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AN INVESTIGATION OF LIQUID-METAL HEAT TRANSFER IN A COCURRENT-FLOW, DOUBLE-PIPE, HEAT EXCHANGER

by

Richard L. Merriam

RELEASED FOR ANNOUNCEMENT
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Richard L. Merriam

Reactor Engineering Division

June 1965

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NOMENCLATURE

Symbol	Description	Units	Symbol	Description	Units
a_1	Radius of circular tube	ft	t_∞	Equilibrium temperature	$^{\circ}\text{F}$
a_2	Width of annular space	ft	t_{wi}	Heat-transfer surface temperature in channel i	$^{\circ}\text{F}$
$A_n^{(i)}$	Eigenfunction, Eqs. (60a) and (60b)	-	t_w	Measured center-tube wall temperature	$^{\circ}\text{F}$
b	Thickness of circular tube wall	ft	Δt	Bulk-fluid temperature difference	$^{\circ}\text{F}$
b'	Effective thickness of circular tube wall, Eq. (13)	ft	T_i	Temperature indicated by thermocouple i	$^{\circ}\text{F}$
c	Heat capacity	$\text{Btu/lb}_m\text{-}^{\circ}\text{F}$	u_i	Local-fluid velocity in channel i	ft/hr
c_i	Heat capacity of fluid in channel i	$\text{Btu/lb}_m\text{-}^{\circ}\text{F}$	U	Overall heat-transfer coefficient	$\text{Btu/hr-ft}^2\text{-}^{\circ}\text{F}$
D_1	Inside diameter of tube	ft	W_i	Mass flow rate of fluid in channel i	lb_m/hr
D_2	Inside diameter of annular space	ft	x_i	Distance normal to heat-transfer surface	ft
D_3	Outside diameter of annular space	ft	x	Normalized distance normal to heat-transfer surface	-
F	Eigenvalue function, Eq. (63)	-	y	$1 - \sigma x$	-
g_i	Function defined by Eqs. (47a) and (47b)	-	Y_0, Y_1	Bessel functions of second kind	-
h_i	Heat-transfer coefficient for channel i	$\text{Btu/hr-ft}^2\text{-}^{\circ}\text{F}$	z	Dimensionless axial distance, Eq. (45)	-
$\bar{h}_{UHf}$	Heat-transfer coefficient appropriate to uniform-heat-flux boundary condition, assumed uniform	$\text{Btu/hr-ft}^2\text{-}^{\circ}\text{F}$	Greek Letters		
H	Heat-capacity flow-rate ratio, Eq. (15)	-	α_i	Thermal diffusivity of fluid in channel i	ft^2/hr
J_0, J_1	Bessel functions of first kind	-	γ	Exponent in Eq. (58a)	-
k	Thermal conductivity	$\text{Btu/hr-ft-}^{\circ}\text{F}$	δ_i	Temperature ratios, Eq. (77a) and (77b)	-
k_i	Thermal conductivity of fluid in channel i	$\text{Btu/hr-ft-}^{\circ}\text{F}$	ϵ	Heat-exchanger efficiency, Eq. (40)	-
k_i^+	Thermal-conductivity multiplier for turbulent flow in channel i	-	ϵ_H	Eddy diffusivity of heat	ft^2/hr
k_t	Effective total thermal conductivity	$\text{Btu/hr-ft-}^{\circ}\text{F}$	ϵ_M	Eddy diffusivity of momentum	ft^2/hr
k_{ti}	Effective total thermal conductivity of fluid in channel i, $k + c\mu_{EH}$	$\text{Btu/hr-ft-}^{\circ}\text{F}$	n	Dimensionless space variable Eqs. (44a) and (44b)	-
k_w	Thermal conductivity of wall	$\text{Btu/hr-ft-}^{\circ}\text{F}$	λ	Eigenvalue, Eq. (63)	-
K	Fluid thermal-resistance ratio, Eq. (50)	-	μ	Dynamic viscosity	$\text{lb}_m/\text{ft-hr}$
K_w	Relative wall thermal resistance, Eq. (51)	-	ξ_i	Dimensionless local temperature of fluid in channel i, Eq. (17)	-
$\mathcal{L}$	Axial distance	ft	ξ_{Bi}	Dimensionless bulk temperature of fluid in channel i	-
L	Heat-transfer length	ft	ξ_∞	Dimensionless equilibrium temperature	-
L_0, L_1	Special Bessel functions defined by Eqs. (62a) and (62b)	-	ρ	Density	lb_m/ft^3
N_i	Normalized Nusselt number for channel i	-	ρ_i	Density of fluid in channel i	lb_m/ft^3
N_1^0	Normalized overall Nusselt number based on channel 1	-	σ	$1 - R$	-
Nu	Nusselt number	-	ν	Kinematic viscosity	ft^2/hr
Nu_i	Nusselt number for channel i	-	ϕ	Effectiveness coefficient, Eq. (67)	-
Nu_1^0	Overall Nusselt number based on channel 1	-	$\bar{\psi}$	Average value of the ratio of the eddy diffusivities of heat and momentum, Eq. (58a)	-
$[Nu]_p$	"Plug-flow" Nusselt number	-	ω^2	$[R/(1+R)](KH/\sigma^2)$	-
Pe	Peclet number	-	ω'^2	$HK/2$	-
Pe_i	Peclet number in channel i	-	Subscripts		
Pr	Prandtl number	-	0	Inlet	
Pr_i	Prandtl number in channel i	-	1	Refers to tube side	
q_i	Heat flux at heat-transfer surface in channel i	Btu/hr-ft^2	2	Refers to annular space	
Q	Total heat transfer	Btu/hr	e	Outlet	
r_w	Radial distance to thermocouple position from center of tube	ft	pp	Parallel plane	
R	Annular ratio, D_2/D_3	-	an	Annulus	
Re_i	Reynolds number in channel i	-	T	Tube	
slope	Slope of dimensionless outside-wall-temperature distribution, Eq. (74a)	-	Superscripts		
(slope) ₀	Extrapolated value of slope	-	1	Refers to tube side	
t_i	Local temperature of fluid in channel i	$^{\circ}\text{F}$	2	Refers to annular space	
t_{Bi}	Bulk temperature of fluid in channel i	$^{\circ}\text{F}$	-	Denotes average value	
t_{i0}	Inlet temperature of fluid in channel i	$^{\circ}\text{F}$			

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ABSTRACT

An analytical and experimental investigation of liquid-metal heat transfer in cocurrent-flow, double-pipe, heat exchangers is presented. The analysis predicts local and fully-developed heat-transfer coefficients, as well as overall rates of heat transfer. It is shown that: (1) heat-transfer coefficients depend upon the operating conditions of the heat exchanger, and their values can be significantly lower than those corresponding to a uniform-surface-temperature boundary condition, (2) use of the customary design equation to predict heat-exchanger performance can lead to significant errors.

The construction of a mercury heat-transfer loop and experiments with cocurrent-flow, double-pipe, heat-exchanger, test sections is described, including application of a new experimental technique to obtain values of fully-developed heat-transfer coefficients. The experimental results showed agreement with the analysis.

I. INTRODUCTION

Since liquid metals have characteristics that make them superior to ordinary fluids in many applications, much attention has been focused on liquid metals as heat-transfer media. The most significant of these characteristics are low viscosity, low vapor pressure, high thermal conductivity, and a wide temperature range between the melting and boiling points.

The design of heat exchangers is customarily based on the assumptions that heat-transfer coefficients can be considered uniform along the length of the exchanger and are relatively insensitive to boundary conditions. The interpretation of data obtained from laboratory experiments with heat exchangers is often based on equivalent assumptions. With fluids of relatively high Prandtl numbers, these assumptions are reasonably valid and

lead to sufficiently accurate results. With fluids in the low-Prandtl-number range (liquid metals), however, the heat-transfer coefficient becomes quite sensitive to the imposed heat-transfer boundary conditions, and may not be sufficiently uniform along the length of the exchanger. Figure 1, taken from Ref. 51, illustrates the relative dependence of the heat-transfer coefficient on the boundary conditions; the dependence is significant only for the low-Prandtl-number fluids. The relative distance for the heat-transfer coefficient to become "fully developed" is illustrated in Fig. 2, also taken from Ref. 51. It is seen that the "thermal-entrance lengths" are much larger for low-Prandtl-number fluids. As a result, and as emphasized by Stein,⁵³ the validity of these assumptions when applied to liquid-metal heat exchangers deserves to be questioned.

Fig. 1. Ratio of Nusselt Number at Uniform Heat Flux to Nusselt Number at Uniform Wall Temperature vs Reynolds Number⁵¹

Fig. 2. Thermal-entrance Length for a Pipe at Uniform Wall Temperature⁵¹

This report investigates turbulent-flow, liquid-metal, heat transfer in a cocurrent-flow, double-pipe, heat exchanger. The analysis is an extension of that of Stein^{53,54} who studied heat transfer in a heat exchanger with a narrow annulus. The analysis in the present work considers the wide annular-space geometry in which the annular ratio, defined as the ratio of the inner to outer diameters of the annulus, becomes an important parameter. The effect of varying the heat-exchanger operating conditions on both the local heat-transfer coefficients and the overall rate of heat transfer is investigated analytically and experimentally.

II. REVIEW OF THE LITERATURE

A. Customary Heat-exchanger Analysis

The customary analysis of a simple, parallel-flow, double-pipe, heat exchanger is based upon the assumption that the overall heat-transfer coefficient is uniform along the length of the exchanger. With this assumption, a simple relationship results for the heat-exchanger efficiency, ϵ ,

$$\epsilon = 1 - \exp \left[- \left(\frac{1}{W_1 c_1} + \frac{1}{W_2 c_2} \right) UA \right], \quad (1)$$

where U is the overall heat-transfer coefficient. The efficiency is defined as the ratio of the actual rate of heat transfer to the rate that would be obtained in an infinitely-long heat exchanger operating under the same conditions. A calculation of a value for the overall coefficient, U , would involve choices for the coefficients for the tube and annulus sides of the exchanger, as well as a determination of the thermal resistance of the separating wall. The details of heat-exchanger design are contained in a standard heat-transfer text, such as Ref. 14.

Basic to this approach to heat-exchanger design are the assumptions that (1) an overall heat-transfer coefficient can be chosen which adequately applies to the operating conditions of the exchanger, and (2) the overall coefficient is uniform along the length of the exchanger. As mentioned in the Introduction, for heat transfer with liquid metals, these assumptions may not always be valid.

B. Recent Theoretical Developments

Ting⁶¹ analyzed a system equivalent to a heat exchanger, based on the assumption that only the heat-transfer coefficient for the fluid flowing in the annulus is known and is independent of length along the exchanger. The analysis was based on a plug-flow model for the fluid in the tube side of the exchanger.* A complicated method of heat-exchanger design was suggested, but the dependence of the heat-transfer coefficient on the heat-exchanger operating conditions was not investigated.

A similar analysis for heat exchange, where one of the heat-transfer coefficients is assumed to be known and to be uniform along length of the exchanger, has been carried out by Stein,⁴⁵ but with emphasis on the influence of the operating conditions on the tube-side coefficient. The results showed that the heat-exchanger operating conditions can have a significant effect on the fully-developed heat-transfer coefficients. It was found that, for cocurrent flow, the fully-developed heat-transfer coefficients are never larger than those corresponding to uniform heat-flux conditions and can be

*Uniform velocity and conductivity distributions, sometimes called "slug" or "rod-like" flow.

significantly less than those corresponding to uniform-surface-temperature conditions, but for countercurrent flow, the fully-developed heat-transfer coefficients are never less than those for uniform-surface-temperature conditions and can be significantly larger than those corresponding to uniform-heat-flux conditions.

Hall et al.²⁰ investigated fully-developed heat transfer to a fluid where the heat flux at the wall is distributed exponentially. They found that the fully-developed heat-transfer coefficient depends upon the "period" of the exponential distribution when the heat flux increases with duct length, and suggested application of this dependence to heat-exchanger design. No account, however, was made of the thermal entrance region.

Bentwich and Sideman⁵ calculated detailed temperature profiles in a system with laminar, two-phase, liquid flow. The two phases were considered to be completely separated, with the less viscous fluid forming an annular shell between the pipe wall and the other fluid. The system can be considered equivalent to a double-pipe heat exchanger, with a separating wall of zero thickness and zero thermal resistance. The heat-transfer boundary condition assumed was that the pipe wall (outer wall of the annulus in a heat exchanger) was at a uniform temperature. Although the heat-transfer boundary condition is different from what would be assumed for a double-pipe heat exchanger, the mathematical solution of the problem is similar. In particular, a separation-of-variables technique is applied to the energy equation yielding an eigenvalue solution, with the orthogonality condition defined over both regions of flow. To simplify the calculations, a plug-flow model was chosen to represent the two fluid streams.

Stein⁵⁴ analyzed laminar-flow heat transfer in a double-pipe heat exchanger in which the thermal resistance of the wall and the effect of the heat-exchanger operating conditions on the heat-transfer coefficients were considered. The analysis also yielded relationships for the heat-exchanger efficiency. A new quantity, $\phi(z)$, called the "effectiveness coefficient," was introduced and was shown to be important. The form of the equation developed for the efficiency is equivalent to that in Eq. (1) when the exponential term is multiplied by the effectiveness coefficient. With laminar flow, the customary design equations can lead to large errors in predictions of heat-exchanger size.

Stein⁵³ also analyzed the problem of turbulent-flow, liquid-metal, heat transfer in a cocurrent-flow, double-pipe, heat exchanger. The analysis was based on a plug-flow model for the heat-exchanging fluids, but included an approximation method to account for the effect of turbulence. The analysis considered both a parallel-plane configuration and a double-pipe configuration with a narrow annular space. Beginning with the energy equation for heat transfer, detailed fluid-temperature distributions were obtained, and, from these temperature distributions, values for the individual

channel and overall heat-transfer coefficients were obtained. The temperature distributions, and, hence, the heat-transfer coefficients, were a function of several dimensionless parameters describing the operating conditions of the heat exchanger. In general, it was found that: (1) values for fully-developed heat-transfer coefficients can be significantly less than those corresponding to the boundary condition of uniform surface temperature, but never larger than those corresponding to the boundary condition of uniform heat flux; (2) the thermal resistance of the wall separating the two channels of the exchanger can have a significant effect on the heat-transfer coefficient, with large wall resistances tending to result in larger values of the individual-channel heat-transfer coefficients; (3) when operating with a constant flow rate in one channel, while varying the flow rate in the adjacent channel, the dependence of the fully-developed heat-transfer coefficient on Peclet number will be less than for the cases of uniform surface temperature or uniform heat flux. Since this report constitutes, in part, an extension of Stein's analysis to a double-pipe configuration with a wide annular space, a more complete treatment of the analysis will be given here.

C. Heat-exchanger Experiments

Most liquid-metal heat-transfer experiments with double-pipe heat exchangers have been performed to determine forced-convection heat-transfer coefficients. The implication was that the heat exchanger could be operated to obtain a specific heat-transfer boundary condition, such as a uniform heat flux or a uniform surface temperature. Operation with equal heat-capacity flow rates in the tube and annulus of a countercurrent heat exchanger has been normally assumed to provide a uniform heat flux; operation with equal heat-capacity flow rates in a cocurrent heat exchanger has been assumed to provide a uniform surface temperature.

In many of the experimental investigations, values for the average, overall, heat-transfer coefficient were obtained. This required a determination of the individual-channel coefficients based on assumptions concerning the ratio of these coefficients. The ratios were computed from various theoretical predictions for heat transfer with a uniform heat flux. The predictions most often used were those of Seban⁴⁸ for parallel-plane channels Eq. (2), Werner et al.⁶⁷ for annular spaces Eq. (3), and Lyon^{33,34} for tubes Eq. (4):

$$Nu_{pp} = 5.8 + 0.020Pe_{pp}^{0.8}, \quad (2)$$

$$Nu_{an} = 0.75(R)^{-0.3}(7.0 + 0.025Pe_{an}^{0.8}), \quad (3)$$

and

$$Nu_T = 7.0 + 0.025Pe_T^{0.8}, \quad (4)$$

where R is the annular ratio. For narrow annular spaces, $0.7 < R \leq 1.0$, the Seban relation was used; for wide annular spaces, $R < 0.7$, the Werner relation was used. The assumed ratios of the tube-side and annulus-side coefficients were then

$$\frac{Nu_T}{Nu_{an}} = \frac{7.0 + 0.025Pe_T^{0.8}}{5.8 + 0.020Pe_{an}^{0.8}}, \quad 0.7 < R \leq 1.0. \quad (5)$$

and

$$\frac{Nu_T}{Nu_{an}} = \frac{7.0 + 0.025Pe_T^{0.8}}{0.75(R)^{-0.3}(7.0 + 0.025Pe_{an}^{0.8})}, \quad R < 0.7. \quad (6)$$

Most of the experiments covered here have been reviewed in detail by Lubarsky and Kaufman.³² This review will be similar but will emphasize the operation of the heat exchanger to provide specific heat-transfer boundary conditions. Where experimental results and theoretical predictions are compared graphically, the Lyon equation, Eq. (4), for uniform heat flux and the Seban and Shimazaki⁴⁹ equation for uniform surface temperature,

$$Nu_T = 5.0 + 0.025Pe_T^{0.8}, \quad (7)$$

will be used to represent heat transfer in a tube. Table I summarizes the experimental investigations. The review presented considers, first,

TABLE I. Summary of Experimental Investigations

Investigation	Type of Heat Exchanger	Heat-transfer Fluid	Heat-transfer Coefficient Determined	Heat-transfer Boundary Condition Approximated
Elser (1949)	Countercurrent	H ₂ O in annulus, Hg in tube	Local, fully developed	Uncertain
Musser and Page (1947) heating data	Condenser	Steam in annulus, Hg in tube	Overall average	Uniform surface temperature
Musser and Page (1947) cooling data	Countercurrent	H ₂ O in annulus, Hg in tube	Overall average	Uncertain
Bailey et al. (1952)	Cocurrent	H ₂ O in annulus, Hg in tube	Local	Uncertain
Doody and Younger (1951)	Cocurrent and Countercurrent	H ₂ O in annulus, Hg in tube	Average	Uncertain
Trefethen (1950)	Countercurrent	Hg	Local, fully developed	Uniform heat flux
Lyon (1949)	Countercurrent	NaK	Overall average	Uniform heat flux
Werner et al.	Countercurrent	NaK	Overall average	Uncertain
Seban (1950)	Countercurrent	Pb-Bi	Overall average	Uncertain
Lubarsky (1951)	Countercurrent	Pb-Bi	Overall average	Uniform heat flux
Baker and Sesonke (1960)	Countercurrent	NaK	Overall average	Uniform heat flux
Sawochka and Schlee (1964)	Cocurrent	K in tube, Na in annulus	Local, fully developed	Uncertain

experiments involving heat exchange between a high-Prandtl-number fluid and a liquid metal, followed by consideration of experiments involving heat exchange between liquid metals only.

Elser¹⁵ (1949) determined heat-transfer coefficients for mercury in countercurrent flow with cooling water in water-jacketed test sections constructed of mild steel and stainless steel. Copper-constantan thermocouples were used to measure the tube-wall and fluid-centerline temperatures; two couples were placed 720 mm apart on the axis of the pipe, and two were placed along the tube wall at the same cross sections. Since the location of the first set of thermocouples was 30 to 40 diameters from the entrance to the test section, it was postulated that thermal and hydrodynamic entrance effects were eliminated and, that, therefore, fully-developed heat-transfer coefficients would be determined. Figure 2 shows this might be a reasonable assumption for Peclet numbers less than 2000. The calculation procedure was based on the assumption that the heat-transfer coefficient was uniform between the two sets of thermocouples.

The bulk-fluid temperatures were evaluated from the measured centerline temperatures according to Martinelli's analysis for the ratio of the difference between the wall and bulk-fluid temperatures to the difference between the wall and centerline temperatures.³⁷ Although not quite certain of the location of the wall thermocouples, Elser assumed them to be at the centerline of the wall. A relaxing of the assumption concerning the radial location of the wall thermocouples could shift Elser's calculations for the heat-transfer coefficient by 4-18%.³²

Fig. 3. Data of Elser for Fully-developed Heat Transfer to Mercury in a Pipe

Figure 3 shows Elser's results as reevaluated by Lubarsky and Kaufman. The accuracy of the methods of obtaining the bulk-fluid and wall-surface temperatures is in doubt. An additional question arises concerning the lack of wetting of the test-section walls by the mercury. A lack of wetting with liquid metals sometimes indicates the existence of an additional thermal resistance at the pipe wall, which could lead to a significant lowering of the experimentally-determined heat-transfer coefficient. (The subject of wetting is covered in greater detail in Section D of this chapter.) In addition, it is not quite clear what heat-transfer boundary conditions were established by the heat-exchanger operating conditions.

Apparently the water flow rate was varied to maintain a constant temperature difference between the water and mercury along the length of the exchanger, thereby approximating a uniform-heat-flux boundary condition.³²

Musser and Page⁴⁰ (1947) determined heat-transfer coefficients for mercury flowing turbulently in a nickel tube under both heating and cooling conditions. (See also Gilliland *et al.*¹⁸) The heating section consisted of a double-pipe heat exchanger, with steam condensing in the annulus and mercury in downflow through the tube. A countercurrent, double-pipe heat exchanger with water flowing through the annulus formed the cooling section. Mercury and water bulk-fluid temperatures were obtained at both ends of the heat-transfer section.

The Wilson-plot method⁶⁸ was used to separate the mercury heat-transfer coefficients from the overall heat-transfer coefficients. This method, however, implies that the heat-transfer coefficients are insensitive to the boundary conditions imposed by the heat-exchanger operating conditions. As discussed in Chapter I, this assumption may be questionable when applied to liquid-metal heat transfer. An attempt was also made to evaluate the mercury heat-transfer coefficient from the overall coefficient by using a heat-transfer correlation for water from McAdams,³⁸ but the results obtained were about 40% below those obtained by the Wilson-plot method. This discrepancy may have been due to a scale formation on the outside of the nickel tube.

The mercury heat-transfer coefficients obtained by Musser and Page *et al.* (Fig. 4) are lower than those predicted for either uniform surface temperature or uniform heat-flux conditions. In the heating section, the heat-transfer boundary condition closely approximated a uniform surface temperature since the thermal resistance of the thin (0.028-in.-thick) nickel-tube wall was small. The heat-transfer boundary conditions in the cooling section are not well-defined.

Fig. 4. Data of Musser and Page for Average Heat Transfer to Mercury in a Pipe

Since overall, average, heat-transfer coefficients were determined, thermal entrance-region effects were present in the results. The method of separating the individual mercury heat-transfer coefficients from the overall coefficient also represents a probable source of large error. Musser and Page report mean temperature differences on the order of 150°F between the

mercury at the tube wall and at the centerline of the heating section. Large changes could occur in the physical properties of the mercury, thereby affecting the calculated values of the Nusselt numbers. In addition, the heat transfer occurred under nonwetting conditions, suggesting another possible explanation for the somewhat low results obtained.

Bailey *et al.*¹ (1952) determined local heat-transfer coefficients for mercury in cocurrent flow with water in a double-pipe heat exchanger. The test section consisted of a mild-steel tube, with the central 18 in. surrounded by a water jacket. Each end of the tube was surrounded by a separate jacket, the mercury flowing between the tube and the jacket at each end.

Temperatures were measured at the inlet and outlet of both the mercury and water conduits, as well as at four places along the tube wall. Since, however, the temperature change of the water was quite small, averaging about 1°F, it was difficult to make accurate heat balances across the test section. In addition, the significance of the mercury exit-temperature measurements is questionable since no provision was made for a mixing of the mercury at the exit. The flow of the mercury through the jacket at the outlet end would tend to flatten the temperature profile of the mercury, however, and, the temperature measured here would be somewhere between a centerline and bulk-fluid temperature. It is not quite clear what calculation procedure was used to determine the heat-transfer coefficients. Presumably, the overall temperature rise of the mercury was used to determine a value for an overall heat-transfer coefficient, which was assumed to be uniform along the length of the exchanger. Use of this overall coefficient and the bulk-fluid and wall temperatures would give the "local" heat-transfer coefficient.

Fig. 5. Data of Bailey *et al.* for Local Heat Transfer to Mercury in a Pipe

Figure 5 shows the results of Bailey *et al.*, as reevaluated by Lubarsky and Kaufman. The results obtained were quite low. Perhaps one explanation for the low results is that the tests were carried out under nonwetting conditions. Additions of magnesium and titanium were made to the mercury to promote wetting, but a check of the test section after completion of the tests showed that wetting did not occur.

Doody and Younger^{9,69} (1951) determined average heat-transfer coefficients for mercury in both cocurrent and countercurrent flow with water in a double-pipe heat exchanger. Fluid temperatures were measured at both ends of the

exchanger. The mercury exit temperature, however, was probably close to the centerline temperature because of poor mixing of the fluid before the point where the temperature was measured. Tube-wall temperatures were also measured along the length of the center tube. Since the wall-temperature measurements did not cover the length of the exchanger, the wall temperatures at the ends of the center tube were determined by an extrapolation procedure. In an attempt to account for the effect of heat conduction along the part of the test section not surrounded by a water jacket, Doody and Younger assumed an effective heat-transfer length between the total length of the center tube and the length of the tube actually surrounded by the water jacket. An average heat-transfer coefficient was then determined for this length of exchanger.

Fig. 6. Data of Doody and Younger for Average Heat Transfer to Mercury in a Pipe

Figure 6 shows the results obtained as reevaluated by Lubarsky and Kaufman. The results for pure mercury are considerably below what was expected. When slight amounts of sodium (0.008-0.096%) were added to the mercury to promote a wetting of the steel tube wall, a considerable increase in the heat transfer was noted, but the results still were below what was expected. No significant differences were noted between the runs with water in cocurrent flow and in counter-current flow.

A good deal of scatter was present in the results. Heat-balance deviations as high as 140% were obtained, large deviations being the rule rather than the exception. An unsuccessful attempt was made to calibrate the system by tests with butanol.

