APPLICATION OF FRACTURE MECHANICS TO GRAPHITE UNDER COMPLEX
STRESS CONDITIONS*

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ABSTRACT

The purpose of this study was to examine the applicability of linear-elastic fracture mechanics to graphite under multiaxial stress conditions. Earlier studies have demonstrated the applicability of fracture mechanics to graphite under a limited range of biaxial states of stress where one principal stress was tensile and the other was compressive.¹ In this paper, the study was extended to consider other combined stress states.

The specimens used in the present study were thick-walled graphite cylinders with flat heads which were internally pressurized. Two series of specimens were used. The first series had complete circumferential notches machined diagonally into the head-cylinder juncture region, while the second series was unnotched.

The methods of linear-elastic fracture mechanics and a finite-element analysis were used to predict pressures to cause fracture for both notched and unnotched specimens. In the latter case, it was necessary to postulate a "size" for a naturally occurring flaw. These predicted pressures were in good agreement with measured

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results for the vessels with machined notches. The unnotched vessels, however, withstood a greater pressure than was predicted, but the agreement is considered good because the flaw sizes used were inferred from tests on other specimens and hence were very approximate. The results from this study indicate that fracture mechanics can be a valuable technique for predicting fracture of graphite under complex stress states, with the method being especially useful where regions of high stress concentration exist.

INTRODUCTION

We begin with a brief review of our prior studies in the series on linear-elastic fracture mechanics. We measured the fracture toughness² of ATJ* and AXM† graphites with several different specimen types including notched beams, compact tension specimens, double cantilever beams, and circumferentially notched tensile specimens. From these tests, it was shown that fracture-mechanics concepts can be applied to these specimens with gross flaws, and specimens and methods for use in determining fracture toughness values were identified. At the same time, extension of fracture-mechanics concepts to graphite components where the flaws are the relatively small naturally occurring "flaws" was examined with positive results.

To illustrate the latter, beams without machined notches, but which were otherwise identical to the notched beams used for fracture toughness determinations, were tested. The effective crack depths were calculated, based on the bending moment at fracture and the K_{Ic} values determined from the beams with notches. The calculated crack depths for naturally occurring "flaws" in ATJ and AXM graphites were 0.010 in. and 0.003 in. respectively. These depths are compatible with dimensions of observed microstructural features in both materials.

Applicability of fracture-mechanics concepts and methods for predicting fracture of graphite under complex stress conditions was then studied through tests on splitting tensile specimens of AXM and ATJ graphites.¹ The splitting tensile specimens were cylindrical or elliptical disks that were loaded in compression along the

*ATJ graphite is a molded graphite that has a maximum grain size of 0.006 in. It has been produced by Union Carbide Corporation, Carbon Products Division, for a number of years, and its mechanical behavior has been well characterized. It was supplied in 14-in.-diam by 15-in.-long billets.

†AXM graphite is an extremely fine grain, isotropic graphite manufactured by POCO Graphite, Inc. This material was supplied in 2-in.-diam by 12-in.-long cylinders.

major axis. Nearly uniform biaxial stress states exist in the central portions of circular or elliptical disks loaded by lateral compressive loads. The maximum tensile stress occurs normal to the loading direction at the center of the disk and is the maximum principal stress at that point. The minimum principal stress at the center is compressive. The ratio of the absolute values of the principal stresses is dependent on the shape of the disk; it ranged from 3.0 to 6.9 in the specimens tested. Slots in the central region that were aligned with the loading direction caused premature failures that were predicted very accurately using fracture-mechanics methods.

Since these tests demonstrated that fracture-mechanics methods can be used for predicting the fracture of graphite under biaxial stress conditions where one principal stress is tensile and the other is compressive, other combined stress states were considered. This paper describes results from an investigation of fracture under the complex stress conditions that exist in an internally pressurized thick-walled cylindrical vessel with a flat head.

