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STEADY TEMPERATURE DISTRIBUTIONS IN THE FAR-FIELD REGION OBTAINED BY SOLUTION OF THE EQUATION OF CONVECTIVE DIFFUSION IN TWO DIMENSIONS

Lawrence Dresner

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DIFFUSION IN TWO DIMENSIONS

Lawrence Dresner

APRIL 1973

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FOREWORD

Oak Ridge National Laboratory is providing technical assistance to the U.S. Atomic Energy Commission Directorate of Licensing in its analyses of the environmental impact of power reactors as required by the National Environmental Policy Act. The assessment of the environmental impact of a nuclear station has required the development of new analytical techniques and the expansion of many existing ones.

This document is one of the spin-off benefits of this effort by the Commission and its National Laboratories to protect the environment. The work reported here, while originally completed in support of the analysis of a particular nuclear plant, has wide application in other areas.

Steady Temperature Distributions in the Far-Field Region Obtained by Solution of the Equation of Convective Diffusion in Two Dimensions

Lawrence Dresner

ABSTRACT

A method based on conformal mapping is suggested for calculating steady, far-field temperature distributions in two dimensions in an arbitrary potential flow field. Two numerical programs for performing the calculations are described and listed, and sample computations are given.

Keywords: diffusion, thermal pollution, heat transfer, convection, model, analytical model, computer program.

INTRODUCTION

A number of authors have used the equation of convective diffusion [Eq. (3) below] to calculate steady-state, far-field temperature rises caused by the release of waste heat into bodies of water. ("Far field" means that region in which transport by ambient turbulence is much greater than transport by turbulence generated by the discharge itself.) A short review of the literature on this subject may be found in the article of Harleman and Stolzenbach.¹ The problem has two aspects: (1) the determination of the appropriate values of the eddy diffusivities and the velocity components and (2) the solution of the equation of convective diffusion for the temperature field. Edinger and Polk² and Csanady³ considered the second aspect under the assumption that the eddy diffusivities and the velocity components were constant and gave simple analytic solutions to the equation of convective diffusion. Both sets of authors ignored longitudinal diffusion, that is, diffusion in the direction of the flow velocity. This report generalizes their results in the *two-dimensional case* to an *arbitrary potential flow field* and includes longitudinal diffusion as well. This latter inclusion is of little practical importance, since longitudinal diffusion is small except close to the source [see discussion following Eq. (24)]; but it is of great theoretical importance, because without the longitudinal diffusion term, the equation of convective diffusion loses the all-important property of conformal invariance. The generalization to an arbitrary two-dimensional potential flow field, on the other hand, is of great practical importance, because situations may arise in which the flow field can hardly be considered uniform. Examples of such situations will be given later.

BASIC EQUATIONS

The heat flux q ($\text{cal cm}^{-2} \text{ sec}^{-1}$) is related to the temperature gradient ∇T and the flow velocity v as follows:

$$q = -k \nabla T + \rho c_p v T, \quad (1)$$

1. D. F. Harleman and K. D. Stolzenbach, "Fluid Mechanics of Heat Disposal," *Ann. Rev. Fluid Mech.* 4; M. van Dyke, W. G. Vincenti, and J. V. Wehausen (eds.), *Annual Reviews, Inc.*, Palo Alto, Calif., pp. 7-32, 1972.

2. J. E. Edinger and E. M. Polk, Jr., "Initial Mixing of Thermal Discharge into a Uniform Current," in Part I of *Heated-Effluent Dispersion in Large Lakes: State-of-the-Art of Analytical Modeling*, A. J. Policastro and J. V. Tokar, ANL/ES-11, Argonne National Laboratory, Center for Environmental Studies, pp. 173-88 (January 1972).

3. G. T. Csanady, "Offshore Outfall and Shoreline Surface Discharge Models for the Prediction of Thermal-Plume Dispersion in the Great Lakes," in Part I of *Heated-Effluent Dispersion in Large Lakes: State-of-the-Art of Analytical Modeling*, A. J. Policastro and J. V. Tokar, ANL/ES-11, Argonne National Laboratory, Center for Environmental Studies, pp. 189-206 (January 1972).

where k is thermal conductivity (including the contribution from turbulent eddies and assumed constant) ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$), ρ is fluid density (g/cm^3), and c_p is heat capacity of the fluid ($\text{cal g}^{-1} \text{ deg}^{-1}$). In steady state, a differential heat balance at any point can be written

$$\nabla \cdot \mathbf{q} = S, \quad (2)$$

where S is heat source density ($\text{cal cm}^{-3} \text{ sec}^{-1}$). Since $\nabla \cdot \mathbf{v} = 0$ in incompressible flow, Eqs. (1) and (2) can be combined to give

$$-k \nabla^2 T + \rho c_p \mathbf{v} \cdot \nabla T = S. \quad (3)$$

A flow may be called two dimensional if, for example, the flow velocity $\mathbf{v}$ depends on only two coordinates, say x and y , and not on the third coordinate z . If the heat source S also depends only on x and y and not on z , and if the bounding surfaces are cylinders whose generators are parallel to the z axis, then the temperature T also depends only on x and y and not on z . Such a temperature field may also be called two dimensional. This report is concerned exclusively with such two-dimensional flow and temperature fields.

One very important heat source is a line parallel to the z axis. The intersection of this source with any plane parallel to the x, y plane is a point. Because we want to reserve the term "line source" for something else, we call such a source a point source after its intersection with the x, y plane. Another important heat source is a segment of a plane parallel to the z axis whose intersection with any plane parallel to the x, y plane is a line segment. We call such a source a line source after its intersection with the x, y plane.

A line source may be approximated by a closely spaced collection of point sources. If the point sources are of equal strengths, the line source is uniform. The point sources need not be of equal strengths, however, in which case the source strength of the line source is not uniform but is weighted in some way along the line. One important weighting is a Gaussian weighting, since the Gaussian distribution arises when we try to join a far-field solution to the near field created by a single jet.

CONFORMAL INVARIANCE

A most important property of Eq. (3) is its invariance to conformal transformations when $\mathbf{v}$ is the velocity of an incompressible, irrotational flow. A conformal transformation is a *two-dimensional* transformation from the coordinates x, y to coordinates ξ, η which obeys certain conditions. The transformation

$$\xi = \xi(x, y), \quad (4a)$$

$$\eta = \eta(x, y), \quad (4b)$$

is conformal if

$$\xi_x = \eta_y, \quad (5a)$$

$$\xi_y = -\eta_x, \quad (5b)$$

where the subscripts denote partial differentiation. Equations (5a and 5b) are called the Cauchy-Riemann equations. It follows from Eqs. (5) that

$$\xi_{x,x} + \xi_{y,y} = 0, \quad (6a)$$

$$\eta_{x,x} + \eta_{y,y} = 0, \quad (6b)$$

$$\xi_x \eta_x + \xi_y \eta_y = 0, \quad (6c)$$

$$\xi_x^2 + \xi_y^2 = \eta_x^2 + \eta_y^2. \quad (6d)$$

Let us now consider any function f of x, y ; f can be considered a function of ξ, η as well because of Eqs. (4). Then, differentiating, we find

$$f_x = f_\xi \xi_x + f_\eta \eta_x, \quad (7a)$$

$$f_y = f_\xi \xi_y + f_\eta \eta_y. \quad (7b)$$

Differentiating again, we get

$$f_{xx} = (f_{\xi\xi} \xi_x + f_{\xi\eta} \eta_x) \xi_x + f_{\xi\xi} \xi_{xx} + (f_{\eta\xi} \xi_x + f_{\eta\eta} \eta_x) \eta_x + f_{\eta\eta} \eta_{xx}, \quad (8a)$$

$$f_{yy} = (f_{\xi\xi} \xi_y + f_{\xi\eta} \eta_y) \xi_y + f_{\xi\xi} \xi_{yy} + (f_{\eta\xi} \xi_y + f_{\eta\eta} \eta_y) \eta_y + f_{\eta\eta} \eta_{yy}. \quad (8b)$$

