Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
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INTERSECTING STORAGE ACCELERATOR NOTES

EFFECT OF LOW SYMMETRY MATCHED INSERTIONS

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ABSTRACT

The effects of low periodicity insertions are considered. The insertions are assumed to have the basic properties suitable for particle experiments in storage rings. They are also assumed to be matched to the main lattice cells with respect to both betatron and momentum dispersion variables at a specific tune and momentum. For a momentum error, however, the tune is shifted and the matching is broken. This mismatching introduces instabilities at tunes directly related to the periodicity of the insertions, N . Gradient instabilities occur at all values of the tune given by $\nu \sim \frac{N}{2} \times \text{integer}$, while dipole resonances occur at $\nu \sim N \times \text{integer}$. Expressions characterizing these instabilities are derived. Specifically, for gradient instabilities, the growth in the β function and the available momentum aperture near the resonances are estimated. The use of sextupoles as correction devices, for reducing the β growth and opening the momentum aperture, is discussed. An idealized lattice with two-fold symmetry is treated as an example.

Introduction

The introduction of a matched insertion¹ of low periodicity into a lattice of high periodicity has the apparent property of not perturbing the main lattice. This is true in the sense that if the betatron parameters at the ends of the insertion are the same as the lattice betatron parameters at the insertion point, then no instability is introduced, provided of course that both lattice and insertion are independently stable. However, it must be recognized that the betatron matching has been obtained for a particular value of momentum. That is, the matching conditions are broken for momenta deviating from the matching momentum. This momentum error can be treated as a perturbation introducing gradient

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instabilities at certain values of the betatron tune, ν . These values are determined entirely by the total lattice symmetry. For a lattice of low periodicity, the ν values at which instabilities are present are bunched closer than for a lattice of higher periodicity. Specifically, for a lattice of period N , the values of ν corresponding to a gradient instability are given by

$$\nu \sim \frac{N}{2} \times \text{integer} .$$

Thus, if $N = 2$, instabilities arising from a momentum error occur at all integer ν values. We will here estimate the magnitude of this instability by calculating ν as a function of $\Delta p/p$, the deviation from the matched momentum, as well as the β function growth.

This type of momentum dependent gradient error can be compensated with local sextupole corrections. However, there are two limitations to this procedure. In the first place, the introduction of sextupoles brings with it nonlinear instabilities; and secondly, the sextupole correction is proportional to the local momentum compaction function. To see the general implications of attempting such corrections, let us consider insertions suitable for use as particle interaction regions in storage rings. This means a low β region in a long drift section where the interactions can take place, as well as zero or small momentum compaction in this region. Given such characteristics, even if we could have a non-zero momentum compaction function in the insertion, the placement of sextupoles near the quadrupoles here would be a questionable procedure in that the values of the β function are large at these points, leading to excessive nonlinear effects. Thus, in general we find that the sextupole corrections are restricted to the main, curved (i.e., where bending magnets are placed) sections of the lattice, that is, outside the insertions.

A higher order dipole effect will be briefly examined in Section V. The main point is that the betatron mismatching causes a mismatching of the momentum dispersion function which is assumed to be matched at $\frac{\Delta p}{p} = 0$. This introduces a distortion of the equilibrium orbit proportional to $\left(\frac{\Delta p}{p}\right)^2$, but one which resonates at integer ν values related to the symmetry of the total lattice. Thus, a two-fold symmetry introduces such resonant conditions at all even integer ν values.

II. Stability Bounds in Terms of Momentum Error

Consider a lattice made up of a basic structure of period n whose symmetry is broken by N matched insertions, leading to a final symmetry of order N . The betatron matching ensures the stability of the lattice no matter what the total ν value, provided that the basic cell as well as the insertion are independently stable. By stability, we refer to the finiteness and boundedness of the betatron oscillations in the basic lattice design. We are not concerned here with imposed perturbing forces.

