AUG 23 1966

MASTER

H.C. 8 1.00, MN 50

CONF-660808-7

Paper submitted to the XIIIth International Conference on
High Energy Physics at Berkeley, 1966

Discussion Group: Baryon Resonances, $S = 0$ (9) or
Hadron Collisions at Very High Energy (12 c)

Title: REGGE RECURRENCE PARITY ASSIGNMENTS FOR THE
 $S = 0$ REGGE RECURRENCES*

V. Barger and D. Cline
Department of Physics
University of Wisconsin
Madison, Wisconsin

ABSTRACT

The $S = 0$ fermion resonances are classified as Regge
recurrences on three straight line trajectories (namely, Δ_δ , N_α , N_γ)
in a Chew-Frautschi plot. These Regge recurrence parity assignments are
shown to be consistent with the structure observed in backward pion-
nucleon scattering.

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

In Reggeization of the pion-nucleon scattering amplitude, separate analytic continuations to complex J are required for partial wave amplitudes of even and odd signature $\tau = (-)^{J-1/2}$ due to the existence of exchange forces. Thereby the physical resonance states associated with the trajectory of a given Regge pole are spaced by two units of total angular momentum J (but have the same parity P , isotopic spin I , and strangeness S). A priori the behavior of the trajectory $\alpha(\sqrt{u})$ as a function of energy $\sqrt{u}$ is unknown, other than the general requirement that $\alpha(\sqrt{u})$ satisfy a dispersion relation with cuts starting at $\pm(M + \mu)$. Consequently, until the spins and parities of the observed resonances are experimentally determined, there exists no definite guideline to follow in the classification of the resonances as recurrences along Regge trajectories. (Recurrences along a trajectory occur where $\text{Real } \alpha(M_R) = J_R$ and $\alpha'(M_R) \neq 0$). In order to construct a plausible empirical classification scheme of the observed baryon resonances according to the Regge recurrence concept, we must at present rely heavily on the low mass resonances for which the spin-parity (J^P) assignments are already experimentally established. The most economical assignment of the known resonances involves only three trajectories (Δ_δ , N_α , N_γ) as shown in Fig. 1. From the masses of the resonances, the resulting forms for $\text{Real } \alpha(\sqrt{u})$ are found to be very well approximated by straight lines in the variable u . On the basis of this Chew-Frautschi plot,² the spin-parity assignments of the $S = 0$ fermion resonances are predicted as listed in Table I.

In this classification the Δ_δ and N_Y trajectories each have five members whereas the (938 MeV, $1/2^+$) nucleon and the (1688 MeV, $5/2^+$) resonance are the only members on the N_α trajectory. A straight line through these two states extrapolates through $9/2^+$ at 2200 MeV and $13/2^+$ at 2610 MeV as shown in Fig. 1. On the basis of this extrapolation, the "parity doublets" $\{N_Y(2190, 7/2^-); N_\alpha(2200, 9/2^+)\}$ and $\{N_Y(2640, 11/2^-); N_\alpha(2610, 13/2^+)\}$ are predicted to occur due to recurrences of the N_Y , N_α trajectories at such similar mass values. The N_Y member of a parity doublet has orbital momentum one unit lower than the N_α member. If the momentum dependence of the elastic width is of the usual form

$$\Gamma_\ell \sim p \left(\frac{p^2}{p^2 + k^2} \right)^{2\ell}$$

then the N_Y member would be expected to have a larger elastic width than the N_α member of the doublet (provided that the radii of interaction and reduced widths are comparable). In such a circumstance the negative parity N_Y resonance should dominate over the positive parity N_α . In total cross section data, the parity doublet would effectively show up as a single bump. The parity doublet in the vicinity of 2200 MeV can presumably be disentangled by a careful phase shift analysis.³ Discovery of the missing N_α recurrences would provide a striking verification of the Regge recurrence assignments based on the straight line approximation to the real part of the trajectory function.