SPECIMENS AND TEST APPARATUS

Thirty thick-walled vessel specimens were tested: fifteen each of AXM and ATJ graphites. The longitudinal axes of the ATJ vessels were oriented in the across-grain direction. The configuration of the thick-walled vessels is shown in Fig. 1. Details of

Fig. 1. Thick-walled vessel with notch.

the circumferential notch are shown in the inset. Some of the specimens were unnotched; in these cases there was a square corner at the intersection of the head and the cylindrical wall.

The rig for testing the vessels is illustrated in Fig. 2. Hydraulic oil was used to pressurize the specimen, and a latex rubber liner prevented the oil from entering the pores in the graphite and also acted as a gasket to seal the vessel.

THEORETICAL ANALYSIS

Finite-Element Analysis

The finite-element method^{3,4} was used for a theoretical analysis of the thick-walled vessel specimens. Since the vessels are axisymmetric, a two-dimensional analysis is sufficient for determining the stresses in each specimen. The axisymmetric finite-element idealization constructed for the unnotched thick-walled vessels consisted of 784 nodal points and 716 discrete elements; the finite-element idealization of the notched vessel was similar. An internal pressure of 100 psi on the entire inner surface of the vessel was assumed in the calculations. In the notched vessels, the pressure was considered to extend into the notch but not to act at the very tip. Even though silicone mold-release grease was put into the notch to transfer the pressure, it was felt that such transfer would not be effective all the way to the tip because the grease has a high viscosity. Analyses showed an appreciable

Fig. 2. Test rig for thick-walled vessels.

difference in results, depending on whether or not there was pressure in the notch.

The uniaxial tensile properties of the two graphites were determined for use in the analyses. Neither of the two graphites exhibited any purely elastic behavior. The stress-strain curves were all nonlinear, even at extremely low stress levels, and the strain ratios* decreased as the stress increased. Since the method used for analyzing the thick-walled vessel specimens was based on linear elasticity, a linear approximation to the stress-strain curve and a constant value of strain ratio were required. The secant modulus and strain ratio at a selected stress level were chosen for use in the analyses. In this way, the nonlinear behavior of the graphite was accounted for, in an approximate fashion, so that linear-elastic analyses could be used. These values are a better approximation of the nonlinear behavior than the tangent modulus and initial strain ratios.

The average strength of three AXM tensile specimens was 7300 psi, and their average strain-to-failure was 0.57%. The secant modulus at a stress of 1000 psi was 1.60×10^6 psi, and the strain ratio at 1000 psi was 0.27. The strength, strain-to-failure, and strain ratio of AXM graphite are all markedly high.

Since ATJ graphite is transversely isotropic, it was necessary to conduct two sets of tensile tests on this material to determine the response to with-grain and across-grain loading. Three across-grain ATJ tensile specimens gave average strength and strain-to-failure values of 2400 psi and 0.41%, respectively. The average secant modulus and average ratio of transverse strain to axial strain at a stress of 500 psi were 0.83×10^6 psi and 0.08, respectively. Four with-grain specimens of ATJ graphite were tested. The average strength was 3500 psi and the average strain-to-failure was 0.55%. The average secant modulus at a stress of 500 psi was 1.39×10^6 psi. The transverse strain varies around the circumference of a with-grain ATJ graphite specimen because of anisotropy. Therefore the transverse strain was measured in both the with-grain and across-grain direction. The ratio of transverse strain in the across-grain direction to axial strain was 0.14 at 500 psi stress. The ratio of transverse strain in the with-grain direction to axial strain was 0.09 at that same stress.

The properties determined from the tensile tests were used in the finite-element analyses of the AXM and ATJ thick-walled vessels. The analyses showed that the maximum stress in the unnotched vessels was at the intersection of the head and wall. For the notched vessels, it was at the tip of the notch.

*Strain ratio is the negative of the ratio of the strain normal to the direction of loading to the strain in the loading direction.