Adding Eqs. (8a) and (8b), we find

$$f_{xx} + f_{yy} = (f_{\xi\xi} + f_{\eta\eta})(\xi_x^2 + \xi_y^2). \quad (9)$$

If we have two such functions, f and g , it follows from Eqs. (7a, b) that

$$f_x g_x + f_y g_y = (f_\xi g_\xi + f_\eta g_\eta)(\xi_x^2 + \xi_y^2). \quad (10)$$

An infinitesimal area in the ξ, η plane is related to one in the x, y plane according to the rule

$$d\xi d\eta = \frac{\partial(\xi, \eta)}{\partial(x, y)} dx dy = (\xi_x^2 + \xi_y^2) dx dy. \quad (11)$$

A point source at position x_0, y_0 has the form

$$S = Q \delta(x - x_0, y - y_0), \quad (12)$$

where Q has the dimensions $\text{cal cm}^{-1} \text{ sec}^{-1}$, and δ is Dirac's delta function. Delta functions transform under change of coordinates according to

$$\delta(x - x_0, y - y_0) dx dy = \delta(\xi - \xi_0, \eta - \eta_0) d\xi d\eta, \quad (13a)$$

which, in view of Eq. (11), can be written

$$\delta(x - x_0, y - y_0) = \delta(\xi - \xi_0, \eta - \eta_0)(\xi_x^2 + \xi_y^2). \quad (13b)$$

When the flow is incompressible and irrotational, v can be written as the gradient of a potential ϕ :

$$v = \nabla \phi, \quad (14a)$$

$$\nabla^2 \phi = 0. \quad (14b)$$

Using Eqs. (14) we can write Eq. (3) in two dimensions for a point source as

$$k(T_{xx} + T_{yy}) - \rho c_p(\phi_x T_x + \phi_y T_y) + Q \delta(x - x_0, y - y_0) = 0. \quad (15a)$$

The equation for ϕ (14b) is then

$$\phi_{xx} + \phi_{yy} = 0. \quad (15b)$$

From Eqs. (9), (10), and (13b), we see that (15a) and (15b) have exactly the same form in terms of ξ and η that they have in terms of x and y ; that is,

$$k(T_{\xi\xi} + T_{\eta\eta}) - \rho c_p(\phi_\xi T_\xi + \phi_\eta T_\eta) + Q \delta(\xi - \xi_0, \eta - \eta_0) = 0, \quad (16a)$$

$$\phi_{\xi\xi} + \phi_{\eta\eta} = 0. \quad (16b)$$

Furthermore, the boundary conditions are likewise conformally invariant. The hydraulic boundary condition is that the velocity be parallel to boundary surfaces; the thermal boundary condition is that the isotherms be perpendicular to boundary surfaces (which are taken impervious to heat). Since the angle between two curves is preserved in conformal mapping, the boundary conditions are conformally invariant. Thus, if we have any solution to a problem of convective diffusion with potential flow, heat-impervious boundaries, and point sources, we get another solution of the same type by conformal transformation.

POINT SOURCE IN A UNIFORMLY FLOWING, INFINITE MEDIUM

We solve Eq. (15a) in an infinite medium. For convenience we locate the point source at the origin ($x_0 = 0, y_0 = 0$). Furthermore, we choose the x axis to point in the direction of flow. Then (15a) becomes

$$k(T_{xx} + T_{yy}) - \rho c_p v T_x + Q \delta(x, y) = 0, \quad (17)$$

where v is the speed of the uniform flow. We try a solution of the form

$$T = T_\infty + A \exp(vx/2\alpha) R(vr/\alpha), \quad (18a)$$

where

$$\alpha = k/\rho c_p = \text{thermal diffusivity (cm}^2/\text{sec)}, \quad (18b)$$

$$r = (x^2 + y^2)^{1/2}, \quad (18c)$$

T_∞ is the temperature of the flowing medium far from the source, and A is a constant of integration. Substitution of Eq. (18a) into (17) yields the differential equation for R :

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} - \left(\frac{v}{2\alpha} \right)^2 R = 0, \quad (19)$$

valid for $r > 0$. Equation (19) is Bessel's equation of order zero when written for the variable $ivr/2\alpha$; hence the solution we seek is a zero-order Bessel function of imaginary argument. It must vanish when $r \rightarrow \infty$, and it must be singular at $r = 0$. This identifies the solution as the zero-order Hankel function of the first kind:

$$R = iH_0^{(1)}(ivr/2\alpha). \quad (20)$$

The factor i outside has been added to make R real.

The constant A is determined by conditions near the source. For

$$\mathbf{q} = Ak \exp(vx/2\alpha)(v/2\alpha) [H_0^{(1)'}(ivr/2\alpha) \mathbf{e}_r + iH_0^{(1)}(ivr/2\alpha) \mathbf{e}_x] + \rho c_p v T_\infty \mathbf{e}_x, \quad (21)$$

where $\mathbf{e}_x$ and $\mathbf{e}_r$ are unit vectors in the x and r directions, respectively. Then

$$Q = \lim_{r \rightarrow 0} (r \int_0^{2\pi} \mathbf{q} \cdot \mathbf{e}_r d\beta) = 4Ak, \quad (22)$$

where $\beta = \arg(\mathbf{r})$ is the polar angle of $\mathbf{r}$. Here we have used the relations

$$iH_0^{(1)}(iy) \approx \frac{2}{\pi} \ln \left(\frac{2}{\gamma y} \right), \quad y \ll 1, \quad (23a)$$

$$H_0^{(1)'}(iy) = -H_1^{(1)}(iy) \approx 2/\pi y, \quad y \ll 1, \quad (23b)$$

where γ is the exponential of Euler's constant. Finally, then

$$T - T_\infty = \frac{Q}{4k} \exp \left(\frac{vx}{2\alpha} \right) iH_0^{(1)} \left(\frac{ivr}{2\alpha} \right). \quad (24)$$

Equation (24) gives the temperature distribution due to a point source of strength Q at the origin in a uniformly flowing infinite medium.

Typical values of α are of the order of magnitude of 10 ft²/sec, and typical values of v are of the order of 1 ft/sec. The characteristic length $2\alpha/v$ is then of the order of 20 ft. Far-field calculations generally apply at distances much greater than 20 ft, so that $vr/2\alpha$ is generally $\gg 1$. Now

$$iH_0^{(1)}(iy) \sim (2/\pi y)^{1/2} e^{-y}, \quad y \gg 1, \quad (23c)$$

so that

$$T - T_{\infty} \sim \frac{Q}{4k} (4\alpha/\pi vr)^{1/2} \exp[-v(r-x)/2\alpha]. \quad (24a)$$

When $vr/2\alpha \gg 1$, $v(r-x)/2\alpha$ is only small when r and x are nearly the same. Now $r-x = y^2/2x$ to lowest order in y , with increasing accuracy for fixed $y^2/2x$ as x increases. Hence, to lowest order in y ,

$$T - T_{\infty} = \frac{Q}{4k} (4\alpha/\pi vx)^{1/2} \exp(-vy^2/4\alpha x). \quad (24b)$$

This is the result quoted by Edinger and Polk² when longitudinal diffusion is neglected [i.e., when the term T_{xx} is dropped in (15a)]. It is only valid far from the source, but this suffices for practical purposes.