The structure observed in backward pion-nucleon scattering provides a sensitive test of the validity of the parity assignments

based on the Regge recurrence classification scheme of Fig. 1. In π^-p elastic scattering at 180° , the detailed structure can be quantitatively interpreted as interference between a $(\Delta_\delta, N_\alpha, N_\gamma)$ direct-channel resonance amplitude and a Δ_δ Regge exchange amplitude.¹ The basic premise of the calculation is that the sum of the direct-channel resonance amplitude and the cross-channel exchange amplitude adequately represents the physical scattering amplitude. At 180° , the π^-p direct-channel amplitude due to the successive recurrences reduces to

$$\frac{1}{k} \left[\frac{1}{3} \sum_{\Delta_\delta} \frac{(-)^l (J + \frac{1}{2}) X_{3/2}}{\epsilon - i} + \frac{2}{3} \sum_{N_\alpha, N_\gamma} \frac{(-)^l (J + \frac{1}{2}) X_{1/2}}{\epsilon - i} \right]$$

where $\epsilon = (M_R^2 - S)/(M_R \Gamma_R)$ and X_I is the I /resonance elasticity. The sums refer to the resonance states listed in Table I. The factor of $(-)^l$ determines whether a particular resonance interferes constructively or destructively with the exchange amplitude. The Δ_δ Regge exchange amplitude has the form

$$\gamma \frac{1 - ie^{-i\pi\alpha}}{\cos\pi\alpha} \left(\frac{E_{\text{LAB}}}{E_0} \right)^{\alpha-1/2}$$

for a straight line trajectory in the variable n .

Assuming that the value of α can be approximately determined by

extrapolation in the Chew-Frautschi plot of Fig. 1a, the effective residue γ is the only free parameter in the Regge exchange amplitude.

As a first approach in investigating the interference model described above, we calculated the 180° π^-p differential cross section using previously reported resonance parameters (M_R, Γ, X_I) for the known resonances on the $(\Delta_\delta, N_\alpha, N_\gamma)$ trajectories. No appreciable variation of the elasticities X_I from their experimental values was

allowed. This calculation produced reasonable qualitative agreement with the data,¹ particularly in light of uncertainties in resonance elasticity parameters. Next, the predicted resonances $N_\alpha(2200, 7/2^+)$, $N(2610, 13/2^+)$ were also included in the direct channel amplitude. The elasticity parameters of all the resonances were then adjusted to yield quantitative agreement with the π^-p 180° differential cross section. The theoretical curve for $\gamma = 0.2 \text{ BeV}^{-1}$ and $E_0 = .4(\text{BeV})^2$ is shown in Fig. 2a along with the 180° experimental data. The calculation is in apparent good agreement with experiment over the entire momentum range, 1.6 to 8.0 BeV/c. It should be emphasized that the theoretical curve in Fig. 2a represents the absolute differential cross section and does not involve an arbitrary normalization.

The final values of the elasticities of the resonances obtained from this calculation are compared with other determinations in the last four columns of Table I. The elasticities from $\pi^-p \rightarrow \pi^0 n$ at 0° are based on a simultaneous analysis of $[\sigma_t(\pi^-p) - \sigma_t(\pi^+p)]$ and the forward charge differential cross section using a Regge ρ -meson exchange background amplitude.⁴ The elasticities from total cross sections were determined without the use of a theoretical model for the smooth background. The phase shift analysis exists only for the low mass resonances and is the only one of the four methods that can completely disentangle two resonances with different quantum numbers at the same position. For such cases of degenerate resonance positions, only a mean resonance height is determined from the other methods. Such a complication arises at the positions of the predicted parity doublets $\{N_\gamma(2190, 7/2^-); N_\alpha(2200, 9/2^+)\}$

and $\{N_Y(2640, 11/2^-); N_\alpha(2610, 13/2^+)\}$. Taking this complication into account, the resonance elasticities determined from $\pi^- p \rightarrow p \pi^-$ at 180° are generally consistent with other determinations.

