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A NUMERICAL METHOD FOR
GENERATING THE COEFFICIENTS

OF $f = (x + x^2 + \dots + x^m)^n$

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Livermore, California

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A NUMERICAL METHOD FOR GENERATING THE COEFFICIENTS OF

$$f = (X + X^2 + \dots + X^m)^n$$

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March 24, 1964

A Numerical Method for Generating the Coefficients of

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ABSTRACT

A numerical routine for generating all the coefficients of the expansion of the function $(X + X^2 + \dots + X^m)^n$ is described. The routine requires only addition and subtraction of whole numbers. A problem of the complexity of $m = 6$, $n = 5$ can be completely solved in a few minutes using pencil and paper. The routine may have applications in the solution of more complex problems since a program for use of a digital computer would be relatively uncomplicated. An analytical solution for calculating each coefficient was published by De Moivre in 1730.

The problem of computing the coefficients of the expansion of $(X + X^2 + \dots + X^m)^n$ and the frequency distribution for the sums of the upper faces of n dice numbered consecutively 1, 2, ..., m and thrown repeatedly, the consecutive throws being assumed to be independent and the individual dice being assumed to behave in a random fashion, are mathematically equivalent. The present description will be based on the dice model, since the method was worked out for the solution of the dice problem. The

method will be described as a recipe and by means of an example, but no proof will be presented.

The frequency distribution is computed using the following rules:

Assume: n = number of dice.

m = number of consecutively numbered sides.

(Numbered 1, 2, 3, . . . , m)

- Recipe:
1. n th order binomial coefficients are selected.
 2. Between each coefficient, $m - 1$ zeroes are inserted and the numbers are entered in a column.
 3. The one at the top of the column is algebraically plus and the signs alternate.
 4. The column is treated as the n th order differences of the frequency distribution.
 5. These differences are added to construct the distribution.

Example: $n = 5$

$m = 6$

Since the frequency distribution is symmetrical about the mean, only the first half will be constructed.

$\Delta 5$	$\Delta 4$	$\Delta 3$	$\Delta 2$	$\Delta 1$	f_i	Sum
1						
	1					
0		1				
	1		1			
0		2		1		
	1		3		1	5, 30
0		3		4		
	1		6		5	6, 29

$\Delta 5$	$\Delta 4$	$\Delta 3$	$\Delta 2$	$\Delta 1$	f_i	Sum
0		4		10		
	1		10		15	7, 28
0		5		20		
	1		15		35	8, 27
-5		6		35		
	-4		21		70	9, 26
0		2		56		
	-4		23		126	10, 25
0		-2		79		
	-4		21		205	11, 24
0		-6		100		
	-4		15		305	12, 23
0		-10		115		
	-4		5		420	13, 22
0		-14		120		
	-4		-9		540	14, 21
10		-18		111		
	6		-27		651	15, 20
0		-12		84		
	6		-39		735	16, 19
0		-6		45		
	6		-45		780	17, 18
0		0		0		

$$\sum f_i = 6^5 = 7776$$

In the example, the 5th-order binomial coefficients are 1, 5, 10, 10, 5, 1. These are listed in a column with $m - 1 = 5$ zeroes between each coefficient and with alternating algebraic signs on the binomial coefficients with the first coefficient positive. Each number in a column is derived by adding to the number above it in the column the one to the left on the line intervening between these two numbers. For example, in the columns headed $\Delta 2$, $\Delta 1$, the following numbers are listed:

<u>$\Delta 2$</u>	<u>$\Delta 1$</u>
.	.
.	35
21	
	56

The 56 is the sum of the 35 above it and the 21 to the left on an intervening line. Thus, through a simple, iterative program of addition the entire frequency distribution can be constructed in a very short time. The close analogy of the method to Pascal's triangle is apparent.

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