Trefethen^{63,64} (1950) determined local heat-transfer coefficients for mercury in a countercurrent-flow, double-pipe, heat exchanger. Several test sections constructed of copper and several constructed of stainless steel were used to carry out tests under both wetting and nonwetting conditions. The heat-transfer loop was a "figure-eight" type of loop in which the mercury leaving the center tube flowed through the annulus, giving the same weight flow rates in both conduits. Temperature measurements along the outside wall of the heat exchanger gave values of the axial temperature gradient, from which the heat flux could be calculated. The value of the local heat-transfer coefficient was determined for the center portion of the exchanger, where any thermal entry effects were considered to be negligible. (The lengths of the exchangers varied from 53 to 127 diameters, based on the inner diameter of the tube.)

The calculation procedure was based on the assumptions that: (1) the bulk-fluid temperature difference between the fluids in the tube and annulus was constant along the exchanger, and (2) the temperature gradient along the outside wall was the same as in the tube and annulus. A method based on the plug-flow values for heat-transfer coefficients with a uniform heat flux was used to separate the individual-channel coefficients from the overall heat-transfer coefficient. The results, shown in Fig. 7 for the tube side, had a maximum error of 15%.

Fig. 7. Data of Trefethen for Fully-developed Heat Transfer to Mercury in a Pipe

Tests under both wetting and nonwetting conditions yielded the same results. Great care was taken throughout the tests to keep all the surfaces in contact with mercury very clean. Trefethen concluded that the effect of non-wetting on heat transfer is negligible if the heat-transfer surfaces in contact with the liquid metal are clean.

The local heat-transfer coefficients, determined near the central portion of the heat exchanger were probably close to being fully developed, with the boundary condition approximating a uniform heat flux. The results are seen, however, to be closer to those predicted for a uniform surface temperature.

Lyon^{33,34} (1949) determined overall, average, heat-transfer coefficients for heat transfer to NaK in a countercurrent-flow, double-pipe, heat exchanger. The test sections, constructed of nickel, varied in length from 76 to 159 diameters, based on the inside diameter of the tube, and had annular ratios of from 0.814 to 0.700. Bulk-fluid temperatures were measured at both ends of the tube and the annulus. An attempt was made to determine the temperature of the inner-tube wall by measuring the thermal expansion of the wall, but the results proved very erratic, and the attempts were abandoned.

Figure 8 shows the results of the tests for the tube-side heat-transfer coefficient, as reevaluated by Lubarsky and Kaufman, for the test section with the greatest length-to-diameter ratio. The individual channel coefficients were determined from the overall heat-transfer coefficient by assuming them to be related as in Eqs. (5) and (6). The results were close to those predicted for fully-developed heat transfer with a uniform surface temperature. The data include, however, entrance-region effects, which could tend to increase the calculated coefficients slightly.

Fig. 8. Data of Lyon for Average Heat Transfer to NaK in a Pipe

taken to cover the flow and temperature ranges for each alloy; no fluctuation of the rate of heat transfer was observed.

The individual-channel heat-transfer coefficients were assumed to be related as in Eq. (6). The results for the tube-side coefficient, as reevaluated by Lubarsky and Kaufman, are shown in Fig. 9. The agreement with what is predicted for a uniform heat flux is quite good. The calculated, overall, heat-transfer coefficients include entrance-region effects, however.

Seban⁵⁰ (1950) determined overall, average, heat-transfer coefficients in a "figure-eight" double-pipe heat exchanger using a lead-bismuth eutectic. Test sections of stainless steel, mild steel, and tinned steel were used to investigate the effect of the type of heat-exchanger surface on the heat-transfer characteristics.

Fluid temperature was measured at both ends of the exchanger.

Lyon suggested that impurities in the liquid metal may have caused a scale formation to take place on the walls. He noted that the results fluctuated, and that frequently the heat-transfer coefficients determined for runs at a low flow rate were considerably higher if taken immediately after a run at a high flow rate.

Werner *et al.*^{66,67} (1949) determined overall, average, heat-transfer coefficients in a "figure-eight" double-pipe heat exchanger, constructed of nickel, using several sodium-potassium alloys. Bulk-fluid temperatures were measured at both ends of the exchanger. To eliminate any effects due to fouling, a continuous series of runs was

Fig. 9. Data of Werner *et al.* for Average Heat Transfer to NaK in a Pipe

The annulus-outlet temperature, however, was probably closer to the fluid-centerline temperature than to the bulk-fluid temperature, since no provision was made for proper fluid mixing before the point where the temperature was measured.

Considerable difficulty was experienced with fouling of the test-section surfaces. Reliable data could be obtained for only about 15 hr after the test-section surfaces were cleaned. An analysis of the coating left on the heat-transfer surfaces revealed an oxygen concentration of about 3%. The results for the overall heat-transfer coefficient compared quite poorly with predictions according to Eqs. (2) and (4). The results for the mild-steel and stainless-steel test sections showed quite a bit of fluctuation, and were only 40-50% of the predicted results; those for the tinned test section were consistent, and were within 15% of the predicted values for the overall heat-transfer coefficients. The discrepancy in the results was attributed to the presence of some additional resistance between the tube wall and the lead-bismuth eutectic. It was postulated that the lead-bismuth wetted the tinned heat-exchanger walls, thereby reducing or eliminating any contact resistance and, hence, allowed for higher heat-transfer rates.

Lubarsky³¹ (1951) used a "figure-eight" type system with a lead-bismuth eutectic. Overall, average, heat-transfer coefficients were calculated from the inlet and outlet bulk-fluid temperatures and the weight flow rate. The individual-channel coefficients were calculated by assuming them to be related, as in Eq. (5).

Fig. 10. Data of Lubarsky for Average Heat Transfer to Lead-Bismuth in a Pipe

The lead-bismuth eutectic failed to wet the stainless-steel test section in the first set of runs. For later sets of runs, 0.04% w/o of magnesium was added to the eutectic to promote wetting. Inspection of the test section showed that this procedure was successful in that the test section appeared to be "tinned" after being in contact with the lead-bismuth. As shown in Fig. 10, for the tube-side heat-transfer coefficients, there appeared to be no difference between the results obtained for "wetting" and nonwetting conditions; however, both sets of data were considerably below what was expected. This suggests that the occurrence of a

"tinning" of the test section might not necessarily indicate the elimination of the additional contact resistance usually found with nonwetting conditions.

The heat-transfer boundary conditions were probably close to a uniform heat flux. The thermal-entry region effects on the average heat-transfer coefficients were probably not large because of the large length-to-diameter ratio of the test section (100 for the tube, and 332 for the annulus).

Baker and Sesonske^{3,4} (1960) determined overall, average, heat-transfer coefficients using NaK in a countercurrent, double-pipe, heat exchanger. Overall, average, heat-transfer coefficients were calculated from the bulk-fluid temperatures and the weight flow rates, and individual channel coefficients were calculated by assuming them to be related as in Eq. (6). An attempt was made to correct for thermal-entrance effects by using the analysis of Lubarsky and Kaufman³² for the tube-side coefficient and the analysis of Poppendiek⁴⁴ for the annular-side coefficient. The result was to lower the calculated heat-transfer coefficients by 0-6%.

Fig. 11. Data of Baker and Sesonske for Average Heat Transfer to NaK in a Pipe

To investigate the effect of the heat-transfer boundary conditions on the heat-transfer coefficient, the relative flow rates for the tube and annulus were varied. However, this variation appeared to be quite small and was probably insufficient to vary the flux distribution significantly. Figure 11 shows the calculated tube-side coefficients for all tests.

Sawochka and Schleef⁴⁶ (1964) determined local heat-transfer coefficients for cocurrent flow of potassium in the tube and sodium in the annulus of a double-pipe, heat-exchanger, test section consisting of a thick-walled nickel tube, surrounded by a Type 316 stainless-steel pipe. (See also Ref. 30.)

Tube-wall surface temperatures were determined at two axial locations by thermocouples embedded at various radial locations in the tube wall. A logarithmic extrapolation of the radial-temperature profile in the wall yielded the inner- and outer-wall-surface temperatures, with which the local heat-transfer coefficients could be calculated. Entrance and exit fluid temperatures were measured for both the tube and annulus; however, since no provision was made for fluid mixing at the outlet of the tube, the potassium-fluid temperature measured was probably closer to the fluid-centerline

temperature than to the bulk-fluid temperature. Calibration of the electromagnetic flow meters after the tests showed a systematic error of 14.5%. After correction for the flowmeter output errors, the test-section heat balances averaged about 7%. Initial runs at low sodium flow rates led to departures of the wall-temperature profile from the expected logarithmic profile because of an eccentric flow condition in the annulus, and subsequent heat-transfer tests were all run with higher sodium flow rates.

The calculation procedure for the local heat-transfer coefficient was based on the assumption that the overall heat-transfer coefficient was uniform along the length of the exchanger. With this assumption, values of the heat-flux and bulk-fluid temperatures were calculated for the two axial locations where the wall thermocouples were located; from these values, the local heat-transfer coefficients were calculated.

Fig. 12. Data of Sawochka and Schleef for Fully-developed Heat Transfer to Potassium in a Pipe

Figure 12 shows the calculated values of the tube-side local Nusselt number corresponding to axial locations of 19 and 38 diameters from the beginning of the heat-transfer section. The coefficients were considerably less than those expected with a uniform-surface-temperature boundary condition, with particularly low values in the low Peclet-number range. Since the thick wall tube controlled the heat transfer, the heat-transfer coefficients at the two axial positions should have been fully developed. The fact that the local values of the heat-transfer coefficient were higher at the second axial location than at the first is an indication of the error in the results. One possible source of

error in the calculation procedure was the assumption that the overall heat-transfer coefficient was uniform along the entire length of the exchanger. Neglect of the thermal entrance region effects would affect the calculated values for the bulk-fluid temperatures at the two axial locations and, hence, the calculated values of the local heat-transfer coefficients. In addition, since the temperature differences were small, any uncertainty in the temperature would also have a significant effect on the results. Since the conditions of the experiment correspond closely to those of this report, the experimental results are evaluated more completely in Section VII-C.

D. Liquid-metal Wetting

Early experimental investigations of liquid-metal heat transfer yielded results considerably below those expected. A possible reason for the low results was that a lack of wetting of the heat-transfer surface by liquid metals led to some sort of additional resistance between the fluid and the surface, which resulted in a lowering of the heat transfer.

Early experience with mercury boilers^{19,52} showed that the addition of agents such as sodium and magnesium-titanium amalgams could increase the heat-transfer capacity of the boilers considerably. Adding sodium to the mercury in the Kearny, New Jersey boiler enabled the boiler to carry its full load, as compared to a 20% load before addition of the sodium. The additives apparently caused a breakdown of the oxides that had built up on the steel heat-transfer surfaces.

Experience with electromagnetic flowmeters³⁵ indicated the existence of an electrical resistance between the pipe wall and the liquid metal with nonwetting conditions. The resistance could, in some cases, be sufficient to impede any current flow completely. Bonilla and Wang⁶ investigated the relationship between electrical and thermal resistance with stagnant mercury and steel plates with both wetting and nonwetting conditions and concluded that a high electrical resistance across an interface does not necessarily indicate the presence of a high thermal resistance.

Experimental heat-transfer investigations with alkali metals, however, seem to have yielded higher heat-transfer coefficients than experiments using heavy metals (mercury, lead-bismuth). Several investigations have been carried out to investigate the effect of wetting on heat transfer with the heavy metals. As discussed earlier in this chapter, the results of Doody and Younger^{9,69} and Seban⁵⁰ seemed to indicate that wetting increases heat transfer, contrary to the results of Trefethen^{63,64} and Lubarsky.³¹ Johnson et al.²⁴ investigated the effect of magnesium-titanium additives to mercury heat transfer in a mild-steel tube and found no increase in the heat transfer. They even questioned whether the visual indication of "tinning" as reported by Seban⁵⁰ and Lubarsky³¹ is a sufficient criterion of "thermal wetting." English and Barrett¹⁶ offered the explanation that the addition of magnesium and titanium to mercury removes dirt and oxide films from the heat-transfer surfaces, rather than causing actual wetting.

Seban⁵⁰ suggested that the resistance to heat transfer could be a film of gas trapped in rough metal surfaces. However, Johnson et al.²⁴ ruled out this possibility after using both argon and helium gas to purge their system before charging it with mercury and getting similar heat-transfer results in both cases, contrary to what would be expected if gases of different thermal conductivities formed an insulating layer at the heat-transfer surface.

In a review of liquid-metal heat-transfer experiments, MacDonald and Quittenton³⁶ suggested that the heat-transfer resistance could be formed by mechanically-entrained gas within the liquid metal. The gas would have to be present in the form of extended gaps, rather than spheres, to cause a significant decrease in the apparent thermal conductivity of the liquid metal. They estimated that if only one-thousandth of the volume of the mixture were a gas present in the form of extended gaps, the thermal conductivity of the mixture would be sufficiently decreased to account for the apparent thermal resistance associated with nonwetting liquid-metal systems.

Using a glass test section for visual observations, Stromquist⁵⁷ noticed local detachments of turbulently-flowing mercury from the tube wall, apparently caused by foreign particles on the wall. The detachments could be made to disappear by throttling the flow on the discharge side of the test section. Addition of 0.02% sodium to the mercury to promote wetting also decreased the tendency of detachments to occur. Stromquist suggested that local detachments of the liquid metal from the test-section walls might be responsible for the low values of the heat-transfer coefficients obtained in experimental investigations; heat-transfer tests carried out under both wetting and nonwetting conditions in a steel tube led Stromquist to conclude that there is no inherent difference between wetted and nonwetted systems.

Chelemer⁸ continued investigations on the heat-transfer system used by Stromquist. He observed similar local detachments, which he concluded were actually bubbles of gas entrained in the mercury. Heat-transfer tests carried out with different pressures maintained on the system showed that the heat-transfer characteristics of the system were not significantly influenced by such gas entrainment. He also concluded that there is no difference per se between wetted and nonwetted systems.

Studies in the Soviet Union^{28, 58-60} have investigated the existence of a thermal-contact resistance by calculating tube-wall inner-surface temperatures from both the tube-wall outer-surface temperatures and from temperature profiles within the liquid metals themselves. Experiments with both alkali and heavy metals yielded heat-transfer coefficients close to those predicted, when the tube inner-wall surface temperature was determined from an extrapolation of the temperature profile in the liquid metal. When the alkali metals were continuously cleaned of oxides, the determination of the wall inner-surface temperatures by the two methods yielded the same values, leading to the conclusion that clean alkali metals have no contact resistance with stainless steel. Tests with heavy metals which were not continuously cleaned, however, showed a contact resistance that was a function of the flow rate. A chemical analysis of the metals showed a high concentration of oxides near the tube wall. The only tests with heavy metals carried out in this country^{7, 23} that yielded heat-transfer coefficients close to those predicted were ones in which the tube-wall surface temperatures were

determined from the fluid-temperature profiles. The results of Isakoff and Drew,²³ however, show agreement between some of the tube-wall surface temperatures calculated by both methods.

In a further analysis of contact resistance with liquid metals, Mizushina *et al.*³⁹ carried out studies with stagnant mercury on copper, nickel, and chromium-plated copper. His tests indicated that no resistance was present with copper, and a slight resistance was present with nickel and chromium-plated copper. As an indication of the effect of the slight contact resistance, Mizushina calculated that for mercury flow inside a 2-cm-diam tube at a Peclet number of 1000, the contact resistance determined for stagnant mercury would cause only a 5% reduction in the heat-transfer coefficient from that calculated when no contact resistance was assumed.

The current opinion concerning the importance of liquid-metal wetting is that there is no significant difference between wetting and nonwetting systems, *per se*. The important consideration is that the heat-transfer surfaces and the liquid metal be clean from dirt and gas films. When wetting is present, such as with copper and mercury, where the mercury amalgamates the copper, the liquid metal apparently chemically attacks the solid metal, thereby eliminating any oxides present at the tube wall. In non-wetting systems, the system must be free of oxides and the heat transfer tests must be run so that no opportunity exists for an oxide formation.

III. ANALYSIS

The analysis presented here follows closely that of Stein,^{53,54} who considered the problem of liquid-metal heat transfer in a cocurrent-flow, double-pipe, heat-exchanger configuration with a narrow annular space. The present work extends Stein's analysis to heat-exchanger configurations with wide annular spaces. As a result, the diameter ratio of the annulus becomes an additional important parameter in the formulation of the problem.

A. Formulation of the Problem

1. Description of Problem

The system under consideration is a double-pipe heat exchanger, consisting of two concentric circular pipes with fluids flowing in parallel flow through the annular space and the central pipe. The fluids enter the heat exchanger with different temperatures and transfer heat through the center pipe wall as they flow along the length of the exchanger. Figure 13 shows the system under consideration.

Fig. 13. Heat-exchanger Configuration

The analysis is based on the following assumptions:

- a) The inlet-temperature distributions within the fluids are uniform.
- b) The physical properties of the fluid are temperature-independent.
- c) Frictional heating is negligible.
- d) Longitudinal heat conduction in the heat-exchanger walls is negligible.
- e) Longitudinal heat conduction in the fluids is negligible.
- f) The velocity distributions within the fluids are uniform.
- g) Heat transport within the fluids by eddy transport is negligible, compared to molecular heat conduction.

The first four assumptions are familiar ones and require little discussion. An investigation by Schneider⁴⁷ of the effect of axial conduction with heat flow through the duct walls to a uniform-temperature environment has shown that, for flow characterized by Peclet numbers greater than 100, axial heat conduction within the fluid may be neglected. Recent extension of Schneider's analysis by Stein⁵⁵ to the uniform-heat-flux case has suggested that axial heat conduction may be neglected for flow with much lower Peclet numbers for other boundary conditions.

Assumptions (f) and (g) comprise what is known as the "plug-flow" model for heat transfer. Dwyer^{11,12} calculated that the eddy transport of heat is negligible compared to molecular conduction for Peclet numbers less than 140 in tubes, and less than 300 in annuli. Poppendiek and Harrison^{42,43} showed that the assumption of a uniform-velocity profile for a liquid metal in turbulent flow leads to reasonably accurate results in calculations for liquid-metal heat-transfer coefficients. As the flow increases, however, the accuracy of the plug-flow model decreases because of the transport of heat by eddy diffusion. An approximation technique, based on the plug-flow model, is introduced in Section B of this chapter to extend the results of the formulation into the higher-Peclet-number range.

With the above assumptions, energy conservation yields the following steady-state convection equations for the two channels:

$$\frac{1}{x_1} \frac{\partial}{\partial x_1} \left(x_1 \frac{\partial t_1}{\partial x_1} \right) = \frac{u_1}{\alpha_1} \frac{\partial t_1}{\partial \ell}, \quad (8a)$$

and

$$\frac{1}{(a_1 + b + a_2 - x_2)} \frac{\partial}{\partial x_2} \left[(a_1 + b + a_2 - x_2) \frac{\partial t_2}{\partial x_2} \right] = \frac{u_2}{\alpha_2} \frac{\partial t_2}{\partial \ell}, \quad (8b)$$

where the subscript i denotes the channel, x_i is the radial coordinate in channel i , and u_i is the velocity in the axial or ℓ direction. The coordinates are chosen so that $x_i = a_i$ represents the heat-transfer surface (see Fig. 13). From assumption (a), the initial conditions are

$$t_i(x_i, 0) \equiv t_{i0}; \quad (9)$$

the boundary conditions at the outer wall and at the center of the tube results in

$$\left. \frac{\partial t_i}{\partial x_i} \right|_{x_i=0} = 0. \quad (10)$$

Continuity of heat flow through the tube wall results in

$$k_1 \left. \frac{\partial t_1}{\partial x_1} \right|_{a_1} = - \left(1 + \frac{b}{a_1} \right) k_2 \left. \frac{\partial t_2}{\partial x_2} \right|_{a_2} \quad (11)$$

Equations (10) and (11) give three boundary conditions for the solution of Eq. (8). Consideration of the heat flow through the tube wall gives a fourth boundary condition:

$$k_1 \left. \frac{\partial t_1}{\partial x_1} \right|_{a_1} = \frac{k_w}{b'} [t_2(a_2, l) - t_1(a_1, l)], \quad (12)$$

where k_w is the wall conductivity and b' is given by

$$b' = a_1 \ln \left(1 + \frac{b}{a_1} \right). \quad (13)$$

2. Dimensionless Formulation

As the heat-exchanger length increases without limit, the fluid temperatures approach a value t_∞ given by

$$c_1 W_1 (t_\infty - t_{10}) = c_2 W_2 (t_{20} - t_\infty). \quad (14)$$

If a dimensionless heat-capacity flow-rate ratio, H , is defined as

$$H = \frac{c_2 W_2}{c_1 W_1}, \quad (15)$$

the following relation is obtained for t_∞ :

$$\frac{t_\infty - t_{10}}{t_{20} - t_{10}} = \frac{H}{1 + H}. \quad (16)$$

This suggests the following form for a dimensionless temperature, ξ_i :

$$\xi_i = \frac{t_i - t_{10}}{t_{20} - t_{10}}, \quad i = 1, 2. \quad (17)$$

The space variables may also be nondimensionalized as

$$x = \frac{x_i}{a_i}, \quad i = 1, 2, \quad (18)$$

and

$$z = \frac{\alpha_1 \ell}{u_1 a_1^2} \quad (19)$$

Note that $0 \leq x \leq 1$, and that $x = 1$ identifies the heat-transfer surface for both channels. The dimensionless axial variable may also be expressed in terms of the Peclet number in channel 1:

$$z = \frac{4}{\text{Pe}_1} (\ell/D_1). \quad (20)$$

As the heat-exchanger length increases without limit, the fluid temperatures, from Eq. (16), approach

$$\lim_{z \rightarrow \infty} \xi_i(x, z) = \frac{H}{1+H} \equiv \xi_\infty. \quad (21)$$

To account for the geometry of the annulus, a dimensionless annular ratio is defined as

$$R = \frac{\text{inner radius}}{\text{outer radius}} = \frac{a_1 + b}{a_1 + b + a_2}. \quad (22)$$

Hence,

$$0 \leq R \leq 1.$$

3. Differential Equations

In terms of the dimensionless variables, the differential equations to be solved become

$$\frac{1}{x} \frac{\partial}{\partial x} \left(x \frac{\partial \xi_1}{\partial x} \right) = \frac{\partial \xi_1}{\partial z}, \quad (23a)$$

and

$$\frac{1}{1 - \sigma x} \frac{\partial}{\partial x} \left[(1 - \sigma x) \frac{\partial \xi_2}{\partial x} \right] = \frac{R}{1 + R} \text{KH} \frac{\partial \xi_2}{\partial z}, \quad (23b)$$

where

$$\sigma = 1 - R, \quad (24)$$

and

$$K = \left(\frac{k_1}{k_2} \right) \frac{a_2}{a_1 + b} = \frac{k_1}{k_2} \frac{\sigma}{R}. \quad (25)$$

The initial and boundary conditions are written as

$$\xi_1(x, 0) = 0, \quad (26a)$$

$$\xi_2(x, 0) = 1, \quad (26b)$$

$$\left. \frac{\partial \xi_i}{\partial x} \right|_{x=0} = 0, \quad i = 1, 2, \quad (27)$$

$$K \left. \frac{\partial \xi_1}{\partial x} \right|_1 + \left. \frac{\partial \xi_2}{\partial x} \right|_1 = 0, \quad (28)$$

and

$$K_w \left. \frac{\partial \xi_1}{\partial x} \right|_1 + \xi_1(1, z) - \xi_2(1, z) = 0, \quad (29)$$

where

$$K_w = \frac{k_1}{k_w} \ln \left(1 + \frac{b}{a_1} \right). \quad (30)$$

The equations that describe the system depend upon the dimensionless parameters H , K , K_w , and R . Once values are assigned to these parameters, the solution of the equations yields dimensionless temperature distributions, $\xi_i(x, z)$, from which the heat-transfer rates and heat-transfer coefficients may be calculated. As discussed above, the parameters H and R are called, respectively, the heat-capacity flow-rate ratio and the annular ratio. The parameters K and K_w are measures of the relative thermal resistances of the fluids and the relative thermal resistance of the tube wall. The case of the annular ratio, R , approaching unity represents the system of equations solved by Stein.^{53, 54}

4. Heat-transfer Coefficients

The individual heat-transfer coefficients, h_i , are given by

$$h_i = \frac{k_i \left. \frac{\partial t_i}{\partial x_i} \right|_{a_i}}{t_i(a_i, l) - t_{Bi}(l)}, \quad (31)$$

where t_{Bi} is the bulk (mixing-cup) temperature of the fluid in channel i , given as

$$t_{B1} = \frac{1}{\pi a_1^2 \bar{u}_1} \int_0^{a_1} 2\pi x_1 u_1 t_1 dx_1, \quad (32a)$$

and

$$t_{B2} = \frac{1}{\pi [(a_1 + b + a_2)^2 - (a_1 + b)^2] \bar{u}_2} \int_0^{a_2} 2\pi u_2 t_2 (a_1 + b + a_2 - x_2) dx_2. \quad (32b)$$

The dimensionless bulk temperatures then become

$$\xi_{B1} = 2 \int_0^1 x \xi_{11} dx, \quad (33a)$$

and

$$\xi_{B2} = \frac{2}{1+R} \int_0^1 (1-\sigma x) \xi_{22} dx, \quad (33b)$$

with the heat-balance relation

$$\xi_{B1} = H(1 - \xi_{B2}). \quad (34)$$

The individual-channel heat-transfer coefficients, expressed as Nusselt numbers, are

$$Nu_i(z) = \frac{2 \left. \frac{\partial \xi_i}{\partial x} \right|_1}{\xi_i(1, z) - \xi_{Bi}(z)}, \quad i = 1, 2. \quad (35a)$$

Integration of Eqs. (23a) and (23b) with respect to x yields

$$\left. \frac{\partial \xi_1}{\partial x} \right|_1 = \frac{1}{2} \frac{d\xi_{B1}}{dz}, \quad (36a)$$

and

$$\left. \frac{\partial \xi_2}{\partial x} \right|_1 = \frac{KH}{2} \frac{d\xi_{B2}}{dz}. \quad (36b)$$

The individual-channel Nusselt numbers can then be expressed as

$$Nu_1(z) = \frac{\frac{d\xi_{B1}}{dz}}{\xi_1(1, z) - \xi_{B1}(z)}, \quad (37a)$$

and

$$Nu_2(z) = \frac{KH \frac{d\xi_{B2}}{dz}}{\xi_2(1, z) - \xi_{B2}(z)}. \quad (37b)$$

The usual overall heat-transfer coefficient, based on the heat-transfer area of channel 1, is given by

$$U_1(\ell) = \frac{k_1 \left. \frac{\partial t_1}{\partial x_1} \right|_{a_1}}{t_{B2}(\ell) - t_{B1}(\ell)}. \quad (38)$$

An overall Nusselt number can be defined as

$$Nu_1^0 = \frac{D_1 U_1}{k_1}.$$

Hence, the equivalent of Eq. (35a) is

$$Nu_1^0(z) = \frac{2 \left. \frac{\partial \xi_1}{\partial x} \right|_1}{\xi_{B2}(z) - \xi_{B1}(z)}. \quad (35b)$$

From the heat-balance relations (34) and (36a), Nu_1^0 may also be written as

$$Nu_1^0(z) = \frac{\frac{d\xi_{B1}}{dz}}{1 - \frac{1+H}{H} \xi_{B1}(z)}. \quad (37c)$$

With Eqs. (37a), (37b), and (37c), it can be shown that the familiar additive thermal-resistance relation holds for the overall Nusselt number:

$$\frac{1}{Nu_1^0(z)} = \frac{1}{Nu_1(z)} + \frac{K_w}{2} + \frac{K}{Nu_2(z)}. \quad (39)$$

5. Heat-exchanger Efficiency

By "heat-exchanger efficiency," ϵ , is meant the ratio of the actual rate of heat transfer to the rate that would be obtained in an infinitely-long heat exchanger operating under the same conditions. With this definition, Eqs. (17) and (21) give

$$\epsilon = \frac{1+H}{H} \xi_{B1}. \quad (40)$$

Equation (37c) yields the following relationship between the efficiency and the overall Nusselt number:

$$\frac{d}{dz} \ln(1 - \epsilon) = - \frac{1+H}{H} \text{Nu}_1^0 z. \quad (41)$$

In conventional analysis, the overall heat-transfer coefficient is usually assumed to be uniform along the length of the exchanger. If Nu_1^0 represents the value of the overall Nusselt number assumed uniform, integration of Eq. (41) would yield the following conventional expression for the heat-exchanger efficiency:

$$\epsilon = 1 - \exp \left[- \frac{1+H}{H} \overline{\text{Nu}_1^0} z \right], \quad (42)$$

where z represents the length of the exchanger.