Let the tune for the matched lattice be given by

$$\nu_n = m + \epsilon \quad (II.1)$$

where m is either an integer or half integer, and ϵ is the ν value deviation from m . The effect of a momentum error is to destroy the matching condition. The ν value is shifted and the β function is distorted. This distortion, given the fact that the matching condition is violated, has the periodicity of the total lattice, N , leading to potential instabilities at values of the perturbed ν given by all integer multiples of $N/2$. If we write the perturbed ν value in terms of its deviation from m , δ -that is,

$$\nu = m + \delta \quad , \quad (II.2)$$

then, to second order in the perturbing momentum error, $\frac{\Delta p}{p}$, we have²

$$\delta^2 = \epsilon^2 - \frac{\epsilon}{2\pi} \left(\frac{\Delta p}{p} \right) J_0 - \frac{1}{16\pi^2} \left(\frac{\Delta p}{p} \right)^2 \left(|J_{2m}|^2 - J_0^2 \right) \quad (II.3)$$

where

$$J_0 = \int_0^c \beta K ds \quad , \quad (II.4)$$

and

$$J_{2m} = \int_0^c \beta K e^{-i2m\phi} ds \quad . \quad (II.5)$$

The quantity $K(s)$ is the gradient forcing function for the lattice,

$$K = \frac{\text{gradient}}{\text{particle magnetic rigidity}} \quad , \quad (II.6)$$

while $\beta(s)$ is the unperturbed, that is matched β function. c is the equilibrium orbit length, s is the azimuthal position along the equilibrium orbit, and ϕ is the physical phase variable,

$$\phi = \frac{1}{v} \int_0^s \frac{ds}{\beta(s)} . \quad (\text{II.7})$$

Actual instabilities occur when

$$\delta^2 < 0 , \quad (\text{II.8})$$

which means that

$$J_{2m} \neq 0 . \quad (\text{II.9})$$

Thus, the instability boundary in $\frac{\Delta p}{p}$ for $v = m$ is defined by

$$\delta = 0, \text{ with } J_{2m} \neq 0 . \quad (\text{II.10})$$

Solving Eq. (II.3) for $\delta = 0$, we find

$$\Delta p = \frac{\Delta p}{p} = \frac{4\pi e}{J_0 \pm |J_{2m}|} = \Delta_p^\pm . \quad (\text{II.11})$$

If $J_0 > 0$, the stability bound is

$$\frac{\Delta p}{p} = \Delta_p^+ = \frac{4\pi e}{J_0 + |J_{2m}|} . \quad (\text{II.12})$$

The magnitude of the instability is reflected by the magnitude of J_{2m} , and we can represent it by giving the maximum distortion of the β function. If $J_0 > 0$, then, near the critical v value, m , this maximum β function distortion as a function of Δp can be written³

$$\left(\frac{\omega\beta}{\beta}\right)_{\max} = -1 + \sqrt{1 + \frac{2 |J_{2m}| \Delta_p}{(\Delta_p^+ - \Delta_p)(J_0 + |J_{2m}|)}} . \quad (\text{II.13})$$

To obtain the critical values for m , we impose the lattice symmetry, N , on the integral for J_{2m} and find the values of m for which $J_{2m} \neq 0$. Using the symmetry of β and K , Eq. (II.5) becomes

$$J_{2m} = \sum_{\ell=0}^{N-1} e^{-i \frac{4\pi m \ell}{N}} \int_0^{c/N} \beta K e^{-i2m\phi} ds .$$

The summation yields a non-zero value only if $\frac{2m}{N}$ is an integer. That is,

$$J_{2m} = N \int_0^{c/N} \beta K e^{-i2m\phi} ds \quad \text{if } m = \text{integer} \times \frac{N}{2} , \quad (\text{II.14})$$

$$J_{2m} = 0, \text{ otherwise.}$$

Thus, for example, if $N = 2$, two-fold symmetry, all integer ν values become critical.

One can generally use sextupole corrections to reduce the ν variation with momentum. In first order, this means finding sextupole strengths which make $J_0 = 0$. However, the instability is not removed. In this case, the ν value is found from

$$\delta^2 = \epsilon^2 - \frac{1}{16\pi^2} \left(\frac{\Delta p}{p} \right)^2 |J_{2m}|^2 , \quad (\text{II.15})$$

while the stability limit in $\frac{\Delta p}{p}$ (assuming it to be in the direction of positive values) and the maximum β function distortion are given by

$$\frac{\Delta p}{p} = \Delta_p^+ = \frac{4\pi\epsilon}{|J_{2m}|} , \quad (\text{II.16})$$

and

$$\left(\frac{\Delta\beta}{\beta} \right)_{\max} = -1 + \sqrt{1 + \frac{2 \Delta_p}{(\Delta_p^+ - \Delta_p^-)}} . \quad (\text{II.17})$$

One should notice from Eq. (II.15) that under these circumstances ($J_0 = 0$), ν approaches the resonant value ($\delta \rightarrow 0$) for $\frac{\Delta p}{p}$ positive as well as negative. The implication of this is that if matching is done too close to a resonant ν value, there is a limit on the correction of J_0 to zero. That is, for J_0 sufficiently small, the total momentum aperture will be bounded by the same resonance.

