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July, August, September, 1965

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ATOMIC ENERGY DIVISION

NATIONAL REACTOR TESTING STATION
US ATOMIC ENERGY COMMISSION

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SPERT PROJECT
JULY, AUGUST, SEPTEMBER, 1965

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SUMMARY

During this quarter, nuclear operation of the Capsule Driver Core (CDC) was initiated in the Spert IV facility with the performance of several low-power experiments. The experiments included: critical and operational core loadings, control-rod and transient-rod worth measurements, fuel-rod worth measurements, and flux distribution measurements. The purpose of these experiments was to provide data for comparison with those obtained for the previous CDC loading in Spert I and to determine the effect of a change in the experiment tube size. The experiment tube is a full-core-length, stainless steel cylinder which separates the fuel region of the core from the test space (centrally located flux trap). The experiment tube used in the Spert I loading had a 4.75-inch OD with a 0.500-inch wall thickness, and the one used in Spert IV has a 4.5-inch OD with a 0.531-inch wall thickness.

The CDC is composed of 0.5-inch-OD, 3-percent-enriched fuel rods loaded on a square pitch of 0.663 inch. Initial criticality was attained in Spert IV with 1279 fuel rods (2046 kg UO_2) loaded in an approximately cylindrical array with no experiment tube in place. After the initial critical experiment, the experiment tube was loaded in the core and fuel rod loading continued until a second critical core was attained with 1423 fuel rods (2277 kg UO_2). The experiment tube was determined to have a reactivity worth of -1.9% from data taken in these critical experiments. Fuel loading was continued until a core with an available excess reactivity of 3.0% and a shutdown margin of approximately 9% was attained. This core is termed the operational core and is the one with which the subassembly testing program will be performed. The operational core contains 1652 fuel rods or a total of 2643 kg of UO_2 .

The control-rod worth, fuel-rod worth, and neutron flux distribution measurements were performed primarily to determine the effects of the change in size of the experiment tube. Within the accuracy of the measurements, the use of the 4.5-inch-OD tube caused no change in the control-rod worth from that measured with the 4.75-inch-OD tube. The total reactivity worth of the transient rod is greater than the 3% excess reactivity of the core. The exact value was not determined since the calibration was performed by the positive-period method using the control rods as a reactivity shim. The fact that the transient rod worth is greater than the excess reactivity of the core is beneficial in that during the initiation of a short-period transient the transient rod can be accelerated for several inches before the reactor becomes critical. This pre-acceleration of the transient rod will permit achievement of the high reactivity insertion rates necessary to the initiation of short-period transients.

Use of the smaller experiment tube resulted in slight changes in fuel-rod worth at core locations near the experiment tube, but for fuel positions greater than four inches from the core center no differences were observed. The reactivity worths per fuel rod removed from the core with the larger and smaller experiment tubes in the test space were, respectively: -4.9 and -4.8% per fuel rod at 2.7 inches from the core center; -1.2 and -0.9% per fuel rod at 3.3 inches from the core center; and $+0.3$ and $+0.45\%$ per fuel rod at 3.8 inches from the core center.

Neutron flux distribution measurements were performed by the irradiation of 0.156-inch-diameter by 0.005-inch-thick gold foils in both the core and the test space. The fluxes measured in the core were almost identical to those

measured with the larger experiment tube in Spert I. The fluxes in the test space, however, were slightly lower than previously measured; they were approximately eight percent lower with only water in the test space and approximately four percent lower with a CDC lead-rod capsule containing either a three-percent or a four-percent-enriched UO_2 fuel rod in the test space. These small changes in the test-space fluxes do not compromise the capabilities of the CDC in its intended use, ie, for the irradiation of fuel samples during short-period transient tests.

The fuel testing program which will be performed in the CDC includes tests in which various fuel types will be subjected to destructive transient energy releases which can result in the generation of explosive pressures. It is, therefore, essential that the CDC test capsules be as strong as possible, consistent with neutron economy in the test space. Recent results from the development of high-strength steels have indicated that a decrease in the carbon content can increase the dynamic strength of the steel. In order to investigate the potential use of low-carbon steels for CDC capsules, a series of tests was performed in which chemical explosives were detonated inside lengths of 2-inch Schedule 80 cylinders made of 304 and 304L (low-carbon) stainless steel. Results from these tests showed that the low-carbon-steel cylinders could contain about 78 percent more explosive energy prior to failure than the higher-carbon-steel cylinders. This result would not have been expected on the basis of static strength properties since both the static yield point and ultimate strength are higher for 304 than for 304L stainless steel. Further tests are planned in order to determine if this result is generally applicable for cylinders of various sizes.

The determination of the amount of mechanical energy which is generated as a consequence of fuel rod failure is one of the objectives of the capsule testing program with the CDC. Part of this mechanical energy may be manifested in the acceleration of a column of water (water hammer) inside capsules which are not completely water-filled. In order to determine the amount of mechanical energy contained in such water hammers, a water-velocity transducer has been developed for use inside CDC capsules. The transducer has a 1/2-inch-OD aluminum tube around which an electrical coil has been wound with the direction of the windings reversed every 1/2 inch of tube length. Mounted around this tube is a buoyant, polyethylene cylinder into which three permanent magnets have been embedded. The polyethylene cylinder is free to move up and down the tube with the surface of the water in the capsule. Therefore, the motion of the magnets as the cylinder moves upward with an accelerated column of water causes a sine wave to be generated in the electrical output of the coil for every inch of vertical travel of the cylinder. Preliminary tests using chemical explosives to generate the water hammers indicate that for pressure pulses with rise times of 30 to 100 μsec and magnitudes of up to 10,000 psi the transducer will provide an accurate measure of the water slug velocity. For pressure pulses with shorter rise times, however, separation of the polyethylene cylinder from the water slug results in significant errors in the measurements. Design changes are being made which are expected to correct this problem.

An analytical study of fuel rod failure mechanisms, presented in the last Spert quarterly report, indicated that the destructive testing of fuel rods of conventional size ($\approx$ 0.5-inch diameter) in the CDC could result in atypical fuel rod failure modes. The UO_2 enrichment in rods of this size must be abnormally high in order that destructive energies be generated in test fuel samples during transients which are nondamaging to the CDC fuel. This high enrichment

can cause severe temperature peaks to occur near the cladding of test fuel samples during short-period transients. These temperature peaks could cause preferential melting and vaporization of fuel near the cladding, thereby resulting in failure which would not be typical of lower enriched fuel. The analytical study also indicated that the use of smaller diameter test fuel rods could eliminate this problem without compromising the validity of the test results. Therefore, tests with scaled-down fuel rods have been included in the CDC experimental program.

The preliminary design and procurement specifications for the scaled-down rods have been completed. The fuel rods have been designed to be as representative as possible of fuel rods currently in use in commercial power plants. The rods will contain UO_2 fuel pellets, and will have a 0.25-inch diameter and an 18-inch active fuel length. Fuel rods clad with both stainless steel and Zircaloy will be procured in order to determine if any significant differences in failure mode or consequences of fuel rod failure can result from the difference in clad material. Also, the UO_2 enrichment will be varied from 3 to 10 percent in order to determine the effects of variation in that parameter on fuel rod failure. Discussions of the design criteria and the procurement specifications for the scaled-down test fuel rods are presented in this report.

One of the important areas of endeavor in the pre-operational planning for the Power Burst Facility (PBF) is the development of an accurate technique for calculating the magnitude of the negative reactivity owing to the increase in the fuel temperature (Doppler reactivity feedback) during short-period transient tests. The use of such a technique is necessary to the specification of the amount of U-238 to be contained in the PBF fuel and to the calculation of the dynamic behavior of the core. Such a technique has been developed and verified for use in analyses of the PBF core. The technique entails the determination of the transient spatial temperature distribution of the core and the calculation of the system reactivity as a function of this spatial temperature distribution. Primary reliance for these calculations is placed in the use of the two-dimensional neutron diffusion code, PDQ. By the use of this code the Doppler reactivity feedback is determined from the reactivity difference between stationary diffusion solutions in which the flux solutions are assumed to be time-independent.

The technique was verified by the comparison of calculated and experimentally determined Doppler feedback for three short-period transient tests which were performed with the Spert I low-enrichment UO_2 core. These transients had asymptotic reactor periods of 5.6, 3.2, and 2.2 msec. The experimental and calculated reactivity compensations owing to the Doppler effect agreed to within a few (< 5) percent for all three transients which were investigated. On the basis of these results, therefore, it has been concluded that the technique can be confidently applied to the PBF core.