USE OF THE VELOCITY POTENTIAL AND THE STREAM FUNCTION AS COORDINATES

Let us now consider a source $Q \delta(x-x_0, y-y_0)$ in an arbitrary prescribed potential flow field. The transformation from x, y to $\phi(x, y), \psi(x, y)$, where ϕ is the (known) velocity potential and ψ is the (known) stream function of the flow, is conformal. Hence we can choose ϕ and ψ to be ξ and η respectively. Equation (16a) then becomes

$$k(T_{\phi\phi} + T_{\psi\psi}) - \rho c_p T_{\phi} + Q \delta(\phi - \phi_0, \psi - \psi_0) = 0. \quad (25)$$

Equation (25) has the same form as Eq. (17) when $v = 1$. We know the solution to the latter equation in an infinite medium [Eq. (24)], but this solution does not immediately apply to practical problems for the following reason. In practical flow problems the flow is usually bounded by one or more solid boundaries; in fact, it is often the presence of these solid boundaries that causes the flow field to be nonuniform. On these solid boundaries the stream function has constant values. If the flow is confined to a simply connected region between two such boundaries, then in the ϕ, ψ plane the flow is confined to a strip parallel to the ϕ axis. It is within this strip that we must solve Eq. (25).

The isotherms are always perpendicular to solid boundaries, the latter being taken impervious to heat. Since angles are preserved by conformal mapping, the isotherms are also perpendicular to the sides of the strip in the ϕ, ψ plane. Let $\psi_1 < \psi_2$ be the values of the stream function on the solid boundaries. Then the boundary conditions on T at $\psi = \psi_1$ and $\psi = \psi_2$ are

$$T_{\psi} = 0 \text{ at } \psi = \psi_1 \text{ and } \psi = \psi_2. \quad (26)$$

We can satisfy the conditions in Eq. (26) if we put image sources at the points

$$\phi = \phi_0, \quad (27a)$$

$$\psi = 2\psi_1 - \psi_0, \psi_0 \pm 2n(\psi_2 - \psi_1), 2\psi_1 - \psi_0 \pm 2n(\psi_2 - \psi_1), \quad (27b)$$

where $n = 1, 2, 3, \dots$. To prove that these images suffice we must show that the system of image sources together with the real source lie symmetrically around the lines $\psi = \psi_1$ and $\psi = \psi_2$. To do this it is enough

to break up the system of sources into pairs, each of whose ψ values add up to $2\psi_1$ or $2\psi_2$. The sets of pairs that add up to $2\psi_1$ are $\psi_0, 2\psi_1 - \psi_0$ and $\psi_0 + 2n(\psi_2 - \psi_1), 2\psi_1 - \psi_0 - 2n(\psi_2 - \psi_1)$, where $n = \pm 1, \pm 2, \dots$. The sets of pairs that add up to $2\psi_2$ are $\psi_0 + 2n(\psi_2 - \psi_1), 2\psi_2 - \psi_0 - (2n - 2)(\psi_2 - \psi_1)$, where $n = 0, \pm 1, \pm 2, \dots$. This means that the solution of Eqs. (25) and (26) we seek in the strip $\psi_1 < \psi < \psi_2$ is the sum of the infinite medium solutions (24) for its real source and all its image sources; that is,

$$T - T_\infty = \sum_{n=-\infty}^{+\infty} \left\{ F\left[\phi - \phi_0, \psi - (\psi_0 + 2n\{\psi_2 - \psi_1\})\right] + F\left[\phi - \phi_0, \psi - (2\psi_1 - \psi_0 + 2n\{\psi_2 - \psi_1\})\right] \right\}, \quad (28)$$

where $F(x, y)$ is the function of x and y defined by the right-hand side of Eq. (24) when $v = 1$.

To facilitate rapid evaluation of temperature rises from Eq. (28), a computer program has been written in the BASIC language for the PDP-10 time-share system at ORNL. The program is named STRIP, and it calculates the quantity $\theta = 4k(T - T_\infty)/Q$ at any point in an arbitrary two-dimensional potential flow field. The program is written in the conversational mode, and an order to RUN will begin a conversation with the computer in which the user is instructed how to proceed.

The program requests the user to supply the following information:

1. Two functions of position specifying, respectively, the velocity potential and the stream function of the flow.
2. The abscissa and ordinate of a point on each boundary. Since the boundaries must be streamlines (on which $\psi = \text{constant}$), one point on each boundary is all that is needed to calculate ψ_1 and ψ_2 .
3. The eddy diffusivity.
4. The position of the source. Either a point source or a line source can be handled. The line source is specified by giving the position of its end points and is actually handled by the program as an array of $N1$ equally spaced point sources each of strength $Q/N1$. (Presently $N1 = 100$, but any value can be used.) Should a weighting factor be desired along the line source, one can be introduced by a comparatively single reprogramming of STRIP.

When the above information has been supplied, the program responds with a query as to whether the user wants the value of θ at a field point. If the answer is yes, the program responds with a request for the abscissa and ordinate of the field point, and upon receipt of these data responds with the value of θ . The user is then queried again as to whether he wants the value of θ at a field point. If the answer is no, the program stops, but a message is presented saying a new case may be run by giving the order RUN.

The program will print an error message if either the source or the field point does not lie between the boundaries.

The location of the origin of coordinates is arbitrary.

Owing to the complexity of formula (28), it cannot be inverted easily, and isotherms cannot be calculated directly. The best way to calculate isotherms is to use STRIP to obtain temperature rises at a lattice of points and then graphically interpolate for the desired temperature rise. Figure 1 shows isotherms obtained in this way for the flow in a 5° converging wedge. Figure 2 shows isotherms for the flow around a half cylinder. A listing of the program and a typical conversation with the computer are given in the Appendix.

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Fig. 1. Isotherms in the converging flow in a 5° wedge.

Fig. 2. Isotherms in the flow around a half cylinder.

SURFACE COOLING

Surface heat exchange can be expressed in terms of an exchange coefficient K ($\text{cal cm}^{-2} \text{ sec}^{-1} \text{ deg}^{-1}$) in the following way according to Harleman and Stolzenbach:¹

$$q = K(T - T_{\infty}), \quad (29)$$

where q is the heat flux *out* of the surface of the body of water, and T is the surface temperature. If we assume complete vertical mixing in the body of water we are studying (in order to keep the problem two dimensional), the heat loss (29) appears as a distributed heat sink with a local strength $(K/d)(T - T_{\infty})$, where d is the (uniform) depth; $(K/d)(T - T_{\infty})$ has the dimensions $\text{cal cm}^{-3} \text{ sec}^{-1}$ as a source or sink should. With this sink term, Eq. (3) for a point source can be written

$$-k\nabla^2(\delta T) + \rho c_p \mathbf{v} \cdot \nabla(\delta T) = Q \delta(x - x_0, y - y_0) - (K/d)(\delta T), \quad (30)$$

where $\delta T = T - T_{\infty}$.

