BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

MASTER

ACCELERATOR DEPARTMENT
(AGS)
Internal Report

PHASE ACCEPTANCE AND BUNCHING IN THE AGS LINAC

J.P. Blewett

March 27, 1963

1. Introduction

In the linear accelerator the initial rate of acceleration is so high that the adiabatic solution often used to represent phase oscillations is not correct. During this stage of acceleration, the symmetrical fish-shaped bucket plotted in energy-phase space for the synchrotron does not give a correct description of the phase acceptance of the linac. To the writer's best knowledge, the correct representation for phase acceptance in the linac was first presented by R. Taylor of the AERE at Harwell ("Acceptance of Axially and Radially Oscillating Particles in the Linac Injector for the Harwell 7 Gev Proton Synchrotron", AERE R-3013, October, 1959) in an analysis of phase acceptance in the 15 Mev Nimrod injector. This machine was designed for preinjection at 600 kev. The pattern presented by Taylor will also be found in the paper by N.D. West in the "Minutes of the Conference on Linear Accelerators for High Energies" held at Brookhaven in August, 1962.

In this report we present a similar analysis for the AGS injector, including an extension of Taylor's diagram to higher energies. Also included in this report is an analysis of bunching for the AGS linac and a proposal for a double buncher which could be more efficient than the single buncher now in use.

APR 22 1963

Facsimile Price \$ 4.60
Microfilm Price \$ 5.20
Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

LEGAL NOTICE
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe upon privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor, is providing, with the Commission, or his employment with such contractor.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

2. Phase Acceptance - Theoretical

The behavior in phase and energy of protons with a variety of initial conditions has been computed step by step through the 124 drift tubes of the AGS injector. The dimensions used are those quoted in the ADD Standards Book, which presumably correspond quite accurately to those actually existing in the linac. Only two assumptions have been made. First, acceleration at each gap has been assumed to take place over an infinitesimal distance at the center of the gap. Second, the assumed rf level was such that the sine of the stable phase angle is 0.9. The results of this analysis are collected in the table on pages 3-5 and are presented in Fig. 1.

The dots in Fig. 1 represent particles which are trapped into stable phase oscillations and accelerated to full energy. The crosses represent particles which failed to be trapped and went into unstable oscillations. The boundary that we have traced between the stable and unstable region is somewhat similar in character to that discovered by Taylor. Two features of this pattern seem to be noteworthy. First, it is neither centered on nor symmetrical about the design injection energy of 750 kev. The center of the pattern lies nearer to 770 kev, which in operation of the AGS linac has been found to be a more satisfactory injection energy. Second, the pattern has a long tail reaching into the region of higher injection energies and higher injection phases. Taylor did not explore this tail above an energy which in our case would correspond to about 900 kev. We have traced the tail over a considerable range with the results indicated in Fig. 2. Somewhat surprisingly, there is an appreciable range of phase acceptance at injection energies as high as 1600 kev and at phases shifted by some 470° from the final stable phase. The tail was not traced beyond this point, although it gave every appearance of going on indefinitely.

Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase	Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase
690.3	60	Unstable		802.6	30	Unstable	
	70	Unstable			35	51.37	75.95
					40	51.73	65.89
702.3	50	Unstable			45	51.67	62.74
	55	51.29	76.05		50	51.62	62.18
	60	51.72	61.92		60	51.61	63.05
	70	51.60	59.20		70	51.63	64.93
	75	51.74	62.49		80	51.63	67.52
	80	Unstable			90	51.58	70.26
					100	51.48	72.39
726.8	35	Unstable			110	51.36	73.29
	40	51.65	72.33		116	51.26	72.89
	45	51.51	59.18		120	51.19	71.88
	50	51.30	60.93		125	51.10	68.74
	60	51.23	65.04		130	51.34	58.58
	70	51.24	67.11		135	Unstable	
	80	51.21	67.78				
	90	51.15	65.11	815.6	35	Unstable	
	95	51.26	60.12		40	51.41	75.73
	100	Unstable			45	51.70	69.42
					135	51.66	59.61
751.6	30	Unstable			140	Unstable	
	35	51.75	66.52				
	40	51.50	59.62	828.7	40	Unstable	
	45	51.34	61.19		50	51.26	75.71
	50	51.31	63.05		60	51.52	74.31
	60	51.34	65.33		70	51.46	74.87
	70	51.39	66.78		80	51.30	75.48
	80	51.41	68.42		90	51.15	73.95
	90	51.35	70.07		100	51.04	72.18
	100	51.20	69.65		110	51.07	72.74
	110	51.40	58.40		120	51.13	73.11
	115	Unstable			130	51.10	71.12
					135	51.06	67.27
776.9	30	Unstable			140	Unstable	
	35	51.73	64.18				
	40	51.55	60.29	835.2	50	Unstable	
	45	51.45	60.83		60	50.97	61.47
	50	51.42	61.82		70	50.93	65.02
	60	51.44	63.35		80	51.92	70.97
	70	51.50	64.74		90	Unstable	
	80	51.53	66.66		100	Unstable	
	90	51.51	69.12		110	50.99	60.52
	100	51.41	71.34				
	110	51.23	71.30				
	116	51.11	67.67				
	120	51.23	60.19				
	125	Unstable					

Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase	Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase
841.9	60	Unstable		937.2	160	Unstable	
	70	Unstable			165	51.00	63.00
	80	Unstable			170	Unstable	
	90	Unstable					
	100	Unstable		951.3	165	Unstable	
	110	Unstable			170	51.02	63.57
	115	Unstable			175	Unstable	
	120	50.98	61.10				
	125	50.97	68.90	965.4	170	Unstable	
	130	51.04	70.87		175	51.08	62.27
	135	51.06	70.19		180	Unstable	
	140	51.06	65.91				
	145	Unstable		979.7	172.5	Unstable	
					175	51.41	54.44
855.2	120	Unstable			177.5	51.03	62.63
	125	Unstable			180	51.51	57.93
	130	51.09	57.79		182.5	Unstable	
	135	50.97	68.05				
	140	51.03	69.06	994.0	177.5	Unstable	
	145	51.08	63.88		180	51.07	59.43
	150	Unstable			182.5	51.12	61.22
					185	Unstable	
868.6	135	Unstable					
	140	50.96	64.60	1008.5	180	Unstable	
	145	51.01	67.91		182.5	51.73	55.59
	150	51.19	60.41		185	51.04	61.91
	155	Unstable			187.5	51.77	66.89
					190	Unstable	
882.1	140	Unstable					
	145	50.99	61.63	1023.0	185	Unstable	
	150	51.01	66.83		187.5	51.10	58.39
	155	51.77	65.09		190	51.17	60.02
	160	Unstable			192.5	Unstable	
895.7	145	Unstable		1037.7	187.5	Unstable	
	150	51.02	60.16		190	51.96	63.4
	155	51.02	65.66		192.5	51.06	61.32
	160	Unstable			195	Unstable	
909.5	150	Unstable		1054.5	192.5	Unstable	
	155	51.02	60.35		195	51.14	57.6
	160	51.06	63.86		197.5	51.25	58.9
	165	Unstable			200	Unstable	
923.3	155	Unstable		1067.3	195	Unstable	
	160	51.00	61.61		197.5	50.93	54.5
	165	51.20	59.94		200	51.08	60.8
	170	Unstable			202.5	Unstable	

Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase	Initial Energy (kev)	Initial Phase	Final Energy (Mev)	Final Phase
1082.3	200	Unstable		1421.0	316	Unstable	
	202.5	51.20	56.6		318	51.59	55.0
	205	51.32	58.2		320	51.33	56.6
	207.5	Unstable			322	51.56	57.5
					324	Unstable	
1097.4	205	Unstable		1508.5	450	Unstable	
	207.5	51.09	60.3		456	51.64	56.2
	210	Unstable			458	51.62	56.8
1174.4	225	Unstable			460	51.73	59.4
	227.5	51.14	59.3		462	51.70	71.8
	230	Unstable			464	Unstable	
1254.0	247	Unstable		1598.5	510	Unstable	
	249	51.25	56.3		520	51.75	57.8
	251	51.56	57.9		530	Unstable	
	253	Unstable					
1336.2	275	Unstable					
	276	51.57	54.7				
	277	51.25	56.8				
	278	51.30	57.5				
	279	51.70	60.0				
	280	Unstable					