In order to test the Regge recurrence parity assignments, the calculation has been repeated with opposite parity assignments to that given in Table I for individual ones of the high mass resonances. Changes in the parities from the recurrence predictions yield qualitative disagreement with the experimental trend of $\frac{d\sigma}{d\Omega}$. We conclude that there is convincing evidence for the parity assignments given in Table I according to the Regge recurrence hypothesis.

A limited amount of experimental data on $\pi^- p$ angular distributions exists at ten different laboratory momenta ranging from 1.7 - 8.0 BeV/c. As a function of momentum, the angular distributions, near 180° alternate between a rapid turn-over and a sharp fall-off. A straightforward extension of the interference model (which invokes no additional parameters) satisfactorily describes the general shapes of these angular distributions. Finally, the model also remains reasonably valid down to very low momenta. In Fig. 2b the available data in the momentum range 600-1900 MeV/c are compared with the theoretical predictions. Here the $N_\beta(\sim 1650)$ resonance has also been included in the direct channel amplitude. In this momentum range the direct channel resonances provide the dominant contribution to the differential cross section.

Additional confirmation for the Δ_δ Regge recurrence parity assignments can be obtained from an analysis of backward $\pi^+ p$ elastic scattering. In this process only the Δ_δ recurrences contribute to the

direct channel amplitude. Hence all the resonances contribute with the same sign to the π^+p amplitude in contrast to the situation in the π^-p reaction where appreciable cancellations occurred between the overlapping Δ_δ and N_Y resonances in the direct channel amplitude. (The resonance contribution to $\pi^-p \rightarrow p\pi^-$ is shown in Fig. 2a and is obviously inadequate to explain the experimental data above 3 BeV/c.) (An additional factor of 3 for the π^+p amplitude is acquired from the isotopic spin Clebsch-Gordan coefficient.) The net result of this circumstance is a large direct channel contribution that dominates the backward π^+p elastic scattering amplitude. Significant contributions from the possible Δ_δ , N_α , or N_Y Regge exchange do not seem to be required to explain the principal features of the π^+p data from 1-5 BeV/c.

The contribution of the Δ_δ resonances to the backward π^+p differential cross section can be immediately calculated from the resonance parameters of Table I obtained from the analysis of backward π^-p scattering. The results of such a calculation are compared with the available data in Fig. 3a. Since only a meager amount of data exists for π^+p elastic scattering it is not possible to compare the model and experimental data at $\cos \theta = -1$ but instead the comparison must be made in the angular region of $\cos \theta = -0.94$ to $\cos \theta = -1.0$. The Δ_δ resonance contribution is in reasonable accord with the experimental differential cross section up to 4 BeV/c. Additional Δ_δ recurrences at high energy may well account for the discrepancy above 4 BeV/c. Alternatively, an exchange amplitude may play a minor role in

the description of π^+p backward elastic scattering. Reversal of the parity of either the $\Delta(2450)$ or $\Delta(2840)$ from its recurrence assignment produces destructive interference with the other Δ resonances and destroys the agreement with the experimental data below 4 BeV/c.

In Fig. 3b the available data is compared with the model in the momentum region of 400 to 1800 MeV/c. The agreement between theory and experiment is qualitatively poor unless the $\Delta_\beta(1690)$ is included. Thus the model is in agreement with the existence of the $\Delta_\beta(1690)$.

Fig. 4 shows the elastic widths of the fermion resonances that have been classified in the Regge recurrence model as a function of the orbital angular momentum of the elastic channel. The uncertainty in the elastic widths is due to the uncertainty in total width and in the elasticity as determined by different methods. It is clear that the elastic width falls rapidly with fermion mass. ~~angular momentum~~. This is to be expected with Regge recurrence model because of the increasing orbital angular momentum with increasing fermion mass. The fall of the elastic width is thus in qualitative agreement with the Regge recurrence spin assignments.

We conclude that the dominant characteristics of the backward pion-nucleon scattering data are accommodated by the $(\Delta_\delta, N_\alpha, N_\gamma)$ fermion resonance model. This does not at all exclude the possible existence of numerous inelastic resonances in lower partial-waves whose presence might have been effectively masked by the contributions of the resonances discussed here.⁵ The analysis of the pion-nucleon scattering data lends strong support to the Regge recurrence parity

assignments for the Δ_δ , N_α and N_γ resonances based on the Chew-Frautschi plot of Fig. 1.