In a uniformly flowing, infinite medium Eq. (30) can also be solved by a substitution of the form (18a). Now the function R satisfies the equation

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} - \left(\frac{v}{2\alpha} \right)^2 \left(1 + \frac{4K\alpha}{\rho c_p d v^2} \right) R = 0. \quad (31)$$

We again identify R with the zero-order Hankel function of the first kind, so that

$$T - T_{\infty} + \frac{Q}{4k} \exp\left(\frac{vx}{2\alpha}\right) i H_0^{(1)} \left[\frac{ivr}{2\alpha} \left(1 + \frac{4K\alpha}{\rho c_p d v^2} \right)^{1/2} \right], \quad (32)$$

the constant A having again been determined by a calculation similar to that following Eq. (21).

According to Harleman and Stolzenbach,¹ typical values for K range from 100 to 200 $\text{Btu ft}^{-2} \text{ day}^{-1} (^\circ\text{F})^{-1}$. Choosing $K = 150 \text{ Btu ft}^{-2} \text{ day}^{-1} (^\circ\text{F})^{-1}$, $\alpha = 10 \text{ ft}^2/\text{sec}$, and $v = 1 \text{ ft/sec}$, we find

$$\frac{4K\alpha}{\rho c_p d v^2} = 4.45 \times 10^{-6} \quad (33)$$

for a typical depth d of 25 ft. When $vr/2\alpha \gg 1$, Eq. (32) becomes

$$T - T_{\infty} + \frac{Q}{4k} (4\alpha/v)^{1/2} \exp[-v(r-x)/2\alpha] \left(1 + \frac{4K\alpha}{\rho c_p v^2 d} \right)^{-1/4} \exp\left(\frac{Kr}{\rho c_p d v} \right) \quad (34)$$

by again using (23c). The last two factors in Eq. (34) represent corrections to (24a) to account for surface cooling. The first correction is very close to 1, and for practical purposes it can be ignored. The second factor is less than a 10% correction when $r \leq \rho c_p d v / 10K = 9.00 \times 10^4 \text{ ft} = 17 \text{ miles}$. At this distance, the center-line temperature rise is $0.00436 (Q/4k)$, and the half-width of the plume $(4\alpha/v)^{1/2} = 1900 \text{ ft}$. If the total rate of heat release is 1000 MW, $Q/4k = 15.2^\circ\text{F}$ with the values of α and d we have been using. The center-line temperature rise is then 0.0661°F . These estimates make it clear that surface cooling may sometimes be neglected in the calculation of isotherms of practical importance.

JOINING THE FAR-FIELD SOLUTION TO THE NEAR-FIELD SOLUTION

The near field is often created by a single jet or by a row of jets. The row of jets produces a distribution of excess temperature that is uniform over a line, and the far field produced by a line source can be directly calculated using STRIP. When the near field is created by a single jet, the distribution of excess temperature can often be described with very good accuracy by a Gaussian function. Such a Gaussian source may be used in STRIP by using a line source with Gaussian weights. This involves a rather simple reprogramming of STRIP as mentioned before. However, even this reprogramming can be avoided under certain circumstances, described below, by replacing the Gaussian temperature distribution by a virtual point source suitably located.

Let us first consider a uniform flow field flowing in the x direction. If the heat source S is distributed, then

$$\delta T(x, y) = \int \delta T_0(x - x', y - y') \frac{S(x', y')}{Q} dx' dy', \quad (35)$$

where δT_0 is the function on the right-hand side of (24). Now

$$\frac{S(x, y)}{Q} = \frac{\exp(-y^2/b^2)}{b\sqrt{\pi}} \delta(x) \quad (36)$$

represents a Gaussian source distributed in the y direction (across the flow) with a half-width at $1/e \times$ maximum of b and centered on the origin of coordinates. If we replace δT_0 by its asymptotic form (24b), then (34) becomes the same as (24b) except that x is replaced by $x + b^2 v/4\alpha$. For the right-hand side of (35) is then proportional to the convolution of two normalized Gaussians, which by DeMoivre's theorem is again a normalized Gaussian whose half-width is the square root of the sum of the squares of the half-widths of the two convoluted Gaussians. Thus the far field can be obtained by placing a virtual point source of total strength Q a distance $b^2 v/4\alpha$ upstream of the center of the Gaussian temperature distribution created by the jet. The same trick can be used in a nonuniform flow field as long as the flow velocity is relatively constant for a distance $\sim b^2 v/4\alpha$ upstream of the source. Typical values of b are of the order of 25 ft, so that $b^2 v/4\alpha = 16$ ft for the figures we have been using. The assumption of the constancy of the flow velocity over such a short distance is often quite good. The use of (24b) implies that the nearest boundary is at least several units of b from the virtual source.

HALF-SPACE WITH A BREAKWATER

The Schwarz-Christoffel transformation

$$(z - a)^2 = [\zeta - (a^2 + c^2)^{1/2}]^2 - c^2, \quad (37)$$

where a and c are real numbers, maps the upper half-plane in the ζ plane ($\zeta = \xi + i\eta$) onto the upper half-plane in the z plane ($z = x + iy$) with an obstacle (breakwater) of length c occurring in the z plane at $x = a$ (cf., e.g., Fig. 3). Let the x axis be the shore, and let the offshore flow be parallel to the x axis for $y \gg c$. We can calculate the temperature at a point (x, y) due to a line source of strength Q at (x_0, y_0) as follows: Using the transformation (37) we find the coordinates of the image points (ξ, η) and (ξ_0, η_0) of (x, y) and (x_0, y_0) respectively. In the ζ plane, the flow is uniform and parallel to the ξ axis everywhere in the upper

Fig. 3. Isotherms in the flow field around a breakwater.

half-plane. The ξ axis, being the image of the x axis (shore) together with the breakwater, is perpendicular to the isotherms in the ζ plane and therefore represents the shore there. If we place an image source of strength Q in the ζ plane at $(\xi_0, -\eta_0)$, the temperature rise we seek is the sum of those produced individually at (ξ, η) by sources at (ξ_0, η_0) and $(\xi_0, -\eta_0)$, the individual contributions being calculated with Eq. (24).

A second BASIC program named FARFLD has been written to facilitate calculations based on the above ideas. Whereas STRIP calculates a dimensionless temperature rise and requires no source strength as input, FARFLD calculates temperature rises in degrees Fahrenheit and requires data on the source as input.

The command RUN causes FARFLD to ask for the following quantities:

1. the volume flow rate through the condensor, ft^3/sec ;
2. the temperature rise in the condenser, $^\circ\text{F}$;
3. the eddy diffusivity, ft^2/sec ;
4. the offshore flow velocity, ft/sec ;
5. the depth, ft ;
6. the position of the foot of the breakwater and the length of the breakwater (if one is present), ft .

Either a point or a line source is allowed. If a point source is elected, FARFLD asks for its coordinates in feet; if a line source is elected, FARFLD asks for the coordinates of its end points.

When the above information has been supplied, the program responds with a query as to whether the user wants the temperature rise at a field point. If the answer is yes, the program responds with a request for the coordinates of the field point and, upon receipt of these data, responds with the temperature rise in

degrees Fahrenheit. The user is then queried again as to whether he wants the temperature rise at a field point. If the answer is no, the program stops with a message saying a new case may be run by giving the order RUN.

