead to reactivity effects greater than one dollar, and hence the superprompt critical condition; 2) having melted a significant portion of the core in a power excursion, the determination of the behavior of molten fuel in space and time. From this information estimates can be made of the possibility and severity of a nuclear incident. The present work is concerned only with the first area.

Basically, the problem of the reactor designer is to introduce features into the reactor design so that credible abnormalities during reactor operation, such as failure of coolant pumps or the insertion of reactivity due to malfunction of the control system, do not initiate events leading to a prompt critical accident. Water-moderated and -cooled thermal reactors can usually be designed so that the reactivity effects due to a reduction in coolant density when boiling occurs are negative. This can be done by adjusting the coolant volume fraction in the core so that the reactor is undermoderated and, hence, the coolant density reduction due to heating and/or void formation always gives a negative reactivity feedback. In a large, sodium-cooled fast breeder reactor, however, a reduction of sodium density can give a negative or positive reactivity effect--depending on the location of the void in the core.²⁻⁴ Therefore, in a determination of the

kinetic behavior of a fast reactor with such a variable sodium void coefficient, a fairly detailed description of the void distributions in space and time is necessary to obtain the proper feedback reactivity. Furthermore, in some cases, the magnitude of positive sodium void effect is large enough so that the other negative-feedback contributions, namely, a Doppler effect and axial fuel expansion, can be overridden if the coolant boils. Therefore, an autocatalytic behavior in transients which lead to coolant boiling is possible under certain circumstances. This relatively small initial perturbation could then lead to a superprompt critical accident and extensive core damage or destruction. In the absence of detailed information, it is assumed in many of the fast reactor safety analyses that once the coolant starts boiling the core is irretrievably lost and that a significant amount of melting of fuel will occur. This may not be true in all instances and, indeed, boiling can give a negative reactivity initially, depending on the region of the core where the boiling starts.

Various theoretical studies of fast reactor transients to date⁵ have not included the reactivity effects due to voids resulting from sodium coolant boiling. A better understanding of the interplay between sodium boiling and core reactivity could lead to design modifications which would improve the safety characteristics of large fast breeder reactors, and this study was undertaken to help provide such knowledge. The objective of this work is to obtain information on sodium expulsion from a reactor core, due to boiling, from an analysis using the basic equations of hydrodynamics. The analysis is limited to transients which are slow enough so that coolant boiling occurs before the fuel or cladding melts. Since there is essentially no experimental data available on the transient boiling of sodium, or any liquid metal confined in a channel, it is the aim of this work to serve the following two purposes: 1) to supply a detailed description of sodium density in a reactor core in space and time to be used as input for reactor kinetics analyses which include the effect of sodium voiding; 2) to aid in the design and analysis of transient sodium boiling experiments now in the preliminary planning stage. All of the work carried out here is analytic and involves obtaining numerical solutions to the coupled fuel element heat transfer and two-phase coolant hydrodynamic equations. The steady-state analyses performed as an initial phase of the work are compared with the limited experimental data available in the literature. Although the problem formulation is quite general, the work is oriented toward the large (1000 MWe) fast breeder reactors having uranium oxide or uranium carbide fuel rods.

II. LITERATURE REVIEW AND SURVEY OF PREVIOUS WORK

According to Fauske,⁶ the recorded work on two-phase flow dates back to the work of Sauvage in 1872. An indication of the tremendous volume of research which has been performed in this field in recent years is given by the number of references in bibliographies by Kepple and Tung⁷ and Van Hoogstraten.⁸ In spite of this large effort, which has been greatly augmented in recent years by the work on boiling water reactors, a satisfactory theory of two-phase flow which predicts both dynamic and steady-state behavior has not yet been developed. Most of the difficulty stems from the large number of variables which are found in a typical boiling system. The effects of relatively nonreproducible and intangible parameters, such as surface roughness, which in turn influence the nucleation sites for bubble formation, are very important in boiling systems. Likewise, the phenomena of liquid superheat and subcooled boiling have so far proved difficult to predict. A factor which complicates attempts at analysis is the multiplicity of possible flow regimes, that is, the relative arrangement of vapor bubbles and liquid drops or slugs, which is possible in a boiling system--each one requiring a different model and mathematical description. Furthermore, it has so far proved impossible to calculate one of the most important parameters in a boiling system, namely, the vapor void fraction (fraction of volume occupied by the vapor phase) from fundamental principles only. This quantity must be obtained by experiment on a system which is physically similar to and operating at essentially identical conditions to the boiling system under design.

In spite of these difficulties and limitations, a great deal of very useful work has been accumulated from research on two-phase flow and heat transfer. The great bulk of the work has been devoted to the steady boiling of water under pressures of tens of atmospheres, since this is the area of greatest interest from a practical standpoint in the design of equipment such as steam boilers and nuclear reactors. In 1949 Lockhart and Martinelli⁹ presented correlations for void fraction and friction pressure losses for two-phase flow which were derived from isothermal experiments with a variety of hydrocarbon fluids and air. Their results were applied to the analysis of forced circulation of boiling water with heat addition by Martinelli and Nelson.¹⁰ In spite of subsequent refinements in the methods for calculating forced- and natural-convection boiling in water systems with heat addition, this method is still useful and was the basis for the design calculations for the first boiling water reactor.

A large portion of the more recent work in dynamic analysis and experiment for two-phase flow is mainly concerned with stability analysis--again in connection with the design of boiling water reactors. Basically, these are treatments based on linear transfer function which frequently include reactivity feedback effects via the reactor kinetics equations. Such studies are aimed at determining the threshold of nuclear and/or hydrodynamic instabilities for specific designs of the reactor core. A good

review of the status of the work in two-phase flow for boiling reactor systems is given by Lottes *et al.*¹¹ More recently Wallis and Collier¹² gave a thorough treatment of the whole area of two-phase flow and heat transfer, starting at an introductory level and progressing to a description of the latest analytic techniques being employed.

The subject area of this work, namely, transient boiling of sodium under forced-convection flow in a channel, has hardly been touched by analysis or experimental work. The obvious reasons for the difficulties in running an experiment are the high temperatures involved and the resulting materials and instrumentation problems. An analytic treatment of the problem requires solution of a system of coupled, nonlinear, partial differential equations and is more difficult than a similar problem for a boiling water system operating at relatively higher pressure than a liquid metal system. This will be discussed in more detail later.

A. Analysis of Fast Reactor Transients

The problems arising from possible coolant boiling under abnormal operating conditions in a fast reactor have been recognized for some time. An early attempt at analysis in this area was made by Nicholson.¹³ Nicholson performed steady-state calculations for the Enrico Fermi Fast Breeder Reactor, wherein he estimated the percentage decrease in flow which would just produce boiling in the reactor core. He also calculated two-phase pressure drops to obtain estimates of the increased pressure drop in the fuel subassemblies which would result from boiling of the sodium coolant. He concluded that substantial increases in channel pressure drop are encountered when the coolant boils, as expected.

Stein¹⁴ outlined in greater detail the considerations relevant to predicting coolant behavior in fast reactor transients. A fairly detailed description of the methods used for calculating two-phase flow pressure drops as well as some representative parameters for boiling sodium is given in his work. Stein also goes into a fair amount of detail in developing analytic Laplace transform techniques which could presumably be used for calculating time-dependent coolant and fuel-rod temperatures. Although the analytic techniques are developed in detail, no numerical results are presented.

A good qualitative review of the considerations in connection with coolant vaporization in fast reactors is given by Goldman.¹⁵ He describes the situations in which coolant vaporization is likely to be important as well as some of the important effects of coolant boiling. He points out that analysis of transients including the effects of coolant boiling is needed, and mentions some of the difficulties involved in performing such an analysis.

A survey of the methods which have actually been applied to the analysis of fast reactors is given by Kazi and Cherry.⁵ A listing is given

of the salient features of each of four reactor codes which have been used to analyze fast reactor transients: ARGUS,¹⁶ FORE,¹⁷ TIGER,¹⁸ and TER-4.¹⁹ The descriptions of these computer codes will not be repeated here except to say that none of the methods treats coolant boiling. Basically, all four are two- or three-dimensional, time-dependent, multiregion, digital computer heat transfer codes which, in two cases (FORE and TER-4), include reactivity feedback effects via the reactor kinetics equations.

An example of the analog computer approach to the problem is given by Bryant and Gopinath.²⁰ These authors take advantage of the inherent ease of solving the point reactor kinetics equations on the analog computer and use a space-discretization technique to write the time-dependent differential equations describing the temperatures of reactor fuel and coolant lumps. For certain configurations of fuel and coolant this method compares favorably with calculations done with ARGUS and has the advantage of easily incorporating the reactivity feedback effects. Again, however, coolant vaporization is not treated.

The only really serious attempt to include sodium coolant boiling in reactor analysis was made by Noyes *et al.*²¹ Their digital computer code, TRANS-FUGUE-I, incorporates a fairly detailed treatment of transient two-phase coolant dynamics and automatically introduces this calculation when the coolant temperature exceeds a specified saturation temperature. Although this work does have some rather serious weaknesses, discussed later, it represents a significant first attempt at developing an analytical model.

B. Transient Two-phase Hydrodynamics

A helpful factor is that a great deal of parallel effort exists in the various analyses and experiments which have been performed for water-moderated and -cooled thermal reactors. As mentioned earlier, a major portion of this work is represented by the linear transfer function type of analyses for boiling water reactors. Such work does not have much direct bearing on the present problem. On the other hand, some of the analyses of pressurized water- and organic-cooled reactors, which include treatment of coolant boiling, is relevant to sodium-cooled reactors. Also, analysis of two-phase transient flow of cryogenic fluids is pertinent to the present problem, in that the mathematical formulation is the same.

A predominant work is the CHIC series of computer programs developed at the Bettis Laboratory for analysis of pressurized water reactor transients. The first in this series was the ART program,²² written for the IBM-704 computer and published in 1959. Various improved versions, each of which introduced more realism and detail into the calculations, have been developed. The most recent version, called CHIC-KIN,²⁴ was published in 1965. A review of the various numerical differencing schemes and

computational techniques used in this series of programs is given by Birkhoff.²³ The work on these programs was motivated by a need to develop analytic techniques which would predict the transient behavior of pressurized water reactors and include in reasonable detail the important self-shutdown mechanism resulting if the coolant boils. Although the application of the ART program is rather severely restricted by certain assumptions used to make the computation manageable, namely, a constant reference pressure during the transient as well as the assumed constancy of mass flow rate with position, the program does comprise a solution of the complete set of coupled hydrodynamic, fuel heat conduction, and reactor kinetics equations. In CHIC-KIN, these deficiencies have been removed; however, several empirical parameters have been introduced which describe the boiling process and very much influence the results of a given calculation. This means that in order for computation with CHIC-KIN to be useful, it is necessary to have available the experimentally determined parameters, preferably measured on a system physically similar if not identical to the one being analyzed. Calculations performed with CHIC-KIN have shown good comparison with experimental results from the SPERT-I reactor performed at low pressure, if the proper values for the so-called "fraction of heat going to voids" and "bubble removal time" are inserted into the calculation. Both ART and CHIC-KIN use the simplifying assumption that the liquid and vapor velocities are equal (homogeneous-flow model).

Calculations for SPERT-I by Fleck²⁵ aimed at the development of a model which included the effects of system pressure changes. Fleck's emphasis was on the numerical solution of the three hydrodynamic conservation equations, which he solved in a spacewise integrated form. Feedback through the reactor kinetics equations was included, but heat conduction in the fuel elements was not treated in detail. Fleck assumed a linear variation of the vapor void-fraction with channel length. Although he did not use the assumption of equal liquid and vapor velocities (homogeneous-flow model) as in the case of the ART and CHIC-KIN programs, he did choose a slip ratio (vapor-to-liquid velocity ratio) which remained constant in space and time. Some results were presented for the SPERT-I core behavior with various ramp rates of reactivity insertion; however, no comparison of the calculations with experimental results is given.

Meyer²⁶ puts the problem of transient two-phase hydrodynamic calculations into perspective by summarizing four different flow models with varying degrees of complexity and detail which he has applied to two-phase transient calculations. Two of these, the single mass velocity model and the momentum integral model, are the basis for the ART and CHIC-KIN codes, respectively. Meyer gives some numerical results and compares the results obtained with the different models.

Meyer and Rose²⁷ applied the momentum integral model to a study of oscillations in boiling channels. This work represents one of the few

nonlinear analyses of this type found in the literature. Since the calculations were used for analysis of test-loop experiments, it was not necessary to include equations for fuel-element heat conduction nor any feedback through the reactor kinetics equations. The analysis did permit the treatment of slip flow with continuously varying slip ratio. Pressure effects were not included, and all the calculations of physical properties were made at a constant reference pressure. The results of the calculations compared well with a number of loop experiments which measured the onset of oscillations in parallel channel-type flow experiments.

An example of the analog method of solution of the transient equations for two-phase flow is given by Anderson *et al.*,²⁸ which was developed for the analysis of natural-circulation loop experiments with boiling water. Due to the limitations imposed by the amount of analog equipment available, this treatment incorporates a relatively coarse spatial description. Thus, rather than being a numerical solution of the fundamental partial differential equations in the usual sense, it gives the solution of a series of ordinary differential equations which represent mass and energy balances written over several major segments of the boiling loop. Some comparisons between analog and experimental results are given for steady-state runs, oscillatory power runs, and power for the inception of oscillation experiments. In general, the comparison between experiment and calculation is quite good.

Halfen and Berganzoli²⁹ also used the analog computer approach in studying transients in organic-cooled reactors which would result if the cooling-system pressurization was lost. They used a three-space node model to describe a forced-convection boiling channel. The agreement between calculated results and flow loop experiments was generally good.

Some of the more recent innovations in the analysis of transient two-phase flow are included in the work of Staub *et al.*³⁰ They formulated the conservation equations in terms of the center of mass of the mixture rather than from the Eulerian (coordinate system fixed in channel) point of view as in the previous references. In order to avoid the assumption of phase equilibrium in the two-phase system, separate continuity equations were written for the gas and liquid phases. A void-propagation equation was introduced which gave the response of the void fraction to variations of power density, pressure, energy storage in the liquid and vapor, compressibilities of the liquid and vapor, flow rate, and gravitational force field. Although this formulation is more satisfying from a theoretical point of view, solutions cannot be obtained without information on quantities such as the liquid and vapor "drift velocities" and the source terms in the liquid and vapor continuity equations. Correlations for the vapor drift velocity for two different flow regimes were used to obtain closed-form analytic solutions for a linear form of the void-propagation equation. However, it was necessary to make the usual assumptions of thermodynamic

equilibrium between phases and incompressibility of the vapor phase. The solutions were found to be in satisfactory agreement with experiments on a forced-convection flow loop using Refrigerant-22 in which the response to small oscillations in power and flow were observed.

Jarvis³¹ gave equations which he derived for the cooldown problem, i.e., the cooling of a warm transfer line when the flow of a cryogenic liquid is started. The problem was formulated in terms of separate continuity, momentum, and energy equations for the vapor and liquid phases. These conservation equations, along with an equation describing the wall temperature and an equation of state for the gas, comprise a system of nonlinear partial differential equations. Numerical calculations were made by use of various methods of integrating the equations, and it was found that numerical instabilities resulted when slip was permitted between the two phases. The equations were solved successfully in terms of a homogeneous-flow model, however. The main objective of this work was to predict the magnitude of the first pressure pulse which occurs when the cryogenic fluid is introduced into a warm pipe. Limited numerical results were given for the flow of liquid nitrogen in a 200-ft-long copper transfer line. An interesting feature of these calculations is the method used to simplify the problem of resolution of the phase boundaries between liquid and two-phase mixture as well as between gas and two-phase mixture. This was done by allowing only two-phase flow everywhere; therefore, quality is always between 0 and 1.0, and a small amount of gas is present in the liquid phase as is a small amount of liquid present in the gas phase.

C. Experimental Work

There is essentially no experimental data on the transient boiling of liquid metals. A number of investigators³²⁻³⁵ have reported results of steady-state experiments with boiling sodium and/or potassium in which they have measured heat transfer rates, pressure drops, and vapor void fraction. All experiments performed with sodium have been at subatmospheric pressures in order to minimize the temperatures and concomitant equipment problems. Boiling of sodium at these low pressures turns out to be a somewhat unpredictable process characterized by high liquid superheat³⁶ and unstable flow under certain circumstances. Some investigators feel that this is not necessarily an indication of basic differences between sodium and water boiling, but rather is a reflection of the extremely large ratios of liquid-to-vapor density encountered for either fluid at low pressure. Some of the steady-state data for liquid metal boiling will be discussed in more detail later in connection with the steady-state two-phase flow calculations in Chapter IV.

Simulation experiments designed to investigate the behavior of coolant and fuel in the FERMI fast breeder reactor for flow and power transients were performed by Smith et al.³⁷ in 1961 and later by Huebotter.³⁸ In these

experiments several electrically heated stainless steel or lead fuel pins were placed in a rectangular glass channel and were cooled with flowing water. The basic assumption here was that the water at low pressure would behave like sodium in transient flow decay or power increase tests in which boiling is initiated. Likewise, if melting of the lead pins should occur, the subsequent motion and dispersal of the molten fuel material would be similar to that expected with uranium. The experiments were essentially qualitative in nature and designed to investigate the likelihood of reassembly of portions of a fast reactor core into a critical assembly in a transient severe enough to cause melting of some of the fuel. The data taken consisted of high-speed motion pictures of flow decay and overpower transients of from 5- to 10-sec duration; hence, very little basic hydrodynamics data are available from the experiments.

The transient boiling experiments with water have been primarily directed toward a better understanding of the shutdown effects due to coolant voiding and pressure pulses observed in experiments with the SPERT-I reactor. Johnson et al.³⁹ reported results of a large number of experiments performed by electrically heating very thin ribbons confined in a narrow channel with both stagnant and flowing water. These experiments were carefully instrumented to measure void fraction by means of an X-ray technique, ribbon temperature, and pressure. Overpower transients were run with initial periods of exponential increase as short as 5 ms. The effect of various parameters such as initial degree of subcooling, heat flux, pressure, and coolant flow rate on the rate of void growth were investigated. This work represents probably the only really quantitative data which are available on the transient boiling of any fluid.

Zivi et al.⁴⁰ reported the results of inpile transient boiling experiments performed in the KEWB reactor. In these experiments the heated element was a disc of uranium fuel material which was placed in a capsule with one transparent face through which the formulation of voids could be observed with high-speed movies. Reactor periods ranged between 1 and 5 ms in these tests, designed to obtain data which would be useful in interpreting the self-shutdown of the SPERT-I reactor in the destructive test of 1962.

III. MATHEMATICAL DEVELOPMENT OF PROBLEM

A. Objectives of This Work

In view of the importance of sodium voiding effects in reactor transients, it is desirable to have available analytic techniques which will successfully predict the time-dependent behavior of boiling sodium. As is evident from the foregoing literature review, the effort in this direction has barely begun, with the work of Noyes et al.,²¹ representing the only previous reactor transient analysis which attempts to include the effects of sodium boiling. This work, however, includes some simplifications which it would be desirable to eliminate:

1. the use of a constant reference pressure during the transient, and
2. the use of a homogeneous-flow model, i.e., no slip permitted between liquid vapor phases.

The first assumption is undoubtedly the most limiting in a boiling system slightly above atmospheric pressure, since even relatively small increases in pressure represent a significant change in total pressure. Such changes will greatly influence pressure-dependent properties of the coolant, such as vapor density and saturation temperature. While the adoption of a homogeneous-flow model is computationally expedient, the limited experimental evidence which is available for liquid metals indicates that the slip velocities between phases tend to be higher than for boiling water systems (values greater than 100 have been reported).³² Therefore, the major goals of the present work were to include a space- and time-dependent pressure, as well as a slip-flow ratio in the hydrodynamics calculations for boiling sodium. The types of transients treated cover a wide range, starting with liquid subcooled by hundreds of degrees, going to the point of incipient coolant boiling in the channel, and extending into two-phase flow with high vapor void fractions. For this reason, the differential equations must be used in the nonlinear forms and, accordingly, numerical solutions will be required. The work summarized in the literature review on the transient boiling in pressurized water reactor coolant channels is useful in developing the mathematical formulation of the problem.

A further objective of this work is to try to develop the model and computations so that it will be possible to include them as part of a more complicated calculation which includes the reactor kinetics equations and reactivity feedback.

B. Reactor Channel and Flow Model

In order to keep the computations within manageable limits the analysis was carried out for a reactor having an upper and lower breeding

Fig. 1. Model of Reactor Channel

blanket with no flux and power gradients in the radial direction (infinite slab), and identical coolant channels (no orificing). The heat transfer and fluid-flow-analysis model of Fig. 1, based on a single fuel rod and its associated coolant, was used to study two basic types of transients:

1. mild overpower transients in which the driving function is an exponentially increasing rate of heat generation in the fuel, and
2. coastdown of the coolant flow resulting from an exponentially decreasing available pressure head at the coolant-channel inlet.

These two cases correspond to situations in which there is an inadvertent insertion of reactivity and a failure of the coolant pump, respectively.

An annular-flow model was used in the treatment of the two-phase fluid. As shown in Fig. 2, this model assumes the vapor to be moving in a central core which is surrounded by an annulus of liquid in contact with the walls of the channel. In addition, the following assumptions were made:

1. No subcooled boiling occurs, and vapor is formed only when the local bulk coolant temperature exceeds the local saturation temperature.

2. There is thermodynamic equilibrium between phases--liquid and vapor temperatures are equal to the saturation temperature, and the effects of possible liquid superheat are neglected.

3. The heat transfer in the boiling region is treated in a simple manner wherein the heat transfer coefficient is calculated by assuming the liquid film stays in contact with the channel wall up to a void fraction of 80 or 90%. No "burnout" heat flux limit is imposed as is commonly done for boiling water systems.

4. All heat input to the coolant is assumed to be transferred through the channel wall; the effects of direct heating of the sodium by neutrons and gamma rays are neglected.

112-6431

Fig. 2. Annular-flow Model

C. Equations

The Eulerian equations for conservation of mass, momentum, and energy for a two-phase fluid flowing upward in a vertical channel are:

continuity:

$$\frac{\partial}{\partial t} \sum_{k=1}^2 \alpha_k \rho_k + \frac{\partial}{\partial z} \sum_{k=1}^2 \alpha_k G_k = 0; \quad (1)$$

momentum:

$$\frac{\partial}{\partial t} \sum_{k=1}^2 \alpha_k G_k + \frac{\partial}{\partial z} \sum_{k=1}^2 \alpha_k \frac{G_k^2}{\rho_k} + \frac{\partial p}{\partial z} + g \sum_{k=1}^2 \alpha_k \rho_k + \frac{\partial \tau}{\partial z} = 0; \quad (2)$$

energy:

$$\frac{\partial}{\partial t} \sum_{k=1}^2 \alpha_k \rho_k H_k + \frac{\partial}{\partial z} \left(G \sum_{k=1}^2 x_k H_k \right) = \gamma \phi(z, t), \quad (3)$$

where

α_k = volume fraction of k^{th} component;

ρ_k = density of k^{th} component;

G_k = mass flow rate of k^{th} component;

H_k = enthalpy of k^{th} component;

$x_k = w_k/w_t$ = weight flow fraction of k^{th} component;

p = pressure;

$\frac{\partial \tau}{\partial z}$ = frictional pressure gradient;

$\phi(z, t)$ = heat flux at channel wall;

γ = ratio of channel perimeter to cross-sectional area;

$G = \sum_{k=1}^2 \alpha_k G_k$ = total mass flow rate;

w_k = flow rate of k^{th} component;

w_t = total flow rate;

g = acceleration of gravity.

These equations, derived by Meyer,⁴¹ are based on the assumption that there is no heat conduction in the fluid parallel to the direction of flow, and neglect pressure and temperature gradients transverse to the flow direction. Also, the terms in the complete energy equation which give the effect of system pressure and frictional dissipation losses on energy content are neglected. This is acceptable since they are negligible compared to the large heat input term in Eq. (3).

