Partial Wave Analyses of $\pi^+p + \pi^0\pi^+p$ and $K^+\Sigma^+$ at 1.28 - 1.84 GeV/c*

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ABSTRACT

Partial wave analyses of $\pi^+p + \pi^0\pi^+p$ and $K^+\Sigma^+$ have yielded values for the s-channel resonance parameters in the $\pi^0\Delta^+(1236)$ and $K^+\Sigma^+$ final states, and information on the t-channel exchanges leading to the $p^+p$ final state. The analysis of $\pi^0\Delta^+(1236)$ also yields confirmation of a prediction, based on the duality hypothesis, that the $\Delta(1236)$ will be in a pure helicity 3/2 state.

INTRODUCTION

This report summarizes the results of a bubble chamber study of the inelastic decay modes of $\Delta$-resonances in the mass interval 1820-2090 MeV, formed in $\pi^+p$ collisions at seven points in this energy range (1-5). Phase shift analysis of the elastic channel shows this mass region to be dominated by the $F_{37}(1950)$ resonance, with possibly resonant states also in the $F_{35}(1890)$, $P_{31}(1910)$, and $D_{35}(1960)$ waves (6). Of primary interest here are the $\pi^0\Sigma$ and $K^+\Xi^-$ final states (1-3). The elastic and $K^+\Xi(1385)$ data from this experiment are discussed in Refs. 4 and 5.

The $\pi^0\pi^+p$ system has been analyzed in terms of the quasi-two-body channels $\pi^0\Delta^+(1236)$ and $p^+p$, using techniques described in Refs. 1 and 2 which assure a clean separation of these states from each other and from other backgrounds (which are small in any case). Both the $\pi^0\Delta^+$ and $K^+\Sigma^+$ data are best described by an s-channel partial wave formalism, while the $p^+p$ state is best described in
terms of t-channel poles. (This treatment of \( \pi N \), using a pure isospin-3/2 state and a quasi-two-body partial wave analysis, should be compared with the analysis of Ref. 7, which includes these data.)

**THE REACTION \( \pi^+ p + \pi^0 \pi^+ p \)**

The total cross sections for the quasi-two-body states \( \pi^0 \pi^+ \) and \( \rho^+ p \) have been obtained from the \( \pi^0 \pi^+ p \) data by making a fit to the Dalitz plots in the manner described in Refs. 1 and 2. The results for these cross sections are shown in Fig. 1 as a function of laboratory momentum. In this figure, the presence of an s-channel resonance in \( \pi^0 \pi^+ \) in the 1920-MeV mass region is quite evident, while there is little evidence for such a state in \( \rho^+ p \). This observation is corroborated by inspection of the production angular distributions, examples of which are shown in Fig. 2. Those for \( \pi^0 \pi^+ \) show a backward peak and rapid variation across the resonance interval, while those for \( \rho^+ p \) show almost no backward peak or variation with energy. These conclusions have been confirmed quantitatively by fits of a Legendre series expansion to the angular distributions. For the \( \pi \Delta \) channel, an expansion of sixth order is required, indicating the importance of the F-wave in the interaction (8). For the \( \rho p \) channel an expansion of fourth order is adequate, suggesting that the F-wave is not strongly present.

An energy-dependent partial wave analysis of the \( \pi^0 \pi^+ \) channel has been carried out as described in Ref. 1, and detailed results.

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**Fig. 1.** Total cross sections for \( \pi^+ p + \rho^+ p \) and \( \pi^+ p + \pi^0 \pi^+ \). All points below 2 GeV/c are from this experiment.
Fig. 2. Production angular distributions for $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ and $\pi^+ p + \rho^+ p$ at three energies from this experiment. The $\Delta^{++}$ density matrix element $\rho_{33}^S$ is also shown. The dashed curves correspond to the best partial wave fit.

...for the masses and widths of the resonant states are given there. The Argand diagram of the best partial wave fit obtained is given in Fig. 3a. It shows that this channel is dominated by the $\Delta(7/2^+, 1950)$, decaying in the F-wave (labeled FF7 in the figure), with a root-product of branching fractions $\sqrt{X_{\pi N N} X_{\pi \Delta}}$ of $0.43 \pm 0.06$ (averaged over several acceptable fits). It also indicates the presence of $\Delta(5/2^+, 1890)$ decaying in the F-wave (labeled FF5) with $\sqrt{X_{\pi N N} X_{\pi \Delta}} = 0.21 \pm 0.03$.