Two of the most likely mechanisms for fuel rod failure during a rapid power excursion are clad rupture from internal pressure and clad melting. Both modes of failure depend upon the amount and rate of energy generation within the fuel; therefore, the energy density in the core is of importance in predicting fuel failure thresholds. If the excursion is limited by prompt-acting mechanisms, such as the Doppler effect, and the reactor period is short enough so that heat transfer from the fuel to the coolant is negligible, a quantity of interest is the energy density in the fuel at the end of the excursion. A method has been developed whereby this quantity can be obtained directly by the use

of existing neutron diffusion computer codes without having to solve for and integrate the core fluxes. The method involves the development and solution of equations for the energy density which are time-independent and can be applied both to small cores and to large cores where flux tilting may be a problem.

Test calculations were performed in which both the conventional method (involving solution for and integration of the core fluxes) and direct solutions for the energy density were obtained. In the most extreme case, the fission cross sections in an end fourth of a 240-cm slab core were perturbed to produce an excursion with an asymptotic period of 0.18 msec. The WIGLE-40 computer program was used to obtain the solutions for both methods. The results from both the conventional and direct solutions were in excellent agreement with each other for this problem; however, the direct solution was accomplished approximately six times faster than the conventional solution on the IBM-7040 computer. It is concluded that the direct solution method is accurate and represents a considerable savings in time over the more conventional solution. The method can also be extended to the solution of multi-dimensional diffusion equations where the computer time involved in the solution for and integration of the transient fluxes might be prohibitive.

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I. SPERT-IV CAPSULE DRIVER CORE SUBASSEMBLY TEST PROGRAM

1. INTRODUCTION

Nuclear operation of the Capsule Driver Core (CDC) was begun in the Spert IV facility this quarter in preparation for the forthcoming experimental studies of UO_2 fuel rod failure mechanisms. Following achievement of the critical and operational core loadings, a program of static nuclear measurements was initiated to provide data for comparison with data previously obtained for the CDC in Spert I^[1] and to determine the effects of minor changes in core hardware made in the Spert IV installation. The measurements completed include control and transient rod worths, fuel rod worths, and neutron flux distributions in the test space. Next quarter, further static measurements (figure-of-merit measurements, detailed flux measurements, and a power calibration) will be completed, and a series of fiducial transient experiments will be initiated to verify the expected transient behavior of the core and to establish the operating range of energy densities and reactor periods within which the CDC fuel can be safely used for repeated transient operation.

A program of out-of-pile explosive tests to obtain data on the strength properties of potential capsule materials has been continued. In particular, an investigation was initiated to determine the relative performances of 304 and 304L (low carbon content) stainless steels under dynamic loads. Also as part of the explosive testing program, a water velocity transducer has been designed and tested. The transducer will be used to aid in determining the mechanical energy of water slugs which may result from fuel failure in sub-assembly capsules.

Specifications have been prepared for the procurement of quarter-inch-diameter UO_2 fuel rods to be used as test samples in the CDC fuels testing program. Tests with these fuel rods will determine the effects of UO_2 enrichment, clad material, and geometrical scaling on fuel rod failure.

The following sections contain the experimental results from the static tests completed in Spert IV, results of out-of-pile explosive testing with potential capsule materials and the velocity transducer, and a description of the criteria used in determining the specifications for the quarter-inch-OD fuel rods.

2. RESULTS FROM CRITICAL AND OPERATIONAL CORE LOADINGS

CDC fuel loading operations were begun with a determination of the critical loading for the three-percent-enriched CDC fuel rods with no experiment tube located in the test space. The fuel rod loading proceeded from the test space outward with an approximately cylindrical array being maintained for each loading step. Criticality was obtained with 1279 fuel rods (2046 kg UO_2) loaded in the geometrical arrangement shown in Figure 1 and with the control rods

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Fig. 1 Plan view of the Capsule Driver Core (CDC).

withdrawn 35.5 inches (upper limit of the control rods is 36.5 inches above the bottom of the core).

After the initial critical experiment, the experiment tube was positioned in the test space and fuel rod loading was continued. A second critical core was attained with 1423 fuel rods (2277 kg UO_2) loaded in the geometrical arrangement shown in Figure 1 and with the control rods withdrawn 36 inches above the bottom of the core. From control-rod worth measurements made with these two critical loadings and fuel-rod worth measurements made during the next fuel loading step, the reactivity worth of adding the experiment tube was determined to be -1.9\$.

The experiment tube used in the loading is slightly smaller than the one used during previous static experiments performed with the CDC in Spert I^[1] (4.5-inch OD by 0.531-inch wall thickness compared with 4.75-inch OD by 0.500-inch wall thickness). This smaller experiment tube was used because results from chemical explosive tests reported last quarter^[2] indicated that explosive energies generated within the test space could possibly cause the larger experiment tube to expand sufficiently to become bound by the core grids, and that the use of the smaller experiment tube would preclude this possibility.

Calculations of the transient behavior of the CDC^[3] had indicated that about three dollars of excess reactivity would be required for the performance of the short-period power excursion tests desired for fuel rod testing. Therefore, following the second critical loading, fuel rod loading was continued until the available excess reactivity in the core, as indicated by control-rod-worth measurements, was 3.0\$. Loading operations were terminated with 1652 fuel rods (2643 kg UO_2) loaded in the core arrangement indicated in Figure 1. The control-rod-bank critical position for this loading, defined as the operational core, was 27.4 inches above the bottom of the fuel rods. A shutdown reactivity margin of about nine dollars was determined by the integral-count, rod-drop technique^[4] with the start-up source removed from the core.

A summary of the design characteristics of the CDC as installed in Spert IV is given in Table I.

TABLE I
DESIGN CHARACTERISTICS OF SPERT IV CAPSULE DRIVER CORE

<u>Vessel</u>	
Type	Free-standing cylindrical tank
Composition	Welded, rolled, Type-304 stainless steel plate
Size	25 feet high, 20 feet in diameter Top: 23 feet, 5/16 inch thick Bottom: 2 feet, 5/8 inch thick
Bottom thickness	1/2 inch
Design pressure	Below gate: 25-foot hydrostatic load plus 50-psi static surcharge Above gate: 6-foot hydrostatic load
<u>Core</u>	
Moderator reflector	Light water
Pitch	0.663 inch, square pitch
Nonmoderator-to-moderator ratio	0.807
<u>Fuel</u>	
Type	Compressed UO ₂ powder
Average length of fuel rods ^[a]	72.8 inches
Active length	67 ± 1 inch
Fuel rod OD	0.500 inch
Clad thickness	0.028 inch
Enrichment	3 wt% U-235
UO ₂ density	9.28 g/cm ³
Mass of UO ₂ per fuel rod	≈ 1600 grams
Mass of U-238 per fuel rod	≈ 1368 grams
Mass of U-235 per fuel rod	≈ 42.3 grams
Cladding	Cold-worked, Type-304 stainless steel
<u>Control Rods</u>	
Number	4 gang-operated with two cruciform-shaped sections per control rod
Composition	12 wt% boron in a sintered dispersion of B ₄ C in Al
Withdrawal rate	0 to 12 inches/minute

TABLE I (Contd.)

DESIGN CHARACTERISTICS OF SPERT IV CAPSULE DRIVER CORE

<u>Control Rods (Contd.)</u>	
Scram time	Approximately 300 msec from upper limit
Length of poison section	61 inches
Total length	107 inches
Width	4-3/4 inches
Thickness	0.245 inch
<u>Transient Rods</u>	
Number	4 gang-operated, cruciform-shaped blades
Composition	12 wt% boron as B ₄ C in Al
Length of poison section	28 inches
Total length	118-1/2 inches
Cruciform dimensions	2-1/8 inches x 3-3/8 inches, 0.245 inch thick
Ejection time from upper limit	Approximately 80 msec
<u>Experiment Tube</u>	
Tube material	Type-304 stainless steel
Length	6 ft
OD	4.50 inches
ID	3.438 inches
Wall thickness	0.531 inch

[a] Typically, the total fuel rod lengths vary from 72.4 inches to 73.1 inches.