It is necessary to have an expression which gives the volume fraction of vapor in a two-phase system. Such an equation may be looked upon as an equation of state for the two-phase fluid, analogous to that required for analysis of single-phase compressible flow. A number of expressions giving the void fraction of vapor which have been derived from steady-state experiments with boiling water are available. Some limited results are also available for boiling of sodium at subatmospheric pressures.^{32,33} The experimental vapor void fraction is most frequently correlated by an expression of the form

$$\alpha_g = f_1(X_{tt}), \quad (4)$$

where

$$X_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\rho_g}{\rho_l} \right)^{0.5} \left(\frac{\mu_l}{\mu_g} \right)^{0.1};$$

$\alpha_g = \alpha$ = vapor volume (void) fraction;

α_l = liquid volume fraction;

$x_g = x$ = vapor weight flow fraction (quality);

ρ_g = vapor density;

ρ_l = liquid density;

μ_g = vapor viscosity;

μ_l = liquid viscosity,

and the quantity X_{tt} is the Lockhart-Martinelli two-phase flow parameter.⁹ It should be mentioned that the correlations represented by Eq. (4) include a vapor-liquid slip velocity since the slip ratio is implicit in it (see Appendix A). Equation (4) and the assumption that the channel is completely filled with fluid either liquid or vapor, that is,

$$\alpha_g + \alpha_l = 1.0,$$

fix the fraction of the channel occupied by the liquid.

Similarly, the pressure gradient due to frictional losses in two-phase flow, $\frac{\partial \tau}{\partial z}$ in Eq. (2), is given by correlations derived from steady-state boiling

experiments. Thus the two-phase frictional pressure gradient is usually represented by an equation of the form

$$\frac{\partial \tau}{\partial z} = f_z(X_{tt}). \quad (5)$$

The appropriate equation for transient heat conduction in the fuel rod is

$$k_f \nabla^2 T_f + Q(z, t) = C_f \rho_f \frac{\partial T_f}{\partial t},$$

which is approximated, since the axial conduction in the rods can be neglected relative to the radial conduction, by

$$k_f \left(\frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r} \frac{\partial T_f}{\partial r} \right) + Q(z, t) = C_f \rho_f \frac{\partial T_f}{\partial t}, \quad (6)$$

where

T_f = fuel temperature;

$Q(z, t)$ = fuel volumetric heat generation rate;

k_f = fuel thermal conductivity;

C_f = fuel heat capacity;

ρ_f = fuel density.

The heat flux at the channel wall as a function of position and time is given in the usual way by an equation of the form

$$\phi(z, t) = h(G, T_c) [T_s(z, t) - T_c(z, t)], \quad (7)$$

where

$T_s(z, t)$ = fuel cladding surface temperature;

$T_c(z, t)$ = bulk coolant temperature,

and the heat transfer coefficient, $h(G, T_c)$, is given by an established correlation for the single-phase region and, in general, by a modified correlation in the two-phase region. An additional thermal resistance to allow for the cladding and cladding-to-fuel contact resistance can be combined with $h(G, T_c)$ to obtain an overall coefficient.

D. Boundary Conditions

The boundary conditions, in addition to Eq. (7), required to complete the problem formulation are:

1. temperatures at $t = 0$ in the coolant and fuel;
2. coolant temperature at the channel inlet as a function of time;
3. channel-outlet pressure and pump-outlet pressure as a function of time;
4. the assumption that the radial temperature gradient at the center of the fuel rod is zero, i.e.,

$$\left(\frac{\partial T_f}{\partial r} \right)_{r=0} = 0.$$

In the reactor model used here it is assumed that all the coolant channels behave identically. For this reason, the momentum of the fluid leg in the piping between the pump outlet and the reactor inlet may be important in the overall momentum balance in some cases. If there is no way for flow to bypass the core through adjacent coolant passages, the inertia of this fluid will have an effect on flow changes during transients. Ideally, the characteristics of the complete coolant loop should be considered in attempting to solve a given problem, but since this unduly complicates the problem at this stage, a compromise has been adopted wherein several of the important loop parameters are considered. This will be discussed in more detail later.

It should be noted that the vapor void fraction, frictional pressure gradient, and convection heat transfer coefficient at the channel wall given by Eqs. (4), (5), and (7) respectively, are quantities that have been measured in steady-state rather than transient experiments. The present use of these steady-state quantities in the calculation of transients is also one of the limitations of all the other analyses of this type performed previously.

IV. STEADY-STATE SOLUTIONS

A. Equations and Method of Solution

The work summarized in this chapter was done prior to solving the more difficult transient problems to obtain some feeling for the ranges of parameters involved in boiling sodium systems. The fluid dynamics equations were solved for a channel, shown in Fig. 1, in steady flow, with the sodium entering the channel subcooled, changing to two-phase mixture at some point in the channel and, in some cases, to superheated vapor still further along the channel. Details of the heat transfer at the channel wall were not treated; it was merely assumed that there was a given spatially dependent heat flux input to the channel. Subcooled boiling was not considered; the transition was made from the single-phase liquid equations to the two-phase flow equations when the liquid temperature exceeded the local saturation temperature. Likewise, if the vapor quality exceeded a value of 1.0, a transition was made to the equations for superheated vapor.

For steady flow the continuity equation becomes

$$\frac{\partial}{\partial z} [\alpha G_g + (1 - \alpha) G_\ell] = 0,$$

which states the constancy of mass flow rate with position. The equations for conservation of momentum and energy for steady flow are:

momentum:

$$\frac{\partial p}{\partial z} = -\frac{\partial}{\partial z} \left[\alpha \frac{G_g}{\rho_g} + (1 - \alpha) \frac{G_\ell}{\rho_\ell} \right] - g [\alpha \rho_g + (1 - \alpha) \rho_\ell] - \frac{\partial \tau}{\partial z}; \quad (8)$$

energy:

$$\frac{\partial}{\partial z} \{ G [x H_g + (1 - x) H_\ell] \} = \gamma \phi(z). \quad (9)$$

These equations were integrated directly for the different phase regions of the coolant channel:

Single-phase liquid region ($T_c < T_S$; $x = 0.0$)

momentum:

$$-P \Big|_0^{z_1} = \frac{G^2}{g \rho_\ell(H)} \Big|_0^{z_1} + \int_0^{z_1} \rho_\ell(H) dz' + \frac{G^2}{2gD_h} \int_0^{z_1} \frac{f_{sp}(G, H)}{\rho_\ell(H)} dz'; \quad (10)$$

energy:

$$H_{\ell}(z_1) - H_{\ell}(0) = \frac{\gamma}{G} \int_0^{z_1} \phi(z') dz'. \quad (11)$$

Two-phase region ($T_c = TS$; $0.0 \leq x \leq 1.0$)

momentum:

$$\begin{aligned} -p \Big|_{z_1}^{z_2} &= \frac{1}{g} \left[\alpha \frac{G_g^2}{\rho_g} + (1 - \alpha) \frac{G_{\ell}^2}{\rho_{\ell}} \right]_{z_1}^{z_2} + \int_{z_1}^{z_2} [\alpha \rho_g + (1 - \alpha) \rho_{\ell}] dz' \\ &+ \frac{G^2}{2gD_h \rho_{\ell}} \int_{z_1}^{z_2} f_{sp}(G, H)(1 - x)^{2-a} \phi_{\ell}^2 dz'; \end{aligned} \quad (12)$$

energy:

$$x(z_2) = \frac{1}{\lambda(z_2)} \left[\frac{\gamma}{G} \int_{z_1}^{z_2} \phi(z') dz' - H_{\ell}(z_2) + H_{\ell}(z_1) \right]. \quad (13)$$

Single-phase vapor region ($T_c > TS$; $x > 1.0$)

momentum:

$$-p \Big|_{z_2}^L = \frac{G^2}{g \rho_g(H, p)} \Big|_{z_2}^L + \frac{G^2}{2gD_h} \int_{z_2}^L \frac{f_{sp}(G, H, p)}{\rho_g(H, p)} dz'; \quad (14)$$

energy:

$$H_g(L) - H_g(z_2) = \frac{\gamma}{G} \int_{z_2}^L \phi(z') dz'; \quad (15)$$

where

f_{sp} = single-phase friction factor;

D_h = hydraulic diameter;

C_{ℓ} = liquid heat capacity;

C_g = vapor heat capacity;

z_1 = distance from channel inlet where boiling starts;

z_2 = distance from channel inlet where superheating of vapor starts;

L = channel length;

λ = heat of vaporization;

ϕ_L^2 = two-phase friction factor multiplier;

a = exponent in friction factor correlation (see Appendix A);

TS = saturation temperature;

g = acceleration of gravity.

The vapor void fraction is given by a Lockhart-Martinelli correlation for turbulent flow in both phases:

$$\alpha = 1 - \frac{1}{\sqrt{1 + (21/X_{tt}) + (1/X_{tt}^2)}} \quad (16)$$

Likewise, the Martinelli correlation was used to calculate the friction factor multiplier, ϕ_L^2 , for two-phase flow in Eq. (12):

$$\phi_L^2 = 1 + \frac{21}{X_{tt}} + \frac{1}{X_{tt}^2} \quad (17)$$

Some of the details of the derivations of Eqs. (16) and (17) are given in Appendix A.

An ALGOL program was written to solve the above set of equations by an iterative technique. First, the energy equations were integrated by Simpson's rule to calculate an enthalpy profile for the channel. This profile was used to calculate pressure- and temperature-dependent properties of the coolant. By means of these properties, a new pressure profile was calculated through integration of the momentum equations by Simpson's rule. Iteration continued between the momentum and energy equations until two successively calculated pressure profiles agreed within a specified convergence criterion. In the calculation, a switch was made from the single-phase liquid equations to the two-phase equations whenever the coolant temperature exceeded the local saturation temperature. Similarly, a switch was made from the two-phase equations to the superheated vapor equations whenever the quality exceeded 1.0. In this manner the point where boiling starts in the channel shifted with each iteration until convergence was obtained. The input quantities required by the program are:

1. channel heat flux profile;
2. channel hydraulic diameter and length;

3. coolant inlet temperature and flow rate;
4. channel exit pressure.

All the equations for calculating the physical properties of sodium are included in the program (see Appendix E). In these calculations, as in the subsequent transient calculations, the pressure boundary conditions were applied to points just outside the channel entrance and exit; therefore, the expansion and contraction losses into coolant plenums were not included. The program output quantities are:

1. the location of the interfaces between phases;
2. channel pressure and temperature profiles;
3. void fraction and quality in the two-phase region.

A listing of the ALGOL program is given in Appendix B.

B. Numerical Results

Some results of steady-state calculations for the coolant channel of a reactor having 0.635-cm-diameter rods and 50 v/o coolant are presented

112-5644 Rev. 1

Fig. 3. Vapor Void and Quality Profiles for Various Flow Rates--
 $\phi_{\max} = 50 \text{ cal/cm}^2\text{-sec}$

ing for the flattening of the quality profiles at the end of the channel. The slight increase in vapor quality shown at the end of the channel occurred because there was flashing of liquid due to the steep pressure gradient there.

Pressure profiles for successively decreasing flow rates in a channel having different exit pressures and a maximum heat flux of $100 \text{ cal/cm}^2\text{-sec}$ are shown in Figs. 5, 6, and 7. Of primary interest is

in Figs. 3 through 8. In these problems the axial heat flux distribution was a chopped cosine with a specified maximum heat flux. In Figs. 3 and 4 the quality and void fraction profiles are given for two different peak heat fluxes at various flow rates and a channel exit pressure of 2.0 atm. A salient feature of the curves is the abrupt increase of the coolant void fraction when boiling starts. This agrees qualitatively with the results obtained by various experimenters studying boiling of liquid metals. The channel has blanket sections at the ends with reduced rates of heat generation, account-

112-5645 Rev. 1

Fig. 4. Vapor Void and Quality Profiles for Various Flow Rates--
 $\phi_{\max} = 100 \text{ cal/cm}^2\text{-sec}$

112-5642 Rev. 1

Fig. 5. Pressure Profiles for Various Flow Rates,
 $P_{\text{ex}} = 6.0 \text{ atm}$

112-5641 Rev. 1

Fig. 6. Pressure Profiles for Various Flow Rates,
 $P_{\text{ex}} = 4.0 \text{ atm}$

112-5643 Rev. 1

Fig. 7. Pressure Profiles for Various Flow Rates,
 $P_{\text{ex}} = 2.0 \text{ atm}$

the appearance of sharp changes in slope in the curves for the exit pressure of 2.0 atm (see Fig. 7). This phenomenon has not been studied in detail, but some comparisons of the flow rates for cases in which the abrupt change was obtained have shown rough agreement with theoretical estimates of critical flow rates by Fauske.⁴² The implication is that the curves having changes in slope are describing physical situations for which the flow rate is close to the critical flow rate.

Figure 8 shows the effect of channel heat flux on the total channel pressure drop as a function of flow rate. Pressure drop vs flow rate curves are shown for maximum channel heat fluxes of 50, 100, and 200 cal/cm²-cm. As shown by the curves, the pressure drops at first and then decreases with decreasing flow rate until boiling starts, whereupon

the pressure drop increases abruptly. These curves are in qualitative agreement with the work for boiling water in the simulation studies reported by Huebotter.³⁸

112-6427

Fig. 8. Total Channel Pressure Drop vs Flow Rate for Various Heat Fluxes

C. Experimental Data for Steady-state Boiling

The experimental data available for boiling potassium³³ indicate that the Martinelli correlations [Eqs. (16) and (17)] overestimate both the void fraction and the friction pressure gradient for boiling liquid metal systems. On the other hand, the limited data available from subatmospheric boiling experiments with sodium agree well with the Martinelli prediction for friction pressure gradient.³² A comparison of values of the vapor void fraction predicted by the Lockhart-Martinelli correlation with experimental

V. TRANSIENT SOLUTIONS FOR CHANNELS CONTAINING ONLY TWO-PHASE MIXTURE

A second phase of the preliminary work found necessary in approaching the solution of the complete problem is presented in this chapter. Numerical instabilities were encountered in the first attempts to solve the complete transient problem wherein coolant enters the channel as liquid and makes a transition to a two-phase mixture at some point in the channel. One possible cause for the difficulty was the method used for integrating across the discontinuity at the single-phase, two-phase boundary. Another possible source of the trouble appeared to lie in the two-phase difference equations themselves. In order to eliminate the first possibility and test the two-phase equations, a somewhat artificial two-phase transient model was considered. In this problem the coolant enters the channel as a two-phase mixture, increasing in quality and void as the flow progresses up the channel. As in the work of the previous chapter, the heat transfer at the channel wall was not treated in detail, and the time- and space-dependent heat flux, $\phi(z,t)$, was specified. Since their sole function was to develop stable two-phase difference equations and a convergent iteration scheme, the solutions developed for this model are not of great practical interest.

A. Derivation of Equations

The conservation equations for the two-phase mixture are:

continuity:

$$\frac{\partial}{\partial t} [\alpha \rho_g + (1 - \alpha) \rho_\ell] + \frac{\partial G}{\partial z} = 0; \quad (18)$$

momentum:

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left[\alpha \frac{G_g^2}{\rho_g} + (1 - \alpha) \frac{G_\ell^2}{\rho_\ell} \right] + \frac{\partial p}{\partial z} + g[\alpha \rho_g + (1 - \alpha) \rho_\ell] + \frac{\partial \tau}{\partial z} = 0; \quad (19)$$

energy:

$$\frac{\partial}{\partial t} [\alpha \rho_g H_g + (1 - \alpha) \rho_\ell H_\ell] + \frac{\partial}{\partial z} \{G[x H_g + (1 - x) H_\ell]\} = \gamma \phi(z,t), \quad (20)$$

where the more compact notation of Eqs. (1), (2), and (3) has been replaced by the specific terms for the liquid and vapor phases. The continuity and energy equations [Eqs. (18) and (20)] are put in a more useful form for

numerical solution by the following manipulations. Performing the differentiation indicated in the first term of Eq. (18) yields

$$(\rho_g - \rho_l) \frac{\partial \alpha}{\partial t} + \frac{\partial G}{\partial z} = 0, \quad (21)$$

if the terms $\alpha \frac{\partial \rho_g}{\partial t}$ and $(1 - \alpha) \frac{\partial \rho_l}{\partial t}$ are neglected. This is permissible since these terms are small with respect to $(\rho_g - \rho_l) \frac{\partial \alpha}{\partial t}$ in a boiling sodium system. Similarly, the energy equation can be written as

$$(\rho_g H_g - \rho_l H_l) \frac{\partial \alpha}{\partial t} + \frac{\partial G}{\partial z} (x\lambda + H_l) + G\lambda \frac{\partial x}{\partial z} = \gamma \phi(z, t), \quad (22)$$

if it is assumed that the terms

$$\alpha \frac{\partial}{\partial t} (\rho_g H_g) \text{ and } (1 - \alpha) \frac{\partial}{\partial t} (\rho_l H_l)$$

can be neglected--also a reasonable assumption for a boiling sodium system. Combination of Eqs. (21) and (22) gives

$$[\lambda \rho_g + \lambda x \rho_l - \lambda x \rho_g] \frac{\partial \alpha}{\partial t} + G\lambda \frac{\partial x}{\partial z} = \gamma \phi(z, t).$$

Since at low pressure $\rho_g \ll \rho_l$, this equation can be approximated as

$$(\rho_g + \rho_l x) \frac{\partial \alpha}{\partial t} + G \frac{\partial x}{\partial z} = \frac{\gamma \phi(z, t)}{\lambda}. \quad (23)$$

The momentum equation is rewritten in the following way. The second term, representing the momentum pressure gradient, can be written in the terms of the total flow rate as

$$\left(\frac{\partial p}{\partial z} \right)_{\text{mom}} = \frac{\partial}{\partial z} \left[\alpha \frac{G_g^2}{\rho_g} + (1 - \alpha) \frac{G_l^2}{\rho_l} \right] = \frac{\partial}{\partial z} \frac{G^2}{\rho_l} \left\{ \left[\frac{\beta x^2}{\alpha} + \frac{(1 - x)^2}{1 - \alpha} \right] \right\}$$

where

$$\beta = \rho_l / \rho_g.$$

Also, the static head term can be written in terms of β as

$$\left(\frac{\partial p}{\partial z} \right)_{\text{stat}} = g[\alpha \rho_g + (1 - \alpha) \rho_l] = g \rho_l [(\alpha / \beta) - \alpha + 1].$$

Further, the friction gradient is

$$\left(\frac{\partial p}{\partial z}\right)_{fr} = \frac{\partial \tau}{\partial z} = \frac{f_{sp} G^2}{2 \rho_\ell D_h} (1-x)^{2-a} \phi_\ell^2.$$

Thus, the momentum equation becomes

$$\begin{aligned} \frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left\{ \frac{G^2}{\rho_\ell} \left[\frac{\beta x}{\alpha} + \frac{(1-x)^2}{1-\alpha} \right] \right\} + g \rho_\ell [(\alpha/\beta) - \alpha + 1] \\ + \frac{f_{sp} G^2}{2 \rho_\ell D_h} (1-x)^{2-a} \phi_\ell^2 + \frac{\partial p}{\partial z} = 0. \end{aligned}$$

This form of the momentum equation is identical to that used earlier for the steady-state solutions with the exception that the time-dependent term $\frac{\partial G}{\partial t}$ has been added.

The most rigorous solution of Eqs. (21), (23), and (24) would be given by the "sectionalized compressible model" as described by Meyer.²⁶ However, a numerical solution in terms of this model requires a knowledge of the maximum signal, or sonic, velocity in the fluid. Since the sonic velocity in two-phase sodium cannot be accurately predicted, sonic effects were omitted and the momentum equation was used in the so-called "momentum integral" form. In this method the momentum equation is used in a space-wise integrated form. Hence, integration of Eq. (24) over the length of the channel gives

$$\begin{aligned} \int_0^L \frac{\partial G}{\partial t} dz + \frac{G^2}{\rho_\ell} v' \Big|_0^L + g \int_0^L \rho_\ell [(\alpha/\beta) - \alpha + 1] dz \\ + \int_0^L \frac{f_{sp} G^2 (1-x)^{2-a}}{2 \rho_\ell D_h} \phi_\ell^2 dz + p \Big|_{P_0}^{P_{ex}} = 0, \end{aligned} \quad (25)$$

where,

$$v' = \frac{\beta x^2}{\alpha} + \frac{(1-x)^2}{1-\alpha}.$$

In this form the transfer of momentum across the control volume boundaries (the ends of the reactor channels) is treated explicitly while the details of momentum transfer within the channel are suppressed. As Meyer stated, this model is analogous to the use of a rigid body in mechanics problems, since pressure and velocity disturbances are propagated instantaneously and an increase in the applied pressure drop across the channel tends to accelerate fluid at all positions.

In contrast to the work of Meyer and others, however, the fluid properties used in evaluating the various integrals in Eq. (25) are pressure dependent in this work, rather than being evaluated at a constant reference pressure. Equation (25) is put in a form convenient for computations:

$$\frac{\partial \hat{G}(t)}{\partial t} = \frac{1}{L} [\Delta p(t) - F], \quad (25a)$$

where

$$\hat{G}(t) = \frac{1}{L} \int_0^L G(z, t) dz;$$

$$\Delta p(t) = p_0(t) - p_{ex};$$

$$F = \frac{G^2}{\rho_\ell} v' \bigg|_0^L + g \int_0^L \rho_\ell (\alpha/\beta - \alpha + 1) dz + \int_0^L \frac{f_{sp} G^2 (1-x)^{2-a} \phi_\ell^2}{2 \rho_\ell D_h} dz.$$

Thus, the time rate of change of average channel velocity, $\hat{G}(t)$, is given as the difference between the imposed channel pressure drop, $\Delta p(t)$, and the three spatial integrals given by F . In considering the inertial effects of the fluid leg in the coolant pipe between the pump and channel inlet, it can be shown that L , the channel length, is replaced by $L_p A_r + L$, and Eq. (25a) becomes

$$\frac{\partial \hat{G}(t)}{\partial t} = \frac{1}{L_p A_r + L} [\Delta p(t) - F], \quad (26)$$

where

L_p = length of liquid leg between pump and channel;

A_r = total flow area of channels/pipe flow area.

The assumption made in deriving Eq. (26) is that only the inertial pressure drop is important in the pipe and that the friction, static, and momentum pressure drop terms can be neglected.

B. Finite Difference Forms of the Equations

Equations (21), (23), and (26) are in forms in which they can be solved numerically. To accomplish this, the channel is divided into n equal segments, as shown in Fig. 1, and the differential equations are approximated by finite difference forms. The finite difference form of the continuity equation [Eq. (18)] uses backward differences to represent both space and time derivatives, thus

$$\frac{\bar{\rho}_j^{i+1} - \bar{\rho}_j^i}{\Delta t} + \frac{G_j^{i+1} - G_j^{i+1}}{\Delta z} = 0, \quad (27)$$

where terms of order Δt and Δz and higher are neglected, and

$$\bar{\rho}_j^i = \rho_j^i \left[(\alpha_j^i / \beta_j^i) - \alpha_j^i + 1 \right]; *$$

i = time node;

j = space node.

The energy equation [Eq. (23)] is represented by means of an implicit characteristic method wherein the space derivatives and dependent variables are averaged over the time step, and the fluid properties are the values calculated at the previous time step; thus,

$$\left[\rho_j^i \left\{ (1/\beta_j^i) + x_j^i \right\} + \rho_j^i \left\{ (1/\beta_j^i) + x_j^{i+1} \right\} \right] \frac{\alpha_j^{i+1} - \alpha_j^i}{\Delta t} + \left[G_j^i \frac{x_j^i - x_{j-1}^i}{\Delta z} + G_j^{i+1} \frac{x_j^{i+1} - x_{j-1}^{i+1}}{\Delta z} \right] = \gamma \frac{\phi_j^{i+1} + \phi_{j-1}^{i+1}}{\lambda_j^i}, \quad (27a)$$

where terms of order $(\Delta t)^2$ and Δz and higher are neglected. This equation on rearranging gives

$$\alpha_j^{i+1} = \alpha_j^i + \left\{ \frac{\gamma \Delta t (\phi_j^{i+1} + \phi_{j-1}^{i+1})}{\gamma_j^i} - \frac{\Delta t}{\Delta z} \left[G_j^i (x_j^i - x_{j-1}^i) + G_j^{i+1} (x_j^{i+1} - x_{j-1}^{i+1}) \right] \right\} / (\Gamma_j^{i+1} + \Gamma_j^i), \quad (28)$$

where terms of order $(\Delta t)^3$ and $\Delta z \Delta t$ and higher are neglected and

$$\Gamma_j^i = \rho_j^i \left[(1/\beta_j^i) + x_j^i \right];$$

$$\Gamma_j^{i+1} = \rho_j^i \left[(1/\beta_j^i) + x_j^{i+1} \right].$$

*Hereafter in the difference forms, ρ refers to the liquid density, vapor density is given by ρ/β .