The stress state in the element with the largest principal stress, along with the orientation of the maximum principal stress, is given in Table 1 for each of the thick-walled pressure vessels. The maximum principal stress is always in the meridional plane; the angle given is the angle between the direction of the maximum principal stress and the longitudinal axis of the cylinder. Because of the large gradient in stress through this area of the vessel's cross section, the calculated stress values are strongly dependent on the sizes of the discrete elements. But the calculated values do give an indication of the triaxiality of the stress state at the point of fracture initiation. The elements at the intersection were made smaller for the AXM graphite than for the ATJ graphite because of the smaller natural flaw size for AXM graphite. Therefore the calculated stresses in the unnotched AXM vessel were higher than in the unnotched ATJ vessels because the element size was smaller in the former.

Fracture Mechanics

The procedure used for determining the relationship among strain-energy release rate, crack length, and pressure was developed by Watwood⁵ and by Mowbray.⁶ It is basically an analog to the compliance method of experimentally determining values of the strain-energy release rate at onset of rapid crack propagation. The finite-element method was used to compute the strain energy stored in several vessels which are identical except for differences in crack lengths. This was done by sequentially removing two elements at the notch tip or wall-to-head intersection. With the strain energy U known as a function of crack area A , Irwin's⁷ definition of the strain-energy release rate

Table 1. Stress states at point of maximum stress in thick-walled pressure vessels

Graphite	Notched	Maximum meridional stress (psi)	Minimum meridional stress (psi)	Orientation of maximum meridional stress ^a (deg)	Circumferential stress (psi)
AXM	No	1328	241	38	443
	Yes	1287	690	25	513
ATJ	No	690	99	44	73
	Yes	1232	529	34	105

^aThe angle between the direction of the maximum principal stress and the longitudinal axis of the cylinder.

$$G = -\partial U / \partial A \quad (1)$$

was used to find the strain-energy release rate for a particular vessel with a given crack length. Since the strain energy in an elastic body is proportional to the square of the pressure p , the relation between the pressure and the strain-energy release rate is

$$p = \sqrt{G/F} , \quad (2)$$

where F is a constant of proportionality for a particular graphite and crack length that is found from the finite-element analyses. The values of F for the notch depths of interest are given in Table 2. A value of F was determined for a notch the depth of the natural flaw size for each graphite as well as for the machined notch.

These values may be used to predict the pressures necessary to break the vessels, if the value of G_{Ic} for the material is known. The values of G_{Ic} are listed in Table 3.

Table 2. Calculated values of F for thick-walled pressure vessels

Graphite	Notch depth (in.)	F (in. ³ /lb)
AXM	0.003	0.83×10^{-6}
	0.156	1.99×10^{-6}
ATJ	0.010	1.55×10^{-6}
	0.156	2.94×10^{-6}

Table 3. G_{Ic} values for ATJ and AXM

Graphite	Loading direction	Cracking direction	G_{Ic} (in.-lb/in. ²)
AXM	(Isotropic)	(Isotropic)	0.81
ATJ	With grain	Across grain	0.48
	Across grain	With grain	0.57

At this point we were confronted with a difficulty in applying fracture mechanics to the ATJ vessels. Since the cracks are not aligned with the principal material axes, we did not know the value of G_{Ic} to use to predict the pressure that will fracture the vessels. In these studies, we assumed that the values of G_{Ic} for cracks aligned with the principal axes of the material are the maximum and minimum values of G_{Ic} and that the value of G_{Ic} varies smoothly between the two extremes at intermediate orientations. Although data do not exist for establishing the validity of this assumption, it was used here to provide a comparison between calculated and experimental results.