One of us (V. B.) would like to thank Professor Geoffrey Chew for the hospitality of the Lawrence Radiation Laboratory.

REFERENCES

*Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Foundation and in part by the U. S. Atomic Energy Commission under contract No. AT(11-1)-881. COO-881- 77

1. V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).
2. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).
3. A. Yokosawa, et al., Phys. Rev. Letters 16, 714 (1966) suggest that another partial wave resonates in the vicinity of 2200 MeV in addition to the $7/2^-$ state.
4. V. Barger and M. Olsson, "Interference of Regge Rho Exchange with Direct Channel Fermion Resonances (to be published).
5. Near the backward or forward direction the resonances with the highest elasticities dominate the amplitude independent of their orbital angular momentum; away from these directions because of the oscillations of $P_\ell(X)$ and $\frac{d}{dx}P_\ell(X)$, lower elasticity resonances can make significant contributions.

(a) References for π^+p Elastic Data

- a1. A. I. Alikhanov et al., Physics Letters 19, 345 (1965).
- a2. A. S. Vovenkov et al., JETP Letters 2, 255 (1965).
- a3. P. M. Ogden et al., Phys. Rev. 137, B1115 (1965).

- a4. H. H. Atkinson et al., Proc. Roy. Soc. A, 289, 449 (1966).
- a5. J. A. Helland et al., Phys. Rev. 134, B1062 (1964).
- a6. F. E. James et al., Physics Letters 19, 72 (1965).
- a7. Frisken et al., Phys. Rev. Letters 15, 313 (1965).
- a8. C. T. Coffin et al., to be published.
- a9. Baker et al., abstracts of International Conference on High Energy Physics, Oxford (1965), reported in paper of Coffin, et al.

(b) References for π^-p Elastic Data

- b1. H. H. Atkinson et al. Proc. Roy. Soc. A, 289, 449 (1966).
- b2. J. A. Helland et al., Phys. Rev. 134, B1079 (1964).
- b3. S. W. Kormanyos et al., Phys. Rev. Letters 16, 709 (1966).

(c) References for Elasticity Determination from
Total Cross-Section and Charge Exchange Reactions

- c1. A. Citron, et al., Phys. Rev. 144, 1101 (1966).
- c2. A. Carroll et al., Proc. Roy. Soc. A, 289, 513 (1966).
- c3. W. Galbraith, Proc. Roy. Soc. A, 289, 521 (1966).
- c4. A. H. Rosenfeld, et al., UCRL-8030--Part I, August, 1965,
corrected April, 1966.
- c5. V. Barger and M. Olsson, to be published.
- c6. For the results of phase shift calculations see reference c4.

TABLE CAPTION

Table I. Experimental parameters for the $S = 0$ fermion resonance states. The predicted values of spin-parity (J^P) are based on the Regge recurrence model discussed in the text. The quoted elasticity parameters X_I are based on the proposed J assignments. A comparison of elasticity determinations from four different methods is presented in the last four columns.