Since Eqs. (27) and (28) evolved from a large amount of experimenting with various differencing schemes to find stable equations, the details of these trials have been omitted. It seemed evident that implicit schemes like those represented by Eqs. (27) and (28) would be more successful in view of the large changes, particularly in flow rates and void fraction, when boiling starts. Consequently, almost all the effort was directed toward implicit schemes, with the work of Birkhoff and Kimes²³ as a guide. The integral momentum equation [Eq. (26)] was evaluated by a simple trapezoidal integration scheme and thus posed no direct stability problems. This integration scheme, which neglects terms of order $(\Delta z)^2$ and higher, is

$$\frac{\hat{G}_j^{i+1} - \hat{G}_j^i}{\Delta t} = \frac{1}{L_p A_r + L} [P_0^i - P_n - F^i], \quad (28a)$$

where

$$\begin{aligned} F^i = & \left(\frac{G^2}{\rho} v' \right)_n^i - \left(\frac{G^2}{\rho} v' \right)_0^i + \frac{g}{2} \sum_{j=1}^n \left[\rho_j^i \left\{ (\alpha_j^i / \beta_j^i) - \alpha_j^i + 1 \right\} \right. \\ & \left. + \rho_{j-1}^i \left\{ (\alpha_{j-1}^i / \beta_{j-1}^i) - \alpha_{j-1}^i + 1 \right\} \right] + \frac{1}{2D_h} \sum_{j=1}^n \frac{f_j^i G_j^i (1 - x_j^i)^{2-a} (\phi_\ell^2)_j^i}{\rho_j^i} \\ & + \frac{f_{j-1}^i G_{j-1}^i (1 - x_{j-1}^i)^{2-a} (\phi_\ell^2)_{j-1}^i}{\rho_{j-1}^i}, \end{aligned}$$

P_n = channel exit pressure, constant in time;

P_0^i = channel inlet pressure, specified in time, by input data.

To develop a practical scheme of solution, the continuity equation [Eq. (27)] was rearranged in a way suggested by Meyer:²⁷

$$G_j^{i+1} = G_{j-1}^{i+1} + \frac{\Delta z}{\Delta t} (\bar{\rho}_j^i - \bar{\rho}_j^{i+1}) = G_{j-1}^{i+1} + \sigma_j^{i+1}, \quad (29)$$

where terms of order $\Delta t \Delta z$, and $(\Delta z)^2$, and higher have been neglected, and

$$\sigma_j^{i+1} = \frac{\Delta z}{\Delta t} (\bar{\rho}_j^i - \bar{\rho}_j^{i+1}).$$

It is then possible to write

$$G_j^{i+1} = G_0^{i+1} + \sum_{\ell=1}^j \sigma_\ell^{i+1} = G_0^{i+1} + \eta_j^{i+1}, \quad (30)$$

where terms of order Δz , Δt , and higher have been neglected, and

$$\eta_j^{i+1} = \sum_{\ell=1}^j \sigma_{\ell}^{i+1}$$

Then the average mass flow rate in a channel is given by

$$\hat{G}^{i+1} = \frac{1}{2L} \sum_{j=1}^n \left(G_j^{i+1} + G_{j-1}^{i+1} \right) \Delta z. \quad (31)$$

Substitution of Eq. (30) into Eq. (31) gives

$$\hat{G}^{i+1} = \frac{1}{2L} \sum_{j=1}^n \left(G_0^{i+1} + \eta_j^{i+1} + G_0^{i+1} + \eta_{j-1}^{i+1} \right) \Delta z,$$

which on rearranging yields

$$G_0^{i+1} = \hat{G}^{i+1} - \frac{1}{2L} \sum_{j=1}^n \left(\eta_j^{i+1} + \eta_{j-1}^{i+1} \right) \Delta z. \quad (32)$$

Thus Eq. (32) gives G_0^{i+1} directly in terms of the known $\hat{G}^{i+1}$ and σ_j^{i+1} 's. Equation (30) can then be used to calculate all the remaining G_j^{i+1} 's. The steps in the iterative numerical solution involving Eqs. (26), (27), (28), (30), and (32) are:

1. By use of values of temperature- and pressure-dependent fluid properties from the previous time step, Eq. (26) is used to calculate $\partial \hat{G} / \partial t$ and thence obtain G^{i+1} .

2. Estimates for the α_j^{i+1} 's are first obtained by use of Eq. (28) and the assumption that

$$\hat{G}^{i+1} = G_j^{i+1}, \quad j = 0, 1, 2, \dots, n$$

and

$$x_j^{i+1} = x_j^i, \quad j = 0, 1, 2, \dots, n.$$

3. The explicit expressions Eqs. (32) and (30) are used to calculate G_0^{i+1} and the new G_j^{i+1} 's, respectively.

4. The Martinelli void correlation is used, inverted to be explicit in terms of x instead of α (see Appendix A), to evaluate the new x_j^{i+1} 's from the α_j^{i+1} 's and fluid physical properties evaluated at i .
5. The cycle of steps from 2 is repeated on until the α_j^{i+1} , x_j^{i+1} , and G_j^{i+1} 's agree between two successive iterations within a specified convergence criterion (see Appendix D).
6. The new pressure profile at $i + 1$ is then calculated by Eq. (28a), and then the cycle of steps starting with 1 is repeated for the next time step.

Initial conditions (void fraction and quality profiles) are obtained by trapezoidal integration of the time-independent forms of the momentum and energy equations as in the steady-state calculations described in the previous chapter. Vapor quality, x , of the two-phase fluid entering the channel is given as an input parameter to the calculation. The ALGOL program which performs this series of computations is listed, along with a block diagram, in Appendix C.

C. Numerical Results

Some results of calculations for transients having exponentially changing power inputs with periods, ω , of 0.10, 0.05 and -0.10 sec are given in Figs. 10 through 21. The parameters for this series of calculations are summarized in Table I. Because the purpose of this series of calculations

Table I. Parameters for Two-phase Transient Problems

Channel length, L	140 cm
Extrapolated boundary, R_2^*	20 cm
Channel equivalent diameter, D_h	0.635 cm
Maximum heat flux at time zero, ϕ_0	10.0 cal/cm ² -sec
Inlet quality, x_0	0.001
Inlet vapor void fraction, α_0	0.245
Exit pressure, P_{ex}	3.0 atm
Initial mass velocity, G_0	200 gm/cm ² -sec
Pump decay constant, λ_p	0.0
Length of pump liquid leg, L_p	1500.0 cm
Flow area ratio, A_r	1.0
Time step, Δt	0.001 sec
Space step, Δz	5.0 cm

*Determines chopped-cosine heat-flux distribution.

was mainly to check the convergence of the foregoing equations and iteration scheme, the results are of less practical interest than those in the following chapter. As stated earlier, the heat flux at the channel wall was specified in space and time, and it is the exponential increase or decrease of the heat flux in time which acts as the driving function for the transient. The heat-flux profile at time zero (and for all subsequent times) was of chopped-cosine form corresponding to the extrapolated boundary, R_2 , given in Table I; the peak value at time zero was chosen to give a reasonable exit void fraction and quality with the specified inlet conditions.

The option in the calculation which considers the momentum of the liquid leg between the pump and the coolant channel inlet [Eq. (26)] was used in all these problems. This insured that the maximum pressure in the channel would occur at the channel inlet and, thus, was a better representation of the case treated in the next chapter wherein the coolant boiling interface and point of maximum pressure always coincide, with the two-phase mixture tending to be forced through the channel by the inertia of the liquid behind it. The pump pressure head was held constant, and the decreases in flow were forced only by the increasing heat input to the channel.

Curves giving the profiles of mass flow rate at different times for the two exponentially increasing transients (see Figs. 10 and 11) show a sharp drop in inlet flow and an increase in exit flow as the power is increased. The channel-averaged mass flow rate decreased from 200 to 107.9 gm/cm²-sec in the transient shown in Fig. 10 and from 200 to 166.4 gm/cm²-sec for the transient shown in Fig. 11, until the calculations were terminated because the inlet flow rate dropped to zero. Figure 12 gives the mass velocity profiles for the decreasing transient and, in contrast to the previous two figures, the channel inlet flow rate increases while the exit decreases initially, reaches a minimum, and then starts increasing. During the time represented in Fig. 12 the channel-averaged mass flow rate increased from 200 to 323.2 gm/cm²-sec.

Corresponding profiles for the vapor void fraction for the transients are shown in Figs. 13, 14, and 15. In the increasing transients the curves show a steadily increasing slope at the inlet while they remain parallel at the outlet. The curves for the decreasing transient shown in Fig. 15 start at the same initial condition and become flatter, finally becoming almost linear.

The quality profiles at different times for the three transients are given in Figs. 16, 17, and 18. Quality x is analogous to temperature in the single-phase transient calculation in that the slope of the profile increases at the channel inlet and decreases at the outlet as the transient progresses. As shown in Fig. 18, the curves for the decreasing transient decreases in slope during the transient and eventually become almost linear.

112-6391

Fig. 10. Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.100$ sec

112-6429

Fig. 11. Mass Flow Rate vs Channel Position at Various Times for $\omega = 0.050$ sec

112-6389

Fig. 12. Mass Flow Rate vs Channel Position at Various Times for $\omega = -0.100$ sec

112-6393

Fig. 13. Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.100$ sec

112-6413

Fig. 14. Vapor Void Fraction vs Channel Position at Various Times for $\omega = 0.050$ sec

112-6430

Fig. 15. Vapor Void Fraction vs Channel Position at Various Times for $\omega = -0.100$ sec

112-6392

Fig. 16. Vapor Quality vs Channel Position at Various Times for $\omega = 0.100$ sec

112-6396

Fig. 17. Vapor Quality vs Channel Position at Various Times for $\omega = 0.050$ sec

112-6395

Fig. 18. Vapor Quality vs Channel Position at Various Times for $\omega = -0.100$ sec

Pressure profiles during the three transients are given in Figs. 19, 20, and 21. As expected, the pressure drop increases with time for the two increasing transients and decreases with the decreasing one. As shown in Fig. 21 the pressure profile is almost linear at the time when the decreasing transient calculation was terminated.

The convergence of the system of two-phase equations was investigated in more detail for the increasing transient with a period of 0.10 sec. Four solutions were obtained to this problem for a constant value of the ratio $\Delta z/\Delta t = 5000$ cm/sec, but with time steps of 0.0005, 0.001, 0.002, and 0.004 sec, respectively. Some calculated values of the variables G_j^i , α_j^i , x_j^i , and p_j^i are plotted against the time-step size in Figs. 22 through 25. In Fig. 22 the values of G_j^i are given at the channel inlet, midpoint, and outlet. Figures 23 and 24 give values of α_j^i and x_j^i at the channel midpoint and exit; values of p_j^i at the channel inlet and midpoint are shown in Fig. 25. As shown by the curves in this series of figures, the convergence of the four variables is approximately linear as would be expected with the differencing schemes used.

112-6390

Fig. 19. Pressure Profiles at Various Times for $\omega = 0.100$ sec

112-6388

Fig. 20. Pressure Profiles at Various Times for $\omega = 0.050$ sec

112-6394

Fig. 21. Pressure Profiles at Various Times for $\omega = -0.100$ sec

112-6403

Fig. 22. Calculated Values of G_j^i vs Time-step Size

112-6421

Fig. 23. Calculated Values of α_j^i vs Time-step Size

112-6419

Fig. 24. Calculated Values of x_j^i vs Time-step Size.

112-6420

Fig. 25. Calculated Values of P_j^i vs Time-step Size

The numerical stability of the computational scheme was studied by a series of calculations in which the power was held constant. Initial conditions were calculated as in the previous problems, and the time-dependent portion of the problem consisted of recalculating the initial conditions repeatedly. In the cases for which numerical instability was present, the result would usually diverge within 10 or 15 time steps. Difficulties would first become evident by the appearance of an alternating gradient in the mass flow rate profiles, followed by failure of the iterations to converge. In the stable cases, typically, there was a slight perturbation in going from the initial conditions to the time-dependent calculation, and very small variations in the mass flow rate with position were calculated for the first several time steps. These variations became less and less as the number of steps increased and, finally, steadied out so that a constant mass flow rate with position was obtained. Further calculations then produced exactly the same values from one step to the next. The stability calculations were allowed to run for 2 min on the CDC-3600 computer and, accordingly, the total number of time steps calculated was a function of the size of the time step in a particular problem. In all but the problems with the very smallest space and time steps, however, the number of time steps calculated was more than 200. The results of the stability calculations are summarized in Table II.

Table II. Summary of Results of Stability Studies
with Two-phase Difference Equations

Δt (sec)	Δz (cm)	$\Delta z/\Delta t$ (cm/sec)	$\left(G_j^i/\bar{\rho}_j^i\right)_{\max}$ (cm/sec)	Result
0.0002	1	5,000	1480	stable
0.0005	↓	2,000	↓	unstable
0.001	↓	1,000	↓	unstable
0.0002	5	25,000	↓	stable
0.0005	↓	10,000	↓	stable
0.001	↓	5,000	↓	stable
0.002	↓	2,500	↓	stable
0.004	↓	1,250	↓	unstable
0.001	10	10,000	↓	stable
0.002	↓	5,000	↓	stable
0.004	↓	2,500	↓	stable
0.008	↓	1,250	↓	unstable
0.002	20	10,000	↓	stable
0.004	↓	5,000	↓	stable
0.008	↓	2,500	↓	stable
0.010	↓	2,000	↓	stable

From these results the following approximate stability criterion was determined:

$$\Delta t < \Delta z \left(\frac{\frac{\bar{\rho}_j^i}{i}}{G_j} \right)_{\max} \quad (33)$$

where $(G_j^i / \bar{\rho}_j^i)_{\max}$ is the maximum velocity in the channel. Actually, this is very close to the velocity of the liquid phase, since the vapor phase contributes little to the total mass flow rate because of the low quality. The physical meaning of Eq. (33) is that the time step must be small enough so that a fluid particle moves less than one space step per time step.

In all cases for which the local liquid temperature was allowed to be a function of the pressure, difficulties were encountered with numerical instability. However, it was possible to obtain the satisfactory solutions reported by evaluating the liquid properties at a constant temperature corresponding to the saturation temperature at the channel outlet pressure. This means that effects such as liquid flashing due to a decrease in enthalpy because of the pressure gradient in the channel (see Chapter IV, B) are suppressed.

The running time of these problems was 2 min or less on the CDC-3600. The convergence of the inner iterations was usually 10^{-4} or better within two iterations in the stable problems.

VI. TRANSIENT SOLUTIONS FOR SINGLE-PHASE, TWO-PHASE CHANNELS WITH HEAT TRANSFER FROM FUEL

In the previous chapter, a satisfactory numerical method for solving the coupled continuity, momentum, and energy equations for two-phase flow was developed. The additional complication which must now be introduced to obtain the complete solution of the problem arises from defining the motion of the interface between the single-phase and two-phase portions of the boiling channel. The approach chosen here was to follow carefully the motion of the interface during the computations, then use a straightforward method of integration across the discontinuity. Addition of the fuel heat-conduction equations to the calculation causes no changes in the basic approach, but means only that the heat flux at the channel wall is specified in a different way.

A. Single-phase Convection Equation

The convection equation for the one-dimensional time-dependent flow of a fluid is

$$\rho \frac{\partial H}{\partial t} + G \frac{\partial H}{\partial z} = \gamma \phi(z, t), \quad (34)$$

where, in general, the liquid mass flow rate G and density ρ are both functions of z and t . In solving problems in which the coolant is greatly subcooled initially, fairly long periods (tens of seconds) of real time may be required in the single-phase part of the calculation before boiling starts. Therefore, a stable numerical representation for Eq. (34) was desired so that the required time steps in this region would not be inordinately small. After a good deal of experimentation with other methods, the so-called "box method" representation²³ was chosen,

$$\begin{aligned} \rho_j^i \frac{H_j^{i+1} - H_j^i}{\Delta t^i} + \rho_{j-1}^i \frac{H_{j-1}^{i+1} - H_{j-1}^i}{\Delta t^i} + G^{i+1} \frac{H_j^{i+1} - H_{j-1}^{i+1}}{\Delta z} \\ + G^i \frac{H_j^i - H_{j-1}^i}{\Delta z} = \gamma(\phi_j^i + \phi_{j-1}^i), \end{aligned} \quad (35)$$

where the density ρ_j^i is evaluated at the previous time step and the heat flux ϕ_j^i is given by

$$\phi_j^i = h_j^i(TC_j^i - T_j^i), \quad (36)$$

and

TC_j^i = cladding surface temperature;

T_j^i = bulk coolant temperature;

h_j^i = convection heat transfer coefficient.

In the single-phase region, the mass flow rate is independent of position, i.e.,

$$\frac{\partial G}{\partial z} = 0;$$

therefore,

$$\hat{G}^{i+1} = G_j^{i+1}; \quad j = 0, 1, 2 \dots n.$$

The value of $\hat{G}^{i+1}$ is obtained from the derivative at i :

$$\hat{G}^{i+1} = \hat{G}^i + \left[\frac{\partial \hat{G}}{\partial t} \right]_i \Delta t^i, \quad (37)$$

where Δt^i is the time step at i and $\left[\frac{\partial \hat{G}}{\partial t} \right]_i$ is obtained from Eq. (28a) where the simpler functions for the single-phase static head, momentum, and friction pressure drops are substituted for the two-phase quantities. Equation (35) can then be rearranged to give an explicit formula for H_j^{i+1} since the G_j^{i+1} 's are determined for all j in the single-phase region by Eq. (37); thus,

$$\begin{aligned} H_j^{i+1} = & [(\rho_j^i - U^i) H_j^i + (\rho_{j-1}^i + U^i) H_{j-1}^i \\ & - (\rho_{j-1}^i - U^{i+1}) H_{j-1}^{i+1} + \Delta t^i \gamma (\phi_j^i + \phi_{j-1}^i)] / (\rho_j^i + U^{i+1}), \end{aligned} \quad (38)$$

where

$$U^{i+1} = G^{i+1} \Delta t^i / \Delta z;$$

$$U^i = G^i \Delta t^i / \Delta z.$$

B. Fuel Heat Conduction Equations

The method used for calculating transient temperatures in the fuel rods is basically the same as that used in the ARGUS¹⁶ code (see Appendix D,2). The present calculations, however, are designed to handle only a

single heated zone (fuel pin) and its associated coolant--in contrast to the large number of options with regard to geometry permitted in ARGUS. The partial differential equation describing one-dimensional (axial conduction is neglected) time-dependent conduction in the fuel rods is

$$k_f \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + Q(z, t) = \rho_f C_f \frac{\partial T}{\partial t}, \quad (39)$$

where

k_f = fuel thermal conductivity;

ρ_f = fuel density;

C_f = fuel heat capacity;

$Q(z, t)$ = fuel volumetric-heat-generation rate;

r = fuel-rod radius.

The derivation of the finite-difference forms which follow is given by Dusenberre⁴³ and will not be repeated here. They are obtained by writing heat balances over the fuel segments indicated in Fig. 26. Since all the equations are written in terms of one radian of arc, the factor 2π is omitted. For the central fuel node the difference form is

$$T_0^{i+1} = \left(1 - \frac{4}{M} \right) T_0^i + \frac{4}{M} T_1^i + \frac{Q_j^{i+1} + Q_j^i}{2} \frac{\Delta t^i}{\rho_f C_f}, \quad \text{all } j, \quad (40)$$

where

$$M = \frac{\bar{\Delta r}^2 \rho_f C_f}{k_f \Delta t^i}.$$

Fig. 26

Mesh Layout for Fuel Heat Conduction Equations

The equation for the general intermediate point is

$$T_m^{i+1} = \frac{1}{M} \frac{(2m-1)}{2m} T_{m-1}^i + \left(1 - \frac{2}{M}\right) T_m^i + \frac{1}{M} \frac{2m+1}{2m} T_{m+1}^i + \frac{Q_j^{i+1} + Q_j^i}{2} \frac{\Delta t^i}{\rho_f C_f}, \quad \text{all } j. \quad (41)$$

In the present treatment, the calculations were simplified by omitting the details of the temperature distribution in the thin fuel cladding since it is not of primary interest. Accordingly, the thermal resistance of the cladding was lumped with the fuel-to-clad thermal resistance and specified as an input parameter. This, in turn, was combined with the calculated convection film coefficient at each point of the clad surface to obtain an overall coefficient [see Eq. (43)] from the fuel surface to the coolant. This approximation neglects the heat stored in the cladding and is acceptable for the type of fuel elements under consideration. In practice it was found that the fuel-to-clad thermal resistance was controlling. This means that the precise value of the convection film coefficient, particularly in the boiling zone, is not of great importance.

The equation for the segment at the surface of the fuel rod was derived using a modification developed by Back (see Ref. 44 for details of derivation). This method has the advantage that it is stable for any value of the film coefficient h , or the cladding and contact resistance R_c . The only requirement for stability of the set of equations is that the modulus M , is large enough. The surface segment equation is

$$T_m^{i+1} = C_1(T_{m-1}^{i+1} + T_{m-1}^i) + C_2 T_m^i + C_3(H_j^{i+1} + H_j^i) + C_4(Q_j^{i+1} + Q_j^i), \quad \text{all } j, \quad (42)$$

where

$$C_1 = \frac{4m-2}{4m-2+4mN+4mM-M};$$

$$C_2 = \frac{2-4m-4mN+4mM-M}{4m-2+4mN+4mM-M};$$

$$C_3 = \frac{4mN}{4m-2+4mN+4mM-M} \frac{1}{C_\ell};$$

$$C_4 = \frac{M(4m-1)}{4m-2+4mN+4mM-M} \frac{\Delta t^i}{2\rho_f C_f};$$

$$m = m_{\max} = n;$$

$$N = \frac{he_j^i \Delta r}{k_f};$$

and he_j^i is the equivalent overall heat transfer coefficient, that is,

$$he_j^i = \frac{1}{(1/h_j^i) + R_c} = \frac{h_j^i}{1 + R_c h_j^i}. \quad (43)$$

The convection film coefficient h is calculated by an expression of the type

$$Nu = K_1(Pe)^{K_2} + K_3, \quad (44)$$

where

$$Nu = \text{Nusselt number} = hD_h/k; \quad Pe = \text{Peclet number} = D_h G/C_\ell,$$

which is the form of correlation used for calculation of convection heat transfer coefficients for fluids of low Prandtl number (such as liquid metals). Equations (40), (41), and (42) comprise a set of explicit expressions for fuel temperatures at the next time step in terms of known values at the previous time and the time-dependent rate of heat generation. The criterion for the numerical stability of the equations is $M \geq 4$.⁴³

The initial steady-state temperatures were determined by numerical integration of the time-independent form of Eq. (39). The equation for the central node point is

$$T_0^0 = T_1^0 + Q_j^0 \frac{\bar{\Delta r}^2}{4k_f}, \quad \text{all } j, \quad (45)$$

and for the remaining fuel node points the finite-difference form is

$$T_{m-1}^0 = T_m^0 + \frac{(2m-1) Q_j^0 \bar{\Delta r}^2}{4k_f}, \quad \text{all } j. \quad (46)$$

The derivations of Eqs. (45) and (46) are given in Ref. 43. The sequence of steps in the calculation of the fuel temperature is as follows:

1. The initial fuel temperatures are calculated from the initial coolant profile, the calculated overall heat transfer coefficient, and Eqs. (45) and (46).
2. The values for the constants in the time-dependent equations [Eqs. (40), (41), and (42)] are calculated from the current value for the time step, Δt^i , and the values of physical properties at i .