Again isotherms are best calculated by the procedure previously outlined in connection with STRIP.

Figure 3 shows the results of a typical calculation done using FARFLD. A listing of the program as well as a typical conversation with the computer may be found in the Appendix.

ADDITIONAL REMARKS

Sometimes the longitudinal and lateral eddy diffusivities are not the same. As we saw from the discussion following Eq. (24), longitudinal diffusion is not important except close to the source (where a far-field calculation does not apply anyway). Little error is then incurred if we replace the true value of the longitudinal eddy diffusivity by the value of the lateral eddy diffusivity. When this replacement is made, the methods of this report can be employed. Thus, when the longitudinal and lateral diffusivities are different, k should have the value of the lateral diffusivity.

The results in this report can be used not only for the diffusion of heat, but also for the diffusion of dissolved substances. Any equation in this report can be transcribed for the diffusion of dissolved substances if the unit of heat (calorie) is replaced in the definition of S and Q by a unit of amount, for example, mole, and if the quantity $\rho c_p \delta T$ is replaced by the concentration of the dissolved substance in the corresponding units, for example, moles/cm³ when the unit of amount is mole. There is no surface exchange of nonvolatile dissolved substances; but if the dissolved substance is radioactive, a distributed sink exists with a strength proportional to the concentration. The surface heat loss (29) was also treated as a distributed sink, and the formulas of the section of surface cooling can be rewritten for a dissolved radioactive substance by making the changes noted above and by replacing $\rho c_p d/K$ by the mean life of the decay. When, typically, $K = 150 \text{ Btu ft}^{-2} \text{ day}^{-1} (\text{°F})^{-1}$ and $d = 25 \text{ ft}$, the mean life of the equivalent decay is about ten days. This means that radioactive decay has much less effect on the diffusion of a dissolved substance than surface cooling has on the diffusion of heat when the mean life of the decay is long compared with ten days. If no distinction is made between radioactive parent and daughter, there is no "surface cooling."

Symbols

a	Number in the conformal mapping given in Eq. (37)
b	Half-width of Gaussian source in Eq. (36)
c	Number in the conformal mapping given in Eq. (37)
c_p	Heat capacity ($\text{cal g}^{-1} \text{ deg}^{-1}$)
d	Depth (cm)
f, g	Any functions of x, y
F	Function of x, y defined by the right-hand side of Eq. (24) when $v = 1$
k	Thermal (eddy) conductivity ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$)
K	Surface heat exchange coefficient ($\text{cal cm}^{-2} \text{ sec}^{-1} \text{ deg}^{-1}$)
q	Heat flux vector ($\text{cal cm}^{-2} \text{ sec}^{-1}$)
Q	Point source strength ($\text{cal cm}^{-1} \text{ sec}^{-1}$)
r	Radius vector (cm)
R	Function defined in Eq. (18a)
S	Heat source density ($\text{cal cm}^{-3} \text{ sec}^{-1}$)
T	Temperature (deg)
δT	$T - T_\infty$
v	Flow velocity vector (cm/sec)
x, y, z	Space coordinates (cm)
α	Eddy diffusivity $= k/\rho c_p$ (cm^2/sec)
β	Polar angle of r
γ	Exponential of Euler's constant = 1.781
δ	Dirac's delta function
ζ	Image of z in the conformal mapping of Eq. (37)
θ	$4k(T - T_\infty)/Q$
ξ, η	Space coordinates after conformal transformation (cm)
ρ	Density (g/cm^3)
ϕ	Velocity potential (cm^2/sec)
ψ	Stream function (cm^2/sec)

Subscripts

0	Position of point source
∞	Ambient temperature
$x, y, \xi, \eta, \phi, \psi$	Partial differentiation
1, 2	Values of stream function on boundaries

APPENDIX

STRIP

```

10 PRINT "USER: TYPE 'LISTNH 100-350' FOR MESSAGE. AFTER MESSAGE"
20 PRINT "SUPPLY FUNCTIONS FNF(X,Y) AND FNP(X,Y) AND TYPE"
30 PRINT "'DELETE 10-40'. THEN TYPE 'RUN'."
40 STOP
50 REM *****
100 THIS PROGRAM CALCULATES THETA AT ANY POINT IN AN ARBITRARY TWO-
110 DIMENSIONAL POTENTIAL FLOW FIELD. FOR HEAT DIFFUSION, THETA =
120  $4 * K * T / S$ , WHERE K = EDDY THERMAL CONDUCTIVITY, T = TEMPERATURE
130 RISE ABOVE AMBIENT, AND S = TOTAL SOURCE STRENGTH PER UNIT
140 DEPTH. THE DIMENSIONS OF THESE QUANTITIES ARE AS FOLLOWS:
150  $[K] = Q L^{-1} T^{-1} DEG^{-1}$ ,  $[T] = DEG$ , AND  $[S] = Q L^{-1} T^{-1}$ 
160 (HERE Q REPRESENTS HEAT). ANY UNITS OF LENGTH AND TIME MAY BE
170 USED, BUT THEY MUST BE USED CONSISTENTLY. FOR EXAMPLE, IF
180 LENGTHS ARE IN FEET AND TIME IS IN SECONDS, VELOCITIES MUST
190 BE GIVEN IN FEET PER SECOND AND DIFFUSIVITIES IN FEET SQUARED
200 PER SECOND, ETC. ANY UNITS OF HEAT AND TEMPERATURE MAY
210 BE USED, BUT THEY MUST ALSO BE USED CONSISTENTLY.
220 FOR THE DIFFUSION OF DISSOLVED SUBSTANCES, THETA =  $4 * A * C / S$ ,
230 WHERE A = EDDY DIFFUSIVITY, C = CONCENTRATION, AND S = TOTAL
240 SOURCE STRENGTH PER UNIT DEPTH. THE DIMENSIONS OF THESE
250 QUANTITIES ARE AS FOLLOWS:  $[A] = L^2 T^{-1}$ ,  $[C] = M L^{-3}$  AND
260  $[S] = M L^{-1} T^{-1}$  (HERE M REPRESENTS AMOUNT).
270 AGAIN, ANY UNITS OF LENGTH AND TIME MAY BE USED AS LONG
280 AS THEY ARE USED CONSISTENTLY. ANY UNITS MAY BE USED FOR
290 AMOUNT, E.G., MOLES, CURIES, GRAMS, ETC.
300 THE SOURCE CAN BE EITHER A POINT OR A LINE.
310 THE USER MUST SUPPLY TWO FUNCTIONS, FNF(X,Y) AND FNP(X,Y),
320 WHICH GIVE THE VELOCITY POTENTIAL AND STREAM FUNCTION, RESPEC-
330 TIVELY, AT THE POINT (X,Y). THESE FUNCTIONS GO IN LINES
340 1000-1999, BUT THE DEFINING LINES MUST USE LINES 1000 AND 1010.
350 NOTE CHOICE OF SIGN: VELOCITY = +GRAD FNF(X,Y).
360 REM *****
1000 DEF FNF(X,Y)=X
1010 DEF FNP(X,Y)=Y
2000 DEF FNB(R)=.63662*LOG(1.1229/R)*(1+1.2438*R3.08)
2010 DEF FNA(R,X)
2020 IF R/2 >= .75 THEN 2050
2030 FNA=EXP(X/2)*FNB(R/2)
2040 GO TO 2090
2050 IF (R-X)/2 > 20 THEN 2080
2060 FNA=.79788*EXP((X-R)/2)/(SQR(R/2)*(1+.0952*(R/2)(-.885)))
2070 GO TO 2090
2080 FNA=0
2090 FNEND
2100 REM *****
2110 REM ***** INITIALIZE. SETS P2, P1, A1. TEMPORARIES USED
2120 REM ***** ARE X5, Y5, X6, Y6, T5, T6, Z1$.
2130 PRINT "ABSCISSA AND ORDINATE OF A POINT ON ONE BOUNDARY";
2140 INPUT X5,Y5
2150 PRINT "ABSCISSA AND ORDINATE OF A POINT ON OTHER BOUNDARY";
2160 INPUT X6,Y6