B. Approximation for Turbulent Flow

As discussed previously, the plug-flow model gives predictions for turbulent-flow liquid-metal heat transfer which are most accurate for flow rates characterized by Peclet numbers less than 100. At higher flow rates, heat transport by eddy diffusion becomes significant, thereby necessitating the inclusion of this mode of heat transport in any model that is representative of the heat-transfer mechanism. An approximation technique, developed by Stein,⁵³ is introduced below to extend the results of the plug-flow formulation into a higher-Peclet-number region.

1. Mathematical Treatment

For turbulent flow, the steady-state convection equations equivalent to Eq. (8) become

$$\frac{1}{x} \frac{\partial}{\partial x} \left[(k_1 + c_1 \rho_1 \epsilon_{H1}) x \frac{\partial t_1}{\partial x} \right] = c_1 \rho_1 a_1^2 u_1 \frac{\partial t_1}{\partial \ell} \quad (43a)$$

and

$$\frac{1}{1 - \sigma x} \frac{\partial}{\partial x} \left[(k_2 + c_2 \rho_2 \epsilon_{H2}) (1 - \sigma x) \frac{\partial t_2}{\partial x} \right] = c_2 \rho_2 a_2^2 u_2 \frac{\partial t_2}{\partial \ell}, \quad (43b)$$

where $c_i \rho_i \epsilon_{Hi}$ expresses the contribution to heat transport by turbulent eddy diffusion. The term $k + c_i \rho_i \epsilon_{Hi}$ can be interpreted as an effective total thermal conductivity, k_{ti} .

Assumption (g) (p. 29) can be removed by formulating the problem in terms of the effective conductivity. A new space variable, $\eta(x)$, is introduced by the differential equations

$$k_1^+ \frac{dx}{x} = \frac{k_{t1}}{k_1} \frac{d\eta}{\eta}, \quad (44a)$$

and

$$k_2^+ \frac{dx}{1 - \sigma x} = \frac{k_{t2}}{k_2} \frac{d\eta}{1 - \sigma \eta}, \quad (44b)$$

where $\eta(0) = 0$ and k_1^+ and k_2^+ are chosen so that $\eta(1) = 1$.

If the dimensionless axial distance, z , is also redefined in terms of the new variable k_1^+ ,

$$z = \frac{\alpha_1 k_1^+}{a_1^2 \bar{u}_1} \ell = \frac{4k_1^+}{Pe_1} (\ell/D_1), \quad (45)$$

the convection Eqs. (43a) and (43b) become

$$\frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial t_1}{\partial \eta} \right) = g_1(\eta) \frac{\partial t_1}{\partial z}, \quad (46a)$$

and

$$\frac{1}{1 - \sigma \eta} \frac{\partial}{\partial \eta} \left[(1 - \sigma \eta) \frac{\partial t_2}{\partial \eta} \right] = \frac{R}{1 + R} KH g_2(\eta) \frac{\partial t_2}{\partial z}, \quad (46b)$$

where

$$g_1(\eta) = \frac{u_1}{\bar{u}_1} \frac{k_{t1}}{k_1 k_1^+} \frac{x^2}{\eta^2}, \quad (47a)$$

$$g_2(\eta) = \frac{u_2}{\bar{u}_2} \frac{k_{t2}}{k_2 k_2^+} \frac{(1 - \sigma x)^2}{(1 - \sigma \eta)^2}, \quad (47b)$$

and the average velocities $\bar{u}_i$ are defined as

$$\bar{u}_1 = 2 \int_0^1 x u_1 dx, \quad (48a)$$

and

$$\bar{u}_2 = \frac{2}{1+R} \int_0^1 (1 - \sigma x) u_2 dx. \quad (48b)$$

For turbulent flow with small Peclet numbers, the quantities $g_i(\eta)$ are close to unity over the cross section of the duct, while for all Peclet numbers, $g_i(\eta)$ have an average value of unity across the duct. As an approximation, then, the $g_i(\eta)$ are assumed to be unity for all values of η . The energy Eqs. (46a) and (46b) then become

$$\frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial \xi_1}{\partial \eta} \right) = \frac{\partial \xi_1}{\partial z}, \quad (49a)$$

and

$$\frac{1}{1 - \sigma \eta} \frac{\partial}{\partial \eta} \left[(1 - \sigma \eta) \frac{\partial \xi_2}{\partial \eta} \right] = \frac{R}{1+R} KH \frac{\partial \xi_2}{\partial z}. \quad (49b)$$

The effect of this variable change is to give temperature distributions in terms of $\xi_i(\eta, z)$. Since $\eta(0) = 0$ and $\eta(1) = 1$, the variable change influences only the heat-flux boundary conditions. The expression for the heat flux can be written in terms of the new variable η , as follows:

$$\begin{aligned} q_1 &= k_1 \left. \frac{\partial t_1}{\partial x} \right|_1 \\ &= k_1 \frac{\partial \eta}{\partial x} \left. \frac{\partial t_1}{\partial \eta} \right|_1 \\ &= k_1 \left(\frac{k_1 k_1^+}{k_{t1}} \right) \left. \frac{\partial t_1}{\partial \eta} \right|_1 \end{aligned}$$

At the boundary, $k_{t1} = k_1$, and hence,

$$q_1 = k_1 k_1^+ \left. \frac{\partial t_1}{\partial \eta} \right|_1.$$

Therefore, the effect of the change in variables from x to η is to replace the thermal conductivities by the effective conductivities, $k_i k_i^+$. As a result, the dimensionless parameters K and K_w become

$$K = \left(\frac{k_1}{k_2} \right) \left(\frac{k_1^+}{k_2^+} \right) \frac{\sigma}{R}, \quad (50)$$

and

$$K_w = \frac{k_1 k_1^+}{k_w} \ln \left(1 + \frac{b}{a_1} \right). \quad (51)$$

Calculation for the Nusselt numbers in terms of the dimensionless temperature distributions defined with respect to the space variable η would involve replacing the thermal conductivities, k_i , everywhere by $k_i k_i^+$. If the Nusselt numbers are calculated according to Eqs. (35a) and (35b) or (37a), (37b), and (37c), where $\xi_i = \xi_i(\eta, z)$, then the actual Nusselt numbers would be related to the calculated plug-flow Nusselt numbers by

$$Nu_i(z) \approx k_i^+ [Nu_i(z)]_p, \quad (52a)$$

and

$$Nu_1^0(z) \approx k_1^+ [Nu_1^0(z)]_p, \quad (52b)$$

where $[Nu_i(z)]_p$ denotes the plug-flow values, and

$$z = \frac{4k_1^+}{Pe_1} (\ell/D_1). \quad (45)$$

The approximation sign in Eqs. (52a) and (52b) results from the assumption that $g_i(\eta)$ are equal to unity for all values of η .

2. Approximation of k^+

Exact calculations for k_i^+ would involve a knowledge of the eddy diffusivity distribution across the duct. The value of ϵ_H not only changes across the duct but is dependent upon both the Reynold's and Prandtl numbers of the fluids. If distributions for ϵ_H were available, the values of k_i^+ could be calculated from

$$\frac{1}{k_1^+} \int_0^1 \frac{dx}{x} = \int_0^1 \frac{1}{1 + \frac{\epsilon_H}{\nu} Pr} \frac{dx}{x},$$

and

$$\frac{1}{k_2^+} = -\frac{\sigma}{\ln R} \int_0^1 \frac{1}{1 + \frac{\epsilon_H}{\nu} \text{Pr}} \frac{dx}{1 - \sigma x}.$$

An additional complication arises in the calculation for k_1^+ , however, because of the indeterminacy at $x = 0$.

An alternate approach to the evaluation of k_1^+ would be to use Eq. (52a) and the various Nusselt-number relations for turbulent liquid-metal flow in pipes and annuli. The values for k_1^+ calculated in this manner are, of course, only as accurate as the liquid-metal heat-transfer relations chosen. The accuracy of the approximation is compared by Stein in Ref. 53.

Stein illustrates the accuracy of the approximation by computing Nusselt numbers in the thermal-entrance region of a parallel-plane duct and comparing the results with the calculations of Poppendiek.⁴⁴ The prediction of thermal-entrance-region Nusselt numbers represents a stringent test of the approximation, since k^+ is involved when computing appropriate values of both the Nusselt number and the axial distance, according to Eqs. (52) and (45). The results of the calculations are shown in Fig. 14, from Ref. 53. The k^+ approximation brings the plug-flow solution into agreement with Poppendiek's results in all but the beginning of the thermal-entrance region. The temperature gradients in this region are localized near the duct wall, where the total conductivity, k_t , is close to the molecular conductivity, k , and hence, use of the k^+ approximation here gives effective conductivities that are too large. Calculations for flows at lower Peclet numbers were stated to give even better agreement with Poppendiek's results.

Fig. 14
Nusselt Number vs Length Predictions, Uniform Surface Temperature in Parallel-plane Channel⁵³

For heat transfer in tubes, a variety of heat-transfer relations have been developed. The most commonly used relations are those of Lyon,^{33,34} Eq. (53), for a uniform-heat-flux boundary condition,

$$\text{Nu}(\infty) = 7.0 + 0.025\text{Pe}^{0.8}, \quad (53)$$

and Seban and Shimazaki,⁴⁹ Eq. (54), for a uniform-surface-temperature boundary condition,

$$\text{Nu}(\infty) = 5.0 + 0.025\text{Pe}^{0.8}. \quad (54)$$

The corresponding plug-flow Nusselt numbers can be shown to be 8.0 and 5.78.¹⁴ Calculation of the quantity k_1^+ according to Eq. (52a) would then yield

$$k_1^+ \approx \frac{1}{8}(7.0 + 0.025\text{Pe}^{0.8}) \quad (55a)$$

from the Lyon equation, and

$$k_1^+ \approx \frac{1}{5.78} (5.0 + 0.25\text{Pe}^{0.8}) \quad (55b)$$

from the Seban and Shimazaki equation.

The values of k_1^+ as computed by Eqs. (55a) and (55b) are shown in Fig. 15, from Ref. 53, where the agreement is good up to Peclet numbers of greater than 1000. Various other

Fig. 15. Comparison of k_1^+ Computed by Various Methods⁵³

theoretical heat-transfer relations could have been chosen to evaluate values of k_1^+ , but experimental results in the Soviet Union^{27, 28, 59, 60} have indicated that the Lyon relation is probably the most accurate prediction for heat transfer with a uniform heat flux.

The plug-flow values for constant-heat-flux heat transfer in an annulus are given by⁶⁴

$$[\text{Nu}(\infty)]_p = \frac{8(D_3/D_2 - 1)[(D_3/D_2)^2 - 1]^2}{4(D_3/D_2)^4 \ln(D_3/D_2) - 3(D_3/D_2)^4 + 4(D_3/D_2)^2 - 1}, \quad (56)$$

where D_3/D_2 is the inverse of the annular ratio, R . Several heat-transfer relations have been developed for liquid-metal heat transfer in annuli with a uniform heat flux. Mentioned previously were those of Seban⁴⁸ for a narrow annulus (represented by parallel planes),

$$\text{Nu}(\infty) = 5.8 + 0.020\text{Pe}^{0.8}, \quad 0.7 < R < 1.0, \quad (2)$$

and Werner et al.⁶⁷ for a wide annulus,

$$\text{Nu}(\infty) = 0.75(R)^{-0.3} (7.0 + 0.025\text{Pe}^{0.8}), \quad R < 0.7. \quad (3)$$

Among the other relations that have been developed are those of Bailey,²

$$\text{Nu}(\infty) = [\text{Nu}(\infty)]_p + 0.0106R^{-0.37}\text{Pe}^{0.86}, \quad (57)$$

and Dwyer,¹⁰⁻¹³

$$\text{Nu}(\infty) = 4.82 + 0.697/R + 0.222(\bar{\psi}\text{Pe})^\gamma, \quad \bar{\psi} > 0.0 \quad (58a)$$

where

$$\gamma = 0.758R^{-0.053},$$

$$\bar{\psi} = 1 - \frac{1.82}{\text{Pr}(\epsilon_M/\nu)_{\max}^{1.4}},$$

$$(\epsilon_M/\nu)_{\max} = 4.0 + 0.029\text{Re}^{0.919},$$

and

$$\text{Nu}(\infty) = 4.98 + 0.662/R, \quad \bar{\psi} < 0.0. \quad (58b)$$

Kays and Leung²⁶ have also presented values for the Nusselt number in tabular form.

Fig. 16. Comparison of k_2^+ Computed by Various Methods

The k^+ approximation for the annular space is also relatively insensitive to the annular ratio, R . This insensitivity to R is illustrated in Fig. 17, where values of k_2^+ were calculated by use of Eqs. (58a) and (58b).

Fig. 17. Comparison of k_2^+ Computed by Dwyer's Equations for Various Values of R .

C. Plug-flow Solutions

1. Temperature Distribution

The system of equations to be solved consists of the energy Eqs. (23a) and (23b) with the initial and boundary conditions given by Eqs. (26) to (29). The parameters K and K_w and the dimensionless variable z are given by Eqs. (50), (51), and (45), respectively. Solution of this set of equations yields values for a "plug-flow" Nusselt

number from which the actual Nusselt numbers may be determined by Eqs. (52a) and (52b).

Two techniques are available for solving the system of equations. As discussed by Stein,⁵⁴ a separation-of-variables technique may be used, leading to a Sturm-Liouville problem where the orthogonality condition is defined over both regions, channels 1 and 2. The details of solution by this technique are given in Appendix A. An alternate method of obtaining the solution to the system of equations is to use the Laplace-transform technique, in which the transform is taken with respect to the axial coordinate, z .

The solutions for the dimensionless temperature distributions, $\xi_i(x, z)$, can be written as

$$\xi_1(x, z) = \frac{H}{1+H} + \sum_{n=1}^{\infty} A_n^{(1)}(\lambda_n, x) e^{-\lambda_n^2 z}, \quad (59a)$$

$$\xi_2(x, z) = \frac{H}{1+H} + \sum_{n=1}^{\infty} A_n^{(2)}(\lambda_n, x) e^{-\lambda_n^2 z}, \quad (59b)$$

$$A_n^{(1)} = -\frac{2L_1(\omega\lambda_n R) J_0(\lambda_n x)}{\lambda_n F'(\lambda_n)}, \quad (60a)$$

and

$$A_n^{(2)} = -\frac{2KJ_1(\lambda_n) L_0[\omega\lambda_n(1-\sigma x)]}{\lambda_n \sigma \omega F'(\lambda_n)}, \quad (60b)$$

where

$$\omega^2 \sigma^2 = \frac{R}{1+R} KH, \quad (61)$$

$$L_0(\omega\lambda_y) \equiv J_0(\omega\lambda_y) Y_1(\omega\lambda) - J_1(\omega\lambda) Y_0(\omega\lambda_y), \quad (62a)$$

and

$$L_1(\omega\lambda_y) \equiv J_1(\omega\lambda_y) Y_1(\omega\lambda) - J_1(\omega\lambda) Y_1(\omega\lambda_y). \quad (62b)$$

The eigenvalues λ_n are the positive, nonzero roots of the eigenvalue equation.

$$F(\lambda) = 0,$$

where

$$F(\lambda) = K_w \lambda J_1(\lambda) L_1(\omega\lambda R) - J_0(\lambda) L_1(\omega\lambda R) + \frac{K}{\sigma\omega} J_1(\lambda) L_0(\omega\lambda R). \quad (63)$$

The term $F'(\lambda)$ represents the derivative of $F(\lambda)$ with respect to λ .

2. Heat-transfer Coefficients

Values for the local Nusselt numbers can now be determined from the dimensionless temperature distributions using Eqs. (35a) and (35b) or Eqs. (37a) to (37c). Near the entrance to the heat exchanger, the Nusselt numbers are complicated functions of the axial coordinate, z . For sufficiently large values of z , however, all but the first terms in the series equations for the temperature distributions, (59a) and (59b), may be neglected, and the following asymptotic or "fully-developed" values for the heat-transfer coefficients result:

$$Nu_1(\infty) = \frac{2\lambda_1^2 J_1(\lambda_1)}{2J_1(\lambda_1) - \lambda_1 J_0(\lambda_1)}, \quad (64a)$$

$$Nu_2(\infty) = \frac{2\sigma^2 \omega^2 \lambda_1^2 L_1(\omega\lambda_1 R)}{\frac{2R}{1+R} L_1(\omega\lambda_1 R) + \sigma\omega\lambda_1 L_0(\omega\lambda_1 R)}, \quad (64b)$$

and

$$Nu_1^0(\infty) = \frac{H}{1+H} \lambda_1^2. \quad (64c)$$

Where the heat-transfer coefficients are fully developed, they are independent of length along the exchanger. The region in which they are not fully developed and, hence, depend upon the distance, z , is called the thermal-entrance region. The subject of the thermal-entrance region will be discussed later in Section E of this chapter.

3. Heat-exchanger Efficiency

Previously, the heat-exchanger efficiency was shown to be

$$\epsilon = \frac{1+H}{H} \xi_{B1}. \quad (40)$$

From Eq. (32a) and the dimensionless temperature distributions,

$$\epsilon = 1 - \frac{1+H}{H} \sum_1^{\infty} \frac{4L_1(\omega\lambda_n R) J_1(\lambda_n)}{\lambda_n^2 F'(\lambda_n)} e^{-\lambda_n^2 z}. \quad (65)$$

As introduced by Stein,⁵⁴ Eq. (65) can be written as

$$\epsilon = 1 - \phi(z) e^{-\lambda_1^2 z}, \quad (66a)$$

or

$$\epsilon = 1 - \phi(z) \exp\left[-\frac{1+H}{H} \text{Nu}_1^0(\infty) z\right], \quad (66b)$$

where

$$\phi(z) = \frac{1+H}{H} \sum_1^{\infty} \frac{4L_1(\omega\lambda_n R) J_1(\lambda_n)}{\lambda_n^2 F'(\lambda_n)} \exp\left[-(\lambda_n^2 - \lambda_1^2) z\right]. \quad (67)$$

Comparison of Eq. (66b) with the customary relation, Eq. (42), shows that the two are equivalent when

$$\overline{\text{Nu}}_1^0 = \text{Nu}_1^0(\infty),$$

and

$$\phi(z) = 1.$$

Stein⁵⁴ has called the quantity $\phi(z)$ the "effectiveness coefficient." For sufficiently large values of z , the effectiveness coefficient will reach a fully-developed value

$$\phi(\infty) = \frac{1+H}{H} \frac{4L_1(\omega\lambda_1 R) J_1(\lambda_1)}{\lambda_1^2 F'(\lambda_1)}. \quad (68)$$

The requirement $\epsilon = 0$ at $z = 0$ gives

$$\phi(0) = 1.$$

It can also be shown⁵⁴ that

$$\phi(\infty) \leq \phi(z) \leq 1.0,$$

and that $\phi(z)$ decreases monotonically as z increases.

D. Numerical Results

The heat-transfer characteristics of the heat exchanger have been shown to be determined by values of the dimensionless parameters H , K , K_w , and R . It is of interest to explore the dependence of the heat-transfer characteristics on these parameters over the range of practical interest.

1. Fully-developed Heat-transfer Coefficients

Figures 18-23 show typical values of fully-developed heat-transfer coefficients as a function of the heat-exchanger operating conditions, where the heat-transfer coefficients have been normalized with respect to the appropriate plug-flow values corresponding to a uniform-heat-flux boundary condition. The uniform-heat-flux plug-flow value for the tube side is 8.0;¹⁴ that for the annular space is given by Eq. (56). Values for overall heat-transfer coefficient are then determined by Eq. (39). Also shown are the normalized plug-flow values corresponding to the uniform surface temperature boundary condition (see Appendix B for the appropriate values).

Fig. 18. Fully-developed, Tube-side, Heat-transfer Coefficients, Normalized, vs Heat-capacity Flow-rate Ratio ($R = 0.5$, $K = 1.0$)

Fig. 19. Fully-developed, Annulus-side, Heat-transfer Coefficients, Normalized, vs Heat-capacity Flow-rate Ratio ($R = 0.5$, $K = 1.0$)

Fig. 20. Fully-developed, Overall, Heat-transfer Coefficients, Normalized, vs Heat-capacity Flow-rate Ratio ($R = 0.5$, $K = 1.0$)

Fig. 21. Fully-developed, Tube-side, Heat-transfer Coefficient, Normalized, vs Fluid Thermal-resistance Ratio ($H = 2.0$, $K_w = 0$)

Fig. 22. Fully-developed, Annulus-side, Heat-transfer Coefficients, Normalized, vs Fluid Thermal-resistance Ratio ($H = 2.0$, $K_w = 0$)

Fig. 23. Fully-developed, Overall, Heat-transfer Coefficients, Normalized, vs Fluid Thermal-resistance Ratio ($H = 2.0$, $K_w = 0$)

Figures 18-20 show the dependence of the fully-developed heat-transfer coefficients on the heat-capacity flow-rate ratio, H , for various values of the relative wall thermal resistance, K_w . It is seen that the heat-transfer coefficients can be significantly lower than those corresponding to a uniform surface-temperature condition while they are never larger than those corresponding to a uniform heat-flux condition. Increasing the

flow rate in the tube or annulus leads to a lower relative heat-transfer coefficient in the respective channel; increasing the thermal resistance of the wall separating the fluids leads to higher relative values of the individual-channel heat-transfer coefficients.

Figures 21-23 show the dependence of the fully-developed heat-transfer coefficients on the fluid thermal-resistance ratio, K , for various values of the annular ratio, R . The parameter R is included since it is a factor in both the parameter K and the energy equation for the annular space, Eq. (23b). The Nusselt number depends less on the fluid thermal-resistance ratio, K , than on the heat-capacity flow-rate ratio, H . In general, increasing the thermal resistance of the fluid in a channel decreases the corresponding heat-transfer coefficient. As expected, the dependence on the annular ratio, R , is greatest for the annulus-side heat-transfer coefficient. This dependence on R will be explored more fully in Section F of this chapter, where the results of the formulation are compared with those of the narrow-annulus approximation of Stein.⁵³

2. Heat-exchanger Efficiency

The heat-exchanger efficiency depends upon the fully-developed overall Nusselt number, $Nu_1^0(\infty)$, and the effectiveness coefficient, $\phi(z)$. As shown by Eq. (67), the effectiveness coefficient is a function of z in the region in which it is not fully developed. For purposes of illustration, however, attention will be focused here on the fully-developed effectiveness coefficient, $\phi(\infty)$.

Figures 24 and 25 show the dependence of the effectiveness coefficient on the four basic parameters H , K , K_w , and R . As with the Nusselt numbers, the dependence is greatest for the heat-capacity flow-rate ratio, H .

Fig. 24. Fully-developed Effectiveness Coefficient vs Heat-capacity Flow-rate Ratio ($R = 0.5$, $K = 1.0$)

Fig. 25. Fully-developed Effectiveness Coefficient vs Fluid Thermal-resistance Ratio ($H = 2.0$, $K_w = 0$)

Values significantly less than unity can occur, and, hence, the effectiveness coefficient can be an important quantity in heat-exchanger design. As the relative wall thermal resistance, K_w , is increased, the effectiveness coefficient approaches unity, where the conventional equation for the efficiency, Eq. (42), and the exact equation, Eq. (66b), become equivalent in form. This is expected, since with large values of K_w , the resistance of the wall controls the heat transfer. Figure 25 also suggests that the effectiveness coefficient becomes most important for heat-exchanger configurations with a narrow annular space.