3. The new coolant temperature profile at $i + 1$ is calculated with Eq. (38) (in subcooled region) using the heat flux at i .
4. Equation (40) is used to calculate the central region fuel temperatures at $i + 1$.
5. Equations (41) and (42) are used to calculate fuel temperatures at the center and surface of the fuel rod, respectively.
6. Steps 1 through 5 are repeated for the next forward step in time, etc.

When coolant boiling starts, the method of calculating fuel temperatures is unchanged.

C. The Complete Calculation

The position of the single-phase, two-phase interface is followed carefully after boiling starts. Each time a new coolant temperature is

$$EB = \Delta Z \frac{TS(J-1) - T(J-1)}{T(J) - T(J-1) + TS(J-1) - TS(J)}$$

112-6397

Fig. 27. Interpolation to Locate Single-phase, Two-phase Interface

calculated it is compared with the saturation temperature at that point. If the local saturation temperature is exceeded, an interpolation is made, as in Fig. 27, to locate the point where boiling starts. A switch is then made to the two-phase equations, and the coolant temperatures are set equal to the local saturation temperature. Because of the complications involved in following the moving boundary between phases, the work was limited at an early stage to the case that coolant boiling starts at the end of the channel and then progresses steadily toward the channel inlet. In problems in which boiling starts at an intermediate point in the channel, two interfaces are generated, one moving toward the channel inlet and the other toward the exit. This is a case of practical interest but has not been considered in the present analysis due to the additional complexity involved. Thus, in the present work, the parameters giving the spatial distribution and time rate of increase of heat flux must be such that the coolant temperature profile is a monotonically increasing function of the space variable at the time boiling starts.

In order for the two-phase continuity and energy-difference equations [Eqs. (27) and (28)] to work properly, it is necessary that the interface move through the mesh at the rate of about one space node per time interval. Since the velocity of the interface is, in general, not constant, it was necessary to

have a variable time step. In Fig. 28, the curved line represents the trace of the interface in the space-time plane. In the cases studied here, the rate of travel is high initially, at the channel exit, and decreases as the point moves down the channel, as shown by the decreasing slope of the curve in the figure. This rate, determined by the relative slopes of the saturation and coolant temperature profiles at the point in question, is highest when the curves are almost parallel and lowest when their slopes differ by the greatest amount. The computer program calculates the slopes of these two curves at the interface and then sizes the time steps so that the interface will move one space step toward the channel inlet. Early difficulties were encountered with this part of the calculation because of the rather abrupt increase in pressure when boiling started at the channel exit. This meant that

112-6402

Fig. 28. Variable Space Mesh in Two-phase Region of Channel

there was a corresponding increase in saturation temperature for the next forward step, which tended to suppress boiling and move the interface back toward the channel exit. To get around this difficulty, some early attempts at solution were based on an iteration of the pressure profile. This caused computational difficulties too, since the boiling interface tended to be forced closer to the exit end of the channel as the iterations proceeded. Although such effects may be present to a greater or lesser degree in real physical systems, it was felt to be beyond the scope of this analysis to treat them in a very realistic way and, therefore, different approaches were sought.

The scheme finally adopted is one in which the pressure profile from the last time step is used for calculating the new step. This method assumes that the profile is not changing rapidly from one time step to the next and, therefore, the error introduced by assuming it to be constant over a time interval is not large. From the results of calculations performed it was found that the assumption is poorest when boiling starts and becomes progressively better as the boiling region of the channel increases. The motion of the boiling interface down the channel, rather than being determined by the saturation temperature corresponding to local temperature, is controlled by an input parameter which specifies the slope of the saturation temperature curve for the whole channel.

The heat transfer coefficient in the boiling region is calculated with the equation used for predicting the single-phase coefficients [Eq. (44)]. Since the mass flow rates increase sharply, the equation gives increasing heat transfer coefficients in the boiling region.

In this work, flow reversal of the coolant at the channel inlet was not treated. Calculations were terminated when the flow rate at the channel inlet dropped to zero. To handle flow reversal properly, the problem should be reformulated with different boundary conditions at the time reversal occurs to permit the calculation to continue.

As shown by results given in the previous chapter, a stability criterion was determined for the two-phase equations, namely,

$$\Delta t < \Delta z \left(\frac{\bar{\rho}_j^i}{G_j^i} \right)_{\max} \quad (33)$$

As the results given in the following section show, there is a large increase in the ratio $G/\bar{\rho}$ when boiling starts. This stability criterion was, therefore, frequently exceeded after the boiling interface had moved a relatively short distance down the channel, and well before the time of zero flow at the channel inlet. The usual result in the computer calculations when this occurred was that the inner iterations with the two-phase equations for continuity and energy failed to converge and the calculation stopped. Since the time step is under program control in order to move the boiling interface the correct amount, it was not possible to make the time step arbitrarily smaller to alleviate this difficulty. Therefore, a variable space mesh was used where in the space steps were increased in size a specified number of mesh points downstream of the boiling interface. In this method, the first steps near the interface are small where the changes are large and then are increased further down the channel where the changes are smaller and less detailed description is necessary. This scheme, shown schematically in Fig. 28, causes some complications in programming the computer, but does make it possible to extend the solution to the time when the flow drops to zero at the channel inlet. The parameter N6 in the figure is the number of small space steps used in the two-phase region adjacent to the interface, while N7 is the maximum number of small steps permitted in the largest step.

As seen by results presented in the following section, the increases in flow rate just beyond the boiling interface are large, as expected. This means that the space step size, Δz , is determined by the need to obtain sufficient detail in this region. In its present form the computer program is somewhat inefficient since the same small space step is used throughout the subcooled single-phase region even though such a fine description is unnecessary there. The ALGOL program which performs the complete transient single-phase, two-phase calculation is listed in Appendix D along with

further details and a block diagram. The program in its present form has no built-in tests for the stability criteria for the various equations; hence, it is up to the user to see that these criteria are met by proper selection of input parameters.

D. Numerical Results

Some results for several cases calculated with the ALGOL program for an oxide-fueled fast breeder reactor are summarized in Figs. 29 through 45. The first two cases are power transients in which the available pump head was kept constant at the channel inlet and the fuel power generation increased exponentially with periods of 0.10 and 0.20 sec, respectively. The third case is a flow decay transient in which the power generation rate was kept constant and the available pump head at the channel inlet decreased exponentially with a half-period of 10 sec. The various input parameters for these cases are given in Table III. All three transients were started from essentially the same steady-state condition, that is, the reactor was operating at normal full power with a coolant velocity of approximately 5 m/sec and an exit coolant temperature of about 600°C.

TABLE III. Input Parameters for Single-phase, Two-phase Transient Problems

Problem Description	Power Transient I	Power Transient II	Pump-coastdown Transient
Reactor period, ω , sec	0.10	0.20	0.0
Pump-head decay constant, λ_p , sec^{-1}	0.0	0.0	-0.069
Core length, L , cm	140	140	140
Fuel-rod diameter, cm	0.635	0.635	0.635
Coolant volume fraction, V_c	0.50	0.50	0.50
Coolant inlet temp, $T_c(0)$, °C	400	400	400
Coolant outlet temp at $t = 0$, °C	615	604	604
Coolant mass velocity at $t = 0$, $G(0)$, $\text{gm}/\text{cm}^2\text{-sec}$	400	400	400
Coolant inlet velocity at $t = 0$, m/sec	4.67	4.67	4.67
Peak fuel heat generation at $t = 0$, $\text{cal}/\text{cm}^3\text{-sec}$	200	200	200
Outlet pressure, P_{ex} , atm	3.0	3.0	3.0
Axial extrapolation distance for cosine power distribution, R_2 , cm	120	70	70
UO ₂ fuel properties			
Thermal conductivity, k , $\text{cal}/\text{sec-cm-}^\circ\text{C}$	0.004	0.004	0.004
Density, ρ_f , gm/cc	10.0	10.0	10.0
Specific heat, C_f , $\text{cal}/\text{gm-}^\circ\text{C}$	0.090	0.090	0.090
Equivalent resistance of gap and cladding, R_c , $\text{sec-cm}^2\text{-}^\circ\text{C}/\text{cal}$	1.5	1.5	1.5
Number of radial nodes in fuel	5	5	5
Number of axial nodes in channel, n	140	140	140
Parameters which control the variable space mesh in two-phase region*	$\begin{cases} N6 & 3 \\ N7 & 3 \end{cases}$	$\begin{cases} 2 \\ 5 \end{cases}$	-
Saturation temp difference across core, °C	10	10	10
Equation for convection film coefficient	$Nu = 0.625 (Pe)^{0.4}$		

*See Fig. 28.

As shown in Figs. 29 and 30, the increase in mass flow rate at the channel exit is abrupt in the case of the two increasing power transients, going from a value of 391 to 1132 gm/cm²-sec in the first time increment after boiling starts in the case of the 0.10-sec transient. The corresponding increase in the slower 0.20-sec transient is from 389 to 761 gm/cm²-sec. The faster transient shows a sharper increase in velocity mainly because of the higher heat flux at the time boiling starts (see Table V, p. 70. Some of the calculated linear velocities for the liquid phase at the channel exit during the two power transients are presented in Table IV. As shown, the highest liquid velocities are obtained at the termination of the calculation (when the inlet coolant velocity has dropped to zero), being 71.6 and 42.9 m/sec, respectively, for the 0.10- and 0.20-sec transients. The curves shown in Figs. 29 and 30 give the mass flow distributions at various times in the two-phase portion of the channel; the value in the single-phase region is represented by the short horizontal segment at the bottom of each curve. The total time from the start of boiling to flow reversal is 2.7 ms for the 0.10-sec transient and 8.3 ms for the 0.20-sec transient. At this time the last 19 and 23 cm, respectively, of the two channels are in two-phase flow. Corresponding decreases in the average channel mass flow rate for the two transients are given in Figs. 31 and 32.

Distribution curves for the vapor void fraction for the two transients are presented in Figs. 33 and 34. These curves, which have a steep slope at the point where boiling starts and which then flatten out toward the channel exit, are similar to those obtained in the steady-state solutions presented in Chapter IV. An interesting feature of the curves are the inflections in the void fraction profiles for the more rapid transient, wherein the gradient again becomes steeper at the channel exit. This is apparently caused by the steeper pressure gradient, which produces a noticeable increase in the vapor void fraction because of the sharply decreasing pressure.

The vapor quality profiles are shown in Figs. 35 and 36. In each case, the maximum quality reached at the channel exit when the calculation was terminated was about 1%.

TABLE IV. Linear Velocities at Channel Exit
during Coolant Expulsion

Reactor Period, ω (sec)			
0.10		0.20	
Time (sec)	v (m/sec)	Time (sec)	v (m/sec)
0.0	4.97	0.0	4.97
0.4203	17.5	0.6977	11.7
0.4205	33.2	0.7002	37.7
0.4208	42.1	0.7030	42.9
0.4230	71.6	0.7060	42.9

112-6412

Fig. 29. Mass Flow Rate vs Channel Position
at Various Times for $\omega = 0.10 \text{ sec}$

112-6426

Fig. 30. Mass Flow Rate vs Channel Position
at Various Times for $\omega = 0.20 \text{ sec}$

112-6417

Fig. 31. Average Mass Flow Rate vs Time for $\omega = 0.10$ sec

112-6418

Fig. 32. Average Mass Flow Rate vs Time for $\omega = 0.20$ sec

Fig. 33
Vapor Void Fraction
vs Channel Position
at Various Times for
 $\omega = 0.10$ sec

112-6423

Fig. 34
Vapor Void Fraction
vs Channel Position
at Various Times for
 $\omega = 0.20$ sec

112-6405

112-6398

Fig. 35. Vapor Quality vs Channel Position
at Various Times for $\omega = 0.10$ sec

112-6422

Fig. 36. Vapor Quality vs Channel Position
at Various Times for $\omega = 0.20$ sec

Some of the pressure profiles during the two transients are shown in Figs. 37 and 38. The profiles at time zero are linear and represent the steady-state single-phase pressure drop. These profiles remain essentially constant until boiling starts, at which time the abrupt increases shown by the other curves in the figures are encountered. In both cases, the peak pressure increases monotonically, attaining a maximum value at the termination of the calculation. The peak always occurs at the point where boiling starts, as expected; the peak values for the two transients are 15.8 and 7.8 atm, respectively.

Axial temperature distributions at the fuel centerline during the two transients are given in Figs. 39 and 40. The maximum fuel center temperature is about 1825°C at the start of the transients. In each case, a major fraction of the fuel at the centerline exceeds the UO_2 melting point of 2750°C at the time boiling starts. As shown by the curves, the fuel temperature increases very little during the boiling portion of the transients, since the time from the start of boiling to the reversal of flow at the channel inlet is quite short. Thus the fuel element is acting as a relatively constant-temperature heat source for the boiling channel during the two-phase fluid expulsion. It will be noted that the fuel temperature profiles for the faster transient are flatter than those for the slow transient; this is because of the requirement that the temperature profile in the coolant be a monotonically increasing function of space at the time boiling starts (see Section C below). Since the coolant temperature profiles tend to peak sooner in the faster transients, it was necessary to alter the heat-generation profile parameters (see Table III) to give a flatter profile at time zero for the fast transient. This assured that the maximum point occurred at the end of the channel when boiling started. A reduced rate of heat generation in the breeding blanket was not used in these problems, since this always resulted in boiling starting at an intermediate point in the channel, usually at the core-blanket interface. This then gave the two-interface problem, discussed earlier, which cannot be treated properly by the present computational scheme.

Coolant and cladding temperatures during the two transients are given by the curves in Figs. 41 and 42. Both transients start with an initial coolant outlet temperature of about 600°C. The increasing separation between the clad and coolant temperature curves before boiling starts is evidence of the increasing heat-flux input to the channel during the transients. As in the case of the fuel center temperatures, the coolant temperature increases very little during the boiling part of the calculation since the time is short. On the other hand, as shown by the curves, the cladding surface temperature increases during flow deceleration in the nonboiling part of the channel since the reduced flow in this section brings about a reduction in the convection heat transfer coefficient. At the point where boiling starts, however, there is a decrease in cladding temperature because of the rapid increase in flow rate in the two-phase region. It is believed that this is a reasonable result and that an increase in cladding temperature would be encountered later when vapor blanketing occurs at a void fraction of 80 to 90%.

112-6424

Fig. 37. Pressure Profiles at Various Times for $\omega = 0.10$ sec

112-6425

Fig. 38. Pressure Profiles at Various Times for $\omega = 0.20$ sec

112-6410

Fig. 39. Fuel Center Temperature Profiles
at Various Times for $\omega = 0.10$ sec

112-6401

Fig. 40. Fuel Center Temperature Profiles
at Various Times for $\omega = 0.20$ sec

112-6428

Fig. 41. Coolant Temperature Profiles at Various Times for $\omega = 0.10$ sec

112-6407

Fig. 42. Coolant Temperature Profiles at Various Times for $\omega = 0.20$ sec

A summary of the power-generation rates and heat fluxes during the transients is given in Table V. As shown, the heat-generation rate in the fuel has increased by a factor of 66.9 up to the time boiling starts for the faster transient, whereas it has increased by a factor of 32.7 for the same period in the slower transient. During this same time the maximum heat fluxes have increased by factors of 7.1 and 5.1, respectively. The increase in stored energy in the fuel in the faster transient accounts for the higher heat flux at the start of boiling which, in turn, produces the more violent expulsion of coolant with higher flow rates and pressures. As shown by the data in Table V, the minimum heat flux is at the channel exit when the boiling starts in both power transients. At the time reversal occurs, however, the heat flux in the single-phase part of the channel has decreased somewhat because of the reduced flow rate while it has increased in the boiling zone because of the increased flow rates.

TABLE V. Heat Fluxes and Fuel Power Generation Rates during Transients

	Heat Flux (cal/cm ² -sec)			Peak Power Generation Rate (cal/cm ³ -sec)
<u>Power Transients:</u>				
$\omega = 0.10$ sec	ϕ_{inlet}	ϕ_{midpoint}	ϕ_{exit}	
t = 0.0	26.6	31.7	26.6	200
Start of boiling	261.1	224.5	168.2	13,380
Flow stoppage at inlet	224.3	186.1	207.3	13,780
$\omega = 0.20$ sec				
t = 0.0	22.5	31.7	22.5	200
Start of boiling	150.7	160.7	87.0	6,540
Flow stoppage at inlet	123.3	124.1	106.7	6,860
<u>Flow Decay Transient:</u>				
t = 0.0	22.5	31.7	22.5	200
Start of boiling	22.3	28.1	16.5	200

Results of calculations for the flow decay transient with an exponentially decreasing available pump head at the channel inlet and with a constant rate of heat generation in the fuel are summarized in Figs. 43 through 45. The input parameters for this problem are also listed in Table III. As shown by Fig. 43, the flow decay is approximately linear during the 18.2-sec duration of the transient. As shown by the curves in Fig. 44, the maximum fuel center temperature increases from 1835 to 1980°C in 18.2 sec and thus, in contrast to the power transients, is still

112-6404

Fig. 43. Mass Flow Rate vs Time for Flow Decay Transient

112-6400

Fig. 44. Fuel Center Temperature Profiles at Various Times for Flow Decay Transient

112-6399

Fig. 45. Coolant Temperature Profiles at Various Times for Flow Decay Transient

well below the melting point when coolant boiling starts. Coolant and cladding temperature profiles up to the start of boiling are shown in Fig. 45. The heat flux remains relatively constant up to the time boiling starts, as shown by the values in Table V. The slight decrease in heat flux shown by the data in the table is due to a reduction in the convection film coefficient resulting from the reduced coolant flow rate.

Because of the much slower time scale in the flow coastdown case, it was not possible to obtain satisfactory calculations in the two-phase region for this problem. Basically, the difficulties arose from the fact that the motion of the interface toward the channel inlet was much slower than in the power transients. Since the size of the time step is controlled by the time required for the interface to move one space step, the time steps are larger when the interface velocity is lower. In the flow coastdown transient the interface velocity was low enough so that the time steps were too large to obtain satisfactory convergence of the two-phase hydrodynamics equations. A similar limitation was found in the case of power transients with slower rates of power increase, that is, the time steps tended to be too large due to the low velocity of the interface toward the channel inlet. In

any case, in the pump coastdown transient, the flow dropped to about 20% of the initial value when boiling started. It is evident, therefore, that the pressures and flow rates obtained when boiling starts would be significantly less than those obtained in the two power-increase transients.

The running time for these problems on the CDC-3600 was 2 to 3 min for the power transients and about 5 min for the flow decay transient. Typically, it required about 10 inner iterations with the two-phase equations per time step to obtain a convergence of 10^{-3} . Further details, including a listing of the ALGOL program and data on the convergence of the inner iterations, are given in Appendix D.

The success of these calculations shows that it is possible to integrate across the discontinuity at the interface when transferring from the single-phase to the two-phase hydrodynamics equations. The results are qualitatively what one would expect, based on reported experience with boiling liquid metals, namely, a sharp increase in pressure and flow rate at the single-phase, two-phase interface due to the large vapor void fractions produced.³²⁻³⁵ However, as already mentioned, the solutions are limited with the present computational method to the cases where: (1) boiling starts at the end of the channel and (2) the velocity of the interface toward the channel inlet is great enough to give a time step small enough so that the two-phase hydrodynamics equation stability criterion [Eq. (33)] is not exceeded.

It is believed that the second limitation could be alleviated by specifying a finer space mesh in the two-phase region of the channel. In the calculations performed so far, it was not practical to go to a space step smaller than about 1 cm since, as mentioned earlier, the size of this step is constant for the whole channel even though such a fine mesh is unnecessary in the single-phase region. This is inefficient since a great deal of time is spent calculating extra points in the single-phase region which could be better spent describing the two-phase region in more detail. While the programming gets more complicated, it is believed that the problem could be set up for a coarser space mesh in the single-phase region and a finer space mesh in the two-phase region. It would then be possible to handle the problems with a lower interface velocity.

VII. SUMMARY OF RESULTS AND CONCLUSIONS

Successful completion of the calculations reported in the previous chapter show that the computational methods developed in the present work can be used to predict the time-dependent behavior of the fluid in a channel containing a single-phase, two-phase interface which progresses toward the channel inlet with time. The implicit iterative scheme for solving the coupled two-phase hydrodynamics equations developed in Chapter V was successfully applied to the moving-interface problem by a straightforward integration across the discontinuity. The method of performing the complete calculation which includes the heat transfer from a fuel rod appears to be basically sound and is also reasonable from the standpoint of computer running time, even when a fairly fine space mesh is required. Furthermore, the method of calculation developed permits slip flow between the liquid and vapor phases as specified by an arbitrary equation of state (vapor void fraction correlation) and also considers the fluid properties to be pressure-dependent. The results of calculations presented for power-increase and flow-decay transients in a typical oxide-fueled fast breeder reactor coolant channel show that: (1) The two-phase coolant mixture is expelled from the exit of the channel at high velocities. (2) The decrease of flow rate in the single-phase portion of the channel is rapid. (3) The peak pressures generated in the transients studied are not excessively high.

The high flow rates calculated at the channel exit introduce the possibility that the critical flow conditions may be encountered during the expulsion of two-phase coolant, especially in view of the low sonic velocities which have been predicted and, in some cases, measured for two-phase mixtures.⁴⁵ The significance of a low sonic velocity in a two-phase fluid is that the velocities calculated in the present work may actually be higher than those which would be encountered in the real situation if critical flow is limiting. This is an aspect of the problem which has not been studied during the present work but which should receive attention, both analytic and experimental.

As pointed out in the previous chapters, there are a number of limitations in the present analysis: some in the original formulation of the problem and others which have arisen unexpectedly during the course of the work. The two most important ones arising during the course of the work are: (1) the requirement that boiling start at the end of the coolant channel only, and (2) the problem of controlling the motion of the single-phase, two-phase interface down the channel at a rate such that stable numerical solutions are obtained in the two-phase section of the channel. The second restriction is particularly important since it is closely related to the whole problem of determining the point at which boiling starts in any computational model in which one attempts to simulate the behavior of a real channel. Since it is widely believed that liquid superheat is large for liquid metals, it is conceivable that a portion of the channel could be above saturation temperature and then, upon formation of the first bubbles of vapor,

could suddenly "explode" into two-phase flow. In such a situation, one does not envision an orderly progression of the single-phase, two-phase interface toward the channel inlet initially. On the other hand, the increase in local saturation temperature produced by the increased pressure when boiling starts tends to suppress further vapor formation and hence slow down the motion of the interface toward the inlet. At the present time, the relative importance of these two counteracting effects is unknown and can only be determined by experiment.

The motion of the interface in the calculations performed in this work is controlled by specifying the slope of a pseudosaturation temperature curve. The fact that the local saturation temperature does not correspond to the local pressure in the calculations can be considered, in a sense, to be a nonequilibrium effect associated with the liquid superheat. In view of these limitations and the others introduced in the formulation of the problem, it should be recognized that the values of flow rates, void fractions, pressures, etc., obtained in the calculations are estimates having some uncertainty associated with the numerical values. It is believed, however, that the qualitative picture presented by the results is correct and that they give a more detailed analysis of transient boiling of sodium in reactor coolant channel than is available in any other published work of which this author is aware.

The development of the computational techniques used was characterized by much trial-and-error effort, particularly in developing stable finite-difference equations in a convergent iterative scheme. Furthermore, the effort expended in this area has by no means exhausted all the possibilities for this problem, and further work may well yield methods of solving the two-phase conservation equations without the stability limitations present in the method developed here. Other areas where future work could be profitably performed on this problem are:

- (1) Remove the limitation that boiling starts at the end of the channel by solving the two-interface problem, resulting when the first bubbles are formed at an intermediate point in the channel. In this connection it might be profitable to investigate possible advantages of the Lagrangian formulation of the problem vs the Eulerian formulation used in this work.