3. EXPERIMENTAL RESULTS FROM STATIC MEASUREMENTS

Since previous CDC static measurements were performed using the larger experiment tube, it was necessary to repeat certain measurements pertaining to characteristics of the core in the immediate vicinity of the smaller experiment tube. Based on the results of one-dimensional diffusion theory calculations, the change in the experiment tube size was expected to result in significant thermal neutron flux changes in the core region surrounding, and in the immediate vicinity of, the experiment tube. The change in the experiment tube size, however, was expected to decrease the figure-of-merit[5] for a CDC lead-rod by only four percent. Following the completion of the critical and operational core

loadings in the Spert IV facility, control and transient rod worth experiments were performed and portions of fuel rod worth and flux distribution measurements were repeated in order to determine the magnitudes of any changes caused by the use of the smaller experiment tube.

3.1 Control and Transient Rod Worths

Differential control-rod-worth measurements were made following each loading step after the initial critical configuration was obtained. Results of these measurements are shown in Figure 2, together with data obtained during the

Fig. 2 CDC differential and integral control rod worth curves.

control rod calibration previously performed at Spert I. As can be observed in the figure, the smaller experiment tube did not, within the accuracy of the measurements, alter the control rod worth.

The transient rod has only two static positions, upper and lower limit, and the incremental worth of the transient rod was not measured at Spert I. Since the incremental transient rod worth must be known in order to determine the reactivity input as a function of time for analytical purposes, these measurements were performed at Spert IV. Vertical positioning of the transient rod was accomplished by the use of a chain-and-collar designed specifically for that purpose. The transient rod worth was determined from the change in the critical position of the control rods for the various transient rod positions using the control rod calibration curve, shown in Figure 2. Results from the

transient rod calibration are shown in Figure 3. With the transient rod poison section raised^[a] 23.6 inches above the bottom of the core, the reactor could not be made critical by pulling the control rods to the upper limit. Since the maximum transient rod poison insertion is 28.3 inches, approximately five inches are available for accelerating the transient rod before the reactor can become critical during the initiation of a transient. This acceleration or "pre-run" distance is important for the performance of short-period tests where high reactivity insertion rates (requiring high transient rod ejection speeds) are necessary so that the reactivity insertion can be completed before the reactor power reaches levels at which inherent reactivity feedback becomes a consideration.

3.2 Fuel Rod Worth

An experiment was performed previously at Spert I to determine the effect of the removal of fuel rods from various positions in the core. Portions of this experiment were repeated at Spert IV to determine if use of the smaller experiment tube caused any changes in the fuel rod worth. Slight changes in fuel rod worth were observed at core locations near the experiment tube, but for fuel rod locations greater than four inches from core center, no differences were observed. The reactivity worth per fuel rod removed from the core with the larger and smaller experiment tube in the test space was, respectively: -4.9 and -4.8¢ per fuel rod at 2.7 inches from the core center; -1.2 and -0.9¢ per fuel rod at 3.3 inches from the core center; and +0.3 and +0.45¢ per fuel rod at 3.8 inches from the core center.

3.3 Neutron Flux Distributions

Axial neutron flux distributions in the core, and axial and radial flux distributions in test space were obtained by the irradiation of 0.156-inch-diameter by 0.005-inch-thick gold foils. The flux distributions in the core were in good agreement with those measured previously in Spert I. The flux distributions in the test space, however, were slightly different from those measured in the Spert I experiments owing to the change in the experiment tube size. The flux distributions in the test space were measured for three different configurations: (a) test space water-filled, (b) test space water-filled and containing a 3-percent-enriched CDC fuel rod inside a water-filled stainless

Fig. 3 CDC transient rod worth curve.

[a] The transient rod poison is inserted from the bottom of the core; therefore, the transient rod is raised to decrease the reactivity of the core, and is rapidly ejected from the core to produce a positive reactivity "step". The transient rod is not used for normal control of the reactor.

steel CDC lead-rod capsule, and (c) test space water-filled and containing a 4-percent-enriched Spert I oxide core fuel rod inside the water-filled CDC lead-rod capsule.

Figures 4 and 5 show the axial and radial neutron flux profiles which were obtained for the three test-space configurations and the axial profile previously

Fig. 4 Axial flux profiles in CDC test space.

Fig. 5 Radial flux profiles in CDC test space.

obtained for configuration (a) using the larger experiment tube. The most significant change resulting from the use of the smaller experiment tube is a decrease of about eight percent in the maximum flux at the test space center for the case in which only water was in the test space. The flux measurements made in Spert I with the capsules and fuel rods in the test space were made using aluminum capsules; consequently, those results cannot be directly compared with the present results. However, calculations have indicated that the smaller experiment tube would cause a four percent decrease in the peak flux relative to the larger experiment tube case for conditions (b) and (c).

3.4 Conclusions

The results of the static experiments performed thus far at Spert IV indicate that use of the smaller experiment tube causes no significant changes in control rod worth and core flux distributions; only slight changes of fuel rod worth in the immediate vicinity of the experiment tube; and slight changes in the neutron flux distributions within the test space. These small changes in the static nuclear characteristics of the core do not compromise the CDC capabilities, and upon completion of the static tests the transient experiments with the CDC will be initiated.

4. OUT-OF-PILE EXPLOSIVE TESTING PROGRAM

The fuel testing program which will be performed in the CDC includes tests in which various fuel types will be subjected to destructive, transient energy releases. Because the violent dispersal of very hot fuel into surrounding water can result in the generation of explosive pressures, it is imperative that the understanding of the behavior of potential capsule materials, core components, and core instrumentation under rapidly applied loads be improved. Therefore, for the past year, an explosive testing program, using chemical explosives, has been conducted (and is continuing) in order to improve this understanding. Results from this program have been reported previously [2,5]. During this quarter, explosive tests were performed to investigate the influence of carbon content on the response of stainless steel cylinders to explosive loads and to develop a transducer for the determination of the mechanical energy of a water slug accelerated by an explosive energy release.

4.1 Explosive Tests with 304 and 304L Stainless Steel Cylinders

Data reported on the development of a new high-strength steel[6] have indicated that a decrease in the carbon content of the steel increases the dynamic strength of the steel. In order to obtain further information on the influence of the carbon content, a series of explosive tests was performed using extra-low-carbon stainless steel (304L) and similar stainless steel with a higher carbon content (304).

The test conditions were similar to the conditions for explosive tests which have been previously reported[5]. In the tests, 3-foot-long cylinders of 2-inch, Schedule 80 pipe contained linear charges of Primacord[a] which extended over the full length of the vertical axes of the cylinders. The charges were held in position with styrofoam plugs which were placed in the ends of the cylinders, and slots were cut in the plugs to allow for water movement. The cylinders were submerged in water and the explosives were detonated by means of electric blasting caps which were placed at one end of the charges. The charge strength of the Primacord was increased in successive tests until failure of the cylinders occurred.

A comparison of the results of these tests is given in Table II. As can be seen from the table, the 304L stainless steel, which failed at a charge loading of 800 gr/ft, apparently possesses considerably greater dynamic strength properties than the 304 stainless steel, which failed at 450 gr/ft. In other words, the 304L cylinders were able to contain about 78 percent more explosive energy than 304 cylinders of the same size. This result would not have been expected from the static strength properties of the materials, also given in Table II, since both the static yield point and ultimate strength are slightly greater for the 304 than for the 304L.

[a] Trade name for detonating cord manufactured by the Ensign Bickford Company, which contains an explosive core of pentaerythritol tetranitrate (PETN). When used in this report, the unit grains per foot (gr/ft) means grains of PETN per foot of Primacord.

TABLE II
RESULTS FROM EXPLOSIVE TESTS WITH
304 AND 304L STAINLESS STEEL CYLINDERS

Material	Cylinder Size	Material Specification	Static Yield Point (psi)	Static Ultimate Strength (psi)	Static Elongation (%)	Carbon Content (%)	Explosive Charge Resulting in Failure (gr/ft)	Elongation as a Result of Explosive Failure (%)
Type 304 SS	2 in. Schedule 80	ASTM-A312	43,850	83,680	67	0.05	450	15
Type 304L SS	2 in. Schedule 80	ASTM-A312	38,200	82,800	53	0.02	800	27

On the basis of these tests, the tentative conclusion is drawn that a reduction in the carbon content of stainless steel can significantly increase the dynamic strength of the material. It should be noted, however, that this conclusion is based upon the results of tests of one cylinder size, using one type of explosive. More experimental data are required before any general conclusions can be justified.