```

STRIP

```

2170 T5=FNP(X5,Y5)
2180 T6=FNP(X6,Y6)
2190 IF T5 >= T6 THEN 2230
2200 P2=T6
2210 P1=T5
2220 GO TO 2250
2230 P2=T5
2240 P1=T6
2250 PRINT "DIFFUSIVITY = ";
2260 INPUT A1
2270 PRINT "IS THE SOURCE A POINT OR A LINE";
2280 INPUT Z1$
2290 IF Z1$="LINE" THEN 2470
2300 REM *****
2310 REM ***** POINT SOURCE. INPUTS COORDINATES (X0,Y0) OF SOURCE
2320 REM ***** POINT. CALCULATES F0, P0, F5, P5.
2330 PRINT "ABSCISSA AND ORDINATE OF SOURCE";
2340 INPUT X0,Y0
2350 F0=FNP(X0,Y0)
2360 P0=FNP(X0,Y0)
2370 GOSUB 3070
2380 F5=FNP(X7,Y7)
2390 P5=FNP(X7,Y7)
2400 GOSUB 2830
2410 GOSUB 3190
2420 GO TO 2370
2430 REM *****
2440 REM ***** LINE SOURCE. INPUTS COORDINATES (X8,Y8) AND (X9,Y9)
2450 REM ***** OF ENDPOINTS. CALCULATES F0, P0, F5, P5. TEMPORARIES
2460 REM ***** USED ARE N1, N2, X3, Y3, T3, J.
2470 PRINT "ABSCISSA AND ORRDINATE OF ONE ENDPOINT";
2480 INPUT X8,Y8
2490 PRINT "ABSCISSA AND ORDINATE OF OTHER ENDPOINT";
2500 INPUT X9,Y9
2510 N1=100      'N1 MUST BE AN EVEN INTEGER.
2520 GOSUB 3070
2530 F5=FNP(X7,Y7)
2540 P5=FNP(X7,Y7)
2550 F0=FNP(X8,Y8)
2560 P0=FNP(X8,Y8)
2570 GOSUB 2830
2580 T3=T
2590 F0=FNP(X9,Y9)
2600 P0=FNP(X9,Y9)
2610 GOSUB 2830
2620 T3=T3+T
2630 N2=1
2640 FOR J=1 TO N1-1
2650 X3=X8+J*(X9-X8)/N1
2660 Y3=Y8+J*(Y9-Y8)/N1

```

STRIP

```

2670 F0=FNFC(X3,Y3)
2680 P0=FNPC(X3,Y3)
2690 GOSUB 2830
2700 T3=T3+(3+N2)*T
2710 N2=-N2
2720 NEXT J
2730 T=T3/(3*N1)
2740 GOSUB 3190
2750 GO TO 2520
2760 REM *****
2770 REM ***** THIS SUBROUTINE CALCULATES THETA (T) AT A POINT
2780 REM ***** (F5,P5) IN THE VELOCITY POTENTIAL-STREAM FUNCTION
2790 REM ***** PLANE. THE SOURCE IS LOCATED AT (F0,P0). THE
2800 REM ***** STRIP IS BOUNDED BY VALUES P2>P1 OF THE STREAM
2810 REM ***** FUNCTION. THE DIFFUSIVITY IS A1. TEMPORARIES USED
2820 REM ***** ARE X1, Y1, Y2, P7, R1, R2, I, N, T1, T2.
2830 IF P5 <= P2 THEN 2832
2831 GO TO 2833
2832 IF P5 >= P1 THEN 2834
2833 PRINT "FIELD POINT DOES NOT LIE BETWEEN BOUNDARIES."
2834 IF P0 <= P2 THEN 2836
2835 GO TO 2837
2836 IF P0 >= P1 THEN 2838
2837 PRINT "SOURCE NOT ENTIRELY BETWEEN BOUNDARIES."
2838 Y1=(F5-F0)/A1
2840 Y1=(P5-P0)/A1
2850 Y2=(F5+P0-2*P1)/A1
2860 P7=2*(P2-P1)/A1
2870 R1=SQR(X12+Y12)
2880 R2=SQR(X12+Y22)
2890 I=FNA(R1,X1)+FNA(R2,X1)
2900 FOR I=-1 TO 1 STEP 2
2910 FOR N=1 TO 1000
2920 R1=SQR(X12+(Y1-I*N*P7)2)
2930 R2=SQR(X12+(Y2-I*N*P7)2)
2940 T1=FNA(R1,X1)
2950 T2=FNA(R2,X1)
2960 IF T1=0 THEN 2980
2970 GO TO 2990
2980 IF T2=0 THEN 3010
2990 T=T+T1+T2
3000 NEXT N
3010 NEXT I
3020 RETURN
3030 REM *****
3040 REM ***** SUBROUTINE TO STOP OR CONTINUE. INPUTS
3050 REM ***** COORDINATES (X7,Y7) OF FIELD POINT. TEMPORARIES
3060 REM ***** USED ARE Z3S.
3070 PRINT
3080 PRINT "DO YOU WANT THETA AT FIELD POINT";

```

STRIP

```
3090 INPUT Z3$
3100 IF Z3$="YES" THEN 3140
3110 PRINT
3120 PRINT "TO RUN ANOTHER CASE TYPE 'RUN'"
3130 STOP
3140 PRINT "ABSCISSA AND ORDINATE OF FIELD POINT";
3150 INPUT X7,Y7
3160 RETURN
3170 REM *****
3180 REM ***** OUTPUT SUBROUTINE.
3190 PRINT "THETA = "T
3200 RETURN
3210 REM *****
3220 END
```