- (2) Extend the calculations beyond the point of flow stoppage at the channel inlet to allow calculation of simultaneous expulsion of coolant from both ends of the channel. As mentioned earlier, this requires a reformulation of the problem with different boundary conditions to continue the calculation.

ACKNOWLEDGMENTS

The author is indebted to Argonne National Laboratory for granting him academic leave to attend Purdue University and for financial support throughout his whole Ph.D. program. He owes special thanks to Dr. Frank Myers and Dr. Merlin Peterson of the Laboratory Director's Office for their aid and suggestions in planning his program.

The author wishes to thank Mr. Raymond Brittan of the Reactor Physics Division at Argonne for his support and encouragement throughout this research. He also wishes to thank Professor Alexander Sesonske, his major professor at Purdue, for his valuable assistance and guidance during the author's graduate career.

The author gratefully acknowledges the aid of Dr. David Taylor of the Applied Mathematics Division at Argonne for his very helpful suggestions during difficult phases of the numerical analyses.

REFERENCES

1. Hummel, H. H., et al., *Sodium Void Effect*, Reactor Physics Division Annual Report, ANL-7010 (Jan 1965), p. 86.
2. Okrent, D., et al., *Some Nuclear and Safety Consideration in the Design of Large Fast Power Reactor*, Proceedings of Third International Conference on the Peaceful Uses of Atomic Energy, Sept 1964, Vol. 6, Session 1.4, P/267.
3. Okrent, D., *Neutron Physics Considerations in Large Fast Reactors*, Power Reactor Technology, Vol. 7, No. 2, Spring 1964.
4. Okrent, D., *A Look at Fast Reactor Safety*, Nuclear Safety, Vol. 6, No. 4, Summer 1965.
5. Kazi, A. H., and Cherry, B. H., *Survey of Current Methods for Analysis of Fast Reactor Transients Including Meltdown*, UNC-5092 (Aug. 28, 1964).
6. Fauske, Hans K., *Contribution to the Theory of Two-phase, One-component Critical Flow*, ANL-6633 (Oct 1962).
7. Kepple, Robert R., and Tung, Thomas V., *Two-phase (Gas-Liquid) System: Heat Transfer and Hydraulics*, ANL-6734 (July 1963).
8. Van Hoogstraten, Anita, *Research on Boiling - A Bibliography*, Report No. N-2806G (Jan 1964).
9. Lockhart, R. W., and Martinelli, R. C., *Proposed Correlation of Data for Isothermal Two-phase, Two-component Flow in Pipes*, Chem. Eng. Progress, Vol. 45, No. 1 (Jan 1949).
10. Martinelli, R. C., and Nelson, D. B., *Prediction of Pressure Drop during Forced-circulation Boiling of Water*, Transactions of the ASME (Aug 1948).
11. Lottes, Paul A., and Cook, William H., *Fluid Dynamics, Stability and Vapor Liquid Slip in Boiling Reactor Systems*, Proceedings of the Third International Conference on the Peaceful Uses of Atomic Energy, Sept 1964, A/CONF. 28/P/230R-1.
12. Wallis, Graham B., and Collier, John G., *Two-phase Flow and Heat Transfer, Summer Course*, June 27-July 9, 1965, Thayer School, Dartmouth College, Hanover, New Hampshire.
13. Nicholson, R. B., *Sodium Boiling Calculations*, AECU-3698 (July 2, 1957).
14. Stein, R. P., *Studies Related to Predicting Coolant Behavior for Fast Reactor Safety*, NDA-2147-8 (Aug. 31, 1961).
15. Goldman, Arthur J., *A Survey of Vaporization Effects Important to Fast Reactor Safety*, UNC-5021 (June 30, 1962).
16. Schoeberle, D. F., et al., *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration. The ARGUS Program, 1089/RE 248, Written in Fortran II.*, ANL-6654 (Nov 1963).
17. Greebler, P., et al., *FORE: A Computational Program for the Analysis of Fast Reactor Excursions*, GEAP-4090 (Oct 1962).
18. Briggs, D. L., *TIGER - Temperatures from Internal Generation Rates*, KAPL-M-EC-29 (1963).

19. Ryan, S. A., *TER-4, Reactor Transient Analysis Program*, KAPL-M-SR-2 (1963).
20. Bryant, Lawrence T., and Gopinath, D. V., *A Fast Reactor Excursion Simulator*, ANL-6798 (Dec 1963).
21. Noyes, R. C., et al., *TRANS-FUGUE-1, A Digital Code for Transient Two-phase Flow and Heat Transfer*, NAA-SR-11008 (July 25, 1965).
22. Meyer, J. E., et al., *ART - A Program for the Treatment of Reactor Thermal Transients on the IBM-704*, WAPD-TM-156 (Nov 1959).
23. Birkhoff, G., and Kimes, T. F., *CHIC Programs for Thermal Transients*, WAPD-TM-245 (Feb 1962).
24. Redfield, J. A., *CHIC-KIN - A Fortran Program for Intermediate and Fast Transients in a Water Moderated Reactor*, WAPD-TM-479 (Jan 1965).
25. Fleck, Joseph A., Jr., *The Influence of Pressure on Boiling Water Reactor Dynamic Behavior at Atmospheric Pressure*, Nucl. Sci. Eng. 9 (Feb 1961).
26. Meyer, J. E., *Hydrodynamic Models for the Treatment of Reactor Thermal Transients*, Nucl. Sci. Eng. 10 (July 1961).
27. Meyer, J. E., and Rose, R. P., *Application of a Momentum Integral Model to the Study of Parallel Channel Boiling Flow Oscillations*, ASME Journal of Heat Trans. (Feb 1963).
28. Anderson, R. P., et al., *Transient Analysis of Two-phase Natural Circulation Systems*, ANL-6653 (Dec 1962).
29. Halfen, F. J., and Berganzoli, *Boiling Depressurization Transients*, NAA-SR-9033 (Aug. 1, 1964).
30. Staub, F. W., et al., *A Program of Two-phase Flow Investigation. Ninth Quarterly Report, April-June 1965*, GEAP-4910 (Aug. 1, 1965).
31. Jarvis, Stephen, *On the Formulation and Numerical Evaluation of a Set of Two-phase Flow Equations Modelling the Cooldown Process*, NBS Technical Note No. 301 (Jan. 4, 1965).
32. Lurie, H., *Steady-state Sodium Boiling and Hydrodynamics*, NAA-SR-11586 (Jan. 15, 1966).
33. Balzhiser, Richard E., et al., *Investigation of Liquid Metal Boiling Heat Transfer*, The University of Michigan 05750-50-19-P (Feb 1965).
34. *Forced Convection Boiling Potassium Experiments*, Pratt & Whitney Aircraft Co. PWAC-430 (Dec. 23, 1964).
35. Tippets, F. E., and Converse, G. L., *Alkali Metals Boiling and Condensing Investigations - Quarterly Progress Report #11*, NASA-CR-54405 (Jan. 1, 1965-March 31, 1965).
36. Krakoviak, A. I., *Superheat Requirements with Boiling Liquid Metals*, Proceedings of 1963 High-temperature Liquid-Metal Heat Transfer Technology Meeting at Oak Ridge National Laboratory, Sept. 4-6, 1963, ORNL-3605, Vol. I, Nov 1964.
37. Smith, J. O., et al., *Model Studies of Reactor Coolant and Fuel Pin Behavior Under Conditions of Coolant Boiling and Overheating of the Fuel Elements*. Engineering Research Dept. Report 59D65, Detroit Edison Co., TID-18229 (Nov. 20, 1961).

38. Huebotter, Paul R., *Study of Fast Reactor Meltdown Accidents Using Simulant Materials*, APDA-155 (Aug. 31, 1963).
39. Johnson, H. A., *et al.*, *Transient Boiling Heat Transfer and Void Volume Production in Channel Flow*, SAN-1007 (March 1963).
40. Zivi, S. M., *et al.*, *Kinetic Studies of Heterogeneous Water Reactors, Annual Summary Report - 1963* (Dec. 30, 1963).
41. Meyer, J. E., *Conservation Laws in One-dimensional Hydrodynamics*, Bettis Technical Review, WAPD-BT-20 (Sept 1960).
42. Fauske, H. K., *Two-phase Critical Flow with Application to Liquid Metal Systems (Mercury, Cesium, Rubidium, Potassium, Sodium, and Lithium)*, ANL-6779 (Oct 1963).
43. Dusenberre, G. M., *Heat Transfer Calculations by Finite Differences*, International Textbook Co., Scranton (1961).
44. Back, L. H., *Technical Brief, Numerical Approximation of the Convective Boundary Condition*, Trans. ASME J. of Heat Trans. (Feb 1962), p. 89.
45. Gouse, S. W., Jr., and Brown, G. A., *Some Remarks on the Velocity of Sound for Two-phase Mixtures and Flows*, MIT-8040-1 (AD-411443) (April 1963).
46. Dunning, E. L., *The Thermodynamic and Transport Properties of Sodium and Sodium Vapor*, ANL-6246 (Oct 1960).
47. Tokar, J. V., Unpublished work.
48. Stone, J. P., *et al.*, *High Temperature Properties of Sodium*, NRL-6241 (Sept. 24, 1965).
49. Weatherford, W. D., Jr., *Properties of Inorganic Energy-conversion and Heat-transfer Fluids for Space Applications*, WADD-TR-61-96 (Nov 1961).
50. Grosse, A. V., *Viscosities of Liquid Sodium, Potassium from Their Melting Points to Their Critical Points*, Science 147, 1438 (1965).

GENERAL BIBLIOGRAPHY

1. Bird, R. B., Stewart, W. E., and Lightfoot, E. N., *Transport Phenomena*, Wiley and Sons, New York (1960).
2. Courant, R. John, and Friedrichs, K. O., *Supersonic Flow and Shock Waves*, Interscience Publishers, New York (1948).
3. Forsythe, G. E., and Wasow, W. R., *Finite Difference Methods for Partial Differential Equations*, John Wiley and Sons, New York (1960).
4. Richtmeyer, R. O., *Difference Methods for Initial Value Problems*, Interscience Publishers, Inc., New York (1957).
5. Robertson, J. M., *Hydrodynamics In Theory and Application*, Prentice-Hall, Englewood Cliffs, N. J. (1965).
6. Shapiro, Ascher H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 2, Ronald Press Co., New York (1954).
7. Smith, G. D., *Numerical Solution of Partial Differential Equations*, Oxford University Press, London (1965).
8. Tong, L. S., *Boiling Heat Transfer and Two-phase Flow*, John Wiley and Sons, New York (1965).

APPENDIX A

Two-phase Flow Correlations1. Friction Correlation

According to the correlation developed by Lockhart and Martinelli,⁹ the two-phase friction pressure gradient can be written as

$$\left(\frac{\partial p}{\partial z}\right)_{tp} = \left(\frac{\partial p}{\partial z}\right)_\ell \phi_\ell^2, \quad (A1)$$

where

$$\left(\frac{\partial p}{\partial z}\right)_\ell = \text{the pressure gradient for all liquid flowing at a flow rate of } 1 - x \text{ times the total flow rate of two-phase mixture;}$$

$$\phi_\ell^2 = \text{Lockhart-Martinelli friction multiplier;}$$

and $(\partial p / \partial z)_\ell$ is represented in the usual way by means of a friction factor, i.e.,

$$\left(\frac{\partial p}{\partial z}\right)_\ell = \frac{f_\ell [G(1-x)]^2}{2\rho_\ell D_h} \quad (A2)$$

The single-phase friction factor, f_ℓ , is given for well-developed turbulent flow in terms of the Reynold number, Re , by the expression

$$f_\ell = \frac{b}{(Re)^a} = \frac{b}{(D_h G / \mu_\ell)^a}, \quad (A3)$$

where the values $a = 0.2$ and $b = 0.1875$ were used throughout the present work. Thus, values of f_ℓ in Eq. (A2) are given by

$$f_\ell = \frac{b}{[D_h G(1-x) / \mu_\ell]^a} \quad (A4)$$

Therefore

$$\begin{aligned} \left(\frac{\partial p}{\partial z}\right)_\ell &= \frac{b}{[D_h G(1-x) / \mu_\ell]^a} \frac{[G(1-x)]^2}{2\rho_\ell D_h} \\ &= \frac{b}{(D_h G / \mu_\ell)^a} \frac{G^2(1-x)^{2-a}}{2\rho_\ell D_h} \\ &= \left(\frac{\partial p}{\partial z}\right)_0 (1-x)^{2-a}, \end{aligned} \quad (A5)$$

where $(\partial p / \partial z)$ is the pressure gradient for all liquid flow at a flow rate equal to the total flow rate of the two-phase mixture.

In steady flow $(\partial p / \partial z)_0$ can be taken as constant in the two-phase portion of the channel. Hence, in this case, Eq. (A1) can be written as the product of a constant and two variable terms:

$$\left(\frac{\partial p}{\partial z} \right)_{tp} = \left(\frac{\partial p}{\partial z} \right)_0 (1-x)^{2-a} \phi_\ell^2 \quad (A6)$$

Lockhart and Martinelli obtained experimental curves of ϕ_ℓ^2 vs the dimensionless two-phase flow parameter X_{tt} :

$$X_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\rho_g}{\rho_\ell} \right)^{0.5} \left(\frac{\mu_\ell}{\mu_g} \right)^{0.1} \quad (A7)$$

where

ρ_g = vapor density;

ρ_ℓ = liquid density;

μ_ℓ = liquid viscosity;

μ_g = vapor viscosity,

and the subscript tt denotes turbulent flow in both phases. Other forms of the flow parameter are given by the authors for different flow conditions in the two phases, i.e., viscous-viscous, viscous-turbulent, etc.

In the present work the Martinelli curve relating X_{tt} and ϕ_ℓ^2 was approximated by the expression

$$\phi_\ell^2 = 1.0 + \frac{20.0}{X_{tt}} + \frac{1.0}{X_{tt}^2} \quad (A8)$$

This expression when substituted in Eq. (A6) can be shown to give the correct expression at the limits of all liquid or all vapor flow, that is,

$$\lim_{x \rightarrow 1.0} \left(\frac{\partial p}{\partial z} \right)_{tp} = \left(\frac{\partial p}{\partial z} \right)_g,$$

and

$$\lim_{x \rightarrow 0} \left(\frac{\partial p}{\partial z} \right)_{tp} = \left(\frac{\partial p}{\partial z} \right)_\ell.$$

2. Vapor Void Fraction Correlation

Lockhart and Martinelli also obtain a correlation for the vapor void fraction α as a function of the two-phase parameter X_{tt} . In the present work, the curve given in their work was approximated by the expression

$$\alpha = 1.0 - \frac{1.0}{\sqrt{1.0 + (21.0/X_{tt}) + (1.0/X_{tt}^2)}} \quad (A9)$$

for the steady-state calculations reported in Chapter IV. In the time-dependent calculations an abbreviated form of Eq. (A9) was used, namely,

$$\alpha = 1.0 - \frac{1.0}{\sqrt{1.0 + (21.0/X_{tt})}} \quad (A10)$$

Since it was necessary to invert the correlation to obtain an expression explicit in X_{tt} instead of α , Eq. (A10) was rearranged to give

$$X_{tt} = \frac{21.0(1.0 - \alpha)^2}{\sqrt{[1.0 - (1.0 - \alpha)^2]}} \quad (A11)$$

The values of X_{tt} corresponding to the given α were then obtained by arranging Eq. (A7) to be explicit in quality x . Computer calculations were run to compare the differences in α predicted by Eqs. (A9) and (A10), which were found to be small.

The relationship between the void fraction and the slip ratio in a two-phase system can be shown as follows. Assuming the annular-flow model (see Fig. 2); the flow rates of the separate phases are given by

$$w_l = (1 - x) w_t = v_l \rho_l (1 - \alpha) A; \quad (A12)$$

$$w_g = x w_t = v_g \rho_g \alpha A, \quad (A13)$$

where

w_l = liquid flow rate;

w_g = vapor flow rate;

w_t = total flow rate;

v_g = vapor velocity;

v_l = liquid velocity;

A = cross-sectional area of channel.

Dividing Eq. (A13) by Eq. (A12) gives

$$\frac{v_g \rho_g}{v_l \rho_l} \frac{\alpha}{1 - \alpha} = \frac{x}{1 - x} \quad (\text{A14})$$

Let $v_g/v_l = K =$ slip velocity. Since $\rho_l/\rho_g = \beta$, Eq. (A14) can be rearranged to give

$$K = \frac{x}{1 - x} \frac{1 - \alpha}{\alpha} \beta \quad (\text{A15})$$

or

$$\alpha = \frac{\beta x}{x + K(1 - x)} \quad (\text{A16})$$

Thus, specifying a vapor void fraction by Eq. (A10) determines the slip ratio between the phases for the annular-flow model.

It is possible, therefore, to calculate the slip ratio implicit in a given vapor void fraction correlation by means of Eq. (A15). This was done for the correlation given by Eq. (A10) for sodium at a temperature of 1000°C for a series of values of the ratio of liquid to vapor densities, β . The results of these calculations, run on CDC-160A computer, are given in Table VI. As shown by these results, the value of the slip ratio for a given quality x decreases as the pressure increases. It can be seen from the results of Table VI that slip ratios in the calculations reported in Chapter VI range from values of about 1 to 5.

TABLE VI. Calculated Slip Ratios Implicit in the Martinelli Void Fraction Correlation

x	X_{tt}	α	K
$\beta = 2500.0$			
0.000100	103.498557	0.088230	2.583740
0.000200	55.458511	0.148330	2.871429
0.000300	38.498777	0.195604	3.085186
0.000500	24.305517	0.267551	3.423703
0.000700	17.951940	0.321122	3.702218
0.000900	14.315390	0.363322	3.946399
0.001000	13.019140	0.381372	4.059336
0.002000	6.970498	0.500791	4.994179
0.003000	4.834926	0.567395	5.735506
0.005000	3.047481	0.644011	6.944307
0.007000	2.247188	0.689090	7.951467

TABLE VI. (Contd.)

x	X_{tt}	α	K
$\beta = 2500.0$ (Contd.)			
0.009000	1.789045	0.719813	8.837646
0.010000	1.625717	0.731946	9.247977
0.020000	0.863276	0.801291	12.652431
0.030000	0.593825	0.834169	15.370922
0.040000	0.454112	0.854512	17.735233
0.050000	0.368003	0.868766	19.875910
0.060000	0.309351	0.879512	21.860609
0.070000	0.266698	0.888015	23.729831
0.080000	0.234207	0.894977	25.510136
0.090000	0.208589	0.900827	27.220093
0.100000	0.187841	0.905843	28.873338
0.200000	0.090537	0.934480	43.820717
0.300000	0.055738	0.948549	58.115798
$\beta = 2000.0$			
0.000100	115.714929	0.080002	2.300156
0.000200	62.004514	0.135707	2.548027
0.000300	43.042955	0.180185	2.730712
0.000500	27.174401	0.248944	3.018463
0.000700	20.070885	0.300937	3.254412
0.000900	16.005097	0.342344	3.460974
0.001000	14.555844	0.360172	3.556457
0.002000	7.793256	0.479747	4.346410
0.003000	5.405613	0.547546	4.972900
0.005000	3.407189	0.626372	5.994924
0.007000	2.512433	0.673112	6.846832
0.009000	2.000214	0.705101	7.596602
0.010000	1.817607	0.717762	7.943815
0.020000	0.965172	0.790378	10.825146
0.030000	0.663917	0.824939	13.126411
0.040000	0.507713	0.846357	15.127849
0.050000	0.411440	0.861378	16.939973
0.060000	0.345865	0.872709	18.620040
0.070000	0.298177	0.881677	20.202326
0.080000	0.261852	0.899024	21.709319
0.090000	0.233210	0.895198	23.156740
0.100000	0.210013	0.900493	24.556130
0.200000	0.101223	0.930739	37.207401
0.300000	0.062317	0.945606	49.305306

TABLE VI: (Contd.)

x	X_{tt}	α	K
$\beta = 1500.0$			
0.000100	133.616056	0.070387	1.981263
0.000200	71.596627	0.120676	2.186423
0.000300	49.701711	0.161562	2.336000
0.000500	31.378288	0.226003	2.569816
0.000700	23.175857	0.275688	2.760579
0.000900	18.481090	0.315821	2.927196
0.001000	16.807637	0.333249	3.004140
0.002000	8.998874	0.452301	3.640024
0.003000	6.241863	0.521327	4.144247
0.005000	3.934282	0.602777	4.967247
0.007000	2.901108	0.651604	5.653652
0.009000	2.309647	0.685221	6.257979
0.010000	2.098792	0.698567	6.537892
0.020000	1.114484	0.775509	8.861493
0.030000	0.766625	0.812329	10.717776
0.040000	0.586256	0.835200	12.332308
0.050000	0.475090	0.851262	13.794140
0.060000	0.399371	0.863388	15.149429
0.070000	0.344305	0.872992	16.425823
0.080000	0.302361	0.880862	17.641454
0.090000	0.269287	0.887479	18.809011
0.100000	0.242502	0.893154	19.937797
0.200000	0.116883	0.925602	30.141706
0.300000	0.071957	0.941663	39.897981
$\beta = 1000.0$			
0.000100	163.645624	0.058581	1.607181
0.000200	87.687622	0.101787	1.765230
0.000300	60.871931	0.137734	1.878667
0.000500	38.430406	0.195857	2.053906
0.000700	28.384519	0.241867	2.195688
0.000900	22.634626	0.279770	2.319012
0.001000	20.585073	0.296429	2.375856
0.002000	11.021328	0.413325	2.844491
0.003000	7.644691	0.483395	3.215741
0.005000	4.818493	0.567993	3.822026
0.007000	3.553117	0.619590	4.328083
0.009000	2.828730	0.655455	4.773873
0.010000	2.570485	0.669764	4.980417
0.020000	1.364960	0.752955	6.695927
0.030000	0.938921	0.793125	8.067045

TABLE VI. (Contd.)

x	X _{tt}	α	K
$\beta = 1000.0$ (Contd.)			
0.040000	0.718015	0.818173	9.259765
0.050000	0.581864	0.835802	10.339726
0.060000	0.489127	0.849130	11.340986
0.070000	0.421686	0.859696	12.283948
0.080000	0.370315	0.868362	13.182004
0.090000	0.329808	0.875652	14.044527
0.100000	0.297003	0.881907	14.878389
0.200000	0.143152	0.917716	22.415343
0.300000	0.088129	0.935353	29.620299
$\beta = 500.0$			
0.000100	231.429858	0.042498	1.126616
0.000200	124.009021	0.075239	1.229329
0.000300	86.085900	0.103397	1.301100
0.000500	54.348799	0.150708	1.409536
0.000700	40.141768	0.189730	1.495764
0.000900	32.010193	0.222922	1.570053
0.001000	29.111688	0.237808	1.604137
0.002000	15.585611	0.347299	1.883125
0.003000	10.811225	0.417028	2.103185
0.005000	6.814378	0.505030	2.462508
0.007000	5.024867	0.560591	2.762739
0.009000	4.000428	0.599982	3.027471
0.010000	3.635215	0.615862	3.150199
0.020000	1.930344	0.709856	4.170760
0.030000	1.327834	0.756135	4.987336
0.040000	1.015426	0.785236	5.697955
0.050000	0.822880	0.805816	6.341507
0.060000	0.691731	0.821424	6.938206
0.070000	0.596355	0.833826	7.500180
0.080000	0.523704	0.844014	8.035392
0.090000	0.466420	0.852596	8.549419
0.100000	0.420026	0.859967	9.046356
0.200000	0.202447	0.902284	13.537225
0.300000	0.124634	0.923188	17.828994

TABLE VI. (Contd.)

x	X _{tt}	α	K
$\beta = 150.0$			
0.000100	422.531083	0.023960	0.611086
0.000200	226.408429	0.043380	0.661681
0.000300	157.170610	0.060770	0.695591
0.000500	99.226862	0.091523	0.744837
0.000700	73.288495	0.118365	0.782630
0.000900	58.442341	0.142295	0.814464
0.001000	53.150420	0.153364	0.828891
0.002000	28.456941	0.241456	0.944348
0.003000	19.738503	0.303927	1.033718
0.005000	12.441293	0.390054	1.178703
0.007000	9.174108	0.448602	1.299699
0.009000	7.303747	0.492014	1.406477
0.010000	6.636963	0.509950	1.456019
0.020000	3.524310	0.620913	1.868972
0.030000	2.424283	0.678294	2.200295
0.040000	1.853906	0.715184	2.489004
0.050000	1.502366	0.741611	2.750650
0.060000	1.262922	0.761824	2.993346
0.070000	1.088790	0.777983	3.221975
0.080000	0.956149	0.791318	3.439752
0.090000	0.851562	0.802590	3.648927
0.100000	0.766859	0.812301	3.851158
0.200000	0.369616	0.868484	5.678671
0.300000	0.227550	0.896464	7.424543

For purposes of comparison, the void fraction correlation for data on boiling potassium reported recently by Balzhiser *et al.*³³ was also analyzed on the computer. The correlation, reported for data over a limited quality range (data of Fig. 9), is

$$\alpha = 1 - 0.574(X_{tt})^{0.342} \quad (A17)$$

This form the correlation, intended for use over the range of the data, does not have the correct functional form to apply over a wider range. The void fraction α does not approach the limit 0 when x approaches 0, and Eq. (A17) gives negative values of void fraction (and slip ratio) for values of X_{tt} greater than 5.1. For sodium at 1000°C this value of X_{tt} corresponds to a quality of about 0.003. To correct this problem, an expression having the functional form of Eq. (A10) but which approximates Eq. (A17) was derived; it is

$$\alpha = 1 - \frac{1}{\sqrt{1 + [3.025/(X_{tt})^{0.684}]}} \quad (A18)$$

The results of calculations run on the CDC-160A using Eq. (A18) are given in Table VII. As can be seen by comparing these results with the values in Table VI, the modified Balzhiser correlation gives lower values for the vapor void fraction and, accordingly, implies higher slip ratios. It also appears that the slip ratio is a more rapidly varying function of quality with the modified Balzhiser correlation.