Preparations are being made to conduct a series of explosive tests using 3-inch-OD thin-wall (0.065-inch-thick) tubing fabricated of Type 304 and 304L stainless steel. Results of this series of tests will provide additional insight into the comparative behavior of these materials at the high rates of strain of interest to the subassembly testing program. Since present capsule design objectives do not include investigation of capsule response in the quasi-static region, no tests are planned at low rates of strain.

4.2 Water Velocity Transducer Development

Determination of the amount of mechanical energy generated as a consequence of fuel rod failure is one of the objectives of the CDC capsule test program. Part of this mechanical energy release may cause a column of water (water hammer) to be accelerated in the capsule which contains the test fuel rod. Measurement of the velocity of this water slug in conjunction with measurement of pressure at the bottom of the capsule would permit determination of the magnitude of the mechanical energy contained in the water slug. Therefore, a device has been designed to measure water slug velocity, and preliminary testing of this device has been completed using chemical explosives. The following discussion contains a description of the velocity transducer and the results of the explosive tests.

The velocity transducer consists essentially of a moveable, buoyant, polyethylene cylinder with attached magnets and a stationary coil. The coil is wound around a 1/2-inch-OD tube, and a hole drilled in the center of the polyethylene cylinder allows it to slide freely up and down the tube. The direction of the coil winding is reversed after each 1/2-inch length of the tube so that the output signal from the coil owing to the motion of the magnets is also reversed, approximating a sine wave for each inch of travel of the polyethylene cylinder and attached magnets. Figure 6 is a drawing of the velocity transducer.

Preliminary testing of the velocity transducer was performed using small chemical explosives ("cherry-bombs" which contain approximately one gram of black powder) which were detonated in the lower portion of a prototype CDC lead-rod capsule. The velocity of the polyethylene cylinder was measured

Fig. 6 Drawing of water velocity transducer.

In the calculations which were made to obtain the data in Table III, the water slug was assumed to move with the same velocity as the polyethylene cylinder, an assumption which could be invalid if the cylinder were to become separated (spall off) from the water slug. Therefore, four additional tests were performed with the capsule top open, and high speed motion pictures of the cylinder and water slug were taken. For these tests the velocity transducer was mounted such that the tube and coil extended above the top of the capsule as shown in the photograph in Figure 7. The volume of water ejected from the capsule was

over 16 inches of travel, and the pressure at the bottom of the capsule was measured using a bonded-strain-gauge pressure transducer. Results of these tests are listed in Table III together with a comparison of the momentum of the water slug (the average velocity over 16 inches of travel times the mass of water above the explosive) with the momentum as determined from the pressure transducer at the capsule bottom (the integral of the pressure pulse times the inside cross sectional area of the capsule). These momenta agree within 20 percent for each test. The kinetic energy of the water slug listed in Table III is one-half the average velocity of the water slug times the momentum indicated by the pressure pulse.

TABLE III

RESULTS FROM WATER VELOCITY TRANSDUCER TESTS

Test Number	Maximum Velocity (ft/sec)	Average Velocity (ft/sec)	Momentum from Velocity Transducer (slug ft/sec)	Momentum from Pressure Transducer (slug ft/sec)	Kinetic Energy in Water Slug (ft-lbs)
1	133	128	8.9	10.5	670
2	136	126	10.7	10.6	670
3	104	99	8.4	10.1	500
4	95	89	7.5	8.2	365
5	142	132	11.2	12.8	845
6	144	136	11.5	12.6	855
7	169	113	9.6	8.9	505
8	156	138	11.7	12.3	850
9	157	108	10.8	11.5	620
10	125	106	10.5	11.4	610

Fig. 7 Photograph of water velocity transducer mounted on simulated CDC lead-rod capsule.

measured for each test. For the first test, the same type of small chemical explosive detonated in previous tests was used. One-foot lengths of 4-gr/ft and 20-gr/ft Pyrocore^[a] were then used in tests two and three, respectively; and finally, a one-foot length of 25-gr/ft Primacord was used in the fourth test.

Motion picture photographs showed that spalling water emerged from the capsule first and was followed closely by the polyethylene cylinder and finally the water slug. Results of the first test indicated that the water slug and polyethylene cylinder moved with about the same velocity, and the momenta indicated by the velocity and pressure transducers agreed to within 25 percent. However, in the three following tests, the polyethylene cylinder moved faster than the water slug; consequently, the momenta indicated by the velocity and pressure transducers differed by almost 50 percent in each case. Results of the tests are listed in Table IV.

TABLE IV
RESULTS FROM MOTION PICTURE
TESTS OF WATER VELOCITY TRANSDUCER

Test Number	Velocity of Spalling Water as it Emerged from Capsule (ft/sec)	Velocity of Polyethylene Cylinder		Velocity of Water Slug		Maximum Pressure (psi)
		As Indicated by Velocity Transducer (ft/sec)	As Indicated by Motion Photography (ft/sec)	Leading Edge (ft/sec)	Tailing Edge (ft/sec)	
1	136	82	87	87	50	11,500
2	135	120	113	76	56	8,300
3	283	157	173	133	80	11,200
4	243	157	165	92	55	6,900

The velocity of the polyethylene cylinder as indicated by the velocity transducer output was in good agreement with its velocity as determined from the motion pictures. However, for the last three tests the water slug velocity was considerably slower than that of the cylinder. This spalling of the polyethylene cylinder could be due to the faster rise time of the pressure pulses produced by Primacord and Pyrocore (about 10 μ sec as compared to about 30 to 100 μ sec for the small chemical explosive).

It can be concluded from the explosive tests performed thus far that for pressure pulses having rise times of 30 to 100 μ sec and magnitudes up to 10,000 psi, the velocity transducer as presently designed will provide a good measure of water slug velocity. However, for pressure pulses having shorter rise times, spalling of the polyethylene slug results in sizable errors in the measurement. Design changes in the velocity transducer will be made to correct this problem, and the explosive tests will be continued.

[a] Du Pont trademark for detonating cord which contains a mixture of pentaerythritol tetranitrate (PETN), aluminum, and beryllium oxide as an explosive core.

5. DESIGN OF 1/4-inch-OD TEST FUEL RODS

One of the conclusions from the analytical studies of UO_2 fuel rod failure mechanisms reported in the last Spert quarterly report[2] was that a considerable advantage could be obtained by testing scaled-down fuel rods of 1/4-inch diameter instead of rods of more conventional size. This conclusion was based on the following inter-related factors:

- (1) The test capsule need not contain as much energy, thereby permitting use of thinner capsule walls with a resultant higher fission density in the fuel
- (2) The enrichment required to obtain the desired fuel fission densities is lower, and thus the flux depression effects are more typical of power reactors
- (3) Failure due to internal pressure or cladding stress considerations is relatively independent of rod size for large, fast rates of energy insertion.

Therefore, a portion of the UO_2 fuel failure studies in the CDC subassembly test program will be performed using 1/4-inch-OD fuel rods. Preliminary specifications have been developed in preparation for procurement of the test fuel rods. In the following sections, the design criteria are discussed and the preliminary design is described.

5.1 Design Criteria

The specifications for the 1/4-inch-OD fuel rods were chosen to obtain a rod design typical of fuel currently used in water-moderated and -cooled commercial power reactors (UO_2 pellets encased in a zirconium alloy or stainless steel). Typical pellets have a density near the theoretical value and a stoichiometric ratio near 2.0. High pellet density and a near stoichiometric ratio are both desirable as these parameters have considerable effect on the thermal conductivity of the pellet.

Commercial fuel rods normally contain a plenum volume to contain fission product gases, and the size varies widely between different reactors. Since it is not likely that the test fuel will be exposed to irradiation prior to testing, the exact size of the plenum in this case is not important. However, a plenum volume was provided to allow for axial fuel expansion during the transient tests. To keep the pellets from moving into the plenum volume prior to loading in a reactor core, a spring is placed at one end of the rod. An insulator pad is also frequently placed between the fuel and the clad end caps in commercial fuel to protect the end cap from excessive temperature and to reduce thermal strain. These devices have been incorporated in the design of the 1/4-inch rods which will be tested.

Zirconium alloys used commercially for fuel rod cladding are Zircaloy-2 or Zircaloy-4. These differ only slightly with Zircaloy-4 exhibiting better corrosion resistance. Zircaloy-2 was specified for the test rods, however, because it is the more common alloy of the two currently in use. The stainless steel most generally used, and specified for the 1/4-inch rods, is Type-304. The type of clad heat treatment or amount of cold working varies for individual reactor

fuel designs, and typical values of these parameters do not exist; therefore, the annealed condition was selected for all cladding of the 1/4-inch-OD test fuel rods.