The final distribution in energy vs phase is shown in Fig. 3. The circled points in Fig. 3 are protons which originated well up in the long tail shown in Fig. 2 and so have little significance with regard to actual linac operation. All circled points originated at injection energies higher than 1000 kev. Evidently the phase and energy spread from protons originating in the high energy tail is much worse than that of protons originating in the main stable region. The areas A and B of Fig. 3 are bounded by arbitrarily drawn ellipses representing energy spreads of 0.3 and 0.6 Mev respectively and phase spreads of 6° and 12° respectively. All of the protons ending in area A of Fig. 3 originated in the area A indicated in Fig. 1. Similarly, protons ending in region B in Fig. 3 originated in region B of Fig. 1. Points in Fig. 3 can be identified with points in Fig. 1 by use of the table above.

3. Phase Acceptance - Experimental

The phase acceptance of the AGS linac as a function of Cockcroft-Walton energy can be derived by drawing a horizontal line on Fig. 1 at the appropriate energy level and measuring the length of that line included within the stable region of Fig. 1. The percentage acceptance curve derived from this procedure is plotted in Fig. 4. An experimental check of this curve was run by disconnecting the buncher and measuring accelerated current as a function of Cockcroft-Walton energy. The Cockcroft-Walton current was measured at the same time and the assumption was made (based on previous measurements) that 60% of the Cockcroft-Walton beam is made up of protons. Measured experimental points are shown on Fig. 4; it is evident that the agreement between experiment and theory is reasonably good. The agreement is particularly striking in view of the fact that the rf level in the linac was assumed to be the same as that used in computation of Fig. 1. A precise verification of this assumption would be extremely difficult.

4. Bunching in the AGS Linac

The position of the buncher in the AGS linac is indicated in Fig. 5.

The action of this buncher on the proton beam is given by

$$\phi_{\text{final}} = \omega s \left(\frac{1}{v_0} - \frac{1}{v_1} \right) + \phi_i \quad (1)$$

$$= \omega s \sqrt{\frac{m}{2eT}} \left\{ 1 - \left(1 + \frac{V \sin \phi_i}{T} \right)^{-\frac{1}{2}} \right\} + \phi_i \quad (2)$$

where

ω = $2\pi \times$ frequency = 1.2629×10^9 for frequency = 201 Mc/sec

s = distance from buncher to linac = 1.626 m

v_0 = velocity of proton entering buncher gap

v_1 = velocity of proton leaving buncher gap

e/m = 0.95793×10^8 coul/kg (for protons)

T = initial kinetic energy = 770 kev

V = peak volts across buncher gap

ϕ_i = phase of buncher field at the instant of arrival of the proton in question.

In Figs. 6 and 7 are shown the patterns of the bunched beam fitted into the acceptance region for buncher voltages of 18 kv and 19 kv.* In both cases the theoretical capture efficiency is about 72%. From Fig. 7 it is evident that buncher voltages higher than 19 kv will result in reduced capture and that 19 kv should be about the empirically determined optimum. Fig. 8 is a plot showing the linac output with and without the buncher. The "without buncher" curve is drawn through the experimental points of Fig. 4. The "with buncher" curve involved optimizing buncher phase at each setting of

---

* On all curves in this report representing energy-phase relations for bunched beams, the points indicated are points which were separated by 20° in phase when the beam entered the bunching system.

Cockcroft-Walton energy. The maximum capture efficiency indicated by this curve is about 64%. The difference between this figure and the theoretical figure of 72% probably is due, at least in part, to interception of a small fraction of the proton beam by the grids in the buncher gap.

Tracing of the two curves to their low energy intercepts gives a measure of buncher voltage. When the buncher is included, the first capture as energy is raised should occur at an energy lower than the minimum of Fig. 1 by an amount equal to the buncher voltage. From Fig. 8 it is evident that the intercepts of the two curves differ by exactly the 19 kv already predicted.