TABLE VII. Calculated Slip Ratios Implicit in the Modified Balzhiser Void Fraction Correlation

x	X _{tt}	α	K
β = 2500.0			
0.000100	103.498557	0.057870	4.070432
0.000200	55.458511	0.084846	5.394055
0.000300	38.498777	0.105231	6.379069
0.000500	24.305517	0.136491	7.912043
0.000700	17.951940	0.160722	9.144726
0.000900	14.315390	0.180755	10.206930
0.001000	13.019140	0.189643	10.693294
0.002000	6.970498	0.254965	14.639767
0.003000	4.834926	0.298041	17.717436
0.005000	3.047481	0.356056	22.720415
0.007000	2.247188	0.395725	26.910962
0.009000	1.789045	0.425708	30.628744
0.010000	1.625717	0.438304	32.361576
0.020000	0.863276	0.520262	47.046336
0.030000	0.593825	0.566468	59.174384
0.040000	0.454112	0.598088	69.999264
0.050000	0.368003	0.621860	80.010116
0.060000	0.309351	0.640772	89.460164
0.070000	0.266698	0.656398	98.501467
0.080000	0.234207	0.669667	107.234238
0.090000	0.208589	0.681171	115.729085
0.100000	0.187841	0.691305	124.038202
0.200000	0.090537	0.754859	202.968327
0.300000	0.055738	0.790550	283.866353

TABLE VII. (Contd.)

x	X _{tt}	α	K
$\beta = 2000.0$			
0.000100	115.714929	0.053955	3.507095
0.000200	62.004514	0.079334	4.642894
0.000300	43.042955	0.098604	5.486585
0.000500	27.174401	0.128307	6.797174
0.000700	20.070885	0.151456	7.849063
0.000900	16.005097	0.170675	8.754216
0.001000	14.555844	0.179225	9.168303
0.002000	7.793256	0.242480	12.521206
0.003000	5.405613	0.284582	15.128887
0.005000	3.407189	0.341745	19.358330
0.007000	2.512433	0.381118	22.894216
0.009000	2.000214	0.411023	26.027329
0.010000	1.817607	0.423623	27.486609
0.020000	0.965172	0.506089	39.834072
0.030000	0.663917	0.552926	50.014003
0.040000	0.507713	0.585105	59.091194
0.050000	0.411440	0.609361	67.480165
0.060000	0.345865	0.628695	75.395257
0.070000	0.298177	0.644693	82.965001
0.080000	0.261852	0.658294	90.274097
0.090000	0.233210	0.670097	97.382079
0.100000	0.210013	0.680504	104.333017
0.200000	0.101223	0.745929	170.304412
0.300000	0.062317	0.782781	237.854005
$\beta = 1500.0$			
0.000100	133.616056	0.049267	2.894915
0.000200	71.596627	0.072690	3.827859
0.000300	49.701711	0.090580	4.519313
0.000500	31.378288	0.118331	5.590943
0.000700	23.175857	0.140102	6.449035
0.000900	18.481090	0.158271	7.186122
0.001000	16.807637	0.166381	7.522957
0.002000	8.998874	0.226887	10.242914
0.003000	6.241863	0.267636	12.350915
0.005000	3.934282	0.323540	15.759844
0.007000	2.901108	0.362414	18.602604
0.009000	2.309647	0.392130	21.117395
0.010000	2.098792	0.404697	22.287595
0.020000	1.114484	0.487606	32.168414
0.030000	0.766625	0.535161	40.295641

TABLE VII. (Contd.)

x	X_{tt}	α	K
$\beta = 1500.0$ (Contd.)			
0.040000	0.586256	0.568011	47.532804
0.050000	0.475090	0.592863	54.215460
0.060000	0.399371	0.612722	60.516294
0.070000	0.344305	0.629189	66.539019
0.080000	0.302361	0.643212	72.351820
0.090000	0.269287	0.655395	78.002632
0.100000	0.242502	0.666151	83.526806
0.200000	0.116883	0.734004	135.895875
0.300000	0.071957	0.772382	189.446827
$\beta = 1000.0$			
0.000100	163.645624	0.043295	2.209917
0.000200	87.687622	0.064163	2.917628
0.000300	60.871931	0.080222	3.440611
0.000500	38.430406	0.105339	4.248666
0.000700	28.384519	0.125217	4.893678
0.000900	22.634626	0.141922	5.446409
0.001000	20.585073	0.149411	5.698615
0.002000	11.021328	0.205928	7.727557
0.003000	7.644691	0.244610	9.292263
0.005000	4.818493	0.298456	11.811937
0.007000	3.553117	0.336406	13.905469
0.009000	2.828730	0.365686	15.753009
0.010000	2.570485	0.378137	16.611523
0.020000	1.364960	0.461238	23.838297
0.030000	0.938921	0.509606	29.761777
0.040000	0.718015	0.543292	35.026200
0.050000	0.581864	0.568915	39.880599
0.060000	0.489127	0.589472	44.453115
0.070000	0.421686	0.606570	48.820341
0.080000	0.370315	0.621166	53.032604
0.090000	0.329808	0.633874	57.125216
0.100000	0.297003	0.645112	61.124229
0.200000	0.143152	0.716396	98.968943
0.300000	0.088129	0.756975	137.591431

TABLE VII. (Contd.)

x	X_{tt}	α	K
$\beta = 500.0$			
0.000100	231.429858	0.034626	1.394121
0.000200	124.009021	0.051650	1.836461
0.000300	86.085900	0.064899	2.161897
0.000500	54.348799	0.085880	2.662365
0.000700	40.141768	0.102706	3.059897
0.000900	32.010193	0.116998	3.399262
0.001000	29.111688	0.123451	3.553734
0.002000	15.586511	0.173033	4.788803
0.003000	10.811225	0.207865	5.733407
0.005000	6.814378	0.257540	7.243440
0.007000	5.024867	0.293363	8.490001
0.009000	4.000428	0.321450	9.585327
0.010000	3.635215	0.333509	10.093013
0.020000	1.930344	0.415702	14.342520
0.030000	1.327834	0.464842	17.803108
0.040000	1.015426	0.499592	20.867314
0.050000	0.822880	0.526299	23.685735
0.060000	0.691731	0.547891	26.335499
0.070000	0.596355	0.565957	28.862533
0.080000	0.523704	0.581453	31.296920
0.090000	0.466420	0.594999	33.659708
0.100000	0.420026	0.607018	35.966398
0.200000	0.202447	0.684088	57.724932
0.300000	0.124634	0.728529	79.848730
$\beta = 150.0$			
0.000100	422.531083	0.023344	0.627604
0.000200	226.408429	0.035122	0.824313
0.000300	157.170610	0.044428	0.968157
0.000500	99.226862	0.059414	1.187905
0.000700	73.288495	0.071659	1.361221
0.000900	58.442341	0.082217	1.508344
0.001000	53.150420	0.087032	1.575067
0.002000	28.456941	0.125039	2.103456
0.003000	19.738503	0.152817	2.502187
0.005000	12.441293	0.193996	3.131715
0.007000	9.174108	0.224841	3.645481
0.009000	7.303747	0.249699	4.093336
0.010000	6.636963	0.260554	4.299942
0.020000	3.524310	0.337441	6.010646
0.030000	2.424283	0.385786	7.386049

TABLE VII. (Contd.)

x	X_{tt}	α	K
$\beta = 150.0$ (Contd.)			
0.040000	1.853906	0.421020	8.594884
0.050000	1.502366	0.448671	9.701082
0.060000	1.262922	0.471380	10.737097
0.070000	1.088790	0.490618	11.722125
0.080000	0.956149	0.507288	12.668667
0.090000	0.851562	0.521986	13.585432
0.100000	0.766859	0.535123	14.478797
0.200000	0.369616	0.621375	22.849962
0.300000	0.227550	0.672560	31.297787

APPENDIX B

ALGOL Program for Calculations of Steady-state Two-phase Flow

A block diagram showing the main steps in the program is presented in Fig. 46. The following is a listing of the variables used in the program:

PX	channel exit pressure
EB	distance between previous mesh point and point where boiling starts
NBL	distance from channel inlet to point where boiling starts
QB	integrated heat input to channel up to point where boiling starts
HLB	enthalpy of liquid at point where boiling starts
ES	distance between previous mesh point and point where superheating of vapor starts
BL	length of two-phase region
QS	integrated heat input to channel up the point where superheating starts
TSB	saturation temperature at point where boiling starts
TSS	saturation temperature at point where superheating starts
FIL	integral of friction loss in liquid phase
FITP	integral of friction loss in two phase
FIG	integral of friction loss in gas phase
MIL	integral of momentum loss in liquid phase
MITP	integral of momentum loss in two phase
MIG	integral of momentum loss in gas phase
PM	maximum pressure parameter; stops calculation if exceeded
CO	convergence parameter
PIN	inlet pressure
DELZ	size of space step
TO	inlet coolant temperature
L	core length
R	reflector length
BLFR	blanket to core heat generation ratio
PI	π

VFC	coolant volume fraction
DR	fuel-rod diameter
DH	hydraulic diameter of channel
CL	heat capacity of liquid
CG	heat capacity of vapor
PHIMAX	maximum heat flux in channel
CONV	required convergence
I	index for problems with different flow rates
J	space-node index
K	index for inner iterations
N1	number of intervals in core
N2	number of intervals in reflector
N3	number of intervals in channel
JB	node immediately upstream of point where boiling starts
JS	node immediately upstream of point where superheating of vapor starts
P	pressure
Q[J]	integral of heat input to channel up to node J
TS	saturation temperature
T	coolant temperature
VIL	liquid viscosity
VIG	vapor viscosity
DL	liquid density
DG	vapor density
FRL	single-phase liquid friction factor
FRG	single-phase vapor friction factor
X	vapor quality
HL	liquid enthalpy
HVAP	heat of vaporization
B	ratio of liquid to vapor densities
MP	Martinelli flow parameter (X_{tt})
A	vapor void fraction

PHISQ	Martinelli two-phase friction multiplier (ϕ_l^2)
FRGRTP	two-phase friction pressure gradient
FRGRG	vapor friction pressure gradient
FRGRL	liquid friction pressure gradient
MOL	liquid momentum
MOTP	two-phase momentum
MOG	gas momentum
STTP	two-phase static pressure gradient
Z	distance from channel inlet
GT	channel mass flow rate

Conversion Factors:

$$2g = 1961.4 \text{ cm/sec}^2$$

$$\frac{P(\text{g/cm}^2)}{1034.0} = P(\text{atm})$$

112-6409

Fig. 46. Block Diagram of Steady-state Two-phase ALGOL Program

TABLE VIII. Listing of Steady-state
Two-phase ALGOL Program

```

#BEGIN# #REAL# K1,K2,K3,K4,K5,K6,PX,EB,NBL,QB,
      HLB,ES,BL,QS,TSS,F1G,M1G,
      F1TP,S1TP,M1TP,PM,CO,PIN,DELZ,
      TO,L,R,BLFR,P1,VFC,DR,DH,
      CG,CL,F1L,M1L,PM1MAX,TSB,CONVS
#INTEGER# I,J,K,N1,N2,N3,JB,JS,J1,J2$
#ARRAY# P(/0..200/), n(/0..200/), TS(/0..200/),
      T(/0..200/), V1L(/0..200/), V1G(/0..200/),
      DL(/0..200/), DG(/0..200/), FRL(/0..200/),
      MOL(/0..200/), FRGRL(/0..200/), X(/0..200/),
      HL(/0..200/), HVAP(/0..200/), B(/0..200/),
      MP(/0..200/), A(/0..200/), PH1SQ(/0..200/),
      FRGRTP(/0..200/), MOTP(/0..200/), STTP(/0..200/),
      FRG(/0..200/), FRGRG(/0..200/), MOG(/0..200/),
      Z(/0..200/), GT(/0..10/)$
      READ(N1,R,L,PM1MAX,BLFR,DR,VFC,TO,PX,
      PM,CG,CL,CONV)$
#FOR# I=1 #STEP# 1 #UNTIL# 10 #DO# READ(GT(/I/))$
WRITE(##STEADY STATE TWO PHASE PRESSURE PROFILE##)$
DELZ=L/N1$ #COMMENT# N1 MUST BE EVEN INTEGERS
N2=ENTIER(R/DELZ)$
N3=N1+2*N2$
PI=3.14159$
DH=CR*VFC/(1.0-VFC)$
K1=4.0*BLFR*PM1MAX*N3*DEI Z*(1.0-VFC)/
      (PI*DR*VFC)$
K2=PI/N3$
K3=K1/BLFR$
K4=COS(PI*N2/N3)$
K5=COS(PI*(N1+N2)/N3)$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
      Z(/J/)=J*DELZ$
#FOR# I=1 #STEP# 1 #UNTIL# 10 #DO#
#BEGIN#
      #FOR# J=0 #STEP# 1 #UNTIL# N2 #DO#
          Q(/J/)=K1*(1.0-COS(K2*J))/GT(/I/)$
          #FOR# J=N2+1 #STEP# 1 #UNTIL# N1+N2 #DO#
              Q(/J/)=Q(/N2/)+K3*(K4-COS(K2*J))/GT(/I/)$
          #FOR# J=N1+N2+1 #STEP# 1 #UNTIL# N3 #DO#
              Q(/J/)=Q(/N1+N2/)+K1*(K5-COS(K2*J))/GT(/I/)$
          #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
              P(/J/)=PX$
              K=0.0$
5..PIN=P(/0/)$
      #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
          TS(/J/)=12130.0/(10.51-LN(P(/J/)))-273.0$
          #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
              #BEGIN#
                  T(/J/)=TO+Q(/J/)/CL$
                  #IF# T(/J/)#GT# TS(/J/) #THEN# #GOTO# 10$
                  V1L(/J/)=0.01*EXP(756.9/(T(/J/)+273.0)-2.410)$
                  #COMMENT# LIQUID VISCOSITY IN POISES$
                  FRL(/J/)=0.1875/EXP(0.200*LN(DH*GT(/I/)/V1L(/J/)))$
                  #COMMENT# LIQUID FRICTION FACTORS$
                  DL(/J/)=0.9514 - 2.392*-04*T(/J/)$
                  #COMMENT# LIQUID DENSITY$
                  FRGRL(/J/)=FRL(/J/)*GT(/I/)**2/(1961.4*
                      DH*DL(/J/))+DL(/J/)$
                  MOL(/J/)=GT(/I/)**2/(980.7*DL(/J/))$
                  X(/J/)=0.0$
                  A(/J/)=0.0$
              #END#
          #END#

```

TABLE VIII. (Contd.)

```

JB=N3+2$
NBL=DELZ*N3$
BL=0.0$
  *GOTO#40$
10. EB=DELZ*(TS(/J-1/)-T(/J-1/))/(T(/J/)-T(/J-1/))*
    TS(/J-1/)-TS(/J/))$
    NBL=(J-1)*DELZ+EB$
    QB=(Q(/J/)-Q(/J-1/))*EB/DELZ+Q(/J-1/)$
    TSB=EB*(TS(/J/)-TS(/J-1/))/DELZ+TS(/J-1/)$
    HLB=0.3084*TSB-30.16$
    *COMMENT# LIQUID ENTHALPY AT START OF BOILINGS
    K6=FRGRL(/J-1/)-DL(/J-1/)$
    *COMMENT# LIQUID FRICTION GRADIENT AT START OF BOILINGS
    *IF# (J/2)*2-J#N0#0# THEN# JB=J+1 #ELSE#
      JB=JS$
      J1=JS$
    *FOR# J=J1 #STEP# 1 #UNTIL# N3 #DO#
      *BEGIN#
        HL(/J/)=0.3084*TS(/J/)-30.16$
        *COMMENT# LIQUID ENTHALPY$
        HVAP(/J/)=1115.5-0.225*TS(/J/)$
        *COMMENT# ENTHALPY OF VAPORIZATION$
        X(/J/)=(Q(/J/)-QB-HL(/J/)+HLB)/HVAP(/J/)$
        *IF# X(/J/)#G0#1.0 #THEN# #GOTO# 20$
        V1L(/J/)=0.01*EXP(756.9/(TS(/J/)+273.0)-2.410)$
        V1G(/J/)=8.75*-08*TS(/J/)+4.70*-05$
        *COMMENT# VAPOR VISCOSITY IN POISES$
        DL(/J/)=0.9514 - 2.392*-04*TS(/J/)$
        B(/J/)=2600.0/P(/J/)$
        *COMMENT# LIQUID/VAPOR DENSITY$
        MP(/J/)=EXP(0.90*LN(1.0/X(/J/))-1.0)+0.10*
          LN(V1L(/J/)/V1G(/J/))*SQRT(1.0/B(/J/))$
        A(/J/)=1.0-1.0/SQRT(1.0+21.0/MP(/J/)+1.0/
          MP(/J/)**2)$
        PH1SQ(/J/)=1.0+20.0/MP(/J/)+1.00/MP(/J/)**2$
        FRGRTP(/J/)=K6*EXP(1.8*LN(1.0-X(/J/)))*PH1SQ(/J/)$
        *COMMENT# TWO PHASE FRICTION PRESSURE GRADIENTS
        MOTP(/J/)=GT(/I/)**2*(B(/J/)*X(/J/)**2/A(/J/)+
          (1.0-X(/J/))**2/(1.0-A(/J/)))/(980.7*DL(/J/))$
        *COMMENT# TWO PHASE VELOCITY HEAD$
        STTP(/J/)=(A(/J/)/B(/J/)+1.0-A(/J/))*DL(/J/)$
        *COMMENT# TWO PHASE STATIC HEAD GRADIENTS
        T(/J/)=TS(/J/)$
      *END#
      JS=N3+2$
      BL=DELZ*N3$
      *GOTO# 80$
20. ES=DELZ*(1.0-X(/J-1/))/(X(/J/)-X(/J-1/))$
    BL=(J-1)*DELZ+ES$
    TSS=ES*(TS(/J/)-TS(/J-1/))/DELZ+TS(/J-1/)$
    QS=(Q(/J/)-Q(/J-1/))*ES/DELZ+Q(/J-1/)$
    *IF# (J/2)*2-J#N0#0# THEN# JS=J+1 #ELSE#
      JS=JS$
      J2=JS$
    *FOR# J=J2 #STEP# 1 #UNTIL# N3 #DO#
      *BEGIN#
        T(/J/)=TSS*(Q(/J/)-QS)/CG$
        V1G(/J/)=8.75*-08*T(/I/)+4.70*-05$
        FRG(/J/)=0.1875/EXP(0.200*LN(DH*GT(/I/)/V1G(/J/)))$
        DG(/J/)=0.3200*P(/J/)/(T(/J/)+273.0)$
        FRGRG(/J/)=FRG(/J/)*GT(/I/)**2/(1961.4*DH*DG(/J/))$

```

TABLE VIII. (Contd.)

```

#COMMENT# VAPOR FRICTION PRESSURE GRADIENTS
MOG(/J/)=GT(/I/)**2/(980.7*DG(/J/))$
#COMMENT# VAPOR VELOCITY HEADS
DL(/J/)=0.0$
X(/J/)=1.0$
A(/J/)=1.0$
#END#
F1G=0.0$
#FOR# J=N3-2 #STEP# -2 #UNTIL# JS #DO#
#BEGIN#
F1G=F1G+(DELZ/3.0)*(FRGRG(/J+2/)+
4.0*FRGRG(/J+1/)+FRGRG(/J/))$
M1G=MOG(/N3/)-MOG(/J/)$
P(/J/)=P(/N3/)+(F1G+M1G)/1034.0$
#IF# P(/J/)#GR# PM #THEN# #GOTO# 50$
#END#
P(/JS-2/)=2.0*P(/JS/)-P(/JS+2/)$
30..F1TP=0.0$
S1TP=0.0$
#FOR# J=JS-4 #STEP# -2 #UNTIL# JB #DO#
#BEGIN#
F1TP=F1TP+(DELZ/3.0)*(FRGRTP(/J+2/)+
4.0*FRGRTP(/J+1/)+FRGRTP(/J/))$
S1TP=S1TP+(DELZ/3.0)*(STTP(/J+2/)+
4.0*STTP(/J+1/)+STTP(/J/))$
M1TP=MOTP(/JS-2/)-MOTP(/J/)$
P(/J/)=P(/JS-2/)+(F1TP+S1TP+M1TP)/1034.0$
#IF# P(/J/)#GR# PM #THEN# #GOTO# 50$
#END#
P(/JB-2/)=P(/JB/)+(P(/JB/)-P(/JB+2/))/2.0$
40..F1L=0.0$
#FOR# J=JB-4 #STEP# -2 #UNTIL# 0 #DO#
#BEGIN#
F1L=F1L+(DELZ/3.0)*(FRGRL(/J+2/)+
4.0*FRGRL(/J+1/)+FRGRL(/J/))$
M1L=MOL(/JB-2/)-MOL(/J/)$
P(/J/)=P(/JB-2/)+(F1L+M1L)/1034.0$
#IF# P(/J/)#GR# PM #THEN# #GOTO# 50$
#END#
#FOR# J=1 #STEP# 2 #UNTIL# N3-1 #DO#
P(/J/)=(P(/J-1/)+P(/J+1/))/2.0$
K=K+1$
#IF# K #GR# 20 #THEN# #GOTO# 50$
CO=ABS((PIN-PX)/(P(/0/)-PX)-1.0)$
#IF# CO #GR# CONV #THEN# #GOTO# 5$
50..WRITE(##PROBLEM PARAMETERS##)$
PRINT(GT(/I/),PH1MAX,T0,VFC,DR,BLFR,TSS,TSS,
L,R,N1,K,NBL,RI,DEL7,CO,CONV)$
WRITE(##PRESSURE PROFILE##)$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
PRINT(Z(/J/),P(/J/),T(/J/),X(/J/),A(/J/),D_(/J/))$
#END#
#END#
#FOP#

```

APPENDIX C

ALGOL Program for Transient Solutions with Channels
Containing only a Two-phase Mixture

A block diagram showing the main steps in the program is presented in Fig. 47. The following is a listing of the additional variables used in this program which are not already listed in Appendix B:

TIME	total time from start of transient
DELT	time step
GO	coolant mass velocity at time zero
FI	integral of friction loss
MI	integral of momentum loss
XO	vapor quality at channel inlet
RIPER	reciprocal of period of exponential power increase
PDEC	constant for exponential pump head decay
DELPO	channel pressure drop at time zero
GAV1	channel average mass flow rate at t
GAV2	channel average mass flow rate at $t + \Delta t$
ESTX	value of X_2 [N3] for the inner iteration convergence test
LP	length of liquid column between pump and channel inlet
AR	ratio of liquid leg area to reactor core flow area
I	index for time steps
R2	extrapolation distance for chopped cosine axial power distribution
Q[J]	integral of heat input to channel up to node J at time zero divided by flow rate at time zero
DL1	liquid density at t
DL2	liquid density at $t + \Delta t$
X1	vapor quality at t
X2	vapor quality at $t + \Delta t$
GPHI	heat flux at time zero
GPHI1	heat flux at t
GPHI2	heat flux at $t + \Delta t$
HVA1	heat of vaporization at t

HVA2	heat of vaporization at $t + \Delta t$
HL1	liquid enthalpy at t
HL2	liquid enthalpy at $t + \Delta t$
G1	local mass flow rate at t
G2	local mass flow rate at $t + \Delta t$
A1	vapor void fraction at t
A2	vapor void fraction at $t + \Delta t$
DAV1	average density of two-phase mixture at t
DAV2	average density of two-phase mixture at $t + \Delta t$

112-6406

Fig. 47. Block Diagram of Transient Two-phase ALGOL Program

TABLE IX. Listing of Transient Two-phase ALGOL Program

```

*BEGIN* *REAL* K2,K6,K7,K8,PX,TIME,M,CO,
      CONV,PIN,DELZ,DELT,U3,L,R1,R2,BLFR,
      PI,VFC,DR,DH,CL,PHIMAX,G0,FI,MI,X0,
      RIPER,PDEC,DELP0,GAV1,GAV2,SSIG,
      ESTX,LP,ARS
*INTEGER* I,J,K,N1,N2,N3,N4,J1,J2$
*ARRAY* P(/0..200/), Q(/0..200/), T(/0..200/), VIL(/0..200/),
      DI1(/0..200/), DL2(/0..200/), FRL(/0..200/), FR3RL(/0..200/),
      X1(/0..200/), X2(/0..200/), GPHI1(/0..200/), HVA1(/0..200/),
      B(/0..200/), MP(/0..200/), PHISQ(/0..200/), FRGR(/0..200/),
      MO(/0..200/), VIG(/0..200/), HVA2(/0..200/), Z(/0..200/),
      HL1(/0..200/), HL2(/0..200/), GPHI2(/0..200/), MINT(/0..200/),
      G2(/0..200/), GPHI(/0..200/), A1(/0..200/), DAV1(/0..200/),
      DAV2(/0..200/), SIG(/0..200/), A2(/0..200/), SUSIG(/0..200/),
      G1(/0..200/), DEP1(/0..200/), DEP2(/0..200/), C(/0..200/)$
*LIST* OUT1 = *FOR* J=0 *STEP* 1 *UNTIL* N3 *DO*
      (Z(/J/),P(/J/),T(/J/),X2(/J/),A2(/J/),G2(/J/),DAV2(/J/))$
READ(N1,R1,R2,L,PHIMAX,BLFR,DR,VFC,X0,PX,
      CL,CONV,G0,DELT,RIPER,PDEC,LP,ARS)$
WRITE(*TWO PHASE CONVERGENCE TEST PROBLEM**)$
DELZ=L/N1$
N2=ENTIER(R1/DELZ)$
N3=N1+2*N2$
N4=ENTIER(R2/DELZ)$
PI=3.14159$
DH=DR*VFC/(1.0-VFC)$
K2=PI/(N3+2*N4)$
K6=4.0*PHIMAX*(1.0-VFC)/(DR*VFC)$
*FOR* J = 0 *STEP* 1 *UNTIL* N2-1 *DO*
      GPHI(/J/)=BLFR*K6*SIN(K2*(J+N4))$
*FOR* J = N2 *STEP* 1 *UNTIL* N1+N2 *DO*
      GPHI(/J/)=K6*SIN(K2*(J+N4))$
*FOR* J=N1+N2+1 *STEP* 1 *UNTIL* N3 *DO*
      GPHI(/J/)=BLFR*K6*SIN(K2*(J+N4))$
*FOR* J=0 *STEP* 1 *UNTIL* N3 *DO*
      P(/J/)=PX$
      X2(/0/)=X0$
      G(/0/)=0.0$
      K=0$
5..FIN=P(/0/)$
*FOR* J=0 *STEP* 1 *UNTIL* N3 *DO*
      *BEGIN*
      T(/J/)=12130.0/(10.51-LN(P(/J/)))-273.0$
      Z(/J/)=J*DELZ$
      DL2(/J/)=0.9514-2.392*-04*T(/J/)$
      *IF* J *GR* 0 *THEN*
      Q(/J/)=Q(/J-1/)+DELZ*(GPHI(/J/)+GPHI(/J-1/))/(2.0*G0)$
      HL2(/J/)=CL*T(/J/)$
      HVA2(/J/)=1115.5-0.225*T(/J/)$
      VIL(/J/)=0.01*EXP(756.9/(T(/J/)+273.0))-2.410$
      VIG(/J/)=8.75*-08*T(/J/)+4.70*-05$
      B(/J/)=2600.0/P(/J/)$
      *IF* J *GR* 0 *THEN*
      X2(/J/)=X2(/J-1/)+DELZ*(GPHI(/J/)+GPHI(/J-1/))/
      (G0*2.0*HVA2(/J/))$
      *IF* X2(/J/)*GR* 1.0 *THEN* *GOTO* 30$
      MP(/J/)=EXP(0.90*LN(1.0/X2(/J/)-1.0)+0.10*LN(VIL(/J/)/
      VIG(/J/)))*SQRT(1.0/B(/J/))$
      A2(/J/)=1.0-1.0/SQRT(1.0+21.0/MP(/J/))$
      DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
      DEP2(/J/)=DL2(/J/)*(1.0/B(/J/)+X2(/J/))$
      PHISQ(/J/)=1.0+20.0/MP(/J/)+1.0/MP(/J/)*2$
      FRL(/J/)=0.1875/EXP(0.200*LN(DH*G0/VIL(/J/)))$
      FR3RL(/J/)=FRL(/J/)*G0*2/(1961.4*DH*DL2(/J/))$
      FRGR(/J/)=FR3RL(/J/)*EXP(1.8*LN(1.0-X2(/J/)))*
      PHISQ(/J/)+DAV2(/J/)$

```


TABLE IX. (Contd.)

```

MO(/J/)=G0**2*(B(/J/)*X2(/J/)**2/A2(/J/)+(1.0-X2(/J/))**2/
(1.0-A2(/J/)))/(980.7*DL2(/J/))$
GPHI2(/J/)=GPHI(/J/)$
G2(/J/)=G0$
#END#
P(/N3/)=PX$
FI=0.0$
#FOR# J=N3-1 #STEP# -1 #UNTIL# 0 #DO#
#BEGIN#
FI=FI+DELZ*(FRGR(/J+1/)+FRGR(/J/))/2.0$
MI=MO(/N3/)-MO(/J/)$
P(/J/)=PX+(FI+MI)/1034.0$
#END#
K=K+1$
#IF# K #GR# 20 #THEN# #GOTO# 30$
CO=ABS((PIN-PX)/(P(/0/)-PX)-1.0)$
#IF# CO #GR# CONV #THEN# #GOTO# 5$
#COMMENT# INITIAL CONDITIONS ESTABLISHED$
WRITE (##PROBLEM PARAMETERS##)$
PRINT(G0,PHIMAX,VFC,DR,BLFR,L,R1,R2,
N1,DELZ,DELT,RIPER,PDEC,K)$
#FOR# J = 0 #STEP# 1 #UNTIL# N3 #DO#
PRINT(Z(/J/),P(/J/),T(/J/),X2(/J/),A2(/J/))$
DELP0=P(/0/)-PX$
GAV1=G0$
I=0$
K7=EXP(DELT*RIPER)$
K8=1.0$
GAV2=G0$
TIME=0.0$
10..I=I+1$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
#BEGIN#
GPHI1(/J/)=GPHI2(/J/)$
G1(/J/)=G2(/J/)$
G2(/J/)=G1(/J/)*GAV2/GAV1$
GPHI2(/J/)=K7*GPHI1(/J/)$
HVA1(/J/)=HVA2(/J/)$
X1(/J/)=X2(/J/)$
A1(/J/)=A2(/J/)$
HL1(/J/)=HL2(/J/)$
DL1(/J/)=DL2(/J/)$
DAV1(/J/)=DAV2(/J/)$
DEP1(/J/)=DEP2(/J/)$
#END#
L3=DELT/DELZ$
#FOR# J = 0 #STEP# 1 #UNTIL# N3 #DO#
#BEGIN#
T(/J/)=12130.0/(10.51-LN(P(/J/)))-273.0$
HL2(/J/)=CL*T(/J/)$
PL2(/J/)=0.9514-2.392*-04*T(/J/)$
HVA2(/J/)=1115.5-0.225*T(/J/)$
VIL(/J/)=0.01*EXP(756.9/(T(/J/)+273.0)-2.410)$
VIG(/J/)=8.75*-08*T(/J/)+4.70*-05$
B(/J/)=2600.0/P(/J/)$
#END#
K=0$
12..ESTX=X2(/N3/)$
SSIG=0.0$
SUSIG(/0/)=0.0$
#FOR# J=1 #STEP# 1 #UNTIL# N3 #DO#
#BEGIN#
A2(/J/)=A1(/J/)+(DELT*(GPHI2(/J/)+GPHI2(/J-1/))/HVA2(/J/)-
U3*(G1(/J/)*(X1(/J/)-X1(/J-1/))+G2(/J/)*(X2(/J/)-X2(/J-1/)))
/(DEP1(/J/)+DEP2(/J/))$
#IF# A2(/J/) #LS# 0.0 #THEN# #GOTO# 30$
#IF# A2(/J/) #GR# 1.0 #THEN# #GOTO# 30$
#END#

```

TABLE IX. (Contd.)

```

#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
  #BEGIN#
    MP(/J/)=21.0*(1.0-A2(/J/))*2/(1.0-(1.0-A2(/J/))*2)$
    C(/J/)=EXP(-1.11*LN(MP(/J/))-0.555*LN(B(/J/))+
      0.111*LN(VIL(/J/)/VIG(/J/)))$
    #IF# J #GR# 0 #THEN#
      X2(/J/)=C(/J/)/(1.0+C(/J/))$
      DEP2(/J/)=DL2(/J/)*(1.0/B(/J/)+X2(/J/))$
      DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
      PHISQ(/J/)=1.0+20.0/MP(/J/)+1.0/MP(/J/)**2$
      FRL(/J/)=0.1875/EXP(0.200*LN(DH*G2(/J/)/VIL(/J/)))$
      FRGRL(/J/)=FRL(/J/)*G2(/J/)**2/(1961.4*DH*DL2(/J/))$
      FRGR(/J/)=FRGRL(/J/)*EXP(1.8*LN(1.0-X2(/J/)))$
      PHISQ(/J/)+DAV2(/J/)$
      MO(/J/)=G2(/J/)**2*(B(/J/)*X2(/J/)**2/A2(/J/)+
        (1.0-X2(/J/))*2/(1.0-A2(/J/)))/(980.7*DL2(/J/))$
    #END#
  #FOR# J=1 #STEP# 1 #UNTIL# N3 #DO#
    #BEGIN#
      SIG(/J/)=(DAV1(/J/)-DAV2(/J/))/U3$
      SUSIG(/J/)=SIG(/J/)+SUSIG(/J-1/)$
      SSIG = SSIG+SUSIG(/J/)+SUSIG(/J-1/)$
    #END#
    G2(/0/)=GAV2-SSIG/(2.0*N3)$
    #FOR# J = 1 #STEP# 1 #UNTIL# N3 #DO#
      G2(/J/)=G2(/0/)+ SUSIG(/J/)$
      K=K+1$
      #IF# K #GR# 20 #THEN# #GOTO# 30$
      CO=ABS(1.0-ESTX/X2(/N3/))$
      #IF# CO #GR# CONV #THEN# #GOTO# 12$
      #IF# K #LS# 2 #THEN# #GOTO# 12$
      FI=0.0$
    #FOR# J=N3-1 #STEP# -1 #UNTIL# 0 #DO#
      #BEGIN#
        FI=FI+DELZ*(FRGR(/J+1/)+FRGR(/J/))/2.0$
        MI = MO(/N3/)-MO(/J/)$
        MINT(/J/)=FI+MI$
      #END#
      GAV1 = GAV2$
      TIME = TIME + DELT$
      K7=EXP((TIME+DELT)*RIPER)$
      K8=1034.0*DELPO*EXP(TIME*PDFC)$
      GAV2=GAV1+980.7*DELT*(K8-MINT(/0/))/(N3*DELT+
        LP*AR)$
      #IF# GAV2 #LS# 0.0 #THEN#
        #BEGIN#
          J2=N3/2$
          WRITE(##NEG. GAV2##)$
          #GOTO# 32$
        #END#
      #FOR# J = N3-1 #STEP# -1 #UNTIL# 0 #DO#
        P(/J/)=(MINT(/J/)+(GAV2-GAV1)*(N3-J)*U3/980.7)/
          1034.0+PX$
        J2=N3$
        #GOTO# 32$
      30.. J2=J$
      32..WRITE(##TIME VARYING PARAMETERS##)$
        PRINT(I,K,CO,CONV,K7,K8,GAV1,GAV2,TIME,DELT)$
        WRITE(##PRESSURE PROFILE##)$
        OUTPUT(61,##(7E12.4)##,OUT1)$
        #IF# J2 #EQ# N3 #THEN# #GOTO# 10$
      40.. #END#
      #EOP#

```

APPENDIX D

ALGOL Program for Transient Solutions with
Single-phase, Two-phase Channel with Heat Transfer from Fuel

1: Program Listing

A block diagram showing the main steps in the program is presented in Fig. 48. The following is a listing of the additional variables used in this program which are not already listed in Appendix B or C:

NBL1	distance from inlet to point where boiling starts at t
NBL2	distance from inlet to point where boiling starts at $t + \Delta t$
QMAX	maximum rate of fuel heat generation at time zero
HO	enthalpy at coolant inlet
TM1, TM2, TM3, TES, TES2	} parameters used for control of motion of boiling interface through the space mesh
DTS	rise in saturation temperature from exit to inlet of channel
C1, C2, C3	constants in convection heat transfer correlation
KF	thermal conductivity of fuel
DF	density of fuel
CF	heat capacity of fuel
DELR	size of radial space increments in fuel rod
CC1, CC2, CC3, CC4, CC5	coefficients in difference equations for fuel heat conduction
RC	combined thermal resistance of fuel cladding and fuel-cladding gap
PO	parameter determining frequency of print-out of results during calculation
M	index for radial nodes in fuel
N5	number of radial nodes in fuel
N6	number of constant-size space steps in two-phase region before starting variable space steps
N7	maximum number of minimum-size space steps in the largest space step in the two-phase region
Q	volumetric heat generation in fuel at $t = 0$

Q1	volumetric heat generation in fuel at t
Q2	volumetric heat generation in fuel at $t + \Delta t$
TF1	fuel temperature at t
TF2	fuel temperature at $t + \Delta t$
SCD, SC1, SC2, SC3, SC4	} coefficients in finite-difference equations at surface of fuel rod
H	heat transfer coefficient for convection film
HE	equivalent overall coefficient between fuel and coolant
TSC	temperature of fuel cladding surface
KL	coolant thermal conductivity

2. Comparison with the ARGUS Code

In order to insure that there were no errors in the part of the program which calculates the fuel and coolant temperatures before boiling starts, a test problem was run using both the ALGOL program listed in Table X and the ARGUS code.¹⁶ The ARGUS code has been in use at Argonne and elsewhere for several years, and its usefulness in calculating transient fuel and coolant temperatures is generally accepted.

In the test problem the coolant flow rate was held constant while the heat-generation rate in the fuel increased exponentially on a period of 0.25 sec. The time up to coolant boiling was about 0.80 sec. During this time the temperature at the coolant outlet increased from 334 to 945°C while the fuel center temperature at the channel outlet increased from 376 to 1176°C. At this time the greatest difference between the two calculations was in the temperatures at the coolant outlet, with the ARGUS value being lower by about 2%. The ARGUS value for the fuel center temperature was 0.3% lower than that given by the ALGOL program. These differences are quite acceptable in view of the fact that a different finite-difference method was used to represent the coolant convection Eq. (34) in the present work than in ARGUS. The "box method" representation used in the ALGOL program has the advantage that the calculated coolant temperature profiles are somewhat smoother than those obtained with the method used in ARGUS.

3. Convergence of Inner Iterations with Two-phase Hydrodynamics Equations

An indication of the rate of convergence of the inner iterations with the two-phase continuity, energy, and vapor void correlation equations is given by the data in Table XI. In the iterative scheme the convergence is

112-6408

Fig. 48. Block Diagram of Transient Single-phase, Two-phase ALGOL Program

TABLE X. Listing of Transient Single-phase, Two-phase ALGOL Program

```

#BEGIN# #REAL# K1,K2,K3,K4,K5,K6,K7,K8,PX,TIME,
E8,NBL1,NBL2,TSR,PM,CO,CONV,PIN,DELZ,DELT,U1,U2,U3,
T0,L,R1,R2,RLFR,PI,VFC,DR,DH,CL,QMAX,H0,LP,AR,TH1,
G0,F1,M1,RIPFR,PDEC,DELPO,GAV1, TES, TES2, TM2, TM3,
GAV2,SSIG,DTS, FSTX,C1,C2,C3,KF,DF,CF,DELR,CC1,CC2,CC3,CC4,
CC5,RC$
#INTEGER# I,J,K,N1,N2,N3,N4,JB,J1,J2,J3,J4,P0,M,N5,N6,N7$
#ARRAY# P(/0..200/),Q(/0..200/),TS(/0..200/),T(/0..200/),
VIL(/0..200/),DL1(/0..200/),DL2(/0..200/),FRL(/0..200/),
FRGR(/0..200/),X1(/0..200/),X2(/0..200/),SCD(/0..200/),
HVA1(/0..200/),R(/0..200/),MP(/0..200/),PHISQ(/0..200/),
FRGR(/0..200/),MO(/0..200/),VIG(/0..200/),HVA2(/0..200/),
Z(/0..200/),HL1(/0..200/),HL2(/0..200/),DEP1(/0..200/),
DEP2(/0..200/),G2(/0..200/),A1(/0..200/),MINT(/0..200/),
DAV1(/0..200/),DAV2(/0..200/),SIG(/0..200/),A2(/0..200/),
SUSIG(/0..200/),C(/0..200/),G1(/0..200/),TF1(/0..10,0..200/),
TF2(/0..10,0..200/),Q1(/0..200/),Q2(/0..200/),SC1(/0..200/),
SC2(/0..200/),SC3(/0..200/),SC4(/0..200/),HE(/0..200/),
TSC(/0..200/),KI(/0..200/),H(/0..200/)$
#LIST# OUT1= #FOR# J=N #STEP# 1 #UNTIL# N3 #DO#
(Z(/J/),P(/J/),T(/J/),TS(/J/),TSC(/J/),TF2(/N5,J/),TF2(/0,J/),X2(/J/),
A2(/J/),G2(/J/))$
READ(N1,R1,R2,L,QMAX,RLFR,DR,VFC,T0,PX,P0,CL,CONV,G0,DELT,RIPFR,PDEC,
C1,C2,C3,KF,DF,CF,RC,N5,LP,AR,TH1,TH2,TH3,N6,N7,DTS)$
WRITE(##TRANSIENT TWO PHASE HYDRODYNAMICS PROBLEM##)$
DELZ=L/N1$
N2=ENTIER(R1/DELZ)$
N3=N1+2*N2$
N4=ENTIER(R2/DELZ)$
PI=3.14159$
DH=DR+VFC/(1.0-VFC)$
K2=PI/(N3+2*N4)$
K4=COS(PI*N2/N3)$
K5=4.0*(1.0-VFC)/(DR+VFC)$
K6=(1.0-VFC)/VFC$
DELR=DR/(2.0*N5)$
#FOR# J=0 #STEP# 1 #UNTIL# N2-1 #DO#
Q(/J/)=QMAX*RLFR*SIN(K2*(J+N4))$
#FOR# J=N2 #STEP# 1 #UNTIL# N1+N2 #DO#
Q(/J/)=QMAX*SIN(K2*(J+N4))$
#FOR# J=N1+N2+1 #STEP# 1 #UNTIL# N3 #DO#
Q(/J/)=QMAX*RLFR*SIN(K2*(J+N4))$
HL1(/0/)=CL+T0$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
#BEGIN#
#IF# J #GR# 0 #THEN#
HL1(/J/)=HL1(/J-1/)+DELZ*K6*(Q(/J/)+Q(/J-1/))/(2.0*G0)$
Z(/J/)=J*DELZ$
T(/J/)=HL1(/J/)/CL$
VIL(/J/)=0.01*EXP(756.9/(T(/J/)+273.0)-2.410)$
#COMMENT# LIQUID VISCOSITY IN POISES$
KL(/J/)=0.215-1.15*-04*T(/J/)$
G2(/J/)=G0$
H(/J/)=KL(/J/)*(C1*EXP(C2*LN(DH*ABS(32(/J/)))*
CL/KL(/J/))+C3)/DH$
TSC(/J/)=T(/J/)+Q(/J/)*DR/(4.0*H(/J/))$
TF2(/N5,J/)=TSC(/J/)+RC*Q(/J/)*DR/4.0$
#FOR# M=N5-J #STEP# -1 #UNTIL# 0 #DO#
TF2(/M,J/)=TF2(/M+1,J/)+Q(/J/)*DELR**2*(2*M+1)/(4.0*KF)$
FRL(/J/)=0.1875/EXP(0.200*LN(DH*G0/VIL(/J/)))$
#COMMENT# LIQUID FRICTION FACTORS$
DL1(/J/)=0.9514-2.392*-04*T(/J/)$
#COMMENT# LIQUID DENSITY$
HL2(/J/)=HL1(/J/)$
DL2(/J/)=DL1(/J/)$
DAV2(/J/)=DL1(/J/)$
FRGR(/J/)=FRL(/J/)*G0**2/(1961.4*DH*DL1(/J/))+DL1(/J/)$
MO(/J/)=G0**2/(980.7*DL1(/J/))$
#END#