The length of the fueled section of the rods was selected to be 18 inches in order to eliminate end effects that could possibly result from the use of shorter fuel samples. Since the vertical location of the flux peak in the core has been calculated to be dependent on the initial CDC reactor period, a relatively long rod need not be repositioned for tests with various periods. The maximum-to-minimum thermal neutron flux over the 18-inch length of fuel is expected to be about 1.4.

Nominal radial dimensions were selected by applying a scaling factor of one-half to fuel rods of typical size. The cladding thickness used varies among the various reactor plants, depending on the particular operating conditions and design assumptions. Since high temperature strengths of Zircaloy and stainless steel differ significantly, Zircaloy clads are normally the thicker of the two. The CDC, however, will operate only at ambient conditions, and since the room temperature strengths of the two materials are similar, identical nominal clad thicknesses (0.014 inch) were specified for each material for the scaled-down rods.

The diametral clearance or gap between the pellets and clad was also scaled down for the specifications. It was shown in the analysis presented in the last Spert quarterly report that fuel rod failure caused by fuel expansion would occur at the same point in the energy and time domain if the diametral gap were scaled in the same ratio as the rod size. The diametral gap for commercial rods does vary but is normally sized to minimize the gap at operating conditions and to provide for easy loading of the pellets during fabrication. Therefore, a typical diametral gap for a 1/2-inch-OD rod (5 mils) was chosen and scaled by a factor of one-half.

The tolerances on commercial fuel rods are specified no closer than necessary to achieve satisfactory performance under normal steady-state conditions. This reduces the cost of the fuel. For the purposes of the CDC fuel failure studies, however, it is important that the fuel rod dimensions be accurately known and more consistent than those of fuel rods normally used in commercial power plants. Uniformity between rods will simplify post-test analysis, the identification of specific failure mechanisms, and the determination of the failure point in the time and energy domains. Once the behavior of the rod is understood, the effects of parameter variations can be predicted and verified by further experiment. Close tolerances will also ensure that the test rods are of high quality and should aid in reducing defects that could cause atypical failure and cast doubt on the interpretation of experimental results.

Most of the test rods will have an enrichment of three percent. A few of the test rods, however, will have enrichments of five or ten percent. These rods, in conjunction with the lower enriched rods, will be used to study the effect of flux depression and initial period on the mechanisms and consequences of fuel rod failure. These higher enriched rods will also be used in tests with rod clusters to determine the effect of fuel rod failure on adjacent rods of lower enrichment (and presumably not at the failure point) in the cluster.

As indicated in a previous section, testing of Type-304L stainless steel in the capsule development program has shown that a reduced carbon content

can improve the dynamic response of the stainless steel. Since in the CDC tests the fuel rod cladding will also be subjected to high strain rates, testing of low-carbon cladding appears desirable. Thus a few rods clad with this material will be ordered and tested in the CDC.

In order to ensure a high quality product and provide information that may be of value in post-test analysis, the procurement specifications for the test rods will require the performance of an extensive testing program to determine the properties and characteristics of the rods. Tests specified include the following:

- (1) Nondestructive tests to ensure clad integrity
- (2) Clad tensile property tests at room and elevated temperatures
- (3) Clad hydrostatic burst tests
- (4) Chemical analysis of all rod components
- (5) Measurement of pellet and inside clad surface roughness.

The information pertaining to surface roughness will aid in prediction of the heat transfer coefficient between the pellet and the clad. This coefficient will vary widely in magnitude during a transient and is of considerable importance in determining fuel rod thermal behavior.

5.2 Description of Fuel Rods

The design criteria outlined in the previous section have led to the selection of the following basic design for the test fuel rods. The fuel rods will consist of uranium dioxide pellets encased in zirconium alloy or stainless steel seamless tubing. The tubing will be closed at each end with caps of similar material to the tube. The upper end of each rod will contain a compression spring. An alumina insulating pad will be placed at each end of the pellet stack to provide thermal shock protection for the lower end cap and the compression spring. A washer will be placed between the top insulating pad and the spring. Figure 8 is a schematic drawing of the 1/4-inch-OD fuel rod.

The pertinent dimensions of the fuel rods are as follows:

Fig. 8 Drawing of typical CDC 1/4-inch test fuel rod.

Clad OD	0.25 inch nominal
Clad wall	0.014 $\pm$ 0.0007 inch for stainless steel 0.014 $\pm$ 0.001 inch for Zircaloy
Clad ID	0.222 $\pm$ 0.0005 inch for stainless steel 0.222 $\pm$ 0.001 inch for Zircaloy
Tube ovality	within 0.001 inch
Tube straightness	0.010 inch per 2-foot length
Pellet OD	0.2195 inch nominal
UO ₂ stoichiometric ratio	2.00 to 2.02
UO ₂ enrichment	3.0 $\pm$ 0.05, 5.0 $\pm$ 0.05, 10.0 $\pm$ 0.05
Pellet density	93 to 96 percent of theoretical
Pellet stack length	18 $\pm$ 0.030 inch
Overall length	21-1/16 inch nominal

The tolerances of the stainless steel and Zircaloy rods reflect the best manufacturing techniques for commercial tubing of this size.

The information which will be obtained from the transient tests in the CDC with the 1/4-inch-OD fuel rods will be used (a) to compare with information from similar tests using larger, full-size fuel rods, (b) to determine the effects of variations in fuel rod design parameters on fuel rod failure, (c) to check the validity of the analysis of UO₂ fuel rod failure mechanisms which was presented in the last Spert quarterly report, and (d) to provide direction for future investigations of fuel rod failure. It is estimated that testing of the 1/4-inch fuel rods in the CDC will begin about the middle of calendar year 1966.

II. POWER BURST FACILITY DOPPLER REACTIVITY FEEDBACK CALCULATIONS

1. INTRODUCTION

An important consideration in establishing the design specifications of the Power Burst Facility (PBF) core is the development of an accurate technique for calculating Doppler reactivity feedback during short-period transient tests. The use of such a technique is necessary to the specification of the amount of U-238 to be contained in the fuel and to the calculation of the dynamic behavior of the core. During this quarter, a generalized two-dimensional diffusion technique has been developed and verified for use in calculating Doppler reactivity feedback in low-enrichment cores similar to the PBF core. The technique entails the determination of the transient spatial temperature distribution in the core and the calculation of the system reactivity as a function of this temperature distribution. Experimental verification of the technique has been provided by the comparison of calculated and experimentally determined reactivity feedback for short-period transient tests performed with the Spert I low-enrichment UO_2 core [7,8].

An analysis of the development and application of the calculational procedure together with comparisons of calculated and experimental results for the Spert I oxide core for a number of short-period power excursion tests is presented below.

2. GENERAL DESCRIPTION OF THE CRITERIA AND APPROACH FOR THE DOPPLER CALCULATIONS

The complete analysis of Doppler reactivity feedback during short-period transients for a system containing a significant amount of U-238 is a complex problem involving several independent calculations. Among the most significant of these calculations are (a) the determination of the transient spatial temperature distribution within the core and (b) the calculation of system reactivity as a function of this space-dependent temperature distribution. Although the specific criteria for the model developed to perform the calculations were several, they will be discussed in three rather broad categories of (a) practicality, (b) general applicability, and (c) accuracy.

2.1 Practicality Criterion

The practicality criterion pertains to the actual mechanics of setting up the model, obtaining the necessary nuclear parameters, and performing the reactivity calculations. The general guide lines applied were that the necessary calculations and manpower requirements should not be excessive or prohibitively expensive. Also, it was decided that the calculational approach should not exceed the capability of the computer facilities presently located at the National Reactor Testing Station (NRTS). Primary reliance for the reactivity calculations was placed on the use of the two-dimensional neutron diffusion computer code, PDQ[9], which has been programmed for use on the IBM-7040 at the NRTS. This particular code is straightforward in its use, well understood, and is consistent with all the requirements of the practicality criterion.