As an illustration of the errors incurred in a calculation of the heat-exchanger efficiency according to the customary design equation,

Fig. 26. Ratio of Heat-transfer Lengths Predicted by Customary Design Equation to Lengths Predicted by Present Analysis vs Efficiency ($H = 1.0$, $K = 1.0$, $K_w = 0$, $R = 0.5$)

Fig. 26 shows the ratio of the required heat-transfer lengths predicted by the customary design equation and the present analysis versus heat-exchanger efficiency. Calculation according to the customary design equation with the usually-assumed uniform-surface-temperature boundary condition can lead to significant errors. For efficiencies greater than 0.7, calculation using a heat-transfer coefficient appropriate to a uniform heat flux gives efficiencies close to those predicted by use of Eq. (66b). The exact analysis for the efficiency accounts for the high rates of heat transfer in the thermal-entrance region, whereas conventional analysis is based on the assumption that the heat-transfer coefficient is uniform along the length of the exchanger. The use of Nu_1^0 appropriate to a uniform-heat-flux condition gives the largest efficiency consistent with the assumption that the heat-transfer coefficient is uniform.

E. Thermal-entrance Region

The subject of the thermal-entrance region is only incidental to this report and, hence, will be considered only briefly.

The "thermal-entrance region" designates the region in which the temperature profile of the heat-exchanging fluid is changing rapidly as a function of length. Mathematically, this corresponds to the region in which more than one term is necessary in the eigenvalue expansion for the dimensionless temperature distribution, $\xi_i(x, z)$. The dimensionless heat-transfer coefficients, $Nu_i(z)$, and the effectiveness coefficient, $\phi(z)$, are also functions

of length in this region. Usually the local heat-transfer coefficients are higher than the fully-developed coefficients in the entrance region. There are situations, however, in which the tube-side or annulus-side local coefficient has a minimum value below that for the fully-developed coefficient (see Fig. 27).

Fig. 27. Ratio of Local to Fully-developed Annulus-side Heat-transfer Coefficient vs Efficiency ($H = 0.5$, $K = 1.0$, $K_w = 0$, $R = 0.5$)

A convenient criterion for the length of the entrance region could be based upon the ratio of the local to fully-developed coefficients. The length based on this criterion varies, however, depending upon the coefficient of interest. Attempts to relate the entrance length to specific values of the four dimensionless parameters, H , K , K_w , and R , have been fruitless, and only general statements can be made. Increasing the flow rate in channel i lengthens the entrance length in that channel relative to the individual-channel heat-transfer coefficient. An increase in the relative wall resistance, however, decreases the length of the entrance region relative to all coefficients.

The local coefficient of most interest to the designer is the local effectiveness coefficient. Figure 28 illustrates local values as a function of the heat-exchanger efficiency. The entrance length based on this coefficient can vary greatly, depending upon the value of the heat-capacity flow-rate ratio. For flow-rate ratios $1 < H < 2$, however, the local value for the effectiveness coefficient may be approximated by its fully-developed value for heat-exchanger efficiencies as low as 0.3. Engineering calculations based on Eq. (66b) with the use of the fully-developed coefficient $\phi(\infty)$ should give sufficiently accurate values for the efficiency, and would certainly represent an improvement over calculations based on the conventional design equation.

Fig. 28

Ratio of Local to Fully-developed Effectiveness Coefficient vs Efficiency ($K = 1.0$, $R = 0.5$, $K_w = 0$)

F. Narrow-annulus Approximation

As mentioned previously, the analysis of Stein⁵³ included the assumption of a narrow annular space. The annular space was represented by the space between two infinitely-wide parallel planes, thus simplifying the mathematical analysis.

Under this assumption, the energy equation for the annular space, corresponding to Eq. (23b), became

$$\frac{\partial^2 \xi_2}{\partial x^2} = (1/2) KH \frac{\partial \xi_2}{\partial x}.$$

As the annular ratio approaches unity, Eq. (23b) reduces to the above equation. It can be shown that the equations for the fully-developed Nusselt numbers, Eqs. (64a) to (64c), reduce to the corresponding equations developed for the narrow-annular-space approximation:

$$Nu_1(\infty) = \frac{2\lambda_1^2 J_1(\lambda_1)}{2J_1(\lambda_1) - \lambda_1 J_0(\lambda_1)},$$

and

$$Nu_2(\infty) = \frac{2\omega'^2 \lambda_1^2 \sin(\omega' \lambda_1)}{\sin(\omega' \lambda_1) - \omega' \lambda_1 \cos(\omega' \lambda_1)},$$

where

$$\omega'^2 = \frac{HK}{2},$$

and the eigenvalue λ_1 is determined from

$$F'(\lambda) = 0 = \frac{K}{\omega} J_1(\lambda) \cos(\omega' \lambda) + J_0(\lambda) \sin(\omega' \lambda) - K_w \lambda J_1(\lambda) \sin(\omega' \lambda).$$

Since these equations for the heat-transfer coefficients are much easier to work with than the corresponding equations for the wide annular space, it is of interest to determine when the narrow-annular-space approximation gives accurate results. Figures 29-31 show the normalized, fully-developed, Nusselt numbers predicted by the two analyses for various values of the annular ratio, R . The tube-side Nusselt numbers are not greatly affected by R over the range shown. For the annulus-side and overall Nusselt numbers, however, the narrow-annulus approximation gives accurate results only for annular ratios greater than 0.7. The annular ratio also determines the value of the fluid thermal-resistance ratio, K . To investigate the effect of K , calculations were made for an annular ratio of 0.1.

The results, shown in Fig. 32, indicate that even for annular ratios as low as 0.1, the narrow-annulus approximation gives accurate results for the overall heat-transfer coefficient when K is less than unity. The narrow-annulus approximation also becomes increasingly accurate as the relative wall thermal resistance, K_w , is increased. Calculations for the effectiveness coefficient, $\phi(\infty)$, show the same trends as those presented for the overall Nusselt number.

Fig. 29. Comparison of Narrow-annulus Approximation and Present Analysis for Fully-developed Tube-side Nusselt Number vs Annular Ratio

Fig. 30. Comparison of Narrow-annulus Approximation and Present Analysis for Fully-developed Annulus-side Nusselt Number vs Annular Ratio

Fig. 31. Comparison of Narrow-annulus Approximation and Present Analysis for Fully-developed Overall Nusselt Number vs Annular Ratio

Fig. 32. Comparison of Narrow-annulus Approximation and Present Analysis for Fully-developed Overall Nusselt Number vs Fluid Thermal-resistance Ratio ($H = 2.0$, $K_w = 0$, $R = 0.1$)

The important parameter, then, which determines when the narrow-annulus approximation yields accurate results, is the parameter K . With the narrow-annulus approximation, the effect of the annular ratio, R , is accounted for in the definition of K . A low value of K also corresponds to a low thermal resistance for the fluid in the annular space. The temperature gradient would be much larger through the wall and in channel 1, and, hence, the combined resistance of the wall and the fluid in channel 1 would control the heat transfer.

In summary, then, the narrow-annulus approximation provides sufficiently accurate estimates of the overall Nusselt number and the effectiveness coefficient for $R > 0.7$ or for $K < 1.0$.

G. Summary

An analysis has been presented for heat transfer with liquid metals in a cocurrent-flow, double-pipe, heat exchanger. In general, it has been shown that:

(1) Heat-transfer coefficients significantly lower than those corresponding to a uniform-surface-temperature boundary condition can be obtained.

(2) The heat-transfer boundary conditions depend upon the operating conditions of the heat exchanger as defined by the four basic parameters H , K , K_w , and R .

(3) The heat-transfer coefficients vary along the length of the exchanger and can lead to higher heat-exchanger efficiencies than those calculated for the usually-assumed boundary condition of uniform surface temperature.

(4) For annular ratios greater than 0.7 or values of K less than 1.0, the narrow-annulus approximation of Stein⁵³ yields sufficiently accurate estimates of the overall Nusselt number and the effectiveness coefficient.

The analysis has been used to calculate heat-transfer coefficients and efficiencies for the practical case of the heat exchangers used in the experimental portion of this report. The results of the analysis are presented in Chapter VII, where they are compared with the experimental results.

IV. EXPERIMENTAL APPARATUS

A. General Considerations

To conduct an experimental investigation of heat-exchanger performance, a liquid-metal heat-transfer loop, with cocurrent-flow, heat-exchanger, test sections, was designed and built. The aim of the experimental investigation was to determine local, fully-developed, heat-transfer coefficients and heat-exchanger efficiencies as a function of the heat-exchanger operating conditions, expressed by the dimensionless parameters, H , K , K_w , and R . Once the test-section materials and the heat-transfer fluid were chosen, the only parameter that could be significantly varied was the heat-capacity flow-rate ratio, H . A careful choice of materials and the heat-exchanger dimensions was necessitated, then, to obtain heat-exchanger operating conditions of experimental interest.

The first choice of materials to be made was the choice of the heat-transfer fluid. Of the liquid metals readily available for experimental work, mercury appeared to be the most suitable because of its ease of handling, relative to the other liquid metals. Considerable heat-transfer work had also been done in the past using mercury, and, hence, a technology had been developed. Equally important, the physical properties of mercury are well-known. Certain disadvantages, however, are inherent in the use of mercury as a heat-transfer fluid. Among these are its toxicity, its difficulty with containment, and its apparent nonwetting of most metal surfaces. Each of these difficulties was considered and dealt with in a manner described below.

B. Experimental Approach

The primary interest lay in the heat-exchanger efficiency and the individual channel and overall, fully-developed, heat-transfer coefficients. A determination of these quantities requires knowledge of the values of the individual wall-surface temperatures, the bulk-fluid temperatures, and the local heat flux. These quantities are usually difficult to obtain with satisfactory accuracy--mainly because of the small temperature differences involved. As a result, most of the past experimental investigations, as described in Chapter II, determined only average heat-transfer coefficients. The only experiment known to the author involving liquid-metal flow on both sides of the separating wall in which a measurement of the temperature of the wall was attempted was that of Sawochka and Schleef.^{30,46} The measurement technique involved the placement of several thermocouples at known radial locations in the center-tube wall, and the calculation of the wall-surface temperatures from the wall-temperature profile. This approach necessitated the use of a thick wall and involved special machining processes.

In addition, the use of a thick wall gave high values of the relative wall thermal resistance, K_w , for which the analysis suggests that the heat-transfer coefficients approach the values appropriate to a uniform heat flux.

As pointed out to the author by Stein,⁵⁶ the analysis suggests an alternate procedure for determining the individual heat-transfer coefficients. Detailed temperature measurements are made along the length of the outer wall. These temperatures are then written in a dimensionless form and plotted on semilog paper. When the heat transfer is fully developed, the plot becomes linear and its slope is related to the overall Nusselt number, Eqs. (59b) and (64c). With the values for the overall Nusselt number and the bulk-fluid temperatures at the exchanger outlet, the local heat flux may be calculated. A single thermocouple embedded at a known radial location in the wall gives a wall-temperature measurement from which the wall-surface temperatures, and, hence, the individual-channel local-heat-transfer coefficients may be determined, once the heat flux is known.

This approach was chosen because of the various advantages inherent in its use compared to the method involving more than one tube-wall thermocouple. Since only one thermocouple need be located in the tube wall, the wall thickness need not be large. With a high wall thermal conductivity, small values of K_w will result, and the radial location of the wall thermocouple need not be known accurately since the temperature drop across the wall will be small. Since many thermocouples may be placed along the outside wall, the technique is potentially more accurate than other methods. Of special importance is that the technique allows one to determine when the heat transfer becomes fully developed. Also, as will be seen later, use of this procedure aided in revealing nonidealities in the apparatus that might otherwise have been undetected. A more detailed discussion of this approach is presented in Chapter V.

C. Description of the Apparatus

1. Summary

The general characteristics of the heat-transfer apparatus are shown in Fig. 33. The mercury flow splits into two streams after leaving the pump and the pump bypass. The "tube-side" stream flows through a flow control valve into the heater section, and after being heated, flows upwards through the tube of the heat-exchanger test section. Upon leaving the test section, the fluid flows past an electromagnetic flowmeter and enters the pump suction line. The "annulus-side" stream flows through a flow control valve into a cooling section and, from there, into the annulus of the heat-exchanger test section. The cooler fluid passes upwards through the annulus in parallel flow with the hotter fluid in the tube. After

leaving the test section, the fluid flows past an electromagnetic flowmeter and enters the pump suction line. The locations at which the bulk-fluid temperature is measured are indicated in Fig. 33. Shown also are an expansion chamber and a storage tank.

LEGEND

1. SIGHT GLASS
2. EXPANSION CHAMBER
3. ELECTROMAGNETIC FLOWMETERS
4. ORIFICE FLANGES
5. ANNULUS OUTLET MIXING CHAMBER
6. TUBE OUTLET MIXING CHAMBER
7. TEST SECTION
8. STORAGE TANK
9. BYPASS CONTROL VALVE
10. CENTRIFUGAL PUMP
11. FLOW CONTROL VALVES
12. HEATERS
13. COOLER
14. COOLER OUTLET MIXING CHAMBER

Fig. 33. Heat-transfer Flow-loop Components

2. Test-section Design

a. General

The design of the test section determines the values of the dimensionless parameters, K , K_w , and R , once the choice of the heat-transfer fluid is made. Hence, the materials and dimensions of the test section were determined to a large extent by the heat-exchanger operating conditions of interest.

Several types of test sections were designed. Two test sections, with different heat-transfer lengths (Figs. 34 and 35) were designed to obtain local heat-transfer coefficients at the outlet end of the exchanger. The choice of test sections of different heat-transfer lengths was made to investigate the effect of the heat-exchanger length on the values of the individual heat-transfer coefficients and the efficiencies. A third test section (Fig. 36) was designed late in the program to investigate heat-exchanger efficiencies with a short heat exchanger in which the heat transfer is not fully developed.

Fig. 34 Design of 20 L/D_1 and 10 L/D_1 Test Sections

Fig. 35. Test-section Center-tube Design

Figure 36
Design of 8 L/D_1 Test Section

The test-section material chosen was pure nickel, primarily because of its high thermal conductivity and its ability to withstand attack by mercury at the temperature range of operation. The literature^{10,12} also indicates that nickel and mercury form a "wetting" system, and, hence, the possibility of the existence of an additional thermal-contact resistance at the heat-transfer surfaces would be minimized.

The following standard nickel pipe was chosen for construction of the test sections:

Tube: 3/4-in. Schedule 160
Annulus: 2-in. Schedule 40.

The choice of the annulus size gave an annular ratio of 0.5, a value that is in the range of practical interest in heat-exchanger design.

b. Test Sections for Determination of Local Heat-transfer Coefficients

To establish the fluid-velocity profiles before the transfer of heat, a hydraulic entrance region, of 20 L/D_1 in length, preceded the

heat-transfer region in the test sections. The literature contains varying estimates as to the length required. If the criterion is that the velocity profile itself be fully established, lengths of 50 to 80 diameters are usually stated as necessary; if the criterion is that the local pressure gradient become fully established, entrance lengths of only 10 to 20 diameters are stated as sufficient. Because of physical limitations, the length of the entrance region in the test sections was $20 L/D_1$. As mentioned previously, the effect of the shape of the velocity profile on the transfer of heat is not as important with liquid metals as with high-Prandtl-number fluids.

The entrance-region detail is shown in Fig. 35. The center pipe was turned down, and a 1-in.-diam nickel tube was slipped over the end of the tube, forming an insulating air gap. Calculations indicated that the air gap formed an excellent thermal insulator, with a heat flow across the air gap of less than 1% of the total heat flow in the heat exchanger. The 1-in.-diam nickel tube was welded to the center pipe at only one end; hence, the thermal stresses resulting from the temperature difference across the center-tube wall in the hydraulic-entrance region would not be concentrated in the test section, since the center tube would be allowed to slip in the 1-in.-diam tube.

The analysis indicated that the heat-transfer lengths required for the heat transfer to become fully developed varied considerably with the operating conditions of the heat exchanger. For the range of conditions of experimental interest, the length would be about 5 to 20 diameters; hence, heat-transfer lengths of $10 L/D_1$ and $20 L/D_1$ were chosen for this investigation. Longer heat-transfer sections would result in higher heat-exchanger efficiencies and, therefore, smaller temperature differences between the fluids at the outlet.

To accommodate the tube-wall thermocouple, a 0.067-in.-diam hole was drilled at a specified location in the tube wall (Fig. 35), the length of the hole being such that the tip of the thermocouple would be located near the outlet of the annulus. Radiographs of the test-section center tubes indicated that the thermocouple hole alignment was maintained for the length of the hole.

c. Test Section in Which Heat Transfer Is Not Fully Developed

Late in the program, a short ($8 L/D_1$) test section was designed and constructed to provide a check on the analysis for heat-exchanger efficiencies in the region where the heat transfer is not fully developed. The test section, shown in Fig. 36, had neither a hydrodynamic entrance region nor a provision for determining the surface temperatures of the separating tube wall. Hence, only heat-exchanger efficiencies were obtained with this test section.

Since there was no provision for a hydraulic entrance region, the design represented a compromise with the fully-developed flow idealization. However, as pointed out previously, this idealization is not as important for heat transfer with liquid metals as with high-Prandtl-number fluids. The heat-exchanger end plates (Fig. 36) were constructed of stainless steel to reduce heat conduction in this region, since tests with the other test sections suggested that end-plate heat conduction could have a significant effect when the end plates were constructed of a high thermal-conductivity material.

3. Heat-transfer Flow Loop

a. General Construction Notes

With the exception of the test sections and the water cooler, all metal surfaces in contact with mercury were stainless steel. Many other metals are rapidly corroded by mercury, making them undesirable for purposes of construction. The test sections and water cooler were constructed of nickel because of heat-transfer considerations.

In consideration of the high tendency of mercury to leak, all connections were welded wherever possible. Since, however, a premium was placed on the ability to easily disassemble and clean the loop as well as to easily replace the various test sections, many connections within the loop were flanged. The flanges used were, for the most part, ring-groove flanges with stainless-steel ring gaskets. Stainless-steel "Flexitallic" gaskets with Teflon filler were used for the flange connections with the pump and with the heaters.

All the valves in contact with mercury were needle-type valves with Teflon packing. The flow-control and bypass valves were welded into the loop while the drain and vent valves were, in general, attached by the use of threaded fittings on which Teflon tape was used.

The above method of construction completely eliminated the mercury-leakage problems normally found with any mercury heat-transfer apparatus.

b. Pump

The flow-loop design called for the use of a pump capable of delivering 15 gal of mercury per minute at a total dynamic head of 6 ft. After a careful consideration of the various types of pumps available for mercury use, a Goulds Model 3196 centrifugal pump with a water-cooled, double mechanical seal was chosen. The pump was operated over about 6 months without any difficulty in the operation of the pump itself or in any leakage of mercury past the mechanical seal.

Since the pump was coupled to a constant-speed driver, flow control was provided by a system of flow-control and bypass valves (Fig. 33). Stainless-steel pressure gauges, on the suction and discharge sides of the pump, assisted in flow regulation.

c. Heaters

Three hairpin-shaped, tubular, immersion heaters with stainless-steel sheaths supplied the heat input to the mercury flowing through the center tube of the test section. Each heater had a maximum output of $3\frac{1}{3}$ kW at a voltage of 440 V, three-phase, giving a total power rating of 10 kW. Power input to the heaters was through a powerstat.

d. Water Cooler

The mercury flowing through the annular space of the test section was cooled by water in a single-pass, countercurrent-flow, heat exchanger. The cooler was designed so that water-temperature rises on the order of 10-20°F could be maintained to obtain accurate heat balances. The flow through the cooler was provided by an auxiliary pump supplied by a constant head tank. This type of arrangement provided extremely steady flow rates. Since the operating conditions required measurement of water flow rates of from 0.15 to 3.60 gal/min, two calibrated orifice plates were used with two manometers, as listed in Table II.

TABLE II. Water-orifice-assembly Characteristics

Orifice Diameter, in.	Pipe Diameter, in.	Specific Gravity of Manometer Fluid	Water Flow Rate, lb/hr
0.093	0.622	2.95	21.1 $\sqrt{\Delta h^*}$
0.093	0.622	13.60	53.6 $\sqrt{\Delta h}$
0.203	0.622	2.95	101.7 $\sqrt{\Delta h}$
0.203	0.622	13.60	256.0 $\sqrt{\Delta h}$

* Δh is in inches of manometer fluid.

e. Mercury-flow-rate Measurements

Two systems were used to determine the mercury-weight flow rates through the tube and the annulus of the test sections. The primary system consisted of simply-constructed, electromagnetic, flow meters. As shown in Fig. 37, copper rods were inserted through compression fittings welded into the return lines from the test section; permanent magnets, with a strength of 5000 Gauss, were placed around the pipes with the magnetic field perpendicular to both the copper rods and the direction of flow. The mercury flow produced an emf across the copper pins,

the strength of which was proportional to the velocity and, hence, the mercury-weight flow rate. The emf's generated were read on a precision potentiometer.

1. Sight Glass
2. Expansion Chamber
3. Electromagnetic Flowmeter

Fig. 37. Expansion Chamber and Electromagnetic Flowmeters

The secondary system for flow-rate determination consisted of calibrated orifice assemblies in both the test-section tube and the annulus discharge lines, the purpose of the orifice assemblies being to allow for a convenient recalibration of the electromagnetic flowmeters. The orifice assemblies were themselves calibrated with water before installation into the mercury loop, the water flow rates over the Reynold's number range corresponding to the mercury flow rates. This procedure of calibration has been used previously in liquid-metal studies.²⁵ Table III lists the results of the calibration.

TABLE III. Mercury-orifice-assembly Characteristics

Test-section Discharge Line	Orifice Diameter, in.	Pipe Diameter, in.	Mercury Flow Rate, lb/hr
Tube	0.213	0.824	$1034 \sqrt{\Delta h}^*$
Annulus	0.375	0.824	$3264 \sqrt{\Delta h}$

* Δh is in inches of mercury.

The electromagnetic flowmeters were preferred to the orifice meters for flow-rate measurements during actual heat-transfer

Fig. 38. Sample Calibration Curve for Electromagnetic Flowmeter

mercury with temperature. A sight glass was also installed in the expansion chamber so that the mercury level could be observed at all times. This proved to be particularly useful when filling the loop.

tests since a wider range of flow rates could be covered. Since, however, the copper electrodes were subject to corrosion by the flowing mercury, the flowmeters frequently had to be recalibrated. Figure 38 shows a sample calibration curve in which the output is linear with respect to the mercury flow rate.

f. Expansion Chamber

An expansion chamber, located at the highest point in the loop (Figs. 33 and 37), allowed for the density change of the mercury with temperature. A sight glass was also installed in the expansion chamber so that the mercury level could be observed at all times. This proved to be particularly useful when filling the loop.

g. Storage Tank

A storage tank of 1-cu-ft capacity was located near the lowest point in the loop. To fill the loop, the supply tank was pressurized, and the mercury was allowed to flow until the desired level was observed in the expansion-chamber sight glass.

h. Mixing Chambers

Fluid-mixing chambers were placed at the exits from the water cooler and the test section to promote a mixing of the mercury before measuring the bulk-fluid temperatures. Temperature measurements during the tests showed that the baffle-plate arrangements used did provide sufficient mixing of the fluids.

i. Insulation

Portions of the loop were heavily insulated with magnesia lagging. However, only those sections across which heat balances were taken were insulated, since it was felt it might be necessary to disassemble portions of the loop for later modifications.

D. Instrumentation and Control

1. Control of Operating Conditions

The system operating conditions were determined by the mercury-weight flow rates, the rate of heat input in the heater section,

and the rate of heat removal in the cooler. Figure 39 shows the instruments associated with the control of the operating conditions. The mercury flow rates were determined by the output from the electromagnetic flowmeters, as read on a precision potentiometer; the water flow rates through the cooler were determined with the orifice assemblies discussed previously. Although the orifice meters provided accurate determinations of the water flow rate, a water flow meter (rotameter) was also used as a general indication of the rate of flow. A rotameter also indicated the rate of flow of the water cooling the pump seal.

1. Water Flow Meters	7. Heater Voltmeters
2. Flow Switch	8. Pump and Heater On-Off Switches
3. Alarm Bell	9. Precision Potentiometer
4. Safety Control Box	10. Thermocouple Switch Box
5. Heater Safety Control	11. Heater Control
6. Watt Meter	

Fig. 39. Instrument and Control Assembly

The rate of power input to the heater was controlled by a powerstat mounted on the control panel. The rate of heat input was determined from a reading of the voltage supplied to each heater, with the appropriate values for the heater resistances. A wattmeter gave a general indication of the power level.

2. Temperature Measurements

The temperature measurements were all made with special-grade ($\pm 3/4^\circ\text{F}$) copper-constantan thermocouples. The thermocouple

output was fed through an insulated switchbox (Fig. 39) to a Leeds and Northrup portable precision potentiometer with an ice-junction reference.

All thermocouples in contact with mercury or water were incased in stainless-steel sheaths, since otherwise the mercury would rapidly attack the bare thermocouple wire. Mercury bulk-fluid temperature was measured at the inlets to the cooler and heater and at the inlets and outlets of the tube and annulus of the test section. Test-section tube- and annulus-inlet temperatures corresponded to heater- and cooler-outlet temperatures, respectively. Bulk-fluid temperatures of the water entering and leaving the cooler were also measured.

The most detailed temperature measurements were those taken on the test sections used to determine local heat-transfer coefficients. The experimental approach was based on a determination of the local heat-transfer coefficients with a measurement of the inner-tube-wall temperature and with detailed measurements of outside-wall temperatures. The inner-wall temperature was measured by a 0.065-in.-diam, stainless-steel-sheathed, special-grade, copper-constantan thermocouple inserted into a 3-in.-long 0.067-in.-diam hole, drilled at a specified radial location in the inner-tube wall (Figs. 34 and 35). The thermocouples used to measure the outside-wall temperatures were taped onto the tube wall (Fig. 40), starting

20 L/D₁ Test Section

8 L/D₁ Test Section

1. Annulus Outlet Mixing Chamber	4. Test Section
2. Test Section Center Tube Wall Thermocouple	5. Heaters
3. Tube Outlet Mixing Chamber	6. Water Cooler Outlet Mixing Chamber

Fig. 40. Test-section Installation

under the annulus outlet pipe and continuing down the length of the outer-tube wall in 1-in. intervals until the beginning of the heat-transfer region of the test section. Couples were also placed at several locations along the hydraulic entrance region as a check on the transfer of heat in this region. In addition, thermocouples were placed at different angular positions at several locations to detect any circumferential temperature variation.