```

TABLE X (Contd.)

```

P(/N3/)=PX$
FI =0.0$
#FOR# J=N3-1 #STEP# -1 #UNTIL# 0 #DO#
  #BEGIN#
    FI =FI +DELZ*(FRGR (/J+1/)+FRGR(/J/))/2.0$
    MI =MO (/N3/)-MO (/J/)$
    MINT(/J/)=FI+MI$
    P(/J/)=PX+(FI +MI )/1034.0$
  #END#
  #COMMENT# INITIAL CONDITIONS ESTABLISHED$
WRITE(##PROBLEM PARAMETERS##)$
PRINT(G0, QMAX,T0,VFC,DR,BLFR,L,R1,R2,N1,DELZ,DELT,RIPER,PDEC,
      TM1,TM2,TM3)$
WRITE(##INITIAL PRESSURE PROFILE##)$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
  PRINT(Z(/J/),P(/J/),T(/J/),TSC(/J/),TF2(/N5,J/),TF2(/0,J/))$
DELP0=P(/0/)-PX$
TS(/N3/)=12130.0/(10.51-LN(PX))-273.0$
#FOR# J=0 #STEP# 1 #UNTIL# N3-1 #DO#
  TS(/J/)=TS(/N3/)+(N3-J)*DTS/N3$
GAV1=G0$
I=0$
K7=EXP(DELT*RIPER)$
GAV2=G0$
JB=0$
TIME=0.0$
5..I=I+1$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
  #BEGIN#
    Q1(/J/)=Q2(/J/)$
    G1(/J/)=G2(/J/)$
    G2(/J/)=G1(/J/)+GAV2/GAV1$
    Q2(/J/)=K7*Q(/J/)$
    HVA1(/J/)=HVA2(/J/)$
    X1(/J/)=X2(/J/)$
    A1(/J/)=A2(/J/)$
    HL1(/J/)=HL2(/J/)$
    DL1(/J/)=DL2(/J/)$
    DAV1(/J/)=DAV2(/J/)$
    #FOR# M=0 #STEP# 1 #UNTIL# N5 #DO#
      TF1(/M,J/)=TF2(/M,J/)$
      DEP1(/J/)=DEP2(/J/)$
    #END#
    NBL1=NBL2$
    U1=G1(/0/)*DELT/DELZ$
    U2=G2(/0/)*DELT/DELZ$
    U3=DELT/DELZ$
    SUSIG(/0/)=0.0$
    #FOR# J=1 #STEP# 1 #UNTIL# N3 #DO#
      HL2(/J/)=((DL1(/J/)-U1)*HL1(/J/)+(DL1(/J-1/)+U1)*HL1(/J-1/))
        -(DL1(/J-1/)-U2)*HL2(/J-1/)+DELT*K5*(H(/J/)*(TSC(/J/)-T(/J/))+
        H(/J-1/)*(TSC(/J-1/)-T(/J-1/)))/(DL1(/J/)+U2)$
    #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
      #BEGIN#
        T(/J/)=HL2(/J/)/CLS
        #IF# T(/J/) #GQ# TS(/J/) #THEN# #GOTO# 10$
        DL2(/J/)=0.9514-2.392# -04*T(/J/)$
        VIL(/J/)=0.01*EXP(756.9/(T(/J/)+273.0)-2.410)$
        FRL(/J/)=0.1875/EXP(0.200*LN(DH*ABS(G2(/J/))/VIL(/J/)))$
        FRGR (/J/)=FRL(/J/)*G2(/J/)*ABS(G2(/J/))/
          (1961.4*DH*DL2(/J/))+DL2(/J/)$
        MO (/J/)=G2(/J/)*2/(980.7*DL2(/J/))$
        SUSIG(/J/)=0.0$
        DAV1(/J/)=DL1(/J/)$
        DAV2(/J/)=DL2(/J/)$
        X2(/J/)=0.0$
        A2(/J/)=0.0$
      #END#

```

TABLE X (Contd.)

```

NBL2=DELZ*N3$
#GOTO# 20$
10..J8=J8+1$
  EB=DELZ*(TS(/J-1/)-T(/J-1/))/(T(/J/)-T(/J-1/)+
    TS(/J-1/)-TS(/J/))$
  NBL2=(J-1)*DELZ+ERS$
  TS8=EB*(TS(/J/)-TS(/J-1/))/DELZ+TS(/J-1/)$
  J1=J$
  J3=J1+N6$
  HVA2(/J1-1/)=1130.0-0.225*TS(/J /)$
  #FOR# J=J1 #STEP# 1 #UNTIL# N3 #DO#
    #BEGIN#
      X2(/J/)=X1(/J/)$
      HL2(/J/)=CL*TS(/J /)$
      #COMMENT# LIQUID ENTHALPY IN BOILING REGIONS
      HVA2(/J/)=1130.0-0.225*TS(/J /)$
      #COMMENT# ENTHALPY OF VAPORIZATIONS
      DL2(/J/)=0.9514-2.392# -04*TS(/J /)$
      VIL(/J/)=0.01*EXP(786.9/(TS(/J /)+273.0))-2.410)$
      #IF# VIL(/J/) #IS# 0.0 #THEN# #GOTO# 30$
      VIG(/J/)=1.18# -07*TS(/J /)+1.22# -04$
      #COMMENT# VAPOR VISCOSITY IN POISES$
      #IF# VIG(/J/) #IS# 0.0 #THEN# #GOTO# 30$
      R(/J/)=2600.0/P(/J/)$
      #COMMENT# LIQUID/VAPOR DENSITY$
      DEP2(/J/)=DL2(/J/)*(1.0/B(/J/)+X2(/J/))$
    #END#
    DEP1(/J1/)=DEP2(/J1/)$
    #IF# J1 #LS# N3 #THEN#
      G2(/J1/)=(G2(/J1+1/)+G2(/J1-1/))/2.0$
    K=0$
12..ESTX=X2(/N3/)$
  SSIG=0.0$
  #FOR# J=J1 #STEP# 1 #UNTIL# J3 #DO#
    #BEGIN#
      #IF# J #GR# N3 #THEN# #GOTO# 14$
      #IF# J #EQ# J1 #THEN# K1=0.0 #ELSE# K1=1.0$
      A2(/J/)=A1(/J/)+(DELT*K5*(1.0+K1)*H(/J/)*(TSC(/J/)-T(/J/)))/
        HVA2(/J/)-U3*(G1(/J/)*(X1(/J/)-X1(/J-1/))+G2(/J/)*
          (X2(/J/)-X2(/J-1/)))/(DEP1(/J/)*K1+DEP2(/J/))$
      #IF# A2(/J/) #GO# 1.0 #THEN# #GOTO# 30$
      #IF# A2(/J/) #LS# 0.0 #THEN# #GOTO# 30$
    #END#
    #IF# J3 #EQ# N3 #THEN# #GOTO# 14$
    J4= N3-ENTIER((N3-J3)/N7)*N7$
    #IF# J4 #EQ# J3 #THEN# #GOTO# 13$
    A2(/J4/)=A1(/J4/)+(DELT*K5*2.0*H(/J4/)*(TSC(/J4/)-T(/J4/)))/
      HVA2(/J4/)-U3*(J4-J3)*(G1(/J4/)*(X1(/J4/)-X1(/J3/))+G2(/J4/)*
        (X2(/J4/)-X2(/J3/)))/(DEP1(/J4/)+DEP2(/J4/))$
    #IF# A2(/J4/) #GO# 1.0 #THEN# #GOTO# 30$
    #IF# A2(/J4/) #LS# 0.0 #THEN# #GOTO# 30$
13.. #FOR# J=J4+N7 #STEP# N7 #UNTIL# N3 #DO#
  #BEGIN#
    A2(/J/)=A1(/J/)+(DELT*K5*2.0* H(/J/)*(TSC(/J/)-T(/J/)))/
      HVA2(/J/)-U3/N7*(G1(/J/)*(X1(/J/)-X1(/J-N7/))+G2(/J/)*
        (X2(/J/)-X2(/J-N7/)))/(DEP1(/J/)+DEP2(/J/))$
    #IF# A2(/J/) #GO# 1.0 #THEN# #GOTO# 30$
    #IF# A2(/J/) #LS# 0.0 #THEN# #GOTO# 30$
  #END#

```


TABLE X (Contd.)

```

14..#FOR# J=J1 #STEP# 1 #UNTIL# J3 #DO#
  #BEGIN#
    #IF# J #GR# N3 #THEN# #GOTO# 16$
    MP(/J/)=21.0*(1.0-A2(/J/))*2/(1.0-(1.0-A2(/J/))*2)$
    C(/J/)=EXP(-1.111*LN(MP(/J/))-0.555*LN(B(/J/))+
      0.111*LN(VIL(/J/)/VIG(/J/)))$
    X2(/J/)=C(/J/)/(1.0+C(/J/))$
    DEP2(/J/)=DL2(/J/)*(1.0/B(/J/)+X2(/J/))$
    DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
    SIG(/J/)=(DAV1(/J/)-DAV2(/J/))/U3$
    SUSIG(/J/)=SIG(/J/)+SUSIG(/J-1/)$
    SSIG=SSIG+SUSIG(/J/)+SUSIG(/J-1/)$
  #END#
  #IF# J3 #EQ# N3 #THEN# #GOTO# 16$
  #IF# J4 #EQ# J3 #THEN# #GOTO# 15$
  MP(/J4/)=21.0*(1.0-A2(/J4/))*2/(1.0-(1.0-A2(/J4/))*2)$
  C(/J4/)=EXP(-1.111*LN(MP(/J4/))-0.555*LN(B(/J4/))+
    0.111*LN(VIL(/J4/)/VIG(/J4/)))$
  X2(/J4/)=C(/J4/)/(1.0+C(/J4/))$
  DEP2(/J4/)=DL2(/J4/)*(1.0/B(/J4/)+X2(/J4/))$
  DAV2(/J4/)=DL2(/J4/)*A2(/J4/)*(1.0/B(/J4/)-1.0)+DL2(/J4/)$
  SIG(/J4/)=(DAV1(/J4/)-DAV2(/J4/))*(J4-J3)/U3$
  SUSIG(/J4/)=SIG(/J4/)+SUSIG(/J3/)$
  SSIG=SSIG+(SUSIG(/J4/)+SUSIG(/J3/))*(J4-J3)$
15.. #FOR# J=J4+N7 #STEP# N7 #UNTIL# N3 #DO#
  #BEGIN#
    MP(/J/)=21.0*(1.0-A2(/J/))*2/(1.0-(1.0-A2(/J/))*2)$
    C(/J/)=EXP(-1.111*LN(MP(/J/))-0.555*LN(B(/J/))+
      0.111*LN(VIL(/J/)/VIG(/J/)))$
    X2(/J/)=C(/J/)/(1.0+C(/J/))$
    DEP2(/J/)=DL2(/J/)*(1.0/B(/J/)+X2(/J/))$
    DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
    SIG(/J/)=(DAV1(/J/)-DAV2(/J/))*N7/U3$
    SUSIG(/J/)=SIG(/J/)+SUSIG(/J-N7/)$
    SSIG=SSIG+(SUSIG(/J/)+SUSIG(/J-N7/))*N7$
  #END#
  #FOR# J=J3+1 #STEP# 1 #UNTIL# J4-1 #DO#
    #BEGIN#
      A2(/J/)=A2(/J3/)+(A2(/J4/)-A2(/J3/))*(J-J3)/(J4-J3)$
      X2(/J/)=X2(/J3/)+(X2(/J4/)-X2(/J3/))*(J-J3)/(J4-J3)$
      DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
      MP(/J/)=21.0*(1.0-A2(/J/))*2/(1.0-(1.0-A2(/J/))*2)$
      SUSIG(/J/)=SUSIG(/J3/)+(SUSIG(/J4/)-SUSIG(/J3/))*(J-J3)/
        (J4-J3)$
    #END#
  #FOR# M=J4 #STEP# N7 #UNTIL# N3-N7 #DO#
    #FOR# J=M+1 #STEP# 1 #UNTIL# M+N7-1 #DO#
      #BEGIN#
        A2(/J/)=A2(/M/)+(A2(/M+N7/)-A2(/M/))*(J-M)/N7$
        X2(/J/)=X2(/M/)+(X2(/M+N7/)-X2(/M/))*(J-M)/N7$
        DAV2(/J/)=DL2(/J/)*A2(/J/)*(1.0/B(/J/)-1.0)+DL2(/J/)$
        MP(/J/)=21.0*(1.0-A2(/J/))*2/(1.0-(1.0-A2(/J/))*2)$
        SUSIG(/J/)=SUSIG(/M/)+(SUSIG(/M+N7/)-SUSIG(/M/))*(J-M)/N7$
      #END#
    #FOR# J=J1 #STEP# 1 #UNTIL# N3 #DO#
      #BEGIN#
        PHISQ(/J/)=1.0+20.0/MP(/J/)+1.0/MP(/J/)**2$
        FRL(/J/)=0.1875/EXP(0.200*LN(DH*ABS(G2(/J/))/VIL(/J/)))$
        FRGRL(/J/)=FRL(/J/)*G2(/J/)*ABS(G2(/J/))/(1961.4*DH*DL2(/J/))$
        FRGR (/J/)=FRGRL(/J/)*EXP(1.8*LN(1.0-X2(/J/)))*PHISQ(/J/)+
          DAV2(/J/)$
        #COMMENT# TWO PHASE FRICTION PRESSURE GRADIENT$
        MO (/J/)=G2(/J/)**2*(B(/J/)*X2(/J/)**2/A2(/J/)+(1.0-X2(/J/))
          **2/(1.0-A2(/J/)))/(980.7*DL2(/J/))$
        #COMMENT# TWO PHASE VELOCITY HEAD$
        T(/J/)=TS(/J/)$
      #END#

```

TABLE X (Contd.)

```

G2(/0/)=GAV2-SSIG/(2.0*N3)$
#FOR# J=1 #STEP# 1 #UNTIL# N3 #DO#
  G2(/J/)=G2(/0/)+SUSIG(/J/)$
K=K+1$
#IF# K #GR# 30 #THEN# #GOTO# 30$
CO=ABS(1.0-ESTX/X2(/N3/))$
#IF# CO #GR# CONV #THEN# #GOTO# 12$
#FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
20..FI=0.0$
  #FOR# J=N3-1 #STEP# -1 #UNTIL# 0 #DO#
    #BEGIN#
      FI=FI+DELZ*(FRGR(/J+1/)+FRGR(/J/))/2.0$
      MI=MO(/N3/)-MO(/J/)$
      MINT(/J/)=FI+MI$
    #END#
  CC1=DELT*KF/(DELR**2*DF*CF)$
  CC2=1.0-2.0*CC1$
  CC3=DELT/(2.0*CF*DF)$
  CC4=4.0*CC1$
  CC5=1.0-CC4$
  #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
    #BEGIN#
      HE(/J/)=H(/J/)/(1.0+RC*H(/J/))$
      SCD(/J/)=4.0*N5-2.0+4.0*N5*HE(/J/)*
      DELR/KF+4.0*N5/CC1-1.0/CC1$
      SC1(/J/)=(4.0*N5-2.0)/SCD(/J/)$
      SC2(/J/)=(2.0-4.0*N5-4.0*N5*HE(/J/)*DELR/KF
      +4.0*N5/CC1-1.0/CC1)/SCD(/J/)$
      SC3(/J/)=4.0*N5*HE(/J/)*DELR/(KF*SCD(/J/)*CL)$
      SC4(/J/)=(4.0*N5-1.0)*DELT/(CC1*DF*CF*2.0*
      SCD(/J/))$
    #END#
  #FOR# J=0 #STEP# 1 #UNTIL# N3 #DO#
    #BEGIN#
      #FOR# M=N5-1 #STEP# -1 #UNTIL# 1 #DO#
        TF2(/M,J/)=CC1*(2*M-1)/(2*M)*TF1(/M-1,J/)+
        CC2*TF1(/M,J/)+CC1*(2*M+1)/(2*M)*
        TF1(/M+1,J/)+CC3*(Q2(/J/)+Q1(/J/))$
        TF2(/0,J/)=CC5*TF1(/0,J/)+CC4*TF1(/1,J/)+
        CC3*(Q2(/J/)+Q1(/J/))$
        TF2(/N5,J/)=SC1(/J/)*(TF2(/N5-1,J/)+TF1(/N5-1,J/))+
        SC2(/J/)*TF1(/N5,J/)+SC3(/J/)*(HL2(/J/)+HL1(/J/))+
        SC4(/J/)*(Q2(/J/)+Q1(/J/))$
        TSC(/J/)=(RC*H(/J/)*T(/J/)+TF2(/N5,J/))/
        (1.0+H(/J/)*RC)$
        KL(/J/)=0.215-1.15*-04*T(/J/)$
        H(/J/)=KL(/J/)*(C1*EXP(C2*LN(DH*ABS(G2(/J/)))*
        CL/KL(/J/))+C3)/DH$
      #END#
    GAV1=GAV2$
    TIME=TIME+DELT$
    #IF# JB #EQ# 0 #THEN#
      #BEGIN#
        TES=T(/N3/)+T(/N3/)-HL1(/N3/)/CL$
        #IF# TES #GR# TS(/N3/) #THEN#
          #BEGIN#
            DELT=0.95*DELT*(TS(/N3/)-T(/N3/))/(TES-T(/N3/))$
            JR=1$
          #END#
          #GOTO# 22$
        #END#
      #IF# JB #EQ# 1 #THEN#
        #BEGIN#
          TES=T(/N3/)+T(/N3/)-HL1(/N3/)/CL$
          TES2=T(/N3-1/)+T(/N3-1/)-HL1(/N3-1/)/CL$
          FR=DELZ*(TS(/N3-1/)-TES2)/(TES-TES2+TS(/N3-1/)-TS(/N3/))$
          DELT=TM1*DELT*DELZ/(DELZ-EB)$
        #END#

```

TABLE X (Contd.)

```

#IF# JB #EQ# 2 #THEN#
  DELT=TM2 *DELT*(DEI Z+ER)/(NBL1-NBL2)$
#IF# JB #GR# 2 #THEN#
  DELT=TM3*DELT*(DELZ+ER)/(NBL1-NBL2)$
22..K7=EXP((TIME+DELT)*R1PER)$
  K8=1034.0*DELPO*EXP( TIME*PDEC)$
  GAV2=GAV1+980.7*DELT*(K8-M.INT(/O/))/(N3*DELZ+LP*AR)$
#IF# GAV2 #LS# 0.0 #THEN#
  #BEGIN#
    J2=N3/2$
    WRITE(##NEG. GAV2##)$
    #GOTO# 32$
  #END#
#FOR# J=N3-1 #STEP# -1 #UNTIL# 0 #DO#
  P(/J/)=(MINT(/J/)+(GAV2-GAV1)*(N3-J)*DELZ/(DELT*980.7))/
  1034.0+PX$
  J2=N3$
  #IF# X2(/N3/) #GR# 0.0 #THEN# #GOTO# 32$
  #IF# JB #EQ# 1 #THEN# #GOTO# 32$
  #IF# I/PO #NQ# ENTIER(I/PO) #THEN# #GOTO# 5 #ELSE# #GOTO# 32$
30..J2=J$
32..WRITE(##TIME VARYING PARAMETERS##)$
  PRINT(I,K,CO,CONV,K7,K8,NRL2,TSB,GAV1,DELT,TIME,JB)$
  WRITE(##PRESSURE PROFILE##)$
  OUTPUT(61,##( 10E12.4)##,OUT1)$
  #IF# J2 #EQ# N3 #THEN# #GOTO# 5$
40.. #END#
  #EOP#

```

TABLE XI. Values for x , α , and G during Inner Iterations with Two-phase Hydrodynamics Equations*

Z	$10^4 x_j^i$			α_j^i			G_j^i		
	Iteration No.								
	1	9	10**	1	9	10**	1	9	10**
127	0	0	0	0	0	0			183.2
128	2.255	2.008	2.008	0.0555	0.0504	0.0504	287.9	366.7	366.7
129	5.774	5.443	5.443	0.1171	0.1120	0.1120	378.5	593.9	593.9
130	9.508	9.153	9.153	0.1690	0.1646	0.1646	598.2	798.4	798.3
131	13.35	12.97	12.97	0.2141	0.2102	0.2102	795.5	979.2	979.1
132	17.49	17.07	17.07	0.2523	0.2486	0.2486	967.4	1115.8	1115.8
133	21.63	21.16	21.16	0.2905	0.2869	0.2869	1111.7	1252.4	1252.4
134	25.78	25.26	25.26	0.3287	0.3253	0.3253	1256.0	1389.1	1389.1
135	30.24	29.67	29.67	0.3625	0.3593	0.3593	1369.0	1494.2	1494.2
136	34.71	34.08	34.08	0.3965	0.3933	0.3933	1481.9	1599.3	1599.3
137	39.18	38.48	38.48	0.4304	0.4273	0.4273	1594.9	1704.4	1704.4
138	44.45	43.71	43.70	0.4811	0.4783	0.4782	1666.6	1772.6	1772.9
139	49.73	48.95	48.92	0.5318	0.5293	0.5292	1738.3	1840.9	1841.5
140	55.00	54.19	54.14	0.5826	0.5802	0.5801	1810.0	1909.2	1910.1

* The values in this table are taken from the calculation for the $\omega = 0.10$ sec power transient reported in Chapter VI.

**Converged values.

based on the value of x_j^i at the channel-exit node point. Hence, when the criterion

$$1.0 - \left| \frac{(x_{j=n}^i)^{k+1}}{(x_{j=n}^i)^k} \right| < \text{CONV},$$

where CONV is the desired convergence, is satisfied the calculation advances one time step. In practice this was found to be satisfactory since, as shown in Table XI, the values at the channel exit converge slowest. Consequently, when the $x_{j=n}^i$ value reaches the desired convergence, all the other x_j^i 's are converged to an even greater accuracy. A value of $\text{CONV} = 10^{-3}$ was used throughout this work.

The speed of convergence of the inner iterations is sensitive to the size of the time steps and, also, the functional relationship between x and α as given by the vapor void fraction correlation Eq. (A10). A limited amount of experimenting was done with the modified Balzhiser void fraction correlation Eq. (A18). It was found that the convergence was generally slower when using this correlation although convergence was still obtained after a greater number of iterations. This is undoubtedly due to the stronger coupling between x and α which it gives and which results in larger changes from iteration to iteration. It was found that the $(x_j^i)^k$ values at a point tended to oscillate above and below the final converged value with steadily decreasing amplitude as the number of iterations increased.

APPENDIX E

Physical Properties of Sodium

In the earlier part of this work the physical properties of sodium were taken from Dunning.⁴⁶ Equations for calculating the physical properties were obtained by plotting the data on the appropriate graph paper (semilog, log-log or cartesian) which gave a linear presentation vs either temperature or pressure. The equation of the straight line representing the particular property was then obtained graphically and used in the computer program. Later, data for sodium compiled by Tokar⁴⁷ from a number of more recent literature sources⁴⁸⁻⁵⁰ became available and, hence, were used to derive corrected equations for the various properties. These corrected equations did not differ greatly from the original ones and, therefore, it was not necessary to repeat any of the calculations performed up to that time. The corrected equations were used, however, in obtaining all the results reported in Chapter VI.

The corrected equations and their ranges of application are:

1. Liquid Density

$$\rho_l (\text{gm/sec}) = 0.9514 - 2.392 \times 10^{-4} T$$

$$\text{for } 100 < T < 1300^\circ\text{C} \quad (\text{E1})$$

2. Ratio of Liquid to Vapor Density (on saturation curve)

$$\beta = 2600/p$$

$$\text{for } 0.10 < p < 20 \text{ atm} \quad (\text{E2})$$

3. Vapor Density (superheated)

$$\rho_g (\text{gm/cc}) = 0.3200 \frac{P}{T + 273}$$

$$\text{for } 900 < T < 1300^\circ\text{C and } 1 < p < 15 \text{ atm} \quad (\text{E3})$$

4. Saturation Temperature

$$TS (^\circ\text{C}) = \frac{12130.0}{10.51 - \ln p} - 273$$

$$\text{for } 0.10 < p < 20 \text{ atm} \quad (\text{E4})$$

5. Liquid Viscosity

$$\mu_l (\text{poises}) = 0.01 \exp\left(\frac{756.9}{T + 273} - 2.410\right)$$

$$\text{for } 100 < T < 1300^\circ\text{C} \quad (\text{E5})$$

6. Vapor Viscosity

$$\mu_g (\text{poises}) = 1.18 \times 10^{-7} T + 1.22 \times 10^{-4}$$
$$\text{for } 400 < T < 1300^\circ\text{C} \quad (\text{E6})$$

7. Heat of Vaporization

$$\lambda (\text{cal/gm}) = 1130.0 - 0.225T$$
$$\text{for } 200 < T < 1300^\circ\text{C} \quad (\text{E7})$$

8. Liquid Heat Capacity

$$C_l (\text{cal/gm-}^\circ\text{C}) = 0.3084$$
$$\text{for } 100 < T < 1300^\circ\text{C} \quad (\text{E8})$$

9. Vapor Heat Capacity

$$C_g (\text{cal/gm-}^\circ\text{C}) = 0.215$$
$$\text{for } 100 < T < 1300^\circ\text{C} \quad (\text{E9})$$

10. Liquid Thermal Conductivity

$$k (\text{cal/sec-cm-}^\circ\text{C}) = 0.215 - 1.15 \times 10^{-4} T$$
$$\text{for } 200 < T < 1200^\circ\text{C} \quad (\text{E10})$$