FARFLD

```

10 DEF FNB(R)=.63662*LOG(1.1229/R)*(1+1.2438*R↑3.08)
100 DEF FNA(R,X)
110 IF R/2 >= .75 THEN 140
120 FNA=FXP(X/2)*FNB(R/2)
130 GO TO 180
140 IF (R-X)/2 > 20 THEN 170
150 FNA=.79788*EXP((X-R)/2)/(SQR(R/2)*(1+.0952*(R/2)↑(-.885)))
160 GO TO 180
170 FNA=0
180 FNEEND
1010 REM ***** INITIALIZE
1020 PRINT "VOLUME FLOW RATE THROUGH CONDENSOR (FT↑3/SEC)=";
1030 INPUT Q
1040 PRINT "TEMPERATURE RISE IN CONDENSOR (DEG FAHR)=";
1050 INPUT T
1070 PRINT "DIFFUSIVITY (FT↑2/SEC)=";
1080 INPUT D
1090 PRINT "OFFSHORE FLOW VELOCITY (X-DIRECTION) (FT/SEC)=";
1100 INPUT U
1102 PRINT "DEPTH (FT)=";
1104 INPUT D1
1110 PRINT "IS A BREAKWATER PRESENT";
1120 INPUT Z1$
1130 IF Z1$="YES" THEN 1160
1140 N1=2
1150 GO TO 1210
1160 N1=0
1170 PRINT "POSITION OF FOOT OF BREAKWATER (FT)";
1180 INPUT D1
1185 B=B1*U/D
1190 PRINT "LENGTH OF BREAKWATER (FT)";
1200 INPUT C1
1205 C=C1*U/D
1210 PRINT "IS THE SOURCE A POINT OR A LINE";
1220 INPUT Z2$
1230 IF Z2$="LINE" THEN 1280
1240 N2=1
1250 PRINT "ABSCISSA AND ORDINATE OF POINT SOURCE (FT)";
1260 INPUT X3,Y3
1270 GO TO 1330
1280 N2=2
1290 PRINT "ABSCISSA AND ORDINATE OF ONE ENDPOINT OF SOURCE (FT)";
1300 INPUT X1,Y1
1310 PRINT "ABSCISSA AND ORDINATE OF OTHER ENDPOINT (FT)";
1320 INPUT X2,Y2
1330 ON N1+N2 GO TO 2000,3000,4000,5000
1999 REM ***** POINT SOURCE WITH A BREAKWATER
2000 GOSUB 6000
2010 X=X5
2020 Y=Y5

```

FARFLD

```

2030 GOSUB 7000
2040 V5=V
2050 W5=W
2060 X=X3*U/D
2070 Y=Y3*U/D
2080 GOSUB 7000
2090 V8=V
2100 W8=W
2110 T1=0
2120 FOR I= -1 TO 1 STEP 2
2130 R=SQR((V5-V8)2+(W5-I*W8)2)
2140 T1=T1+FNA(R, V5-V8)
2150 NEXT I
2160 GOSUB 6100
2170 GO TO 2000
2999 REM ***** LINE SOURCE WITH A BREAKWATER
3000 GOSUB 6000
3010 X=X5
3020 Y=Y5
3030 GOSUB 7000
3040 V5=V
3050 W5=W
3070 X=X6=X1*U/D
3080 Y=Y6=Y1*U/D
3090 GOSUB 7000
3100 V6=V
3110 W6=W
3120 X=X7=X2*U/D
3130 Y=Y7=Y2*U/D
3140 GOSUB 7000
3150 V7=V
3160 W7=W
3165 N3=100 'N3 MUST BE AN EVEN INTEGER.
3170 T1=0
3180 FOR I=-1 TO 1 STEP 2
3190 R=SQR((V5-V6)2+(W5-I*W6)2)
3200 T1=T1+FNA(R, V5-V6)/(3*N3)
3210 R=SQR((V5-V7)2+(W5-I*W7)2)
3220 T1=T1+FNA(R, V5-V7)/(3*N3)
3230 N4=1
3240 FOR T2=1 TO N3-1
3250 X=X7+T2*(X6-X7)/N3
3260 Y=Y7+T2*(Y6-Y7)/N3
3270 GOSUB 7000
3280 R=SQR((V5-V)2+(W5-I*W)2)
3290 T1=T1+(3+N4)*FNA(R, V5-V)/(3*N3)
3300 N4=-N4
3310 NEXT T2
3320 NEXT I
3330 GOSUB 6100

```

FARFLD

```

3340 GO TO 3000
3999 REM ***** POINT SOURCE
4000 GOSUB 6000
4080 X8=X3*U/D
4090 Y8=Y3*U/D
4100 T1=0
4110 FOR I=-1 TO 1 STEP 2
4120 R=SQR((X5-X8)2+(Y5-I*Y8)2)
4130 T1=T1+FNA(R,X5-X8)
4140 NEXT I
4150 GOSUB 6100
4160 GO TO 4000
4999 REM ***** LINE SOURCE
5000 GOSUB 6000
5030 X6=X1*U/D
5040 Y6=Y1*U/D
5050 X7=X2*U/D
5060 Y7=Y2*U/D
5070 N3=100 'N3 MUST BE AN EVEN INTEGER.
5075 T1=0
5080 FOR I=-1 TO 1 STEP 2
5090 R=SQR((X5-X6)2+(Y5-I*Y6)2)
5100 T1=T1+FNA(R,X5-X6)/(3*N3)
5110 R=SQR((X5-X7)2+(Y5-I*Y7)2)
5120 T1=T1+FNA(R,X5-X7)/(3*N3)
5130 N4=1
5140 FOR T2=1 TO N3-1
5150 X9=X7+T2*(X6-X7)/N3
5160 Y9=Y7+T2*(Y6-Y7)/N3
5170 R=SQR((X5-X9)2+(Y5-I*Y9)2)
5180 T1=T1+(3+N4)*FNA(R,X5-X9)/(3*N3)
5190 N4=-N4
5200 NEXT T2
5210 NEXT I
5220 GOSUB 6100
5230 GO TO 5000
5998 REM ***** SUBROUTINE TO STOP OR CONTINUE
6000 PRINT
6002 PRINT "DO YOU WANT TEMPERATURE AT FIELD POINT";
6010 INPUT Z3$
6020 IF Z3$="YES" THEN 6040
6028 PRINT
6029 PRINT "TO RUN ANOTHER CASE TYPE 'RUN'"
6030 STOP
6040 PRINT "ABSCISSA AND ORDINATE OF FIELD POINT (FT)=";
6050 INPUT X0,Y0
6060 X5=U*X0/D
6070 Y5=U*Y0/D
6080 RETURN
6099 REM ***** OUTPUT SUBROUTINE

```

FARFLD

```
6100 T1=(Q*T*T1)/(4*D*D1)
6110 PRINT "TEMPERATURE="T1;"DEGREES FAHRENHEIT"
6120 RETURN
6999 REM ***** CONFORMAL MAPPING SUBROUTINE: X,Y TO V,W
7000 A1=(X-B)^2-Y^2+C^2
7010 A2=2*Y*(X-B)
7020 A3=SQR(A1^2+A2^2)
7030 V=SGN(A2)*SQR(.5*(A3+A1))+SQR(B^2+C^2)
7040 W=SQR(.5*(A3-A1))
7050 RETURN
99999 END
```


A typical conversation with the computer using the STRIP program.
All entries typed by the user are underlined. Comments not a part
of the actual conversation are typed in lower case letters.

G
IACC/50641 16:29:14

PLEASE LOGIN OR ATTACH.

.LOGIN 6534,17
JOP 24 IACC/50641 TTY14
PASSWORD: _____
1629 19-MAR-73 MON

3-16-73 NEW LOADER(56A) AND COMMAND INTERPRET₀

.R BASIC

{ Message of the day-delete by
typing a control-Ø.

READY, FOR HELP TYPE HELP.
OLD STRIP

READY
RUN

STRIP 16:30 19-MAR-73

{ This initial message instructs the
user how to proceed.

USER: TYPE 'LISTNH 100-340' FOR MESSAGE. AFTER MESSAGE
SUPPLY FUNCTIONS FNF(X,Y) AND FNP(X,Y) AND TYPE
'DELETE 10-40'. THEN TYPE 'RUN'.