Attempts to calibrate the thermocouples in a temperature-controlled, water-stirred bath failed. However, when measurements were taken in the system with the system at ambient temperature, the temperature agreement was within 1°F.

3. Safety Control

Because of the high toxicity of mercury vapor, the heat-transfer loop was constructed in an enclosure which was ventilated and exhausted to the outside of the building. The enclosure was constructed on a stainless-steel pan so that any mercury leakage would be confined to within the enclosure. The expansion tank was also vented to the atmosphere. A mercury-vapor detector monitored the level of mercury vapor present in the enclosure.

Several safety measures were taken to prevent overheating of the mercury in the heaters. The heater power source was interlocked with the pump control to allow heat input only when the pump was operating, and a temperature controller (Fig. 39) shut the heaters off if the heater-wall temperature exceeded a set temperature level (chosen to be 270°F for most runs). A mercury-level probe was also installed in the top of the expansion chamber and connected to a safety-control unit (Fig. 39). With the safety control on, if the mercury level dropped below the desired level, the pump and heater would be shut off, and an alarm bell (Fig. 39) would be activated. The safety-control box allowed the mercury level probe to be bypassed, however, if desired.

To prevent the mechanical seal of the pump from overheating because of a lack of cooling-water flow, the entire system was interlocked through a water-flow switch (Fig. 39). The pump, and hence the heaters, could not be turned on until a water flow in excess of 1 gal/min was maintained through the pump seal.

V. PROCEDURE

A. Experimental Procedure

1. Preparation for Tests

Care was taken to maintain a clean system both before and during the heat-transfer tests. The flow-loop components were thoroughly cleaned with acetone and distilled water before and after assembly of the loop; before being charged with mercury, the loop was evacuated and purged with nitrogen gas. Once the loop was filled, the mercury level was maintained in the loop to minimize the possibility of an oxide formation on the heat-transfer surfaces. The mercury used for the tests had been cleaned in a Bethlehem Oxifier and filtered through a Gold Seal filter. Samples of mercury taken occasionally from the loop failed to show any mercury contamination throughout the entire series of heat-transfer tests.

Considerable effort was expended in attempts to eliminate any thermal contact resistance between the mercury and the nickel heat-transfer surfaces. Operation with the electromagnetic flowmeters indicated that an electrical contact resistance existed between untreated nickel electrodes and pure mercury. This resistance could be removed, however, if electrodes were treated with a 50% solution of hydrochloric acid. The test sections were treated, therefore, with acid and immediately filled with mercury to promote amalgamation of the nickel surfaces. Following preparation of the metal surfaces, the test section used was installed and the loop was filled with clean mercury. Subsequent heat-transfer tests indicated that the thermal contact resistance normally found with heat-transfer investigations using mercury was absent in the present investigation.

2. Startup and Shutdown Procedure

Startup consisted of applying electrical power to the circulating pump, the heaters, and the water-cooler auxiliary pump, and making the proper power and flow-rate adjustments for the particular run. After water flow was provided through the pump seal, the safety controls were set on, and the mercury circulating pump was started. The flow control valves were then adjusted until the desired mercury-weight flow rates were established, as determined by the electromagnetic flowmeters. The auxiliary water pump was started, and the cooling water manometer lines were bled. After adjustment of the cooling-water flow rate, the heaters were turned on, and the power adjustment was made.

Shutdown was initiated by reducing the power input to the heaters and by turning off the safety-control unit, which automatically shut off the circulating pump and heaters. The water flow was stopped, and the manometer lines were secured.

3. Taking of Data

Once the operating conditions were set, the system was allowed to stabilize. Conditions were considered stable when the temperature of the fluid leaving the heater remained within 1.0°F of a steady temperature over a period of 30 min. For the first run after loop startup, it usually took about 4 hr for the system to stabilize. Subsequent tests required about 2 hr for stabilization.

When stable conditions were reached, the water-cooler manometer and heater voltmeters were read and the values recorded. The potentiometer was rezeroed, and the flowmeter emf's were read. The individual thermocouple emf's were then read on the potentiometer and recorded. A quick recheck of the thermocouple readings was made at the end of the run to ensure the accuracy of the recorded values.

If the next run required a change in a flow rate, the pump was shut off and a check was made for flowmeter thermal emf. The flowmeters were then recalibrated by use of the mercury-orifice assemblies. Where the next run was made with the same mercury flow rates, a recalibration of the flowmeters was not made and the results of the previous calibration were used. In general, the change in the flowmeter characteristics was quite gradual, and one calibration a day would suffice.

B. Analysis of the Data

1. Initial Data Reduction

The first step in the data reduction consisted of converting the thermocouple emf's to temperatures by using a standard, copper-constantan-thermocouple, conversion table. Using values of the physical properties of mercury (Appendix C) based on the average tube- and annulus-side temperatures, water-cooler and test-section heat balances were calculated. Initially, heat balances were also calculated across the heaters, but accurate voltmeters were not readily available for the range of operation, and the calculation of these heat balances was discontinued. The method of data analysis, including calculation of heat balances, is illustrated in Appendix D.

With the weight flow rates and the physical properties of mercury, the tube and annulus Peclet numbers were calculated according to

$$Pe_1 = \frac{4}{\pi} \frac{c_1}{k_1} \frac{W_1}{D_1} = 24.9 \frac{c_1}{k_1} W_1, \quad (69a)$$

and

$$Pe_2 = \frac{4}{\pi} \frac{c_2}{k_2} \frac{W_2}{D_2 + D_3} = 4.91 \frac{c_2}{k_2} W_2. \quad (69b)$$

The heat-exchanger efficiency was calculated from

$$\epsilon_1 = \left(\frac{1+H}{H} \right) \frac{\Delta t_1}{\Delta t_0}, \quad (70a)$$

and

$$\epsilon_2 = (1+H) \frac{\Delta t_2}{\Delta t_0}, \quad (70b)$$

where H is the heat-capacity flow-rate ratio, Δt_0 is the inlet-temperature difference, and Δt_1 and Δt_2 are the bulk-fluid temperature differences over the length of the tube and annulus, respectively. The difference in the efficiencies calculated from Eqs. (70a) and (70b) is equivalent to the test-section heat-balance error. The efficiency was taken as the average of these two calculations, except where there was reason to believe that either the tube-side or annulus-side data were in error.

2. Overall, Fully-developed, Nusselt Number

The determination of the individual-channel, local, heat-transfer coefficients depended upon the values for the local heat flux and the wall-surface temperatures. These quantities, in turn, were calculated from the outlet-bulk-fluid temperatures and the center-wall temperature, once the value of the local, overall, heat-transfer coefficient was determined.

As mentioned in Chapter IV, the analysis suggested a method of obtaining the value for the fully-developed overall Nusselt number from the outside-wall-temperature profile. The fully-developed overall Nusselt number is related to the first eigenvalue, λ_1^2 , in the expression for the dimensionless temperature Eq. (59b) by

$$Nu_1^0(\infty) = \frac{H}{1+H} \lambda_1^2. \quad (64c)$$

When the heat transfer is fully developed, the outside-wall-temperature distribution is given by

$$\xi_2(0, z) = \frac{H}{1+H} + A_1^{(2)} e^{-\lambda_1^2 z}. \quad (71)$$

Hence, if the outside-wall temperature, suitably expressed, were plotted as a function of distance on semilog paper, the plot should be a straight line

when the heat transfer is fully developed, and the slope would be proportional to λ_1^2 . It was found most convenient to plot the quantity $t_\infty - t_2(0, \ell)$, equivalent to

$$\Delta t_0 \left[\frac{H}{1+H} - \xi_2(0, z) \right],$$

as a function of the axial position ℓ . The dimensionless length z is related to ℓ according to

$$z = \frac{k_1}{c_1} \frac{\pi}{W_1} \ell. \quad (72)$$

Note that t_∞ is the equilibrium temperature that would be obtained if the heat exchanger were infinitely long, and is easily calculated from

$$t_\infty = \frac{t_{10} + H t_{20}}{1 + H}. \quad (73)$$

If the plot yielded a straight line, the heat transfer was judged to be fully developed and the slope was calculated from

$$\text{slope} = \ln \left[\frac{t_\infty - t_2(0, z_2)}{t_\infty - t_2(0, z_1)} \right] / (z_2 - z_1), \quad (74a)$$

where

$$z_2 - z_1 = \frac{k_1}{c_1} \frac{\pi}{W_1} (\ell_2 - \ell_1), \quad (74b)$$

and ℓ_2 and ℓ_1 were two convenient points on the graph. The sample calculation in Appendix D illustrates the calculation procedure for the determination of the slope.

Ideally, the slope calculated according to Eq. (74a) should be directly related to the fully-developed, overall Nusselt number as in Eq. (64c). Distortions of the outside temperature distributions were often noted, however. The nature of this temperature distortion and its influence on the actual procedure used for estimating heat-transfer coefficients are discussed in detail in the following chapter.

3. Individual-channel, Local Nusselt Numbers

The values of the fully-developed, individual-channel, Nusselt numbers are related to the overall Nusselt number according to

$$Nu_1(\infty) = \frac{\Delta t_e}{t_{B1} - t_{w1}} Nu_1^0(\infty), \quad (75a)$$

and

$$Nu_2(\infty) = \frac{k_1}{k_2} \frac{D_3 - D_2}{D_2} \frac{\Delta t_e}{t_{w2} - t_{B2}} Nu_1^0(\infty), \quad (75b)$$

where t_{B1} and t_{B2} are the outlet bulk-fluid temperatures for the tube and annulus respectively, t_{w1} and t_{w2} are the corresponding tube-wall surface temperatures, and Δt_e is the outlet bulk-fluid temperature difference. The wall-surface temperatures are related to the measured wall temperature, t_w , and the local heat flux by

$$t_{w1} = t_w + \frac{q_1}{2} \frac{D_1}{k_w} \ln(2r_w/D_1),$$

and

$$t_{w2} = t_{w1} - \frac{\ln(D_2/D_1)}{\ln(2r_w/D_1)} (t_{w1} - t_w),$$

where r_w is the radial distance to the thermocouple position from the center of the tube.

The local heat flux is, itself, dependent upon the overall heat-transfer coefficient according to

$$q_1 = \frac{k_1}{D_1} \Delta t_e Nu_1^0(\infty).$$

For the actual test sections used, the expressions for the wall-surface temperatures were

$$t_{w1} = t_w + 0.137 \frac{k_1}{k_w} \Delta t_e Nu_1^0(\infty), \quad (76a)$$

and

$$t_{w2} = t_w - 0.95(t_{w1} - t_w). \quad (76b)$$

The calculated value for the overall heat-transfer coefficient were used, and the following quantities were calculated:

$$\delta_1 \equiv \frac{\Delta t_e}{t_{B1} - t_{w1}}, \quad (77a)$$

and

$$\delta_2 \equiv \frac{\Delta t_e}{t_{w2} - t_{B2}}. \quad (77b)$$

The individual-channel Nusselt numbers are related to δ_1 and δ_2 according to Eqs. (75a, b). The actual calculation of individual heat-transfer coefficients will be discussed further in the following chapter.

VI. ANALYSIS OF EXPERIMENTAL MEASUREMENTS

The analysis upon which the procedure for determining fully-developed heat-transfer coefficients is based, included certain idealizations (see Chapter III). Among these were the assumptions of negligible axial conduction in both the fluids and the walls, and negligible thermal-convection effects. Preliminary experimental results indicated that these idealizations might not always be sufficiently accurate. As a result, the actual method of data analysis required inclusion of an additional experimental procedure to be described in the following paragraphs.

A. Tests with 10 L/D₁ Test Section

The experimental program was begun with the 10 L/D₁ test section. Both wall and bulk-fluid temperature measurements were taken, and calculations were made for the overall heat-transfer coefficients. The heat-exchanger analysis indicated that for low flow rates in the annular side of the heat exchanger, the heat transfer should have been fully developed over a significant portion of the heat-transfer length. Outside-wall-temperature measurements showed a considerable distortion of the temperature distribution, however, suggesting the presence of extraneous heat conduction through the heat-exchanger end plates; plots, on semilog paper, of the dimensionless outside-wall temperatures, $\xi_2(0, z) - H/(1+H)$, also failed to yield the straight-line plots associated with fully-developed heat transfer.

Calculations were also made for the ratio of the annular-side and tube-side Nusselt numbers according to

$$\frac{Nu_2(\infty)}{Nu_1(\infty)} = \frac{k_1}{k_2} \frac{D_3 - D_2}{D_2} \frac{t_{B1} - t_{w1}}{t_{w2} - t_{B2}}, \quad (78)$$

from Eqs. (75a, b), with the assumption that the individual tube-wall surface temperatures, t_{w1} and t_{w2} , could be approximated by the measured center-wall temperature, t_w . In general, the ratio of the Nusselt numbers was considerably higher than what was expected. Temperature measurements around the outside wall also showed a circumferential temperature variation, which might be attributed to end-plate heat conduction. Finally, thermocouples placed at several positions along the outside wall in the hydraulic entrance region indicated wall temperatures significantly higher than inlet, annular-side, bulk-fluid temperatures, suggesting significant heat conduction through the end plate at the inlet end of the heat-exchanger test section.

The heat-exchanger end plates for the 10 L/D₁ and 20 L/D₁ test sections were originally constructed of solid pieces of nickel stock. While preliminary investigations with the 10 L/D₁ test section continued, the

end plates on the 20 L/D_1 test section were milled out to the form shown in Fig. 34. It was hoped that by removing as much stock as possible from the end plates, without structurally damaging the test section, the extraneous heat-conduction effect could be minimized. When the reworked 20 L/D_1 test section was ready, the preliminary investigations with the 10 L/D_1 test section were terminated, and the other test section was installed.

Because of the extraneous heat-conduction effect, no meaningful numerical comparisons could be made between the analytical predictions for heat-exchanger performance and the actual heat-exchanger performance.

B. Tests with 20 L/D_1 Test Section

1. Initial Results

Wall and bulk-fluid temperatures of the 20 L/D_1 test section were measured, and overall heat-transfer coefficients and heat-exchanger efficiencies were computed. It was immediately apparent that the reworking of the end plates had reduced the extraneous heat-conduction effects. Plots of the dimensionless outside-wall temperature on semilog paper appeared linear over portions of the graph corresponding to the regions of the test section where the heat-transfer was expected to be fully developed, and the ratios of the individual-channel Nusselt numbers, calculated from Eq. (78), were more nearly in agreement with what was expected than those calculated for the tests with the 10 L/D_1 test section.

Initial tests with low flow rates in the two channels yielded experimentally-determined heat-transfer coefficients close to those predicted analytically, but at higher flow rates the slope of the outside-wall-temperature plot led to calculated values for the heat-transfer coefficients that were considerably higher than expected. In particular, for tests with high flow rates, the calculation technique seemed to indicate heat-transfer coefficients in excess of those appropriate to a uniform heat-flux boundary condition. As with the shorter test section, a circumferential temperature variation was noted near the outlet end, and the outside-wall temperatures in the hydraulic entrance region were higher than the annulus-inlet bulk-fluid temperature. These results seemed to indicate the continued presence of end-plate conduction effects which seriously influenced the accuracy of the computed results according to the calculation procedure used.

To investigate further the influence of these extraneous heat-conduction effects on the calculated values of the heat-transfer coefficients, experiments were performed in which the inlet bulk-fluid temperature difference was varied while the remaining heat-exchanger operating conditions were held fixed. Ideally, the heat-transfer coefficients and efficiency should be independent of the operating-temperature difference, and any extraneous

heat-transfer effects might depend directly upon the heat-exchanger temperature differences. The results of the experiments indicated that both the computed values of the slope of the outside-wall-temperature distribution and the heat-exchanger efficiencies depended on the inlet-temperature difference in a consistent manner. This dependence is illustrated in Figs. 41 and 42 for series of runs in which the mercury-weight flow rates in individual channels were held fixed while only the inlet-temperature difference

Fig. 41. Slope of Dimensionless Outside-wall-temperature Distribution vs Inlet Temperature Difference with 20 L/D₁ Test Section ($Pe_2 = 100$)

Fig. 42. Heat-exchanger Efficiency vs Inlet-temperature Difference with 20 L/D₁ Test Section ($Pe_2 = 100$)

was varied. The range of operation illustrated covered tube-side flow rates characterized by Peclet numbers of from 100 to 1400 for a fixed annular flow rate of Peclet number 100. The consistency of the dependence upon the inlet-temperature difference made it possible to extrapolate the computed values of the slopes of the outside-wall-temperature distribution and

the heat-exchanger efficiencies to an inlet-temperature difference of zero, thereby minimizing extraneous effects that depend directly upon temperature differences.

Fig. 43. Percentage Change of Slope with Inlet-temperature Difference vs Tube-side Peclet Number

As illustrated in Fig. 43, the percentage change in the calculated slope with inlet-temperature difference,

$$\Delta \left[\frac{\text{slope}}{(\text{slope})_0} \right] / \Delta t_0,$$

showed a slight increase with tube-side Peclet number, where the quantity $(\text{slope})_0$ is the extrapolated value of the slope. Attempts to predict the trends of the computed slopes versus inlet-temperature difference on the basis of simple models failed, indicating that the actual heat-transfer mechanism responsible for these trends may be quite complicated.

2. Modification of the Calculation Procedure

The calculation procedure was based on this extrapolation technique. With the extrapolated values of the slopes of the dimensionless outside-wall-temperature distribution, the overall Nusselt number was calculated for each series of runs according to

$$\text{Nu}_i^0(\infty) = \frac{H}{1 + H} \cdot (\text{slope})_0,$$

from Eq. (64c), with values of the heat-capacity flow-rate ratio, H , appropriate to the average Peclet numbers characterizing the series.

In some instances, there was uncertainty concerning how to extrapolate the computed values for the slope because of the scatter of the data. An error analysis indicated that the value of the slope calculated could be affected greatly by an uncertainty in the weight-flow measurements. This was particularly true for the series of tests performed for a Peclet number of 100 in both channels, where the two calculated values for the slope seemed to indicate a trend with inlet-temperature difference opposite to the trends indicated by the other series of tests. However, the

test performed with an inlet-temperature difference of 74°F had a test-section heat-balance deviation of 12%, while the deviation for the test with an inlet-temperature difference of 159°F was only 2%. The indicated flow-meter output for the tube-side flow was only 0.08 mV, a value sufficiently low that the tube-side flow rate could be uncertain by as much as 12%. A recalculation for the dimensionless outside-wall-temperature slope, using a tube-side flow rate corrected from a heat balance, yielded a slope nearly equal to that resulting from the test performed with an inlet-temperature difference of 159°F; the value of the slope was therefore taken to be the value computed for the test with the inlet-temperature difference of 159°F.

As mentioned in Chapter V, the computed values of the individual-channel Nusselt numbers depended, in part, upon the values of the quantities δ_1 and δ_2 , where

$$\delta_1 = \frac{\Delta t_e}{t_{B1} - t_{w1}},$$

and

$$\delta_2 = \frac{\Delta t_e}{t_{w2} - t_{B2}}.$$

These quantities were in turn influenced by the value of the local heat flux at the outlet end, since the heat flow determined the temperature gradient through the separating wall. The local heat flux was calculated from

$$q_1 = \frac{k_1}{D_1} \Delta t_e \text{Nu}_1^0(\infty),$$

where the value for $\text{Nu}_1(\infty)$ used in the computation was the extrapolated value mentioned above. The ratio of the local heat flux to the inlet-bulk-fluid temperature difference, $q_1/\Delta t_0$, should be fairly constant, and, hence, calculation of the local heat flux, using the value of $\text{Nu}_1^0(\infty)$ determined by the extrapolation procedure, should yield a more accurate estimate of this quantity than the use of a fictitious Nusselt number, based upon the computed slope of the outside-wall-temperature distribution for the particular run. Because of the high thermal conductivity of the nickel wall, the temperature drop across the wall was small, and the quantities δ_1 and δ_2 were actually influenced very little by the accuracy of the estimation for the local heat flux.

The values of δ_1 and δ_2 showed no consistent variation with inlet-temperature difference (see Appendix F for the values). For each series of experiments, the values of δ_1 and δ_2 were taken to be the average values for the series, and the individual-channel Nusselt numbers were calculated from these values and the overall Nusselt number according to

$$Nu_1(\infty) = \delta_1 Nu_1^0(\infty),$$

and

$$Nu_2(\infty) = \frac{k_1}{k_2} \frac{D_3 - D_2}{D_2} \delta_2 Nu_1^0(\infty),$$

from Eqs. (75a, b).

3. Further Tests with 20 L/D₁ Test Section

Attempts were made to determine heat-exchanger performance for an extended range of heat-exchanger operating conditions where tests were performed for various flow rates in the tube with a fixed flow in the annulus corresponding to an annular-side Peclet number of 200. The influence of the extraneous heat-conduction effects, however, was found to be more severe for these tests with higher flow rates in the annulus. In particular, the outside-wall-temperature distribution showed more distortion, and it was not always clear how to determine an appropriate value for the slope.

Computed values of the slopes and the heat-exchanger efficiencies are illustrated in Fig. 44. Clearly consistent trends with inlet-temperature difference are not apparent. Although additional experiments might have clarified the situation, the tests were terminated in favor of experiments with the 8 L/D₁ test section.

Fig. 44. Slope of Outside-wall-temperature Distribution and Heat-exchanger Efficiency vs Inlet-temperature Difference with 20 L/D₁ Test Section (Pe₂ = 200)

C. Tests with 8 L/D₁ Test Section

The short, 8 L/D₁ test section was designed late in the program to enable a further check on extraneous heat-conduction effects. To minimize these effects, the test section was built without a hydraulic entrance region and with heat-exchanger end plates of stainless steel rather than nickel. Since the length of the test section was such that the heat transfer was not fully developed for most of the tests, only heat-exchanger efficiencies were determined.

Series of experiments were performed in which only the inlet bulk-fluid temperature difference was varied for various fixed values of the individual-channel flow rates. As illustrated in Fig. 45, the computed heat-exchanger efficiencies were independent of the inlet fluid-temperature difference. This independence is consistent with the hypothesis concerning the influence of end-plate heat-conduction effects on heat-exchanger performance; with this test section, the effects should be considerably reduced because of the relatively high thermal resistance of the stainless-steel end plates compared to that of the nickel end plates.

Fig. 45. Heat-exchanger Efficiency vs Inlet-temperature Difference with 8 L/D₁ Test Section

VII. DISCUSSION OF RESULTS

Detailed heat-exchanger calculations using the analytical solutions of Chapter III were made for the heat-exchanger test sections described in Chapter IV. The data from the experimental studies were treated as described in Chapters V and VI, and the results were compared with those predicted analytically.

A. Heat-exchanger Calculations

Calculations were made for the efficiency and fully-developed overall and individual-channel Nusselt numbers for the experimental heat-exchanger test sections according to the k^+ approximation, with the values of k_1^+ and k_2^+ given by Eq. (55a) and Eqs. (58a, b), respectively. The analytical solutions for the efficiency and heat-transfer coefficients depend upon the four basic dimensionless parameters: H , K , K_w , and R . For each heat-exchanger test section, the parameter R was fixed, and the parameters K and K_w depended only upon the values of the quantity k_1^+ . The heat-capacity flow-rate ratio, H , was determined by the individual-channel Peclet numbers, in accordance with the following equation:

$$H = \frac{k_2}{k_1} \frac{1 + R}{D_1/D_3} \frac{Pe_2}{Pe_1} \quad (79)$$

Table IV lists the important parameters for the heat-exchanger test sections.

TABLE IV. Test-section Parameters

Parameter	Test Section		
	20 L/D ₁	10 L/D ₁	8 L/D ₁
D ₁ (in.)	0.614	0.614	0.614
D ₂ (in.)	1.000	1.000	1.050
D ₃ (in.)	2.067	2.067	2.067
L (in.)	13.250	7.125	5.875
R	0.483	0.483	0.508
K	1.12(k_1^+/k_2^+)	1.12(k_1^+/k_2^+)	1.02(k_1^+/k_2^+)
K_w	0.06 k_1^+	0.06 k_1^+	0.06 k_1^+
H	4.76(Pe_2/Pe_1)	4.76(Pe_2/Pe_1)	4.84(Pe_2/Pe_1)

Computations for the heat-exchanger efficiency and local Nusselt numbers were made on an IBM-704 computer for which a simple FORTRAN program was written. The effectiveness coefficient, $\phi(z)$, was calculated according to Eq. (67), using the first four eigenvalues, λ_1 to λ_4 ; trial calculations indicated that the effect of terms corresponding to the fifth and

higher eigenvalues was negligible. The calculation procedure is given in Appendix E. The results of the calculations for the 20 L/D₁ test section and the 8 L/D₁ test section are listed in Tables V and VI respectively.