It has been pointed out by Gell, Horn, Jacob, and Weyers that the decay of $\Delta(7/2^+)$ or $\Delta(5/2^+)$ into $\pi\Delta(3/2^+)$ provides a test of...
the duality hypothesis (9). They show that the constraint of reproducing with the t-channel helicity amplitudes the zeroes that occur in the s-channel (resonant) amplitudes leads to the prediction that the $\Delta(3/2^+)$ will be produced with an s-channel helicity of $\pm 3/2$. This idea may be tested experimentally simply by inspection of the density matrix element $\rho_{33}$ of the $\Delta(3/2^+)$ in the region in which the partial wave analysis shows the $\Delta(7/2^+)$ to be dominant. The values obtained for $\rho_{33}$ at three energies are shown in Fig. 2. At 1950 MeV the $\Delta(7/2^+)$ accounts for about 85% of the pp $\rightarrow \pi\Delta$ cross section, and Fig. 2 shows that, at this energy, $\rho_{33}$ has essentially its maximum allowed value of 0.5 over the entire angular range. This indicates pure helicity $\pm 3/2$ for the $\Delta(3/2^+)$, just as predicted by Gell, et. al. Further confirmation of this prediction is obtained by noting that a resonance of spin greater than 1/2 can couple to the $\pi\Delta(3/2^+)$ channel in two different partial waves. The requirement that $\rho_{33}$ be 0.5 for each s-channel resonance can be expressed in terms of the ratio

$$\sqrt{\frac{T_{L+2}}{T_L}}$$

of the couplings of the resonance to the two allowed partial waves. The duality constraint on $\rho_{33}$ implies that both

(a) $\pi^+p \rightarrow \pi^0\Delta^{++}$

(b) $\pi^+p \rightarrow K^+\Sigma^+$

![Argand diagrams](image)

Fig. 3. Argand diagrams of the partial waves found in this experiment for (a) $\pi^+p \rightarrow \pi^0\Delta^{++}$ and (b) $\pi^+p \rightarrow K^+\Sigma^+$. The notation in (a) is LL'(2J) where L is the orbital state of $\pi^+p$, L' is the orbital state of $\pi^0\Delta^{++}$, and J is the total angular momentum. For (b) the notation is L(2J).
### Fig. 4. Schematic representation of the spin relationships found in the reaction $\pi^+ p + p^2\Delta^+$. The $\Delta(5/2^+)$ state decays preferentially via the F-wave, contrary to the expectation from centrifugal barrier considerations, but in accord with a prediction of the duality hypothesis (see text).

The $\Delta(7/2^+)$ and $\Delta(5/2^+)$ will decay predominantly via the F-wave, even though for the latter this is contrary to that expected from centrifugal barrier considerations. For the $\Delta(7/2^+)$ the ratio $\sqrt{r_H/r_F}$ is found to be $0.06 \pm 0.02$, indicating almost no H-wave decay. For the $\Delta(5/2^+)$ the P-wave decay is not found by the fit, and the ratio $\sqrt{r_P/r_F}$ can be given a limit of greater than 2. These results are summarized diagrammatically in Fig. 4, and suggest that the important consideration is not the relative centrifugal barrier, but the magnitude of the spin-flip occurring in the reaction. This is further confirmation of the prediction of Gell, et. al.

As noted above, the absence of a backward peak in the angular distributions for $\pi^+ p + p^2\Delta$ (Fig. 2) indicates that this reaction is not dominated by a single s-channel resonance, or by two s-channel resonances of the same parity. On the other hand, these
distributions have the strong forward peak and dip at $-t = 0.5$ GeV$^2$ which are characteristic of this reaction at higher energy. This suggests that the reaction may be more simply described in terms of t-channel poles, even at these low energies. A t-channel partial wave analysis has been carried out, and shows that the reaction is dominated by $\pi$- and $\omega$-exchange (2). The results for the density matrix of the $\rho^+$ lend support to strong absorption models of this reaction, as reported in Ref. 2. From the point of view of s-channel resonances, it can be understood that they are less prominent in $\pi^0 + \rho^0$ than in $\pi^0 + \pi\Delta$, since the pion-exchange amplitude (believed to be real) cannot be dual to s-channel resonances.

THE REACTION $\pi^0 p \rightarrow K^+ \Sigma^+$

An energy-dependent partial wave analysis of the $K^+\Sigma^+$ channel has been carried out as described in Ref. 3, and shows that this channel is dominated by the $\Delta(7/2^+)$ with $\sqrt{s_{KNK\Sigma}} = 0.09 \pm 0.01$. The Argand diagram of a satisfactory fit, with only the $F_7$ amplitude parameterized as resonant, is shown in Fig. 3b. The assumption of a resonant form for the $F_5$ amplitude gives $\sqrt{s_{KNK\Sigma}} = 0.03 \pm 0.01$ for the $\Delta(5/2^+)$, but does not improve the fit.

FOOTNOTES AND REFERENCES

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