The form of the variation was assumed to be

$$G_{Ic}^{\theta} = G_{Ic}^r - (G_{Ic}^r - G_{Ic}^z) \sin \theta, \quad (3)$$

where θ is the angle that the crack makes with the axis of the cylinder, and G_{Ic}^z and G_{Ic}^r are the values of G_{Ic} in the direction of the axis of the vessel and in the radial direction respectively. G_{Ic}^r is the value for with-grain loading. The value to be used for θ is the complement of the orientation of the maximum meridional stress given in Table 1. The calculated values for G_{Ic}^{θ} are 0.54 in.-lb/in.² for the unnotched ATJ vessels and 0.55 in.-lb/in.² for the notched vessels.

TESTS

AXM Graphite

Fracture data for the AXM thick-walled vessels are given in Table 4. The predicted failure-pressure values were calculated by substituting the appropriate F values from Table 2 and a G_{Ic} value of 0.61 in.-lb/in.² into Eq. (2). All of the unnotched vessels failed at higher pressures than predicted. The average maximum pressure is 21% higher than predicted. In the case of the notched vessels, the predicted failure pressure is in good agreement with the experimental results. The average maximum pressure is only 3% higher than predicted.

Strain gages were mounted in the center of the head and on the outside of the wall, 17/32 in. from the head and oriented in the axial direction, to monitor crack initiation. These gages did not give any indication of crack growth prior to complete fracture in the unnotched AXM vessels. The gages did indicate that the cracks grew some in the notched AXM vessels prior to complete fracture.

Table 4. Data from AXM thick-walled vessels

Nominal notch depth (in.)	Predicted failure pressure (psi)	Specimen No.	Maximum pressure (psi)
0	990	3	1120
		4	1190
		5	1180
		6	1210
		7	1430
		8	1190
		9	1080
		Average	1200
5/32	640	1	680
		2	690
		10	610
		11	680
		12	670
		13	640
		14	680
		15	680
		Average	660

All the unnotched AXM vessels failed at the intersection of the head and wall. All the heads were broken off, with the separation being associated with one circumferential crack. All the notched AXM vessels failed at the tip of the notch. Again the heads were all separated from the rest of the vessel by a circumferential crack.

ATJ Graphite

The ATJ graphite specimens were geometrically identical to the AXM graphite specimens and were tested in the same manner. The ATJ vessels all failed in the same manner as the AXM vessels: the heads broke off with a circumferential crack. Data from the ATJ thick-walled vessels are given in Table 5. The predicted failure pressures were calculated with Eq. (2), using the F values given in Table 2 and G_{IC} values of $0.54 \text{ in.-lb/in.}^2$ for the unnotched specimens and of $0.55 \text{ in.-lb/in.}^2$ for the notched vessels. The average pressure at fracture for the unnotched vessels is 19% higher than predicted and for the notched vessels it is 7% lower than predicted. Strain data from the crack-detection gages indicated crack growth prior to fracture in both the notched and unnotched ATJ vessels.

Table 5. Data from ATJ thick-walled vessels

Nominal notch depth (in.)	Predicted failure pressure (psi)	Specimen No.	Maximum pressure (psi)
0	590	4	700
		5	710
		6	690
		7	700
		8	680
		9	690
		10	720
		Average	700
5/32	430	1	420
		2	440
		3	410
		11	380
		12	340
		13	400
		14	410
		15	420
		Average	400

CONCLUSIONS

The failure pressures for the notched thick-walled vessels were in good agreement with predictions based on fracture mechanics. The unnotched vessels were stronger than predicted, but the agreement there is reasonably good considering the approximation of flaw size used in the calculation. The tests on ATJ graphite specimens illustrate the complexities involved in applying fracture mechanics to anisotropic materials under general states of stress. In this particular case, there is a need for additional basic data, that is, fracture toughness values for cracks that are not aligned with the principal axes of the material are required.

When the results of these tests and the earlier tests on splitting tensile specimens are considered, it is concluded that fracture-mechanics methods can be used to predict fracture of these graphites under general stress conditions at room temperature. These studies also demonstrate that the finite-element method can be used effectively in predicting failure. This demonstration is of importance because of the versatility of the finite-element method in handling varied and complex geometries.

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