TABLE V. Heat-exchanger Calculations for 20 L/D₁ Test Section

Pe ₁	Pe ₂	H	K	K _w	k ₁ ⁺	k ₂ ⁺	ε	Nu ₁ (∞)	Nu ₂ (∞)	Nu ₁ ⁰ (∞)	N ₁ (∞)	N ₂ (∞)	N ₁ ⁰ (∞)
100	100	4.76	1.13	0.06	0.998	0.996	0.932	7.024	4.131	2.248	0.880	0.648	0.712
200	100	2.38	1.23	0.07	1.090	0.996	0.824	7.417	4.957	2.551	0.851	0.778	0.818
400	100	1.19	1.41	0.08	1.249	0.996	0.721	8.129	5.403	2.766	0.814	0.848	0.848
700	100	0.68	1.64	0.09	1.460	0.996	0.655	8.983	5.594	2.918	0.770	0.878	0.853
1000	100	0.48	1.86	0.10	1.654	0.996	0.626	9.666	5.668	3.010	0.730	0.889	0.850
2000	100	0.24	2.51	0.14	2.231	0.996	0.595	11.208	5.751	3.173	0.628	0.902	0.832
100	200	9.52	1.13	0.06	0.998	0.996	0.913	7.304	2.841	1.776	0.915	0.446	0.587
200	200	4.76	1.23	0.07	1.090	0.996	0.791	7.739	4.074	2.296	0.887	0.639	0.737
400	200	2.38	1.41	0.08	1.249	0.996	0.647	8.646	4.891	2.660	0.865	0.767	0.816
700	200	1.36	1.64	0.09	1.460	0.996	0.550	9.871	5.270	2.898	0.846	0.827	0.845
1000	200	0.95	1.86	0.10	1.654	0.996	0.504	10.965	5.423	3.040	0.829	0.851	0.859
2000	200	0.48	2.51	0.14	2.231	0.996	0.445	14.042	5.598	3.306	0.788	0.878	0.866

TABLE VI. Heat-exchanger Calculations for 8 L/D₁ Test Section

Pe ₁	Pe ₂	H	K	K _w	k ₁ ⁺	k ₂ ⁺	ε
100	100	4.84	1.03	0.06	0.998	0.994	0.759
200	100	2.42	1.12	0.07	1.090	0.994	0.611
400	100	1.21	1.28	0.08	1.249	0.994	0.495
700	100	0.69	1.50	0.09	1.460	0.994	0.438
1000	100	0.48	1.70	0.10	1.654	0.994	0.412
100	200	9.68	1.03	0.06	0.998	0.994	0.753
200	200	4.84	1.12	0.07	1.090	0.994	0.583
400	200	2.42	1.28	0.08	1.249	0.994	0.444
700	200	1.38	1.50	0.09	1.460	0.994	0.366
1000	200	0.97	1.70	0.10	1.654	0.994	0.328

B. Experimental Results

1. Tests with 20 L/D₁ Test Section

Local heat-transfer coefficients and heat-exchanger efficiencies were determined for the 20 L/D₁ test section, for heat-exchanger operating conditions corresponding to tube-side Peclet numbers of from 50 to 1400 and annular-side Peclet numbers of 100 and 200. The operating conditions are summarized in Table VII where the quantities listed for each run are the heat-capacity flow-rate ratio, H; the individual-channel Reynold's number, Re₁ and Re₂; the inlet-fluid temperatures, t₁₀ and t₂₀; the average heat flux, $\bar{q}_1$; the efficiency, ε; and the cooler and test-section heat-balance deviations. In several instances, the data obtained were considered unreliable and the values were not included in the table. The basic data for the experiments are listed in Appendix F.

TABLE VIII. Summary of Operating Conditions for Tests with 20L/D₁ Test Section

Run No.	H	Re ₁	Re ₂	t ₁₀ , °F	t ₂₀ , °F	$\bar{q}_1$, Btu/hr-ft ²	ϵ	Heat-balance Deviation, %	
								In Cooler	In Test Section
14	4.94	5,300	4,450	239.5	80.1	17,000	0.978	1.7	2.0
15	1.18	22,700	4,740	218.0	99.4	28,850	0.814	1.0	4.7
16	0.69	40,350	4,940	226.4	110.7	34,700	0.768	5.5	2.8
17	2.48	10,780	4,695	219.0	94.0	21,210	0.897	3.5	3.9
18	0.51	58,100	5,275	223.9	119.0	35,500	0.730	8.7	1.2
19	2.37	10,960	4,600	214.7	92.5	20,870	0.887	1.0	6.5
20	0.35	80,000	4,900	218.8	109.6	39,200	0.737	7.5	8.4
21	9.75	5,200	8,990	202.5	87.0	13,010	0.927	0.0	13.8
22	2.45	10,130	4,500	221.4	101.0	19,460	0.910	6.6	0.5
23	3.77	13,700	9,220	205.6	102.4	22,650	0.811	2.5	2.2
24	2.45	24,450	9,340	225.8	112.7	30,800	0.735	5.7	4.3
25	1.48	35,900	9,270	226.0	110.3	38,700	0.653	0.9	4.6
26	1.05	50,100	9,300	219.3	109.4	41,400	0.608	0.7	1.2
27	0.69	75,400	9,180	202.8	107.9	40,900	0.578	0.5	6.9
28	2.45	10,510	4,460	218.3	77.8	23,550	0.897	15.1	1.8
29	3.07	83,900	4,590	209.9	95.7	16,900	0.932	0.4	4.2
30	3.58	7,160	4,400	230.1	78.6	19,930	0.943	9.6	0.6
31	1.54	16,290	4,410	210.2	87.8	25,320	0.841	6.9	7.2
32	1.10	22,600	4,360	212.2	87.6	29,500	0.815	5.1	7.7
33	0.49	53,000	4,610	211.6	100.7	34,300	0.734	4.9	0.3
34	3.48	7,420	4,480	234.3	84.9	20,160	0.942	6.3	1.0
35	6.92	7,510	9,310	225.7	92.3	20,350	0.928	6.4	5.0
36	4.83	10,680	9,160	192.4	92.3	19,280	0.850	8.8	1.9
37	5.45	4,220	4,570	261.2	93.6	15,080	0.959	3.4	8.2
38	10.22	2,540	4,440	253.1	84.0	-	-	-	-
39	7.23	3,560	4,430	240.0	84.5	-	-	-	-
40	0.49	46,600	4,380	98.0	81.5	4,850	0.675	2.4	29.0
41	0.48	50,700	4,550	139.4	97.2	12,600	0.685	7.5	1.9
42	0.48	54,300	4,750	193.3	107.7	26,200	0.744	1.2	5.1
43	0.67	34,800	4,390	124.7	85.7	10,600	0.712	-	1.3
44	0.67	36,500	4,430	158.0	90.0	18,810	0.732	7.9	4.0
45	0.67	33,800	4,290	108.3	81.5	7,050	0.694	4.5	4.6
46	1.26	18,220	44,600	115.4	78.0	8,200	0.774	1.3	3.0
47	1.30	19,000	4,455	165.4	84.7	18,000	0.798	7.6	2.3
48	1.75	13,360	4,390	119.4	80.0	7,545	0.817	2.5	0.0
49	1.75	14,030	4,500	156.7	87.6	13,510	0.829	3.4	1.1
50	1.75	13,480	4,400	128.9	80.6	9,280	0.819	1.8	1.3
51	2.50	9,170	4,330	114.3	77.0	5,830	0.886	1.0	8.0
52	2.98	7,500	4,210	115.6	78.7	5,410	0.897	2.8	7.1
53	3.06	7,700	4,310	152.9	85.3	9,760	0.918	2.0	3.1
54	3.45	6,800	4,320	150.2	86.3	8,790	0.936	2.8	6.4
55	3.47	7,150	4,410	204.5	89.6	15,600	0.944	2.4	3.9
56	5.55	4,180	4,245	153.6	79.3	7,150	0.942	3.5	12.1
57	5.54	4,840	4,340	212.4	84.5	12,800	0.969	2.9	7.9
58	0.36	65,400	4,370	130.1	84.4	14,460	0.679	9.3	1.0
59	0.36	64,000	4,325	117.2	85.1	10,230	0.662	1.4	7.0
60	0.36	62,550	4,270	104.2	82.7	6,660	0.626	3.8	12.3
61	0.36	61,600	4,255	96.5	80.9	4,925	0.612	1.3	21.5
62	0.85	26,400	4,320	99.3	81.0	4,620	0.697	8.1	8.6
63	0.85	26,900	4,355	109.2	79.8	7,450	0.709	1.0	13.7
64	0.84	36,150	4,250	89.6	76.0	3,410	0.672	7.8	12.5
65	4.88	9,445	8,550	142.6	93.2	8,835	0.881	7.5	7.5
66	4.88	9,850	8,680	184.8	96.1	1,625	0.878	4.1	2.4
67	1.63	27,720	8,500	128.2	93.0	10,680	0.647	0.5	7.6
68	1.64	36,700	8,360	108.2	84.5	7,310	0.631	1.8	8.3
69	1.63	26,180	8,200	97.4	80.0	5,270	0.622	3.7	11.0
70	9.44	4,705	8,280	131.0	81.4	5,485	0.935	6.5	7.4
71	9.55	4,880	8,590	176.4	87.4	9,850	0.970	3.9	4.7
72	6.27	7,450	8,500	155.7	85.4	10,850	0.894	2.1	1.1
73	6.42	7,000	8,420	124.0	81.2	6,490	0.891	2.9	2.3
74	0.35	60,000	4,100	88.1	73.9	4,080	0.630	2.2	2.9
75	0.46	46,200	4,230	89.6	74.8	4,100	0.652	4.3	3.4
76	0.46	45,900	4,100	87.4	73.7	3,740	0.648	4.3	1.0

As mentioned in Chapter VI, extraneous end-plate heat-conduction effects limited successful determination of local heat-transfer coefficients to operation with an annular-side, weight flow rate characterized by a Peclet number of 100. The result obtained for the various heat-exchanger operating conditions are summarized in Table VIII.

TABLE VIII. Summary of Results with 20 L/D₁ Test Section

Pe ₁	Pe ₂	Nu ₁ ⁰ (∞)	Nu ₁ (∞)	Nu ₂ (∞)	ε
100	100	2.40	7.45	4.52	0.910
140	100	2.36	7.25	4.44	0.929
165	100	2.15	5.33	4.60	0.880
200	100	2.81	7.98	5.66	0.881
300	100	2.27	6.06	4.57	0.800
400	100	2.50	6.15	5.35	0.755
600	100	2.83	7.70	5.89	0.655
740	100	3.02	7.91	6.40	0.675
1000	100	3.04	8.79	6.05	0.655
1400	100	3.26	9.52	6.39	0.615
100	200	-	-	-	0.890
150	200	-	-	-	0.800
600	200	-	-	-	0.615

The experimental results and those predicted by both the present analysis and customary design methods are compared here. The customary design equation is based on the assumption that the heat-transfer coefficients are uniform along the length of the heat exchanger. When boundary conditions are important, as with liquid metals, traditional design methods imply use of heat-transfer coefficients appropriate to the boundary condition of uniform surface temperature as most applicable to a cocurrent-flow heat exchanger. Calculations using heat-transfer coefficients appropriate to the boundary condition of uniform heat flux are also made here. The Lyon equation, Eq. (53), and the Seban and Shimazaki equation, Eq. (54), are used to predict the tube-side Nusselt numbers appropriate to the uniform-heat-flux boundary condition and the uniform-surface-temperature boundary condition, respectively; the predictions for the annular-side Nusselt number correspond to the Dwyer equations, Eqs. (58a, b), for a uniform-heat-flux boundary condition, and the plug-flow value for the uniform-surface-temperature boundary condition (Appendix B). The plug-flow solution for the annular-side Nusselt number is used since the author knows no other prediction for liquid-metal heat transfer in an annulus with a uniform-surface-temperature boundary condition; since the annular-side Peclet number, Pe₂, is low, the plug-flow solution should be applicable.

The experimentally-determined, fully-developed, Nusselt number is compared with the various predictions in Figs. 46-48. As illustrated in Fig. 46, the experimental data for the overall Nusselt numbers, computed from the extrapolated values of the slope of the dimensionless, outside-wall-temperature distribution, Eq. (74a), agree equally well with the present analysis and the prediction based upon a

uniform-surface-temperature boundary condition. The computed values show scatter, however, in particular, those values corresponding to tube-side Peclet numbers of 165-400. Since the individual-channel Nusselt numbers were computed using the corresponding values of the overall Nusselt number, the experimental data for the individual-channel Nusselt numbers should reflect the same uncertainty in this range.

Fig. 46. Experimental Results for Overall Nusselt Number ($Pe_2 = 100$)

Fig. 47. Experimental Results for Tube-side Nusselt Number ($Pe_2 = 100$)

Fig. 48. Experimental Results for Annulus-side Nusselt Number ($Pe_2 = 100$)

may vary as a function of the tube-side Peclet number for operation with a constant, annular-side, Peclet number.

The agreement between the present analysis and the experimental data is more evident for the individual-channel Nusselt numbers, as illustrated in Figs. 47 and 48. While there is some scatter in the data, the trends are well-defined. The data for both channels indicate that fully-developed heat-transfer coefficients may, in agreement with the analysis, be lower than those predicted for a uniform-surface-temperature boundary condition. The annular-side data agree with the present analysis in that the fully-developed, annular-side, Nusselt number

As a further check of the consistency of the experimental results with the analytical predictions, the ratio of the individual annular-side to tube-side Nusselt numbers was computed, since the computed value of the overall Nusselt number is needed only for a determination of the wall-surface temperatures from the measured tube-wall temperature. As illustrated in Fig. 49, the ratio follows the trends of the present analysis.

Fig. 49. Ratio of Annulus-side and Tube-side Nusselt Numbers ($Pe_2 = 100$)

heat-flux boundary condition predicts heat-exchanger efficiencies close to those predicted by the present analysis. As mentioned in Chapter III, the assumption of a heat-transfer coefficient appropriate to a uniform heat flux tends to correct for the high rates of heat transfer occurring in the thermal-entrance region, which are not otherwise accounted for by the customary analysis.

Fig. 50. Experimental Results for Efficiency with $20 L/D_1$ Test Section, $Pe_2 = 100$

Fig. 51. Experimental Results for Efficiency with $20 L/D_1$ Test Section, $Pe_2 = 200$

2. Tests with $8 L/D_1$ Test Section

Heat-exchanger efficiencies were determined for the $8 L/D_1$ test section over a range of operating conditions corresponding to tube-side Peclet numbers of from 90 to 1000 and annular-side Peclet numbers of 100 and 200. The operating conditions are summarized in Table IX, as calculated from the basic data given in Appendix F.

TABLE IX. Summary of Operating Conditions for Tests with 8 L/D₁ Test Section

Run No.	H	Re ₁	Re ₂	t ₁₀ , °F	t ₂₀ , °F	q ₁ , Btu/hr-ft ²	ε	Heat Balance Deviation, %	
								In Cooler	In Test Section
77	4.95	9,055	8,360	125.1	82.9	12,400	0.669	1.1	2.1
78	5.08	8,695	8,350	113.5	81.6	9,570	0.675	2.7	7.3
79	5.08	8,500	8,300	98.3	79.5	5,365	0.651	2.4	4.2
80	1.24	37,250	8,645	137.7	99.2	18,920	0.428	4.0	5.9
81	1.23	36,600	8,460	122.7	90.2	16,090	0.428	5.4	2.3
82	1.25	35,300	8,330	109.1	84.3	12,400	0.436	6.1	3.6
83	2.41	18,410	8,345	109.7	81.9	11,040	0.520	4.7	9.4
84	2.42	18,080	8,350	102.4	81.9	8,200	0.524	1.0	2.6
85	2.41	17,840	8,300	93.5	79.1	5,550	0.507	3.0	2.3
86	2.40	18,870	8,440	123.6	86.3	14,910	0.524	1.0	13.1
87	2.38	19,800	8,620	153.9	96.7	23,180	0.529	3.2	7.3
88	3.40	13,840	8,595	156.1	93.4	21,800	0.590	1.0	6.9
89	3.45	13,080	8,430	122.6	84.9	12,880	0.585	2.0	1.3
90	3.49	12,580	8,330	105.2	81.0	8,410	0.599	0.0	4.4
91	9.80	4,520	8,300	119.7	77.8	8,520	0.841	1.0	2.4
92	9.69	5,000	8,500	199.4	87.7	22,780	0.837	2.5	4.8
93	5.41	4,250	4,135	158.9	76.1	14,400	0.864	1.0	0.0
94	5.40	4,550	4,250	225.6	82.2	25,180	0.873	1.0	1.1
95	2.42	9,670	4,200	162.6	82.7	20,950	0.697	1.1	4.3
96	2.47	8,970	4,110	120.4	75.0	11,720	0.694	0.0	1.0
97	1.21	18,620	4,150	129.0	80.3	16,500	0.583	1.6	10.7
98	1.18	18,000	4,025	106.0	76.3	9,850	0.573	2.2	9.5
99	0.68	32,050	4,065	115.4	82.8	13,100	0.535	1.0	9.1
100	0.48	46,700	4,150	136.1	91.2	19,300	0.510	1.8	4.5

Table X summarizes the experimentally-determined heat-exchanger efficiencies, which are illustrated in Figs. 52 and 53, for operation with constant annular-side Peclet numbers of 100 and 200, respectively. As seen, the experimental results show better agreement with the present

TABLE X. Summary of Results with 8 L/D₁ Test Section

Pe ₁	Pe ₂	ε	Pe ₁	Pe ₂	ε
89	100	0.869	100	200	0.839
200	100	0.695	200	200	0.665
406	100	0.578	287	200	0.591
713	100	0.535	414	200	0.521
984	100	0.510	790	200	0.431

Fig. 52. Experimental Results for Efficiency with 8 L/D₁ Test Section, Pe₂ = 100 Fig. 53. Experimental Results for Efficiency with 8 L/D₁ Test Section, Pe₂ = 200

analysis than with the traditional analysis. The relatively high values of the efficiencies obtained experimentally are probably due to annular-side flow distortion at the entrance and exit of the heat exchanger. As with the $20 L/D_1$ test section, the predictions according to the customary design equation using a heat-transfer coefficient appropriate to a uniform-heat-flux boundary condition tends to correct, in part, for the relatively high rates of heat transfer in the thermal-entrance region.

3. Summary of Experimental Results

Within the range of operating conditions successfully covered, the experimental results showed better agreement with the present analysis than with the traditional analysis. In particular, fully-developed heat-transfer coefficients, which were significantly lower than those appropriate to a uniform-surface-temperature boundary condition, were obtained experimentally. Experimentally-determined heat-exchanger efficiencies also showed better agreement with the present analysis than with the traditional analysis. In addition, for certain ranges of operating conditions, calculations using the customary design equation, with heat-transfer coefficients appropriate to a uniform-heat-flux boundary condition, gave nearly the same results as the present analysis.

As an illustration of the accuracy of the various predictions for heat-exchanger performance, calculations have been made to predict the heat-transfer lengths necessary to obtain the actual, experimentally-determined efficiencies for operation with a tube-side Peclet number of 1000 and an annular-side Peclet number of 100. The results are summarized in Table XI. In both cases, the present analysis gives the best estimate as to the actual heat-transfer length required. For the $20 L/D_1$ test section, the present analysis overestimates the required length by 10%, and the traditional analysis overestimates the length by 27%. Use of the customary design equation with a heat-transfer coefficient appropriate to a uniform-heat-flux boundary condition overestimates the required length by 13% and, hence, gives approximately the same results as does the present analysis. For the $8 L/D_1$ test section, the present

TABLE XI. Comparison of Heat-transfer Lengths Predicted for Experimentally-obtained Efficiencies at $Pe_1 = 1000$ and $Pe_2 = 100$

Test Section	ϵ	Actual	Present Analysis	Customary Design Equation	
				Uniform Temp Coefficients	Uniform Flux Coefficients
$20 L/D_1$	0.655	13.25	14.60	16.88	15.04
$8 L/D_1$	0.510	5.88	8.56	10.77	9.45

Note: Dimensions are in inches.

analysis overestimates the required length by 46%, and the traditional analysis overestimates the length by 83%. Here, use of the customary design equation with a heat-transfer coefficient appropriate to a uniform-heat-flux boundary condition overestimates the required length by 61%.

Although computation according to all analyses leads to an overestimation of the length required, the present analysis is clearly more accurate than the traditional analysis. The end-conduction effects experienced with the test sections might well be present with other heat exchangers. Even in this case, the present analysis would give the most accurate estimate of heat-exchanger performance. A more detailed analysis might be necessary, however, to consider these effects directly.

C. Discussion of Results of Sawochka and Schleef

Sawochka and Schleef^{30,46} performed the only other experiment known to the author involving cocurrent, liquid-metal flow in a double-pipe heat exchanger in which a measurement of the wall temperature was attempted. As discussed in Chapter II, the results for the tube-side Nusselt numbers were considerably below those expected. Calculations have been made to see if the present analysis could explain these results. The predictions of the analysis have been compared with the results reported for the annular-side Nusselt number.

As discussed in Chapter II, their test section consisted of a cocurrent-flow, double-pipe, heat exchanger in which the flow rates of the tube-side stream (potassium) and the annular-side stream (sodium) could be individually varied. The procedure for determining values of the local heat-transfer coefficients was quite different from the new technique used in this report. The procedure was based on the assumption that, for the purpose of computing local bulk temperatures and heat fluxes, the overall heat-transfer coefficient can be considered uniform along the length of the exchanger and equal to the average value of the length. Errors introduced by this assumption were considered negligibly small because of the relatively high thermal resistance through the inner-tube wall. The assumption allowed for a simple computation of local bulk temperatures and heat fluxes from measured inlet and outlet temperatures. The local surface temperatures were obtained by extrapolating the measured wall temperatures with the assistance of the computed local heat flux. Local heat-transfer coefficients were then calculated from the local bulk-fluid temperatures, surface temperatures, and heat fluxes at axial positions corresponding to L/D_1 ratios of 19 and 38.

The present analysis has been used to predict local heat-transfer coefficients and heat-exchanger efficiencies according to the test-section parameters listed in Table XII. The local Nusselt numbers presented by Sawochka and Schleef show substantial disagreement with the values predicted, as illustrated in Fig. 54. The present analysis indicates that the

tube-side Nusselt numbers should be between those predicted for boundary conditions of uniform heat flux and uniform surface temperature, while the reported values lie about 40% below the expected results. The analysis also shows that the local Nusselt numbers should be fully developed; hence, contrary to the experimental data, there should be no consistent difference between the values determined at the different axial positions. The computed values of the local annular-side Nusselt number show considerable scatter, especially those obtained at an L/D_1 of 19, and no definite trends can be discerned.

TABLE XII. Sawochka-Schleef Test-section Parameters

Parameter	Value	Parameter	Value
D_1 (in.)	0.625	R	0.697
D_2 (in.)	1.720	K	$0.249(k_1^+/k_2^+)$
D_3 (in.)	2.469	K_w	$0.675k_1^+$
L (in.)	36.0	H	$11.7(Pe_2/Pe_1)$

Fig. 54. Experimental Results of Sawochka and Schleef for Local Nusselt Numbers ($Pe_2 \approx 120$)

It is concluded that the analysis of the present investigation cannot explain the results obtained for the local Nusselt numbers.

Calculations were also made for the heat-exchanger efficiency and compared with the experimental data. The comparison is shown in Fig. 55; agreement is quite good. Because of the relatively high thermal resistance of the separating wall ($K_w \approx 0.68$, compared to a value of about 0.06 with the heat exchanger used in the present investigation), the present analysis and the traditional analysis yield nearly equal predictions for the efficiency.

The wall comprised about 65% of the total thermal resistance. Hence, the contributions of the individual-channel, heat-transfer coefficients to the total thermal resistances were considerably less than with the heat exchangers used in the present investigation.

Fig. 55

Experimental Results of Sawochka and Schleef for Efficiency ($Pe_2 \approx 120$)

VIII. SUMMARY AND CONCLUSIONS

Liquid-metal heat transfer in a cocurrent-flow, double-pipe, heat exchanger has been analyzed. It has been shown that the local and fully-developed heat-transfer coefficients depend upon the operating conditions of the heat exchanger, and that fully-developed heat-transfer coefficients that are significantly lower than those predicted for uniform-surface-temperature boundary conditions can be obtained.

The analysis was an extension of a previous analysis of Stein,⁵³ who considered a heat-exchanger configuration with a narrow annular space. It was found that the narrow-annulus approximation of Stein yields sufficiently accurate estimates of the heat-exchanger efficiency for an annular ratio greater than 0.7 or a fluid thermal-resistance ratio, K , less than 1.0.

Because of the high rates of heat transfer in the thermal-entrance region, resulting heat-exchanger efficiencies may be significantly higher than those calculated with the customary design equation for the usually-assumed uniform-heat-transfer coefficients appropriate to a uniform-surface-temperature boundary condition. Although the actual boundary condition more closely approximates a uniform surface temperature than a uniform heat flux, it was found that for efficiencies greater than 0.7 and with nearly equal heat-capacity flow rates in both channels, use of the customary design equation, with an overall heat-transfer coefficient appropriate to a uniform heat flux, will give approximately the same results as the present analysis.

An experimental investigation was performed with various double-pipe heat-exchanger test sections, and the results were compared with those predicted by the analysis. Fully-developed overall and individual-channel Nusselt numbers and heat-exchanger efficiencies were determined for various heat-exchanger operating conditions. Although successful determination of the fully-developed heat-transfer coefficients was limited to operation with an annular-side Peclet number of 100 for the various tube-side Peclet numbers, the results showed agreement with the present analysis where significantly different results were predicted by traditional analysis. Only general agreement between the analysis and the experimental results was obtained, however, but it is clear that the differences present could be accounted for, at least partially, by the failure to achieve the idealizations desired in the test-section design. The test-section design could be improved considerably by using insulating end plates to eliminate extraneous end conduction effects, and by including an adiabatic exit region, similar to the hydraulic entrance region, to eliminate flow distortion in the annulus of the heat-exchanger heat-transfer section.

A new technique for determining values of the fully-developed overall Nusselt numbers by a measurement of the outside-wall-temperature distribution has also been introduced. The technique appears very promising because of its relative simplicity and potential accuracy compared to other methods of determining values of the overall heat-transfer coefficient. Use of the technique also allows one to determine when the heat transfer is actually fully developed. Hence, it is recommended that this technique be more fully developed and applied to other experimental investigations of liquid-metal heat transfer.

The heat-exchanger performance was found to be related to the inlet-fluid temperature difference where ideally this should not be the case. Other heat-exchanger experimental investigations could also check for consistent trends with varying inlet-temperature differences as a means of detecting the presence of any extraneous heat-conduction effects.

The analysis, in agreement with the experimental results, indicates that many of the usual assumptions made in the calculation procedures for experimental investigations of liquid-metal heat transfer deserve to be questioned. The use of a criterion based upon the L/D_1 ratio alone to determine when the heat transfer is fully developed is questionable, since no account is made of the actual heat-transfer boundary condition. With a cocurrent-flow, double-pipe, heat exchanger, the thermal-entrance length is a complicated function of the various heat-exchanger operating conditions, but the exact relation to the operating conditions is not immediately obvious and deserves further investigation.