2.2 General Applicability Criterion

The criterion of general applicability was intended to permit a confident extrapolation of the model from Spert I to PBF and, hence, allow a verification of the calculational technique. This criterion established the general philosophy of the model development which was:

- (1) The calculational technique was to be formulated by the use of equations and relationships consistent with the basic definitions of neutron balance as provided in transport theory.
- (2) The parameters and constants necessary to solve these equations were to be evaluated in a manner which is theoretically consistent and well verified. Here it was considered important that the energy (temperature) dependence of all the parameters in the reactivity equations be investigated. A priori assumptions which relate the total dynamic effect of a system reactivity perturbation to one or two of these parameters were to be avoided.
- (3) Only assumptions and techniques which are consistent, justifiable, and applicable to both the Spert I and PBF cores were to be utilized in representing the geometrical space dependence of the physical system.

It was recognized that although the Spert I and PBF cores are similar in geometry, the techniques involved in representing various components of the Spert I core may not be exactly applicable to the PBF core. It was required, however, that the general assumptions upon which the techniques were devised be applicable to both cores.

The use of the PDQ code is consistent with the requirements of the general applicability criterion. PDQ provides for direct flux and eigenvalue solutions of the two-dimensional diffusion equation for up to four neutron energy groups. Thus, from a theoretical point of view, this approach is entirely adequate in that the calculation of core reactivity from the eigenvalue solutions is consistent with and directly derived from basic transport definitions of neutron balance. Also, the geometrical representation and setup of a particular core in PDQ is entirely dependent upon the physical characteristics of the system being described.

2.3 Accuracy Criterion

The limits of acceptable accuracy were based upon the assumption that a calculational model could be developed which would rate as high a degree of confidence when applied to PBF as when applied to the Spert I data. The model when applied to the Spert I data for three individual short-period transients would satisfy the accuracy criterion if the calculated Doppler reactivity feedback for each test were within ± 20 percent of the experimentally determined value. The experimental data with which the calculations were to be compared would be obtained from 5.6-, 3.2-, and 2.2-msec-period tests performed with the Spert I oxide core.

For any particular reactor system, the calculational accuracy is not only a function of several independent parameters and variables particular to the system, but also a function of the ability of the calculational technique to provide

accurate solutions. The following comments regarding the accuracy criterion of PDQ apply only to the latter, ie, the ability of PDQ to provide accurate solutions to the problem it computes. The degree of confidence placed on the reactivity solution of a PDQ problem is determined by the convergence limits imposed on the eigenvalue solutions. Theoretically, the convergence limits can be specified so as to ensure any degree of accuracy desired. Practically, however, the convergence is set to allow solution confidence consistent with the error limitations of the problem at hand. For the present work, the convergence criteria were specified such that the calculational error limits were within ± 5 percent for all cases which were considered.

Intrinsic in the objectives of this project was the development of a generalized means of describing the space-dependent effect of the reactivity feedback. A consistent means of determining the parameters and constants to be used in the reactivity equations is vital to the generality of such a development, and equally vital to the calculational accuracy afforded by the model as developed. The study, however, was not intended to provide a better approach to calculating these necessary parameters and constants. Therefore, the most up-to-date, available techniques and computer codes to calculate cross-section data were used in the development.

The detailed procedure used to obtain the temperature-dependent cross-section data for the Spert I Doppler calculations will be omitted in lieu of the following general outline. In view of the wide range of temperatures to be represented in the core, it was decided to calculate constants for three temperatures, 20, 600, 1000°C, and interpolate the constants for all other temperatures. The ambient (20°C) constants were calculated employing standard techniques utilizing the TEMPEST[10], TOPIC[11], and GAM[12] computer codes. The only temperature effect considered was that arising from the Doppler broadening of the U-235 and U-238 resonances. The effect of this phenomenon on the nuclear cross sections was calculated using the ZUT TUZ[13] resonance integral computer program with the DANCOFF JR[14] correction. Heating of the moderator and change in fuel volume were neglected. Thus, the thermal group constants did not change as a function of temperature.

3. APPLICATION OF MODEL TO SPERT I OXIDE CORE

In the performance of a PDQ calculation, the geometry of the core must be simulated by means of a two-dimensional coordinate system (the grid). Once the grid which defines the geometric space points in the core has been established, and the parameters and constants which define the core have been obtained, the code calculates group-dependent flux solutions at each mesh point in the grid. From these flux solutions, PDQ calculates the source strength at each mesh point in the active fuel region. From the source strength at each mesh point a straightforward calculation of the spatial temperature distribution is performed. Once the spatial temperature dependence has been obtained, the PDQ code can be used to calculate the space-dependent reactivity perturbation with the various fuel regions of the core re-defined in terms of their temperature-perturbed fuel constants.

3.1 Physical Representation of the Spert I Core

There are two primary methods of geometrically representing a physical reactor core in the two-dimensional PDQ analysis: x-y which represents the horizontal space coordinates of the core and leaves the vertical direction (core height) to be accounted for by a geometric buckling factor, and r-z which is a vertical representation of the core with azimuthal symmetry assumed. Although both representations have been utilized in the Doppler analysis, the final temperature and reactivity calculations were performed using the r-z approach. The primary reason that the calculations were performed using the r-z geometry is a result of the Spert I control rod positions. For example, in the 2.2-msec-period test performed with the Spert I core, approximately 30 inches of control rod poison remained in the 72-inch-long core during the transient. Since the Doppler effect is a function of the spatial flux distribution and this distribution is strongly dependent on the control rod positions, a partially rodged core must be represented in the calculations. The representation of a partially rodged core is possible only in r-z geometry using PDQ.

In the representation of the Spert I core in r-z PDQ, the actual z-axis geometry was obtained directly from schematic core layouts and blueprints, thus providing a realistic axial representation of the core. The specification of the radial geometry, however, was not as simple as that of the axial geometry, since the two-dimensional horizontal plane of the reactor was to be reduced to a one-dimensional representation. In order to accomplish this, the one-dimensional representation was obtained by the normalization of one-dimensional, radial FOG^[15] solutions to two-dimensional, x-y PDQ solutions.

All components of the Spert I core, except the control rods, lent themselves to a fairly accurate cylindrical, or annular, representation. The control rods, however, consisted of four, flat blades which were inserted in the four quadrant radial boundaries (with the rods inserted, the core was essentially divided into four pie-shaped fuel sections). The control rods were represented in an annular region equal in volume to the actual control rod complex. The radial position of this annular region was determined from the normalized, radial FOG solutions. Thus, representation of the Spert I core control rods in cylindrical geometry was a fairly gross approximation. This approximation can be justified, however, when it is considered that over 90 percent of the energy was generated in the region of the core below the control rod poison sections for any of the short-period transients which were investigated. Thus, although the annular representation of the rods is considered a fairly rough geometrical approximation, the gross effect of the rods on the spatial flux distribution can be reasonably well represented.

The number, spacing, and placement of the grid lines in the r-z geometry is, of course, determined by the physical reactor system and the type of calculation to be performed. For the Spert I calculations, the mesh was kept fine in order to determine as closely as practical the core flux and source distributions and, hence, the spatial temperature distribution. There were 37 mesh lines used to represent the radial (r) space coordinates and 139 mesh lines used to represent the axial (z) coordinates, thus providing a total of 5143 space-dependent flux solutions. The active core region was represented by a 22 x 108 mesh providing 2376 flux (and, consequently, space-dependent source) solutions.

At space points where the flux gradient was expected to be steep (such as interfaces with the transient rod, control rods, intermediate core grid, or reflector), the grid spacing was smaller than 1 cm. In the hot region of the active core, ie, that portion below the control rods, the mesh spacing did not exceed one-half inch (1.27 cm).

Horizontal mesh lines were placed at the experimentally determined control rod critical position and at the transient control rod positions for each of the transients which was investigated. Thus, changes in the control rod position from case to case simply entailed a re-definition of the boundary between the control rod poison section and the follower section.

For the Spert I Doppler calculations the physical system was represented using 178 volume regions, which, in keeping with PDQ nomenclature, will hereafter be referred to as compositions. A composition is defined geometrically by its bounding mesh lines and neutronicly by its group dependent parameters: D , diffusion coefficient; Σ_a , absorption cross section; Σ_r , removal (due to scattering) cross section and; $\nu\Sigma_f$, the number of neutrons liberated for every neutron absorbed in a fission reaction times the fission cross section. Ten of the compositions represented core structural components and water; the remaining 168 represented fuel regions of which six also contained representations of the core intermediate grid.