TABLE I

S = 0 Fermion Resonances

	Resonance (Mass in MeV)	Spin-Parity (J^P) - = established	Width (BeV)	Elasticity (x_I) Determinations			
				$\pi^- p \rightarrow p \pi^-$ at 180°	$\pi^- p \rightarrow p \pi^0$ at 0°	Total cross sections	Phase shifts
REGGE RECURRENCES	$\Delta_\delta(1236)$	$3/2^+$	0.12	1.0	1.0	1.0	1.0
	$\Delta_\delta(1929)$	$7/2^+$	0.17	0.35-0.5*	0.49	0.33-0.41	...
	$\Delta_\delta(2452)$	$11/2^+$	0.28	0.12	0.12	0.11	...
	$\Delta_\delta(2840)$	$15/2^+$	0.40	0.05	0.03	0.03	...
	$\Delta_\delta(3220)$	$19/2^+$	0.44	0.01-0.02	0.003	0.006	...
	$N_Y(1512)$	$3/2^-$	0.12	0.60	0.77	0.76	0.50
	$N_Y(2190)$	$7/2^-$	0.24	0.20	0.25	0.15-0.25	...
	$N_Y(2640)$	$11/2^-$	0.42	0.05	0.08	0.07	...
	$N_Y(3020)$	$15/2^-$	0.40	0.015	0.011	0.007	...
	$N_Y(3350)$	$19/2^-$	0.10	0.003-0.01	0.003	...	...
	$N_\alpha(938)$	$1/2^+$	...	...	...	...	...
	$N_\alpha(1688)$	$5/2^+$	0.10	0.60	1.04	0.80	0.60
	$N_\alpha(2200)$	$9/2^+$	~0.24	0.07	...	...	...
	$N_\alpha(2610)$	$13/2^+$	~0.42	0.02	...	...	...
	$N_\alpha(2970)$	$17/2^+$	...	...	...	...	...
UNCLASSIFIED STATES	$N_\alpha(\sim 1400)$	$1/2^+(P_{11})$	...	...	...	...	0.70
	$N_\beta(\sim 1550)$	$1/2^-(S_{11})$	...	...	...	...	small
	$N_\beta(\sim 1650)$	$5/2^-(D_{15})$	0.10	0.33	...	...	0.33
	$\Delta_\beta(\sim 1688)$	$1/2^-(S_{31})$	...	0.3*	...	...	0.25
	$N_\beta(\sim 1715)$	$1/2^-(S_{11})$	...	...	...	...	0.90

*Elasticity used in $\pi^+ p \rightarrow \pi^+ p$ analysis

FIGURE CAPTIONS

Fig. 1. Chew-Frautschi plot of the $S = 0$ fermion resonances. The open squares denote recurrences predicted from straight line extrapolations of the trajectories.

Fig. 2(a,b) Theoretical curves for the $180^\circ \pi^- p$ elastic differential cross section as a function of laboratory momentum. The theoretical model is based on interference of the direct channel resonance amplitude ($\Delta_\delta, N_\gamma, N_\alpha$) with the amplitude due to fermion Regge exchange (Δ_δ) in the crossed channel. The arrows indicate the positions of the Δ_δ, N_α and N_γ Regge recurrences. See the references for the origin of the experimental data. The dotted line shows the contribution to $\frac{d\sigma}{d\Omega}$ from the resonances alone whereas the solid line and dashed line indicate different elasticity assignments for the conjectured $N_\gamma(3350)$. For additional discussion of the $N(3350)$ see reference 4.

Fig. 3(a,b) Theoretical curves for the backward $\pi^+ p$ elastic differential cross section as a function of laboratory momentum. The theoretical model is based on the direct channel (Δ_δ) resonance amplitude. The arrows indicate the positions of the Δ_δ recurrences. See the references for the origin of the experimental data. The data points have a center of mass scattering angle between $\cos \theta = -0.94$ and $\cos \theta = -1.0$.

$I = 3/2, Y = +1$ Regge Recurrences

■ Known Resonance

□ Predicted Resonance

$\delta(P = +, \tau = -)$

$I = 1/2, Y = 1$ Regge Recurrences

- Known Resonance
- Predicted Resonance
- $\alpha(P = +, \tau = +)$
- $\gamma(P = -, \tau = -)$

$$\pi^- p \rightarrow p \pi^-$$

$$\cos \theta = -1.0$$

$$\pi^- p \rightarrow p \pi^-$$

$$-1.0 \leq \cos \theta \leq -0.94$$

Fig. 2b

$\pi^+ p \rightarrow \pi^+ p$

fig. 3a

$-1.0 < \cos \theta < -0.94$

Fig. 3b

$$\pi^+ p \rightarrow \pi^+ p$$
$$-1.0 < \cos \theta < -0.94$$

Elastic Widths of
Regge Recurrences