The use of an overall, average, heat-transfer coefficient to determine values of the fully-developed, heat-transfer coefficients may be questionable, since the average coefficients include the thermal-entrance effects, whereas the fully-developed coefficients do not. The assumptions frequently made concerning the ratio of individual-channel, heat-transfer coefficients may also be questionable, as was shown with one of the test sections used in the current investigation, since the individual coefficients depend strongly upon the heat-transfer boundary conditions.

The present work has demonstrated that many of the usual assumptions concerning heat transfer in heat-exchange devices may not always be justified, and a new method of predicting heat-exchanger performance for a liquid-metal, cocurrent-flow, heat exchanger has been presented. More experimental work to further verify the accuracy of the analysis would be interesting. It would also be of interest to apply the same methods of analysis to other heat-exchanger configurations.

APPENDIX A

Solution of Plug-flow Equations by Separation of Variables

The solution presented here follows the procedure presented by Stein.⁵⁴

Application of the separation of variables technique to the differential Eqs. (23a) and (23b) leads to a solution in the form

$$\xi_1(x, z) = \sum_n C_n E_{1n}(x) e^{-\lambda_n^2 z}, \quad (A-1)$$

and

$$\xi_2(y, z) = \sum_n C_n E_{2n}(y) e^{-\lambda_n^2 z}, \quad (A-2)$$

where

$$y = 1 - \sigma x. \quad (A-3)$$

The functions E_{1n} and E_{2n} must satisfy

$$\frac{1}{x} \frac{\partial}{\partial x} (x E'_{1n}) + \lambda_n^2 E_{1n} = 0, \quad (A-4)$$

$$\frac{1}{y} \frac{\partial}{\partial y} (y E'_{2n}) + \omega^2 \lambda_n^2 E_{2n} = 0, \quad (A-5)$$

$$E'_{1n}(0) = 0, \quad (A-6)$$

$$E'_{2n}(1) = 0, \quad (A-7)$$

$$K E'_{1n}(1) - \sigma E'_{2n}(R) = 0, \quad (A-8)$$

and

$$K_w E'_{1n}(1) + E_{1n}(1) - E_{2n}(R) = 0, \quad (A-9)$$

where

$$\omega^2 = \frac{R}{1+R} \frac{KH}{\sigma^2}. \quad (A-10)$$

1. The Eigenvalue Equation

If $A(\lambda) F(\lambda, x)$ and $B(\lambda) G(\lambda, y)$ are solutions satisfying Eqs. (A-4) and (A-5), where A and B are constants which might depend upon λ , and $F_x(\lambda, 0) = 0$ and $G_y(\lambda, 1) = 0$, then Eqs. (A-8) and (A-9) yield

$$KF_x(\lambda, 1) A - \sigma G_y(\lambda, R) B = 0, \quad (\text{A-11})$$

and

$$[K_w F_x(\lambda, 1) + F(\lambda, 1)] A - G(\lambda, R) B = 0. \quad (\text{A-12})$$

Equations (A-11) and (A-12) give the eigenvalue equation for λ :

$$F(\lambda, 1) G_y(\lambda, R) + K_w F_x(\lambda, 1) G_y(\lambda, R) - \frac{K}{\sigma} F_x(\lambda, 1) G(\lambda, R) = 0. \quad (\text{A-13})$$

2. The Eigenfunctions

Equation (A-11) gives the following relation between A and B :

$$B(\lambda) = \frac{K}{\sigma} \frac{F_x(\lambda, 1)}{G_y(\lambda, R)} A(\lambda).$$

Since either $A(\lambda)$ or $B(\lambda)$ can be chosen arbitrarily, $A(\lambda)$ is chosen for convenience to be

$$A(\lambda) = G_y(\lambda, R). \quad (\text{A-14})$$

The eigenfunctions E_{1n} and E_{2n} can then be represented by

$$E_{1n}(x) = G_y(\lambda_n, R) F(\lambda_n, x), \quad (\text{A-15})$$

and

$$E_{2n}(y) = \frac{K}{\sigma} F_x(\lambda_n, 1) G(\lambda_n, y). \quad (\text{A-16})$$

3. Orthogonality Conditions

A treatment of Eqs. (A-4) and (A-5) in the usual manner to obtain Sturm-Liouville relations yields

$$(\lambda_j^2 - \lambda_i^2) \int_0^1 x E_{1i} E_{1j} dx = E_{1j}(1) E'_{1i}(1) - E_{1i}(1) E'_{1j}(1), \quad (\text{A-17})$$

and

$$\frac{\omega^2}{R} (\lambda_j^2 - \lambda_i^2) \int_1^R y E_{2i} E_{2j} dy = E_{2j}(R) E'_{2i}(R) - E_{2i}(R) E'_{2j}(R). \quad (A-18)$$

The integrals of Eqs. (A-17) and (A-18) can be related to each other by the boundary conditions (A-8) and (A-9), as follows:

$$E'_{2n}(R) = \frac{K}{\sigma} E'_{1n}(1),$$

and

$$E_{2n}(R) = K_w E'_{1n}(1) + E_{1n}(1).$$

From these conditions results the relation

$$E_{2j}(R) E'_{2i}(R) - E_{2i}(R) E'_{2j}(R) = \frac{K}{\sigma} [E_{1j}(1) E'_{1i}(1) - E_{1i}(1) E'_{1j}(1)].$$

Use of this relation, with Eqs. (A-17) and (A-18), gives

$$\int_0^1 \left[2xE_{1i}E_{1j} + \frac{2H}{1+R} (1-\sigma x) E_{2i}E_{2j} \right] dx = 0, \quad i \neq j. \quad (A-19)$$

Equation (A-19) is the orthogonality condition for the eigenfunctions E_{1n} and E_{2n} . The normalizing factor is

$$N_n = \int_0^1 \left[2xE_{1n}^2 + \frac{2H}{1+R} (1-\sigma x) E_{2n}^2 \right] dx. \quad (A-20)$$

Since $E_{10} = E_{20} = 1$ are solutions of the problem corresponding to $\lambda_0 = 0$,

$$\int_0^1 \left[2xE_{1i} + \frac{2H}{1+R} (1-\sigma x) E_{2i} \right] dx = 0, \quad i \neq 0, \quad (A-21)$$

and

$$N_0 = \int_0^1 \left[2x + \frac{2H}{1+R} (1-\sigma x) \right] dx,$$

or

$$N_0 = 1 + H. \quad (A-22)$$

4. Expansion Coefficients

Let $f_1(x)$ and $f_2(x)$ represent the initial conditions for ξ_1 and ξ_2 . Then, from (A-1) and (A-2),

$$f_1(x) = \sum_n C_n E_{1n},$$

and

$$f_2(x) = \sum_n C_n E_{2n}.$$

Application of Eqs. (A-19) and (A-20) yields the following expression for the expansion coefficients, C_n :

$$C_n = \frac{1}{N_n} \int_0^1 \left[2xf_1(x) E_{1n} + \frac{2H}{1+R} (1-\sigma x) f_2(x) E_{2n} \right] dx. \quad (A-23)$$

5. Evaluation of Dimensionless Temperatures

The initial conditions for the dimensionless temperatures are $\xi_1(x, 0) = f_1(x) = 0$ and $\xi_2(x, 0) = f_2(x) = 1$. Then,

$$C_n = \frac{2}{1+R} \frac{H}{N_n} \int_0^1 (1-\sigma x) E_{2n} dx, \quad (A-24)$$

and

$$C_0 = \frac{H}{1+H} \frac{2}{1+R} \int_0^1 (1-\sigma x) dx,$$

or

$$C_0 = \frac{H}{1+H}. \quad (A-25)$$

The functions $F(\lambda, x)$ and $G(\lambda, y)$ satisfying Eqs. (A-4) to (A-7) can be shown to be

$$F(\lambda, x) = J_0(\lambda x), \quad (\text{A-26})$$

and

$$G(\lambda, y) = L_0(\omega \lambda y), \quad (\text{A-27})$$

where

$$L_0(\omega \lambda y) = J_0(\omega \lambda y) Y_1(\omega \lambda) - J_1(\omega \lambda) Y_0(\omega \lambda y). \quad (\text{A-28})$$

From Eqs. (A-15) and (A-16),

$$E_{1n} = -\omega \lambda L_1(\omega \lambda R) J_0(\lambda x), \quad (\text{A-29})$$

and

$$E_{2n} = -\frac{K\lambda}{\sigma} J_1(\lambda) L_0(\omega \lambda y), \quad (\text{A-30})$$

with

$$L_1(\omega \lambda y) = J_1(\omega \lambda y) Y_1(\omega \lambda) - J_1(\omega \lambda) Y_1(\omega \lambda y). \quad (\text{A-31})$$

Equation (A-13) gives the following eigenvalue relation for λ_n :

$$\begin{aligned} F(\lambda_n) = 0 = & K_w \lambda_n J_1(\lambda_n) L_1(\omega \lambda_n R) - J_0(\lambda_n) L_1(\omega \lambda_n R) \\ & + \frac{K}{\sigma \omega} J_1(\lambda_n) L_0(\omega \lambda_n R). \end{aligned} \quad (\text{A-32})$$

Application of Eqs. (A-20) and (A-24) can then be shown to lead to values of C_n given by

$$C_n = \frac{2}{\omega \lambda_n^2 F'(\lambda_n)}. \quad (\text{A-33})$$

Therefore, the solutions for the dimensionless temperatures may be written as

$$\xi_1(x, z) = \frac{H}{1+H} + \sum_{n=1}^{\infty} A_n^{(1)}(\lambda_n, x) e^{-\lambda_n^2 z},$$

and

$$\xi_2(y, z) = \frac{H}{1+H} + \sum_1^{\infty} A_n^{(2)}(\lambda_n, y) e^{-\lambda_n^2 z},$$

where

$$A_n^{(1)} = - \frac{2L_1(\omega\lambda_n R) J_0(\lambda_n x)}{\lambda_n F'(\lambda_n)},$$

$$A_n^{(2)} = - \frac{2KJ_1(\lambda_n) L_0(\omega\lambda_n y)}{\sigma\omega\lambda_n F'(\lambda_n)},$$

and λ_n is given by Eq. (A-32).

APPENDIX B

Nusselt Numbers for Uniform-surface-temperature Plug Flow

The plug-flow values for the temperature distributions in a tube and an annulus under a uniform-surface-temperature boundary condition would be given by the solutions of the energy equation in the respective conduits:

$$\nabla^2 t_1 = \frac{u_1}{\alpha_1} \frac{\partial t_1}{\partial \ell}, \quad 0 \leq r \leq r_1, \quad (\text{B-1a})$$

and

$$\nabla^2 t_2 = \frac{u_2}{\alpha_2} \frac{\partial t_2}{\partial \ell}, \quad r_2 \leq r \leq r_3, \quad (\text{B-1b})$$

with boundary conditions

$$\left. \frac{\partial t_1}{\partial r} \right|_0 = 0 = \left. \frac{\partial t_2}{\partial r} \right|_{r_3}, \quad (\text{B-2})$$

and

$$t_1(r_1, \ell) = T = t_2(r_2, \ell), \quad \ell > 0. \quad (\text{B-3})$$

A separation-of-variables technique gives the temperature distributions, $t_1(r, \ell)$ and $t_2(r, \ell)$, from which the Nusselt numbers may be determined to be

$$\text{Nu}_1 = \mu^2, \quad (\text{B-4a})$$

and

$$\text{Nu}_2 = \frac{\sigma^2(1+R)}{R} \nu^2, \quad (\text{B-4b})$$

where the eigenvalues μ and ν satisfy

$$J_0(\mu) = 0, \quad (\text{B-5a})$$

and

$$F(\nu) = 0 = J_0(\nu R) Y_1(\nu) - J_1(\nu) Y_0(\nu R). \quad (\text{B-5b})$$

The value for the tube-side Nusselt number is

$$Nu_1 = 5.784;$$

the value for Nu_2 depends upon the annular ratio, R . Table XIII lists some values of Nu_2 for various values of R .

TABLE XIII. Annulus Nusselt Numbers

Annular Ratio, R	Nusselt Number, Nu_2	Annular Ratio, R	Nusselt Number, Nu_2
1.00	4.935	0.50	5.555
0.90	4.990	0.40	5.907
0.80	5.067	0.25	6.966
0.70	5.174	0.10	10.833
0.60	5.328		

APPENDIX C

Physical Properties of Mercury

Several sources were consulted to obtain values for the various physical properties of mercury. Some of these sources are summarized in Refs. 35 and 65.

The thermal-conductivity values were taken from Ewing *et. al.*¹⁷ whose data have been generally accepted as the most accurate values reported. The values presented can be represented by

$$k = 4.710 + 3.756 \times 10^{-3} T \text{ Btu/hr-ft-}^{\circ}\text{F},$$

where T is in $^{\circ}\text{F}$.

The density values result from a linear interpolation of data reported in the Liquid Metals Handbook.³⁵

$$\rho = 845.69 - 0.0841 (T - 68) \text{ lb}_m/\text{ft}^3.$$

The Liquid Metals Handbook also lists the values presented here for the heat capacity. Values reported by the Handbook of Chemistry and Physics²¹ agree to within 0.1%.

The International Critical Tables²² give the following interpolation equation for the dynamic viscosity:

$$\mu = (0.2256 + 6.33T \times 10^{-4} + 4.34T^2 \times 10^{-7})^{-1} \text{ lb}_m/\text{hr-ft}.$$

The above physical properties are shown in Fig. 56.

Fig. 56, Physical Properties of Mercury

APPENDIX D

Sample Calculation of a Heat-transfer Test

For purposes of illustration, the results of Run No. 46 are analyzed. The temperature data for this run (reproduced from Appendix F) are listed below:

<u>Thermocouple</u>	<u>Temp, °F</u>	<u>Thermocouple</u>	<u>Temp, °F</u>
28	115.4	5	86.7
20	99.0	6	86.0
19	90.6	7	85.7
18	78.0	8	84.9
29	96.1	9	83.7
22	61.0	10	82.7
21	54.7	11	81.3
2	95.3	12	80.7
3	88.0	13	80.2
4	87.0	14	80.2

The positions of these thermocouples in the loop are listed in Tables XIV and XV of Appendix F. The water and mercury weight-flow data are

$$W_{H_2O} = 309 \text{ lb}_m/\text{hr},$$

$$W_1 = 2540 \text{ lb}_m/\text{hr},$$

and

$$W_2 = 3200 \text{ lb}_m/\text{hr}.$$

1. Initial Data Reduction

The average tube- and annulus-side bulk-fluid temperatures are, respectively,

$$\bar{t}_1 = \frac{T_{28} + T_{20}}{2} = 107.2,$$

and

$$\bar{t}_2 = \frac{T_{19} + T_{18}}{2} = 84.3,$$

where T_i refers to the temperature indicated by thermocouple i . From Fig. 56 (Appendix C), the mercury physical properties are

$$k_1 = 5.11, \quad k_2 = 5.02,$$

$$c_1 = 0.03310, \quad c_2 = 0.03318.$$

A water-cooler heat balance yields

$$Q_{H_2O} = W_{H_2O}(T_{22} - T_{21}) = 1947 \text{ Btu/hr},$$

and

$$Q_{Hg} = W_2 c_2 (T_{29} - T_{18}) = 1922 \text{ Btu/hr},$$

while a test-section heat balance yields

$$Q_1 = W_1 c_1 (T_{28} - T_{20}) = 1379 \text{ Btu/hr},$$

and

$$Q_2 = W_2 c_2 (T_{19} - T_{18}) = 1338 \text{ Btu/hr},$$

giving heat-balance deviations of 1.3 and 3.0% across the water cooler and test section, respectively.

The Peclet numbers are calculated from Eqs. (69a, b) to be

$$Pe_1 = 24.9 \times \frac{0.03310}{5.11} \times 2540 = 409,$$

and

$$Pe_2 = 4.91 \times \frac{0.03318}{5.02} \times 3200 = 104.$$

With the value for the heat-capacity flow-rate ratio,

$$H = \frac{W_2 c_2}{W_1 c_1} = 1.262,$$

the heat-exchanger efficiency is calculated, according to Eqs. (70a, b):

$$\epsilon_1 = \frac{1+H}{H} \frac{T_{28} - T_{20}}{T_{28} - T_{18}} = 0.786,$$

and

$$\epsilon_2 = (1+H) \frac{T_{19} - T_{18}}{T_{28} - T_{18}} = 0.763.$$

The efficiency is then taken to be the average of these two values,

$$\epsilon = 0.774.$$

2. Overall Nusselt Number

The procedure for determining the overall Nusselt number requires a plot of the quantity $t_{\infty} - t_2(0, z)$ on semilog paper as a function of the heat-transfer length, L . Calculation according to Eq. (73) gives the following value for t_{∞} :

$$t_{\infty} = \frac{T_{28} + H T_{18}}{1 + H} = 94.6.$$

The values of the quantity $t_{\infty} - t_2(0, z)$ for the various values of L are then (from T3 to T14):

$t_{\infty} - t_2(0, z)$	L	$t_{\infty} - t_2(0, z)$	L
6.6	1.020	10.9	0.521
7.6	0.938	11.9	0.437
7.9	0.854	13.3	0.354
8.6	0.771	13.9	0.271
8.9	0.687	14.4	0.187
9.7	0.604	14.4	0.104

These values are plotted in Fig. 57. After a corresponding heat-transfer length of 0.3 ft, the plot approximates a straight line, indicating that the heat transfer has become fully developed. Since the slope drawn passes through the values of the quantity $t_{\infty} - t_2(0, z)$ calculated from the temperatures indicated by thermocouples 5 and 12, the appropriate data points are

$$t_{\infty} - t_2(0, z_2) = 7.9, \quad L = 0.854;$$

$$t_{\infty} - t_2(0, z_1) = 13.9, \quad L = 0.271;$$

and

$$z_2 - z_1 = \frac{5.11}{0.0331} \times \frac{\pi}{2540} (0.854 - 0.271) = 0.1113.$$

The slope of the plot is then calculated to be, by Eq. (74a),

$$\text{slope} = -\ln \left[\frac{7.9}{13.9} \right] / 0.1113 = 5.05.$$

The value of the slope is used to estimate the overall Nusselt number, as described in Chapter VI:

$$\text{Nu}_1^0(\infty) \approx 2.50.$$

Fig. 57

Plot of $t_{\infty} - t_2(0,z)$ vs Heat-transfer Length

3. Individual-channel Local Nusselt Numbers

With the appropriate value for the overall Nusselt number, the wall-surface temperatures are calculated, according to Eqs. (76a, b),

$$\begin{aligned} t_{w1} &= T2 + 0.137 \frac{k_1}{k_w} (T20 - T19) Nu_1^0(\infty) \\ &= 95.3 + 0.137 \times \frac{5.11}{51} \times 8.4 \times 2.50 = 95.6^\circ\text{F}, \end{aligned}$$

$$t_{w2} = 95.3 - 0.95(95.6 - 95.3) = 95.0^\circ\text{F},$$

since

$$k_w = 51 \text{ Btu/hr-ft-}^\circ\text{F.}^{62}$$

Then,

$$t_{B1} - t_{w1} = T20 - t_{w1} = 3.4^\circ\text{F},$$

$$t_{w2} - t_{B2} = t_{w2} - T19 = 4.4^\circ\text{F},$$

and

$$\delta_1 = \frac{8.4}{3.4} = 2.47,$$

and

$$\delta_2 = \frac{8.4}{4.4} = 1.91,$$

by Eqs. (77a, b). The quantities δ_1 and δ_2 are used to calculate the individual-channel Nusselt numbers as described in Chapter VI.

APPENDIX E

Calculation of Efficiency and Nusselt Numbers

A simple FORTRAN program was written to calculate the heat-exchanger efficiency and fully-developed, heat-transfer coefficients as a function of the dimensionless parameters, H , K , K_w , and R , and the dimensionless axial distance, z . The k^+ approximation solution was used, with the values for k_1^+ and k_2^+ calculated from Eqs. (55a) and (58a, b), respectively. The data input to the program were the quantities R , k_1/k_2 , D_3/D_1 , L/D_1 , and $(k_1/k_w) \ln (D_2/D_1)$. For each Peclet number, the basic dimensionless parameters were calculated according to

$$H = \frac{k_2}{k_1} \left(\frac{1 + R}{D_1/D_3} \right) \frac{Pe_2}{Pe_1}, \quad (E-1)$$

$$K = \frac{k_1}{k_2} \left(\frac{1 - R}{R} \right) \frac{k_1^+}{k_2^+}, \quad (E-2)$$

$$K_w = k_1^+ \frac{k_1}{k_w} \ln (D_2/D_1), \quad (E-3)$$

and

$$z = k_1^+ \frac{4}{Pe_1} (L/D_1), \quad (E-4)$$

where Eqs. (E-1) to (E-4) are repetitions of Eqs. (79), (50), (51), and (45), respectively. The appropriate values of k_1^+ depend upon the individual-channel Peclet number.

The first four eigenvalues, λ_n , were calculated from

$$F(\lambda) = 0 = K_w \lambda J_1(\lambda) L_1(\omega \lambda R) - J_0(\lambda) L_1(\omega \lambda R) + \frac{K}{\sigma \omega} J_1(\lambda) L_0(\omega \lambda R), \quad (E-5)$$

where the quantities σ and ω and the functions L_0 and L_1 are defined by Eqs. (24), (61), (62a, b), respectively. The plug-flow Nusselt numbers were then calculated according to

$$[Nu_1(\infty)]_p = \frac{2\lambda_1^2 J_1(\lambda_1)}{2J_1(\lambda_1) - \lambda_1 J_0(\lambda_1)}, \quad (E-6a)$$

$$[Nu_2(\infty)]_p = \frac{2\sigma^2 \omega^2 \lambda_1^2 L_1(\omega \lambda_1 R)}{\frac{2R}{1+R} L_1(\omega \lambda_1 R) + \sigma \omega \lambda_1 L_0(\omega \lambda_1 R)}, \quad (E-6b)$$

and

$$\left[\text{Nu}_1^0(\infty) \right]_p = \frac{H}{1+H} \lambda_1^2, \quad (\text{E-6c})$$

from Eqs. (64a-c). In terms of the k^+ approximation, the actual Nusselt numbers were calculated according to

$$\text{Nu}_1(\infty) = k_1^+ \left[\text{Nu}_1(\infty) \right]_p, \quad (\text{E-7a})$$

$$\text{Nu}_2(\infty) = k_2^+ \left[\text{Nu}_2(\infty) \right]_p, \quad (\text{E-7b})$$

and

$$\text{Nu}_1^0(\infty) = k_1^+ \left[\text{Nu}_1^0(\infty) \right]_p. \quad (\text{E-7c})$$

The calculation for the heat-exchanger efficiency required a calculation for the quantity $F'(\lambda_n)$:

$$\begin{aligned} F'(\lambda_n) = & \frac{L_1(\omega\lambda_n R) J_0^2(\lambda_n)}{J_1(\lambda_n)} + L_1(\omega\lambda_n R) J_1(\lambda_n) \left[1 - \frac{RK}{\sigma} \right] \\ & + \frac{K}{\sigma R} J_1(\lambda_n) \frac{L_2^2(\omega\lambda_n R)}{L_1(\omega\lambda_n R)} - \frac{RK}{\sigma} J_1(\lambda_n) \frac{L_0^2(\omega\lambda_n R)}{L_1(\omega\lambda_n R)}, \end{aligned} \quad (\text{E-8})$$

where

$$L_2(\omega\lambda R) \equiv R \left[J_0(\omega\lambda R) Y_1(\omega\lambda R) - J_1(\omega\lambda R) Y_0(\omega\lambda R) \right]. \quad (\text{E-9})$$

The expression for $F'(\lambda_n)$ can be shown to be equivalent to the more cumbersome expression resulting from a differentiation of $F(\lambda_n)$ with respect to the eigenvalue λ_n . The effectiveness coefficient, $\phi(z)$, was calculated using Eq. (67) and, with this value, the heat-exchanger efficiency was calculated according to

$$\epsilon = 1 - \phi(z) e^{-\lambda_1^2 z}. \quad (\text{E-10})$$

APPENDIX F

Tabulation of Data

The basic data for the heat-exchanger experiments performed are listed here, along with computed values of the individual-channel Peclet numbers and inlet-temperature difference for tests

TABLE XIV. Location of Bulk-fluid-temperature Thermocouples

Couple Number	Location
18	Annulus inlet
19	Annulus outlet
20	Tube outlet
21	Water inlet
22	Water outlet
23	Annulus inlet
28	Tube inlet
29	Cooler inlet

with both the 20 L/D₁ and 8 L/D₁ heat exchangers, and computed values of the slope of the plot of the dimensionless outside-temperature distribution and the quantities δ_1 and δ_2 for tests with the 20 L/D₁ test section. The temperatures listed are associated with the specific thermocouples, the locations of which are given in Tables XIV and XV. The temperatures are in °F, and the weight flow rates in units of lb/hr.

TABLE XV. Location of Wall-temperature Thermocouples

Couple Number	Distance (in ft) from Beginning of Heat-transfer Region	Angular Placement with Respect to Annulus Outlet
1	1.085	180°
2	Inner-tube wall	-
3	1.020	0°
4	0.938	0°
5	0.854	0°
6	0.771	0°
7	0.687	0°
8	0.604	0°
9	0.521	0°
10	0.437	0°
11	0.354	0°
12	0.271	0°
13	0.187	0°
14	0.104	0°
15	0.020	0°
16	-0.331	0°
17	-0.708	0°
31	1.020	90°
32	1.020	180°
33	0.854	90°
34	0.854	180°

In some instances, a great degree of uncertainty was involved in the calculation of the value of the slope of the temperature profile. For these instances, the values of the slope and the quantities δ_1 and δ_2 were considered unreliable and were therefore not listed.

The tabulation (Tables XVI and XVII) begins with Run No. 14. The first 13 runs were with the 10 L/D₁ test section, the results of which were considered unreliable, as discussed in Chapter VI.