The 168 fueled compositions were sized and placed so as to give the best representation of the core temperature distribution possible, consistent with the criteria established for the calculations. The volume of any particular fuel composition was sized according to the flux gradient existing at that core spatial position, being made smaller as the flux gradient increased. For example, the least severe transient investigated, the 5.6-msec-period transient, was set up with 126 of the 168 fuel compositions in the hot area of the core below the control rods; for the 2.2-msec-period transient there were 138 fuel compositions representing this area.

3.2 Reactivity and Temperature Calculations for the Spert I Oxide Core

The magnitude of the Doppler reactivity feedback is a function of the change in the energy content of the fuel between an initial reference condition with no feedback present and a transient condition of interest with feedback present. For the Spert I calculations, the initial reference condition was specified to be that condition corresponding to zero (low) power at a fuel temperature of 20°C. Calculations performed with these conditions are termed Base Case calculations. The transient condition for which the feedback was determined was that condition of the reactor which exists with the power at its maximum value during an excursion. Calculations performed at this condition are termed Perturbed Case calculations.

By the use of the physical representation as has been described, four Base Case calculations were performed to determine the reactivity (or eigenvalue) and the fission distribution of the Spert I core at various experimental control rod positions. The control rod positions used were the experimentally determined zero-power critical position and the positions for the 5.6-, 3.2-, and 2.2-msec-period excursion tests. With the control rods at the experimentally determined critical position, the calculated eigenvalue solution to the two-dimensional diffusion equation was 1.019. This value compares favorably with the experimental critical eigenvalue of 1.013^[16]. With the simulated control rods raised

to experimentally determined positions corresponding to 5.6-, 3.2-, and 2.2-msec-period transients, positive reactivities of 1.70\$, 2.17\$, and 2.57\$ were calculated for the core. These reactivities correspond to experimental values of 1.61\$, 2.10\$, and 2.62\$, respectively. From these results it was concluded that the representation was adequate to describe the Spert I core.

The next step following the Base Case calculations was to determine the core temperature distribution. In the calculation of core temperature distribution, the basic assumption was that the primary energy source was the fission process; hence, the core spatial temperature was assumed to be directly related to and derivable from the spatial fission source distribution. This assumption is mathematically stated and expanded as:

$$T(r) = \left[\frac{\frac{1}{\lambda} \sum_{i=1}^4 \left(v \Sigma_f(r) \right)^i \phi^i(r)}{\frac{1}{V\lambda} \int_V \sum_{i=1}^4 \left(v \Sigma_f(r) \right)^i \phi^i(r) dr^3} \right] \frac{E_f(t)}{C(E_f)} \quad (1)$$

where the numerator is the source strength at any point (r), the denominator is the core average source, $E_f(t)$ is that part of the total core energy release which remains in the fuel at time t, and $C(E_f)$ is the heat capacity of the fuel as a function of energy in the fuel.

In order to simulate the core spatial temperature distribution, the average temperature of each fuel composition must be calculated. To accomplish this, Equation (1) is modified as:

$$\bar{T}_{\text{(fuel composition)}} = \left[\frac{\int_{\text{composition}} \frac{1}{\lambda} \sum_{i=1}^4 \left(v \Sigma_f(r) \right)^i \phi(r)^i dV_{\text{Comp.}}}{\frac{1}{\lambda} \frac{1}{V_{\text{CORE}}} \int_{\text{CORE}} \sum_{i=1}^4 \left(v \Sigma_f(r) \right)^i \phi(r)^i dV_{\text{CORE}}} \right] \frac{E_f(t)}{C(E_f)} \quad (2)$$

or

$$\bar{T}(\text{fuel composition}) = \left[\frac{\text{Average Source in Composition}}{\text{Core Average Source}} \right] \frac{E_f(t)}{C(E_f)} \quad (3)$$

The average source in a fuel composition and the core average source are both obtained from the Base Case PDQ calculations. For Spert I, $C(E_f)$ has been calculated and $E_f(t)$ is taken to be the experimentally determined total energy generated in the fuel up to the time of peak power less a correction for prompt energy transfer.

Once the average temperatures for the fuel regions have been calculated using the above approach, the fuel constants appropriate to these temperatures are substituted in place of the 20°C constants and the problem is re-run. This

calculation is performed to obtain the eigenvalue for the core with the reactivity feedback owing to the increased fuel temperatures taken into account.

In a development^[17] of methods to compute the parameters which appear in the reactor kinetic equations, the definition of reactivity is given as:

$$\rho(t) = \frac{\lambda(t) - \lambda_c}{\lambda_c} \lambda_c^m \lambda_c^n(t) \quad (4)$$

where the lambdas are eigenvalue solutions of the transport equation and m and n are integers or zero. Results of the Spert I calculations are reported in terms of Equation (4) using m = 1 and n = 0. For the purpose of the Doppler reactivity feedback, however, it is necessary to determine the reactivity difference between stationary diffusion solutions in which the flux solutions are assumed to be time independent. Equation (4) thus becomes

$$\rho = \frac{\lambda - \lambda_c}{\lambda}$$

and the expression for $\Delta\lambda$ between two reactor conditions is

$$\Delta\rho = \frac{\lambda_1 - \lambda_c}{\lambda_1} - \frac{\lambda_2 - \lambda_c}{\lambda_2} = \frac{(\lambda_1 - \lambda_2) \lambda_c}{\lambda_1 \lambda_2}$$

where in the present case:

λ_1 = eigenvalue solution of Base Case transients investigated

λ_2 = eigenvalue solutions of Perturbed Case transients investigated

λ_c = eigenvalue solution of steady state critical case (= 1.0195).

The experimental and calculated results of each of the three transient tests reviewed appear in Table V. At the time of peak power the experimental values of energy release and the total reactivity feedback (taken to be the total reactivity inserted minus \$1.00) are those quoted in previous reports^[7, 8] on the transient behavior of the Spert I oxide core. The values used for the energy in the moderator and cladding at the time of peak power owing to conductive and prompt radiative heat transfer were calculated by Spano in a previous analysis^[18]. Error limits for each of the calculations were based solely on the eigenvalue convergence of each particular calculation and in no way reflect upon the validity of the experimentally determined input. In order to express the results in dollars, a β_{eff} of 0.0074 was used.

As can be seen from the results presented in the table, the calculated Doppler reactivity feedback is in good agreement with the experimentally determined total reactivity feedback. However, if the non-Doppler feedback, owing to the energy transferred from the fuel before peak power, is considered the results would be in even closer agreement. For example, depending upon the calculational procedure and the assumptions made, the non-Doppler feedback coefficient for the 2.2-msec-period transient would be calculated to be between one and two cents per MW-sec. The application of this value to the 5.2 MW-sec

lost from the fuel before peak power yields a total calculated feedback (Doppler + non-Doppler) of from 1.59 to 1.64 dollars which brackets the experimentally determined reactivity feedback.

On the basis of the accuracy of these results and in view of the generality inherent in the calculational approach, it is concluded that the model is valid and can be used with confidence in the performance of Doppler feedback calculations for the PBF core.

TABLE V
COMPARISON OF CALCULATED AND
EXPERIMENTAL REACTIVITY FEEDBACK FOR
SHORT-PERIOD TRANSIENTS WITH SPERT I OXIDE CORE

τ (msec)	E_{tm} (MW-sec)	Prompt Energy Transfer (MW-sec)	$E_{Doppler}$ (MW-sec)	$\bar{T}$ (avg) (°C)	λ_{Base}	$\lambda_{Perturbed}$	ρ_{exp}^{Total} (\$)	$\rho_{calc}^{Doppler}$ (\$)
2.2	86.0	5.2	80.8	311.1	1.0394	1.0274	1.62	1.57 ± 0.07
3.2	51.3	3.3	48.0	192.1	1.0361	1.0279	1.10	1.10 ± 0.06
5.6	24.0	1.7	22.3	93.6	1.0324	1.0278	0.61	0.60 ± 0.03

$\tau \equiv$ Reactor period

$E_{tm} \equiv$ Total energy generated up to time of peak power

$E_{Doppler} \equiv$ Energy contained in fuel at time of peak power

$\bar{T} \equiv$ Average core fuel temperature at time of peak power

$\lambda_{Base} \equiv$ Eigenvalue of Base Case calculation (zero power, 20°C)

$\lambda_{Perturbed} \equiv$ Eigenvalue of core for Perturbed Case calculation (at time of peak power)

$\rho_{exp}^{Total} \equiv$ Total experimentally determined negative reactivity feedback at time of peak power

$\rho_{calc}^{Doppler} \equiv$ Total calculated negative Doppler reactivity feedback at time of peak power.