{ This command causes a message to be
printed that describes the program.

TIME: 0.90 SECS.

READY
LISTNH 100-340

100' THIS PROGRAM CALCULATES THETA AT ANY POINT IN AN ARBITRARY TWO-
110'DIMENSIONAL POTENTIAL FLOW FIELD. FOR HEAT DIFFUSION, THETA =
120' $4 \cdot K \cdot T / S$, WHERE K = EDDY THERMAL CONDUCTIVITY, T = TEMPERATURE
130'RISE ABOVE AMBIENT, AND S = TOTAL SOURCE STRENGTH PER UNIT
140'DEPTH. THE DIMENSIONS OF THESE QUANTITIES ARE AS FOLLOWS:
150'[K] = $Q \cdot L^{-1} \cdot T^{-1} \cdot DEG^{-1}$, [T] = DEG, AND [S] = $Q \cdot L^{-1} \cdot T^{-1}$
160'(HERE Q REPRESENTS HEAT). ANY UNITS OF LENGTH AND TIME MAY BE
170'USED, BUT THEY MUST BE USED CONSISTENTLY. FOR EXAMPLE, IF
180'LENGTHS ARE IN FEET AND TIME IS IN SECONDS, VELOCITIES MUST
190'BE GIVEN IN FEET PER SECOND AND DIFFUSIVITIES IN FEET SQUARED
200'PER SECOND, ETC. ANY UNITS OF HEAT AND TEMPERATURE MAY
210'BE USED, BUT THEY MUST ALSO BE USED CONSISTENTLY.

220' FOR THE DIFFUSION OF DISSOLVED SUBSTANCES, $\theta = 4 \cdot A \cdot C / S$,
 230' WHERE A = EDDY DIFFUSIVITY, C = CONCENTRATION, AND S = TOTAL
 240' SOURCE STRENGTH PER UNIT DEPTH. THE DIMENSIONS OF THESE
 250' QUANTITIES ARE AS FOLLOWS: $[A] = L^2 T^{-1}$, $[C] = M L^{-3}$ AND
 260' $[S] = M L^{-1} T^{-1}$ (HERE M REPRESENTS AMOUNT).
 270' AGAIN, ANY UNITS OF LENGTH AND TIME MAY BE USED AS LONG
 280' AS THEY ARE USED CONSISTENTLY. ANY UNITS MAY BE USED FOR
 290' AMOUNT, E.G., MOLES, CURIES, GRAMS, ETC.
 300' THE SOURCE CAN BE EITHER A POINT OR A LINE.
 310' THE USER MUST SUPPLY TWO FUNCTIONS, $FNF(X,Y)$ AND $FNP(X,Y)$,
 320' WHICH GIVE THE VELOCITY POTENTIAL AND STREAM FUNCTION, RESPEC-
 330' TIVELY, AT THE POINT (X,Y) . THESE FUNCTIONS GO IN LINES
 340' 1000-1999, BUT THE DEFINING LINES MUST USE LINES 1000 AND 1010.

READY

1000 DEF FNF(X,Y) = -2500*LOG(X+2+Y+2)

1010 DEF FNP(X,Y) = -5000*ATN(Y/X)

DELETE 10-40

{ Definition of velocity potential
and stream function. These entries
are appropriate to the problem
described in Fig. 1.

READY

RUN

{ Deletes initial message so that second
and succeeding RUN commands cause
execution of computational part of
program as shown below.

STRIP

16:35

19-MAR-73

ABSCISSA AND ORDINATE OF A POINT ON ONE BOUNDARY ?5000,0
 ABSCISSA AND ORDINATE OF A POINT ON OTHER BOUNDARY ?5000,437.45
 DIFFUSIVITY = ?5
 IS THE SOURCE A POINT OR A LINE ?POINT
 ABSCISSA AND ORDINATE OF SOURCE ?5000,0

DO YOU WANT THETA AT FIELD POINT ?YES
 ABSCISSA AND ORDINATE OF FIELD POINT ?5000,10
 THETA = 0.53547

DO YOU WANT THETA AT FIELD POINT ?YES
 ABSCISSA AND ORDINATE OF FIELD POINT ?4000,25
 THETA = 0.144372

DO YOU WANT THETA AT FIELD POINT ?YES
 ABSCISSA AND ORDINATE OF FIELD POINT ?3000,100
 THETA = 5.79259E-2

DO YOU WANT THETA AT FIELD POINT ?NO

TO RUN ANOTHER CASE TYPE 'RUN'

{ If a new velocity field is desired at
this point, the functions FNF and FNP
defined in lines 1000 and 1010 must be
changed before typing "RUN."

TIME: 1.15 SECS.

{ Log off with this command.

READY

BYE

JOB 24, USER [6534,17] LOGGED OFF TTY14 1637 19-MAR-73
 SAVED ALL 22 FILES (315. DISK BLOCKS)
 RUNTIME 0 MIN, 05.00 SEC

A typical conversation with the computer using the FARFLD program.

•LOGIN 6534/17
JØP 21 IACC/50641 TTY14
PASSWORD: _____
1638 19-MAR-73 MØN

•R BASIC

READY, FØR HELP TYPE HELP.
ØLD FARFLD

READY
RUN

FARFLD 16:39 19-MAR-73

{The answers shown below are appropriate
to the problem described in Fig. 3.

VØLUME FLOW RATE THROUGH CØNDENSØR (FT³/SEC)= ?3780
TEMPERATURE RISE IN CØNDENSØR (DEG FAHR)= ?18
DIFFUSIVITY (FT²/SEC)= ?5
ØFFSHØRE FLOW VELOCITY (X-DIRECTION) (FT/SEC)= ?1
DEPTH (FT)= ?50
IS A BREAKWATER PRESENT ?YES
POSITION ØF FOOT ØF BREAKWATER (FT) ?2000
LENGTH ØF BREAKWATER (FT) ?1800
IS THE SOURCE A POINT ØR A LINE ?POINT
ABSCISSA AND ØRDINATE ØF POINT SOURCE (FT) ?0,1300

{With these choices,
 $\delta T = 68 \times 0^\circ \text{F}$.

DØ YØU WANT TEMPERATURE AT FIELD POINT ?YES
ABSCISSA AND ØRDINATE ØF FIELD POINT (FT)= ?2000,2000
TEMPERATURE= 2.28558 DEGREES FAHRENHEIT

DØ YØU WANT TEMPERATURE AT FIELD POINT ?YES
ABSCISSA AND ØRDINATE ØF FIELD POINT (FT)= ?3500,1500
TEMPERATURE= 2.41896 DEGREES FAHRENHEIT

DØ YØU WANT TEMPERATURE AT FIELD POINT ?YES
ABSCISSA AND ØRDINATE ØF FIELD POINT (FT)= ?5000,1750
TEMPERATURE= 0.211027 DEGREES FAHRENHEIT

DØ YØU WANT TEMPERATURE AT FIELD POINT ?NO

TØ RUN ANØTHER CASE TYPE 'RUN'

TIME: 1.48 SECS.

READY
BYE

JØP 21, USER (6534,17) LØGGED ØFF TTY14 1641 19-MAR-73
SAVED ALL 22 FILES (315. DISK BLOØKS)
RUNTIME 0 MIN, 03.39 SEC

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