TABLE XVI. Data for 20 L/D₁ Test Section

Run No.	14	15	16	17	18	19	20	21	22	23	24	25	26	27
W ₁	653	2760	4780	1330	6860	1365	9480	669	1247	1710	2590	4280	6000	9150
W ₂	3220	3250	3307	3295	3480	3240	3275	6525	3055	6450	6352	6320	6320	6270
W _{H₂O}	275	290	280	280	282	277	634	361	119	355	463	1049	1306	1431
T28	239.5	218.0	226.4	219.0	223.9	214.7	218.8	202.5	221.4	205.6	225.8	226.0	219.3	202.8
T20	110.7	164.4	189.6	137.3	197.9	135.6	197.2	105.3	143.5	138.6	165.1	179.8	184.6	179.7
T19	106.6	142.5	162.4	125.5	169.5	123.5	166.7	98.5	132.8	119.7	136.2	140.0	141.7	139.2
T18	80.1	99.4	110.7	94.0	119.0	92.5	109.6	87.0	101.0	102.4	112.7	110.3	109.4	107.9
T29	104.8	142.3	166.9	123.1	177.8	121.0	180.2	99.8	125.7	120.6	138.5	149.9	156.1	157.3
T22	57.6	64.9	69.3	67.8	75.3	65.9	66.2	60.9	79.8	64.9	66.8	60.7	60.0	57.9
T21	48.3	49.2	48.8	56.9	53.5	54.8	53.6	53.3	57.5	54.3	54.4	52.9	52.6	50.8
T2	109.3	154.0	178.2	132.2	186.2	131.4	184.9	102.7	139.4	129.7	152.0	160.8	164.8	162.8
T3	103.8	134.7	154.2	120.9	162.4	120.5	159.6	96.7	129.7	115.7	130.6	133.2	134.5	131.8
T4	103.2	132.8	151.2	120.0	161.1	119.2	155.0	95.9	127.9	114.2	127.7	130.0	130.5	127.9
T5	102.8	131.8	149.6	119.3	157.0	118.6	152.6	95.3	126.9	113.5	126.9	127.8	128.1	125.0
T6	101.1	128.4	145.5	117.2	153.1	116.3	148.2	94.0	124.2	110.7	124.0	124.2	124.6	121.8
T7	100.9	127.6	144.5	116.3	151.5	115.1	145.9	93.1	123.5	109.5	122.2	122.2	122.5	119.8
T8	99.7	125.5	141.2	114.4	146.5	112.6	141.4	92.4	121.6	107.9	119.3	119.1	119.6	116.8
T9	97.9	122.5	137.8	112.0	143.4	110.2	137.5	91.7	120.0	106.9	117.7	116.6	117.0	114.3
T10	95.3	117.4	134.8	108.6	139.1	106.7	131.8	91.0	117.3	105.4	116.4	114.5	115.2	112.6
T11	91.2	113.0	126.1	103.7	132.2	102.6	124.0	90.3	113.0	104.8	115.3	113.5	113.0	111.2
T12	88.0	106.9	120.2	100.6	125.5	99.3	118.7	89.9	108.8	104.1	114.5	112.4	111.7	110.3
T13	86.7	105.3	117.5	99.5	123.9	97.7	115.4	89.9	106.7	104.4	114.8	112.4	111.9	110.3
T14	86.3	104.7	116.2	98.8	123.3	97.0	116.1	89.9	105.9	104.4	114.8	112.4	111.8	110.3
T15	85.9	104.3	115.6	98.2	122.6	96.7	114.9	89.9	105.3	104.1	114.3	112.0	111.3	109.7
T16	84.8	103.3	114.9	97.8	122.0	96.0	114.0	89.2	104.8	103.7	113.9	111.6	111.0	109.5
T17	84.3	103.0	114.5	97.2	122.0	95.6	114.2	89.0	104.3	104.0	114.3	111.7	111.2	109.5
T1	104.7	142.3	163.4	124.6	170.8	124.2	169.3	98.6	131.6	119.7	137.7	142.7	145.0	144.6
T31	101.0	136.7	155.7	121.2	164.3	119.5	159.9	96.8	128.8	114.6	130.0	132.8	134.9	132.0
T32	103.0	139.3	159.7	123.0	168.5	120.6	162.6	97.0	129.8	114.7	130.8	134.2	136.1	133.5
T33	100.1	131.1	149.5	118.3	157.3	115.6	151.0	95.2	126.0	112.1	124.7	125.9	127.0	124.7
T34	101.8	132.0	150.8	119.0	158.6	116.7	152.6	95.3	125.9	111.2	125.1	126.8	128.1	124.9
Pe ₁	99	416	711	203	1018	209	1410	104	189	262	404	639	900	1380
Pe ₂	103	102	103	105	107	103	101	210	96	205	199	198	198	197
Slope	2.88	6.05	8.97	-	14.96	4.30	17.95	-	4.36	5.83	6.74	10.27	13.64	18.87
δ ₁	3.31	2.28	2.67	-	2.76	3.27	2.82	-	2.89	2.49	2.60	2.59	2.79	3.26
δ ₂	1.65	2.05	1.86	-	1.83	1.64	1.82	-	1.73	2.15	2.08	2.29	2.27	2.10
Δt ₀	159.4	118.6	115.7	125.0	104.9	122.2	109.2	115.5	120.4	103.2	113.1	115.7	109.9	94.9

TABLE XVI. (Contd.)

Run No.	28	29	30	31	32	33	34	35	36	37	38	39	40	41
W ₁	1313	1049	892	2020	2770	6380	919	939	1362	513	315	443	6640	6700
W ₂	3220	3225	3190	3116	3055	3130	3200	6490	6570	3240	3220	3200	3220	3210
W _{H₂O}	950	145	593	591	674	587	505	511	522	123	249	256	97	127
T28	218.3	209.9	230.1	210.2	212.2	211.6	234.3	225.7	192.4	261.2	253.1	240.0	98.0	139.4
T20	127.6	131.0	118.4	145.2	156.7	184.7	125.2	120.2	122.3	120.6	102.0	106.8	95.4	130.1
T19	113.8	122.4	109.8	126.7	133.8	155.1	116.4	108.3	107.0	117.7	101.1	104.8	89.0	116.9
T18	77.8	95.7	78.6	87.8	87.6	100.7	84.9	92.3	92.3	93.6	84.0	84.5	81.5	97.2
T29	114.2	119.1	109.2	127.4	136.4	165.1	116.7	108.6	108.3	114.5	102.4	104.5	93.0	122.2
T22	56.8	71.5	53.7	59.6	57.3	60.0	57.5	59.5	60.1	72.0	62.6	74.3	69.1	74.7
T21	52.0	54.2	47.7	52.2	49.6	48.1	50.4	52.2	52.8	54.5	54.5	54.0	56.8	55.4
T2	121.9	127.6	115.2	137.5	147.0	173.6	121.6	115.0	115.3	120.2	102.1	106.4	92.7	125.0
T3	110.8	119.7	107.5	122.9	129.3	150.6	113.7	105.1	103.6	116.2	100.3	103.7	87.0	112.6
T4	108.8	118.4	106.1	120.6	126.2	145.9	112.3	104.0	102.0	115.9	100.0	103.2	86.3	111.2
T5	107.3	117.6	105.4	118.8	124.0	143.5	111.5	102.8	101.5	115.4	99.7	103.0	86.3	110.2
T6	105.2	115.5	103.6	116.4	120.9	139.1	109.5	101.1	99.7	113.9	98.7	101.8	85.4	109.3
T7	103.6	114.6	102.6	115.4	118.9	136.7	108.4	99.7	98.4	113.4	98.5	101.4	85.4	108.7
T8	101.5	112.6	100.7	112.5	116.3	132.7	106.3	98.4	97.2	111.7	97.5	100.0	85.1	107.4
T9	99.2	110.6	98.1	109.8	113.2	129.1	104.1	97.5	96.8	110.3	96.6	98.3	84.3	105.2
T10	95.9	107.8	94.9	106.1	108.9	124.3	100.5	96.5	95.6	107.3	94.6	95.8	84.0	104.1
T11	91.0	104.7	90.6	100.7	103.7	117.7	96.6	95.5	94.6	103.7	91.9	93.1	83.3	102.5
T12	87.4	101.5	86.3	96.1	97.9	111.3	92.8	94.9	94.1	101.0	90.3	91.2	83.0	101.1
T13	84.9	100.2	84.6	94.0	94.9	108.2	91.0	94.9	94.2	99.4	89.5	90.2	82.7	100.1
T14	83.5	99.3	84.0	92.9	93.5	106.8	90.3	94.8	94.2	98.9	89.0	89.6	82.7	99.8
T15	83.1	99.3	83.6	92.5	92.9	106.2	90.0	94.6	93.9	98.5	88.7	89.2	82.7	99.5
T16	82.3	98.7	82.8	92.0	92.0	105.1	89.2	94.1	93.5	97.7	88.0	88.5	82.3	99.5
T17	81.7	98.3	82.2	91.0	91.4	104.7	88.3	94.1	93.5	97.1	87.2	87.9	82.2	98.0
T1	114.3	121.2	109.4	127.6	135.7	158.0	115.1	107.9	107.2	116.2	99.9	105.0	89.7	118.4
T31	109.8	118.1	106.6	121.1	130.2	152.0	112.1	104.1	103.3	115.7	98.5	102.3	86.4	113.7
T32	110.3	119.0	107.2	121.9	130.5	152.6	112.8	104.4	103.7	116.2	98.7	102.4	86.3	114.1
T33	105.7	115.7	104.0	117.0	123.7	144.0	109.8	101.4	100.5	114.5	98.0	101.4	85.0	110.4
T34	106.0	115.9	104.0	116.8	123.7	144.0	110.0	102.0	100.3	114.7	98.3	101.6	85.4	110.7
Pe ₁	201	161	136	308	420	957	140	144	210	77	48	68	1080	1058
Pe ₂	104	103	103	99	97	98	100	205	210	103	104	103	104	102
Slope	4.34	4.44	4.14	5.85	7.33	14.39	4.20	5.08	-	2.39	-	-	10.47	10.86
δ ₁	2.71	2.77	2.96	2.64	2.57	3.02	2.67	2.71	-	9.68	-	-	2.67	2.94
δ ₂	1.82	1.75	1.69	1.81	1.85	1.72	1.80	1.98	-	1.21	-	-	1.88	1.74
Δt ₀	140.5	114.2	151.5	122.4	124.6	110.9	149.4	133.4	100.1	167.6	169.1	155.5	16.5	42.2

TABLE XVI. (Contd.)

Run No.	42	43	44	45	46	47	48	49	50	51	52	53	54	55
W ₁	6670	4740	4730	4720	2540	2465	1850	1850	1838	1285	1052	1025	911	905
W ₂	3220	3180	3140	3150	3200	3200	3230	3230	3225	3210	3130	3130	3140	3140
W _{H₂O}	332	303	512	274	309	558	291	285	285	210	219	219	138	250
T28	193.3	124.7	158.0	108.3	115.4	165.4	119.9	156.7	128.9	114.3	115.6	152.9	150.2	204.5
T20	172.6	113.6	137.7	101.0	99.0	128.5	99.0	120.0	103.8	91.6	91.7	106.7	105.2	121.7
T19	148.2	102.4	119.2	92.9	90.6	112.3	91.6	108.3	95.0	36.8	87.3	100.8	100.2	114.3
T18	107.7	85.7	90.0	81.5	78.0	84.7	80.0	87.6	80.6	77.0	78.7	85.3	86.3	89.6
T29	156.4	109.0	128.2	99.1	96.1	117.8	97.3	112.5	100.3	91.7	93.1	104.0	102.6	114.1
T22	69.3	74.9	61.5	62.0	61.0	61.0	61.9	63.5	59.8	60.1	61.9	65.0	69.2	65.3
T21	53.9	55.0	53.1	55.0	54.7	54.3	55.4	54.5	52.3	52.6	54.9	56.0	57.3	55.4
T2	163.5	108.7	129.8	97.2	95.3	121.7	95.5	115.5	100.0	89.4	89.4	104.3	103.3	119.4
T3	140.7	98.0	114.5	90.7	88.0	107.8	89.6	105.0	92.7	85.0	85.7	98.3	97.8	111.0
T4	137.2	97.3	112.2	89.3	87.0	105.9	88.6	103.6	91.7	84.5	85.2	97.5	97.4	110.5
T5	135.6	96.6	110.6	89.3	86.7	105.0	88.1	102.8	91.1	84.2	84.9	97.1	97.0	109.9
T6	135.0	95.9	110.1	88.0	86.0	104.0	87.7	102.3	90.2	83.5	84.3	96.5	96.3	109.0
T7	133.7	95.3	108.7	87.7	85.7	102.9	87.1	101.3	89.7	83.0	84.0	95.8	95.8	107.9
T8	131.2	94.4	107.1	87.0	84.9	101.3	86.8	100.0	88.8	82.4	83.3	94.8	94.7	106.2
T9	126.3	92.4	103.5	85.7	83.7	98.2	85.1	97.6	87.0	81.6	82.3	92.7	93.0	103.6
T10	123.7	91.0	101.2	84.7	82.7	95.9	84.2	96.3	86.0	80.8	81.9	91.3	92.2	102.2
T11	118.1	89.7	97.0	83.7	81.3	92.9	82.9	94.0	84.5	79.8	81.0	89.7	90.6	98.5
T12	114.2	88.7	95.0	83.2	80.7	90.7	82.3	92.6	83.7	79.2	80.7	89.0	89.7	96.1
T13	112.5	88.1	94.2	82.8	80.2	89.0	81.9	91.1	82.8	78.9	80.3	88.3	89.3	94.8
T14	112.1	87.9	93.6	82.7	80.2	88.5	81.7	90.7	82.6	78.7	80.2	88.1	89.1	94.3
T15	111.7	87.7	93.3	82.4	80.0	88.2	81.7	90.5	82.4	78.7	80.0	87.9	88.8	94.1
T16	111.7	87.2	92.9	82.2	79.6	87.8	81.2	90.1	82.1	78.2	79.8	87.6	88.7	93.5
T17	109.2	86.6	91.9	82.0	78.9	86.7	80.7	89.8	81.6	77.8	79.4	86.9	88.0	92.1
T1	153.2	103.9	123.3	93.7	91.5	115.0	92.4	110.1	96.4	87.0	87.4	101.4	101.0	115.7
T31	144.7	97.9	115.5	88.8	87.9	108.6	88.3	106.5	93.1	85.7	84.0	97.4	97.7	112.2
T32	145.9	98.2	115.4	89.2	88.1	109.1	88.8	106.7	93.5	85.8	84.2	97.4	97.8	112.9
T33	138.2	95.4	110.1	86.8	86.2	104.2	86.7	103.5	91.2	84.6	83.1	95.7	96.2	109.9
T34	138.2	95.4	110.5	87.1	86.2	105.0	86.7	103.7	91.1	84.5	83.1	95.9	96.4	110.1
Pe ₁	1012	756	737	745	409	384	298	290	294	208	170	162	145	139
Pe ₂	101	98	100	102	104	103	104	104	104	104	102	101	101	101
Slope	12.52	8.15	8.70	-	5.05	5.67	-	4.60	4.85	4.11	3.38	3.74	3.55	3.82
δ ₁	3.05	2.55	2.61	-	2.47	2.52	-	2.86	2.51	2.40	2.00	2.68	2.94	3.70
δ ₂	1.72	1.93	1.89	-	1.91	1.82	-	1.72	1.87	2.00	2.20	1.79	1.73	1.51
Δt ₀	85.6	39.0	68.0	26.8	37.4	80.7	39.4	69.1	48.3	37.3	36.9	67.6	63.9	114.9

TABLE XVI. (Contd.)

Run No.	56	57	58	59	60	61	62	63	64	65	66	67	68	69
W ₁	564	565	8780	8780	8795	8780	3750	3760	3790	1265	1266	3750	3740	3740
W ₂	3125	3130	3160	3140	3130	3140	3190	3190	3185	6160	6180	6125	6145	6090
W _{H₂O}	250	248	546	326	199	144	107	243	132	131	313	307	273	269
T28	153.6	212.4	130.1	117.2	104.2	96.5	99.3	109.2	89.6	142.6	184.8	128.2	108.2	97.4
T20	94.3	108.0	121.9	111.8	100.9	94.3	93.7	99.9	85.7	107.9	121.0	114.6	99.3	91.0
T19	91.4	104.8	107.2	101.3	93.3	88.7	88.2	91.4	81.3	100.9	109.5	102.0	90.4	84.3
T18	79.3	84.5	84.4	85.1	82.7	80.9	81.0	79.8	76.0	93.2	96.1	93.0	84.5	80.0
T29	95.7	105.4	116.3	108.7	99.2	93.7	92.8	96.7	86.3	103.0	111.3	106.9	94.8	88.5
T22	61.4	62.7	60.4	62.6	64.0	65.9	67.1	61.1	62.9	72.5	66.4	64.8	61.5	60.2
T21	54.4	54.0	53.7	55.0	55.1	56.8	56.4	53.7	54.5	58.4	56.9	55.7	54.0	53.6
T2	93.2	107.6	116.5	107.8	97.8	92.2	91.5	96.4	83.7	105.0	116.2	108.8	95.5	88.3
T3	89.9	102.4	103.7	98.3	91.4	87.2	87.0	89.7	80.4	98.7	106.5	99.4	88.9	83.3
T4	89.7	102.4	101.7	96.9	90.3	86.3	86.3	88.3	79.8	98.0	105.1	98.0	88.2	82.8
T5	89.4	102.4	100.6	96.2	89.4	86.0	86.0	88.1	79.7	97.6	104.4	97.1	87.9	82.7
T6	88.8	101.5	99.4	95.2	89.0	85.3	85.3	87.2	79.1	97.0	103.3	96.4	87.1	82.0
T7	88.0	100.4	97.9	94.2	88.5	84.9	84.9	86.5	78.8	96.7	102.3	96.1	86.6	81.8
T8	87.3	99.5	96.5	93.0	87.4	84.5	84.5	85.5	78.6	96.1	101.3	95.7	86.4	81.5
T9	86.5	97.0	94.0	91.1	86.4	83.7	83.9	84.2	78.7	95.3	99.8	94.8	86.0	81.3
T10	85.2	95.7	91.8	90.1	85.9	83.5	83.6	83.6	78.0	95.4	99.8	95.0	86.1	81.3
T11	83.5	93.3	89.4	88.7	84.9	82.7	82.7	82.3	77.3	95.0	99.0	94.4	85.4	80.6
T12	82.7	91.0	88.1	87.7	84.4	82.3	82.3	81.9	77.0	94.7	98.5	94.0	85.3	80.6
T13	82.3	89.8	87.3	87.2	84.0	82.0	82.2	81.6	76.8	94.5	98.2	93.9	85.1	80.6
T14	82.0	89.3	87.0	87.1	84.0	81.9	82.1	81.4	76.8	94.6	98.2	93.9	85.3	80.6
T15	82.0	89.1	87.0	86.9	84.0	81.9	82.1	81.4	76.8	94.5	98.0	93.9	85.3	80.5
T16	81.4	88.4	86.3	86.4	83.4	81.5	81.7	81.0	76.4	94.2	98.0	93.6	84.8	80.3
T17	80.7	87.2	85.7	86.1	83.1	81.3	81.4	80.6	76.2	94.1	97.5	93.4	84.7	80.1
T1	91.4	105.3	110.3	102.7	94.3	89.4	89.0	92.9	81.8	101.3	110.3	102.7	91.3	84.9
T31	89.8	103.9	104.5	98.3	90.6	86.6	86.8	89.8	80.5	98.3	106.7	100.0	89.7	83.2
T32	89.8	103.9	105.6	98.7	90.8	86.8	87.0	90.2	80.6	98.0	106.5	99.6	89.7	83.2
T33	88.9	102.4	100.5	95.3	88.4	85.3	85.5	87.8	79.6	97.1	104.0	97.7	88.2	82.3
T34	88.9	102.5	101.2	95.5	88.7	85.2	85.5	87.8	79.6	97.1	104.5	97.7	88.3	82.3
Pe ₁	90	87	1395	1406	1417	1429	610	608	621	201	197	597	604	610
Pe ₂	100	101	101	101	101	102	104	103	104	198	198	197	199	198
Slope	4.04	-	17.68	16.75	15.69	14.50	7.72	8.48	7.17	3.25	5.97	-	8.11	10.32
δ ₁	2.90	-	3.13	3.00	2.72	2.95	2.75	2.66	2.75	2.69	2.88	-	2.78	3.19
δ ₂	1.71	-	1.69	1.72	1.81	1.70	1.77	1.81	2.00	1.79	1.95	-	1.98	1.97
Δt ₀	74.3	127.9	45.7	32.1	21.5	15.6	18.3	29.4	13.6	49.4	88.7	35.2	23.7	17.4

TABLE XVI. (Contd.)

Run No.	70	71	72	73	74	75	76
W ₁	650	643	995	971	8710	6690	6660
W ₂	6130	6130	6230	6230	3080	3082	3090
W _{H₂O}	248	250	291	236	201	200	202
T28	131.0	176.4	155.7	124.0	88.1	89.6	87.4
T20	89.1	100.2	101.3	91.4	85.8	86.5	84.6
T19	86.2	95.8	94.0	86.4	80.6	81.3	79.8
T18	81.4	87.4	85.4	81.2	73.9	74.8	73.7
T29	88.9	97.0	95.8	89.0	85.5	86.3	84.7
T22	63.5	64.5	61.9	61.5	61.0	61.7	61.2
T21	57.2	56.4	54.4	54.5	55.0	55.6	55.4
T2	87.7	98.6	98.3	89.4	84.1	84.5	82.7
T3	84.9	94.0	92.2	85.3	79.8	80.5	79.2
T4	84.6	93.3	91.4	84.8	79.0	79.7	78.4
T5	84.3	93.0	90.8	84.5	78.7	79.2	78.0
T6	84.0	92.2	90.0	83.7	77.9	78.9	77.4
T7	83.7	91.5	89.6	83.5	77.3	78.2	77.0
T8	83.5	90.8	88.8	83.3	76.9	77.7	76.5
T9	82.9	89.9	87.9	83.3	76.3	77.3	76.0
T10	82.9	90.1	87.9	83.3	76.1	76.7	75.7
T11	82.8	89.3	87.3	82.3	75.2	76.0	75.0
T12	82.8	89.0	86.9	82.1	75.0	75.6	74.7
T13	82.8	88.9	86.7	82.1	74.7	75.4	74.5
T14	82.8	88.9	86.7	82.1	74.6	75.4	74.3
T15	82.7	88.9	86.7	82.1	74.6	75.2	74.3
T16	81.9	88.5	86.3	81.8	74.4	75.0	74.2
T17	81.9	88.3	86.3	81.8	74.3	75.0	74.0
T1	85.9	95.9	94.6	86.3	81.3	82.2	80.7
T31	83.2	93.7	92.6	85.0	79.8	80.0	78.5
T32	83.0	93.7	92.6	85.0	79.8	80.0	78.6
T33	82.6	92.5	91.1	84.0	78.4	78.6	77.4
T34	82.3	92.3	91.1	84.0	78.4	78.9	77.4
Pe ₁	105	101	158	156	1431	1098	1092
Pe ₂	199	196	202	202	100	101	101
Slope	2.50	2.65	3.21	-	-	4.01	-
δ_1	2.33	2.93	2.70	-	-	2.89	-
δ_2	2.07	1.63	1.83	-	-	1.73	-
Δt_0	49.6	89.0	70.3	42.8	14.2	14.8	13.7

TABLE XVII. Data for 8 L/D₁ Test Section

Run No.	W ₁	W ₂	W _{H₂O}	T28	T20	T19	T23	T29	T22	T21	Pe ₁	Pe ₂	Δt ₀
77	1248	6180	247	125.1	101.2	87.6	82.9	91.3	63.4	56.5	200	201	42.4
78	1220	6195	214	113.5	95.5	85.4	81.6	89.3	64.2	57.0	197	202	31.9
79	1220	6195	214	98.3	88.1	81.6	79.5	85.6	63.7	57.7	199	201	19.3
80	4950	6150	216	137.7	128.3	106.3	99.2	113.8	68.0	54.8	781	197	38.5
81	4990	6150	336	122.7	115.1	96.5	90.2	104.3	62.1	54.0	795	198	18.6
82	4910	6130	334	109.1	103.2	89.2	84.3	96.0	60.5	53.6	794	198	24.8
83	2567	6190	259	109.7	98.9	85.9	81.9	90.5	62.4	55.9	415	201	27.8
84	2560	6195	196	102.4	94.7	85.0	81.9	89.2	64.0	56.4	416	201	20.5
85	2565	6190	196	93.5	88.3	81.2	79.1	85.3	63.0	56.3	420	201	14.4
86	2575	6190	258	123.6	108.9	91.6	86.3	96.2	63.7	55.9	412	200	37.3
87	2587	6155	258	153.9	131.7	105.3	96.7	110.7	66.8	56.1	405	197	57.2
88	1813	6170	259	156.1	126.4	101.5	93.4	105.8	65.6	55.9	284	198	62.7
89	1796	6195	260	122.6	105.4	89.8	84.9	94.2	63.1	55.6	288	201	37.7
90	1777	6190	215	105.2	94.2	84.3	81.0	88.3	62.7	55.7	288	201	24.2
91	634	6200	215	119.7	88.1	81.1	77.8	84.2	61.3	55.1	102	202	41.9
92	640	6195	215	199.4	112.4	96.2	87.7	97.3	63.9	54.5	99	200	111.7
93	568	3070	236	158.9	98.3	87.2	76.1	89.7	61.8	56.0	90	100	82.8
94	568	3065	236	225.6	119.1	101.6	82.2	100.0	64.1	56.5	87	99	143.4
95	1268	3060	238	162.6	122.3	98.6	82.7	101.2	64.2	56.4	199	99	79.9
96	1241	3070	238	120.4	98.0	84.1	75.0	88.5	61.2	55.4	200	100	45.4
97	2530	3050	238	129.0	112.6	92.4	80.3	98.0	62.4	55.0	403	99	48.7
98	2535	3000	204	106.0	96.3	83.7	76.3	89.4	62.4	56.2	410	98	12.6
99	4415	2990	204	115.4	108.0	92.7	82.8	100.0	65.6	57.3	713	97	32.6
100	6210	2980	205	136.1	128.5	106.3	91.2	116.2	68.8	57.0	984	96	44.9

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