III. ANALYSIS ENERGY DENSITY CALCULATIONS

A problem of concern in reactor safety is that of a large insertion of reactivity in a reactor resulting in a rapid power excursion which generates heat in sufficient quantity to damage the fuel. Two of the most commonly postulated mechanisms for fuel damage in such a situation are clad rupture and clad melting. Both modes of failure depend upon the amount and rate of energy generation in the fuel. Therefore, knowledge of the core energy density is of importance in predicting the failure thresholds of the fuel. If the excursion is limited by a prompt-acting reactivity-compensating mechanism, such as the Doppler effect in metal-clad UO_2 rod-type cores, and the excursion is terminated rapidly enough that heat transfer from the fuel to the coolant and delayed-neutron contributions can be neglected, then a quantity of interest is the energy density in the fuel at the end of the excursion. The following discussion describes a method whereby this quantity can be obtained directly by the use of existing diffusion computer codes without having to solve for and integrate the core fluxes. This method has been shown to be considerably faster than the more conventional method, thereby resulting in a saving of computer time.

The energy density attained at the end of an excursion is a function of core position and is proportional to the time integral of the power density with the integration carried out over all time. Thus the energy density can be written

$$E(r) = c \int_0^{\infty} \phi(r,t) dt$$

where $\phi(r,t)$ is the reactor power and c is a constant of proportionality. The energy density can be obtained by time-integrating the power history of a transient calculated by a neutron diffusion code. The calculation of the time behavior of the flux, however, is time-consuming. Ergen[19] has shown that it is possible to obtain an analytic solution for the final energy distribution without explicitly calculating the time behavior of the flux. Ergen's method involves solving the time-independent equation for the energy density.

With the assumption that the negative reactivity owing to the prompt-acting mechanism is directly proportional to the energy generated during the transient, the one-velocity diffusion equation for the flux is

$$D\nabla^2 \phi + \left(\Sigma - b \int_0^t \phi \, d\tau \right) \phi = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad (1)$$

where b is the reactivity feedback coefficient, and Σ is the lumped cross section. Since $E(r) = c \int_0^{\infty} \phi(r,t) dt$, Equation (1) can be integrated with respect to time from 0 to ∞ to give the ordinary differential equation for the energy density;

$$D\nabla^2 E + \left(\Sigma - \frac{b}{2} E \right) E = - \frac{1}{v} \phi(r,0) \quad (2)$$

where $E = 0$ at the surface of the reactor.

Ergen^[19] has obtained an analytic solution of Equation (2) in terms of Weierstrass P functions for the slab-geometry case in which the transient is initiated from zero power by perturbing the cross section Σ over the entire reactor. An analytic solution of Equation (2) for other cases (for example, one in which Σ is perturbed over only part of the core) can be obtained by Ergen's methods, but the computation can become extremely complex.

A simpler method of obtaining a solution for the multi-region case can be realized by the use of currently available diffusion codes. In particular, the ODD-1^[20] general one-dimensional diffusion transient and/or steady-state code is capable of solving the one-velocity-group, steady-state-like Equation (2) for perturbations in only part of the core. However, Equation (2) has two solutions which vanish at the boundary, and the iterations of the steady-state part of the ODD-1 program converge to the one which is not always positive. Therefore, the steady-state portion of the code cannot be used. It was determined, however, that the transient problem

$$D\nabla^2 E + \left(\Sigma - \frac{b}{2} E \right) E + S = \Sigma \frac{\partial E}{\partial t}, \text{ where } S = \frac{1}{v} \phi(r, o) \quad (3)$$

can be made equivalent to Equation (2) by the use of a pseudo time step, $\Delta t \equiv 1$, in the numerical solution, and that the solution always converges to the non-negative one. In this case, if E_n is the value of E at the n th time step, then the above equation becomes

$$D\nabla^2 E_n + \left(\Sigma - \frac{b}{2} E_{n-1} \right) E_n + S = \Sigma \left(E_n - E_{n-1} \right) \quad (4)$$

or

$$D\nabla^2 E_n + \Sigma E_{n-1} - \frac{b}{2} E_{n-1} E_n + S = 0 \quad (5)$$

which is equivalent to Equation (2).

So, at each pseudo time step, $E_n \approx \frac{S + \Sigma E_{n-1}}{\frac{b}{2} E_{n-1}} > 0$

for values of S , Σ , and b of interest in reactor kinetics.

Since the sequence $\{E_n\}$ defined above with $E_1 > 0$ converges for all positive values of S , Σ , and b , this method of solving Equation (2) can be used with all one-dimensional diffusion codes capable of solving equations of the form

$$\nabla \cdot D\nabla u + f(u) u + S = \alpha \frac{\partial u}{\partial t}.$$

Another case of interest is that in which the perturbation is in only part of a large core, thereby giving rise to a tilt in the flux. A realistic appraisal of this situation requires multi-velocity group equations since the tilt in the flux does not permit a one-velocity equation approximation with flux weighted averages of the diffusion coefficient and cross sections.

By use of the Wigle-40 program[21], the two-group energy density equations can be solved by iterating the solution with pseudo time steps as was done for the one-group case with the ODD-1 program. The transient equations for the fast and slow fluxes solved by Wigle-40 with delayed neutrons ignored, are

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 + \nu \Sigma_{f_1} \phi_1 + \nu \Sigma_{f_2} \phi_2 + S_1 = \frac{1}{v_1} \frac{\partial \phi_1}{\partial t} \quad (6)$$

and

$$D_2 \nabla^2 \phi_2 - \Sigma_2 \phi_2 + \Sigma_{r_1} \phi_1 + S_2 = \frac{1}{v_2} \frac{\partial \phi_2}{\partial t} \quad (7)$$

By integrating these equations with respect to time from 0 to ∞ and setting $E_1 = \int_0^\infty \phi_1(r, t) dt$ and $E_2 = \int_0^\infty \phi_2(r, t) dt$, the steady-state equations for the energy densities E_1 and E_2 are obtained. The equations for E_1 and E_2 determined to be most suitable for use in iterating to the solution using pseudo time steps are

$$D_1 \nabla^2 E_1 - \Sigma_1 E_1 + \nu \Sigma_{f_1} E_1 + \nu \Sigma_{f_2} E_2 + S_1 = \Sigma_1 \frac{\partial E_1}{\partial t} \quad (8)$$

and

$$D_2 \nabla^2 E_2 - \Sigma_2 E_2 + \Sigma_{r_1} E_1 + S_2 = 0 \quad (9)$$

where the pseudo time step is $\Delta t \equiv 1$ and $S_i = \frac{1}{v_i} \phi_i(r, 0)$ are the initial flux shapes.

In order to provide a check on the validity of this technique, test calculations were performed in which both the conventional and direct solutions for the energy density at the end of a rapid power excursion in a 240-cm slab core were obtained. In the calculations, $\nu \Sigma_{f_1}$ and $\nu \Sigma_{f_2}$ were perturbed in an end fourth of the core to produce an excursion with an asymptotic period of 0.18 msec. The excursion was terminated by an increase in the absorption cross section which was proportional to the energy density; ie, $\Sigma_2 = \Sigma_2 + \frac{b}{2} \overline{E_2}$ where $\overline{E_2}$ is the average energy density in the region considered, and b is the reactivity feedback coefficient for the region. The WIGLE-40 computer program was used to obtain the results for both methods. Figure 9 shows the results of the calculations. As can be seen from this figure, the two methods are in excellent agreement. The transient solution for the neutron fluxes required 495 time steps, and then the result had to be integrated in order to obtain the energy density. The direct solution for the energy density, however, required only 80 time iterations. The direct solution, therefore, was more than a factor of 6 faster than that which involved the transient flux solution.

From these results it is concluded that the use of the transient part of a diffusion code to iterate to a solution of the corresponding energy density equations appears to be accurate and represents a saving in time over integrating the transient solution for the flux. The method also has possibilities of being

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Fig. 9 Comparison of calculated energy densities for direct and time-integrated-flux solutions.

extended to multi-dimensional diffusion equations where the solution and the integration of the transient problem might be prohibitive in terms of the amount of computer time involved. Analysis of this technique is continuing in order to include the effects of delayed neutrons in the representations.

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