5. Proposed Double Buncher

In Fig. 9 a possible configuration of two bunchers is shown in which it is possible to obtain somewhat higher bunching efficiency. Across the first gap is applied an rf voltage of 15 kv peak. Across the second gap the voltage is 45 kv and is applied in such phase that the proton arriving at the second buncher at 0° is the same proton which reached the first buncher at 0° phase. In Fig. 10 we show plots of energy vs phase at various stages of the bunching process. The diagram is reasonably self-explanatory; evidently more than 90% of the beam has been bunched into a 60° phase angle. In Fig. 11 the final curve of Fig. 10 has been replotted in the stable region of Fig. 1. About 91% of the protons lie within the capture region. In practice this figure would probably be reduced somewhat by the two sets of grids in the two bunchers. Also, it should be pointed out that the final energy spread from double bunching of this type will probably be slightly greater than that obtained from a single buncher. Nevertheless, a significant gain in synchrotron intensity should result from the use of the double buncher.

6. Space-Charge Effects

When the bunching process is in progress, rather high local concentrations of charge density can arise. For example, between the two upper curves of Fig. 10 there is a stage in which the curve has a vertical section indicating a theoretical infinity in charge density. It would seem that this could result in some debunching action, which would be dependent on beam intensity. In view of the good agreement observed between theory and experiment in the preceding sections of this report, it would seem that space-charge effects are not important with beams of the intensities in use at the AGS. However, for future reference, we present the following simplified analysis of space-charge effects.

In Fig. 12 the density pattern of the beam is shown at successive stages of the bunching process. The shaded regions indicate beam density over the distance traveled by the beam during one period of the radiofrequency signal (about 6 cm at 770 kev). The uppermost plot shows the uniform beam that enters the buncher; the final plot indicates the density pattern as the beam enters the linac. From these plots it appears that, over a distance of the order of 75 cm a bunch exists in the center of the pattern having an extent of the order of 1 cm and a charge density of about three times the average beam density. If we assume a beam diameter of the order of 1 cm, the debunching fields at the surface should be of the order of the fields at the surface of a sphere of charge having charge density three times the average beam density.

For a 50 ma beam of 1 cm^2 cross section at 770 kev, the density is $4.2 \times 10^{-5} \text{ coul/m}^3$; hence in the bunch the density will be $1.26 \times 10^{-4} \text{ coul/m}^3$. The electric field at the surface of a sphere having this density is $2.4 \times 10^4 \text{ v/m}$. Acting over a distance of 0.75 m this field could change the energy of a proton by 18 kev.

At first glance the fact that the space-charge fields are capable of changing proton energies by as much as 18 kev might seem somewhat disturbing. The situation is mitigated, however, by three factors:

1. Individual protons do not remain at the surface of the bunch and so do not continually experience the highest fields.
2. The space-charge fields initially oppose the bunching process and later aid it, so their effect is, to some extent, self-cancelling.
3. The action of the space-charge fields is mainly during the latter part of the bunching process, hence although it may introduce some additional energy spread it will not make major alterations in the bunch shape.

If beam intensities much higher than 50 ma are reached, effects of the space-charge fields will be observed. What these effects will be in detail can be determined only by a complete numerical analysis. If the effects are undesirable, as probably they will be, it will be necessary to proceed in one or both of the following directions. First, during the bunching process the diameter of the beam should be kept as large as possible. Second, bunching would be improved by the use of higher buncher voltages and shorter drift lengths.

7. Conclusion and Acknowledgments

From the theoretical and experimental data presented above it appears that we have a satisfactory understanding of phase acceptance in the linac and of the bunching process. There is as yet no evidence of space-charge effects.

The writer is indebted by Dr. A. van Steenberg for the experimental measurements quoted here, and to Mr. K. Jellett for the computed data used in plotting Figs. 1, 2 and 3.

JPB/yew

4/2/63

Distr.: AD B1, B2

FIG 4

PRESENT BUNCHER CONFIGURATION
FOR A.G.S. LINAC

FIG 5

FIG 8

FIG 9

FIG 12