III. A Lattice With Two-Fold Periodicity

As an example, let us treat the case of two-fold symmetry, $N=2$. Take a lattice⁴ of 52 identical cells and insert two sections as shown in Fig. 1.

Fig. 1 Lattice With Two-Fold Symmetry

Let each cell have the following parameters:

$$\begin{aligned} \text{phase advance} &= v\Delta\phi = \pi/2, \\ \beta_{\max} &= 40 \text{ m}, \\ k \text{ (inverse focal length)} &= \frac{5}{36} \text{ m}^{-1}. \end{aligned}$$

Let each insertion be characterized by two doublets, symmetrically placed around the center, each having the following properties:

$$\begin{aligned} |k| \text{ (inverse focal length)} &= \frac{5}{36} \text{ m}^{-1} \\ \beta_+ &= 1000 \text{ m} \\ \beta_- &= 480 \text{ m} \\ v\phi_D \text{ (phase advance from beginning of insertion)} &= \frac{3}{4} \pi. \end{aligned}$$

If the unperturbed v value⁺ is 15.25, we have

$$J_0 = 52 \beta k + 4k (\beta_+ - \beta_-),$$

$$J_{2m} = 52 \beta k + 14k (\beta_+ - \beta_-), \text{ if } m = 16 \text{ (even)}$$

$$J_{2m} = + 4k (\beta_+ - \beta_-), \text{ if } m = 15 \text{ (odd)}$$

⁺Although the expressions derived in Section II are not strictly valid for ϵ and large as 0.25, they do give a rough estimate of the actual effects.

where we neglect any variation of parameters for the different m values. If correction by sextupoles placed in the curved sections of the lattice is applied, we obtain

$$J_o = 0 ,$$

$$J_{2m} = -52 \beta k + i4k (\beta_+ - \beta_-), \text{ if } m = 16 \text{ (even)}$$

$$\text{and } J_{2m} = + 4k (\beta_+ - \beta_-), \text{ if } m = 15 \text{ (odd)}$$

which gives

$$J_o = 0 ,$$

$$|J_{2m}| = 290 \sqrt{2} , \text{ if } m = 16 \text{ (even)}$$

$$|J_{2m}| = 290 , \text{ if } m = 15 \text{ (odd)} .$$

If we assume that the sign of ϵ determines the direction of the stability limit in $\frac{\Delta p}{p}$, the stability boundaries can be found from Eq. (II.16), and are

$$\frac{\Delta p}{p} < \frac{4\pi(0.25)}{290} = .011$$

$$\text{and } \frac{\Delta p}{p} > - \frac{(4\pi)(0.75)}{\sqrt{2} (290)} = - .024 ;$$

$$\text{that is, } -0.024 < \frac{\Delta p}{p} < + 0.011 \text{ (no limit on } \beta \text{ growth).}$$

However, one may desire a more stringent requirement, such as a limit on the β function growth. Suppose we set a 25% limit on the β function growth. From Eq. (II.17) we see that this reduces the allowable momentum range available by a factor of 5, leading to

$$- 5 \times 10^{-3} \lesssim \frac{\Delta p}{p} \lesssim + 2 \times 10^{-3} \text{ (limit on } \beta \text{ growth-25%).}$$

If we further reduce the limit on β function growth to 10%, the reduction in this case is by a factor of 11. That is, the available momentum aperture becomes

$$- 2 \times 10^{-3} \lesssim \frac{\Delta p}{p} \lesssim + 1 \times 10^{-3} \text{ (limit on } \beta \text{ growth-10%).}$$

IV. Consequences of Sextupole Corrections

The placement of a sextupole at each quadrupole position in the lattice can remove the chromaticity in both vertical and horizontal planes.⁵ If we define b to be the sextupole strength,

$$b = \frac{1}{2} \times \frac{B'' \times \text{length}}{\text{magnetic rigidity}} ,$$

and k to be the gradient strength,

$$k = \frac{\text{gradient} \times \text{length}}{\text{magnetic rigidity}} ,$$

then the correction strength, b , is

$$b = \frac{k}{2\gamma_p} , \quad (\text{IV.1})$$

where γ_p is the local value of the momentum compaction function.

The use of this correction device in the insertions tends to lead to a non-linear stability limit which is severe. Consider for example the effect of one sextupole placed at a quadrupole whose β function is around 500 m and whose inverse focal length is $\frac{5}{36} \text{ m}^{-1}$. (See the example of Section III.) The unstable fixed point has an amplitude, A_u , given by⁶

$$A_u = \frac{\sqrt{B4\pi} |\Delta\nu|}{\beta_s^{3/2} b} \quad (\text{IV.2})$$

where β_s is the value of the β function at the quadrupole, and $|\Delta\nu|$ is the ν shift from resonance. We have assumed a symmetry of 2, which means that the resonant ν values are those corresponding to every even third integer. Using an average value for β , and finding b from Eq. (IV.1) with a reasonable value for $\gamma_p = 1$ m, we obtain

$$A_u = \frac{\sqrt{20} (4\pi) |\Delta\nu|}{(500)^{3/2} (5/72)} \text{ meters} = 7.2 |\Delta\nu| \text{ cm} .$$

Thus, for a large ν shift of 0.1, we have a nonlinear stability limit of 0.7 cm.

In order to see the effects of sextupole corrections applied to the curved, highly periodic lattice sections, we use the parameters given in the example in

Section III. The required sextupole correction in each cell is twice that given by Eq. (IV.1), since to make $J_0 = 0$, this correction must compensate the contribution to the chromaticity due to the insertions, which in this example is equal to the contribution from the curved lattice sections. Since the phase advance per cell is about $\pi/2$, the effective sextupole strength can be found to be given by⁶

$$b_{\text{eff}} = \sqrt{2} \, b \quad .$$

The stability limit at each quadrupole is therefore,

$$A_u = \frac{4\pi}{\beta_Q} \frac{|\Delta v|}{\sqrt{2} \, b} \quad . \quad (\text{IV.3})$$

With the required b value of 0.14, we have

$$A_u = \frac{2/2\pi}{(40)(0.14)} \frac{|\Delta v|}{\text{meters}} = 150 \, |\Delta v| \, \text{cm} \quad .$$

Thus, for a v shift from resonance of 0.1, the stability limit is 15.0 cm. However, at a value of v which is 0.01 from resonance, the stability limit is reduced to 1.5 cm.

V. Effect of Momentum Error on the Equilibrium Orbit

Consider an insertion which not only matches betatron parameters but also matches the boundary values of the momentum compaction function and its derivative.¹ This matching is valid to first order in $\frac{\Delta p}{p}$. By including the effects of the distortion of the betatron parameters, we can obtain the dominant contribution to the equilibrium orbit of order $\left(\frac{\Delta p}{p}\right)^2$. Including the two-fold symmetry of the model lattice of Section II, we obtain for the equilibrium orbit² near an even integer v value, k ,

$$\Delta x = \gamma_p \frac{\Delta p}{p} + \frac{v f_k}{v^2 - k^2} \left(\frac{\Delta p}{p}\right)^2 e^{ik\phi} \quad (\text{V.1})$$

$$\text{where } f_k = \frac{\sqrt{\beta}}{2\pi} \int_0^c \beta^{\frac{1}{2}} K \gamma_p e^{-ik\phi} ds \quad .$$

The second term in Eq. (V.1) is the deviation from the matched condition and shows resonant behavior at all even integer v values. The amplitude of the

oscillation due to this term is

$$A = \frac{\sqrt{2}}{2\pi} \beta_Q k \gamma_p^Q \left(\frac{\Delta p}{p} \right)^2 \frac{1}{|v-k|} \quad (V.2)$$

or, using $\beta_Q = 40$ m, $k = \frac{5}{36} \text{ m}^{-1}$, $\gamma_p^Q = 1.5$ m, we obtain

$$A \approx 188 \left(\frac{\Delta p}{p} \right)^2 \frac{1}{v-k} \text{ cm} .$$

Thus, for a momentum error (i.e., mismatch) of 1%, and a v deviation from resonance of 0.1, we find

$$A \approx 0.2 \text{ cm} .$$

VI. Conclusions

We have examined some of the effects of low symmetry insertions. The significant point here is that a momentum error causes a mismatching of the insertions to the main, curved sections. Thus resonant conditions which we sought to avoid by matching become visible. Gradient and dipole resonances are inherently excited by the momentum error since we are considering a linear system. By attempting to correct the chromaticity with sextupoles, we have also excited third integer resonances, and we have estimated stability limits for these.

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M. Goldhaber, L.J. Haworth, G.H. Vineyard

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