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NUMERICAL SOLUTION OF THE THREE
DIMENSIONAL BOUNDARY LAYER ON
A SPINNING SHARP BODY AT ANGLE
OR ATTACK

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NUMERICAL SOLUTION OF THE THREE-DIMENSIONAL
BOUNDARY LAYER ON A
SPINNING SHARP BODY AT ANGLE OF ATTACK

by

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Symposium
on
Application of Computers
to
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ABSTRACT

Numerical solutions for the laminar, compressible, three-dimensional boundary layer on general, sharp bodies of revolution at angle of attack with moderate axial spin are described. Unique solutions to the boundary layer equations are obtained by a finite-difference technique which properly treats the elliptic nature of the flow in a crossflow plane. Results are presented and compared with experiment for a tangent ogive-cylinder at four degrees angle of attack with and without spin. The results indicate that viscous flow solutions for spinning bodies at angle of attack can be obtained within the framework of conventional boundary layer theory. These solutions are free from the anomalies observed in a previous investigation.

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NOMENCLATURE

c_p	= specific heat at constant pressure $\text{cm}^2/(\text{sec}^2\text{-}^\circ\text{K})$
d	= diameter of cylinder, cm
f'	= nondimensional tangential velocity ratio, u/u_e
g'	= nondimensional crossflow velocity ratio, w/u_e
k	= thermal conductivity, $\text{gm}/(\text{sec } ^\circ\text{K})$
$\bar{l}$	= nondimensional density-viscosity-product $\rho\mu/(\rho\mu)_r$
M	= Mach number
Pr	= Prandtl number $c_p\mu/k$
r	= perpendicular distance from body axis, cm
Re_x	= local Reynolds number $\rho_\infty V_\infty x/\mu_\infty$
Re_∞	= free stream Reynolds number per cm of length, $\rho_\infty V_\infty(1)/\mu_\infty$
T	= temperature, $^\circ\text{K}$
u,v,w	= velocity components in x,y and ϕ directions, cm/sec
V	= transformed normal velocity, see equation (2d)
$V(u,v)$	= velocity component in x, ϕ plane, see Figure 4
V_∞	= free stream velocity, cm/sec .
x	= distance along body, cm
y	= distance along normal from surface, cm
α	= angle of attack, deg
δ^*	= displacement thickness, cm, see equation (9)
η	= transformed normal coordinate, see equation (1b)

- θ = nondimensional temperature ratio, T/T_e
- μ = viscosity, gm/(cm sec)
- ρ = density, gm/cm
- ϕ = angular circumferential coordinate
- ξ = transformed coordinate along body, see equation (1a)
- Ω = angular spin velocity about body axis, rad/sec

Subscripts

- e = conditions at outer edge of boundary layer
- r = reference conditions
- ∞ = free stream conditions

I. Introduction

The theoretical determination of Magnus forces and moments on spinning bodies at angle of attack requires the solution of the three-dimensional boundary layer about the body.[1] The present analysis treats the laminar, compressible, three-dimensional boundary layer on general sharp bodies of revolution at angle of attack with moderate axial spin. The analysis is complicated by difficulties encountered because of the simultaneous effects of angle of attack and spin.

Most of the three-dimensional boundary layer solutions for bodies of revolution which have appeared in the literature to date have been obtained by resorting to approximate formulations, or are solutions to limiting cases where partial similarity or symmetry reduces the number of independent variables. This includes the first reported solution for a spinning body at angle of attack, a perturbation analysis for a sharp cone by Sedney.[2] Noteworthy exceptions to this trend are the fully numerical solutions. Numerical solutions of the three-dimensional boundary layer equations for nonspinning bodies at angle of attack have been obtained by Der[3] and in the Russian literature by Vvendenskaya[4] and Shevelev[5]. Yet until the present work, it was thought that numerical solution of the spinning problem by conventional three-dimensional boundary layer methods was not possible.[6,7]

The three-dimensional boundary layer equations are classified as equations of the parabolic type. They are elliptic in the direction normal to the surface of the body and hyperbolic in the circumferential

and longitudinal directions.[8] Therefore, these equations pose a boundary value problem in the normal direction and initial value problems in the circumferential and longitudinal directions. However, the introduction of surface spin in the circumferential direction causes a reverse crossflow which seems to simulate the effects of an elliptic-type flow in this direction. In view of these facts two basic approaches to handling the numerical solution of this problem have been proposed.[6,7]

The first is to pose the problem in the crossflow plane at a discrete axial location as an initial value problem in the circumferential direction. The initial values are given by the solution of the boundary layer at the inviscid-flow windward plane of symmetry. This limiting solution can be obtained numerically by independent quasi-two-dimensional analysis for small spin rates.[9] If the derivatives in the boundary layer equations are discretized in the proper manner, a circumferential marching procedure can then be used to obtain the solution for the particular crossflow plane under consideration. The solution progresses longitudinally by successive solution of a series of crossflow planes. This approach was attempted for a spinning sharp cone at angle of attack by Dwyer[6] with somewhat disappointing results. Dwyer's solutions near the leeward plane of symmetry exhibited anomalous behavior which he attributed to nonuniqueness. This led Dwyer to conclude erroneously that solutions could not be obtained within the framework of conventional boundary layer theory. This contention was later supported by Clark[7]

The second approach was suggested by Dwyer[6] as an alternative and is akin to the method successfully applied to a yawed spinning ellipsoid in incompressible flow by Clark.[7] This approach involves retaining in the boundary layer approximation second-derivative Navier-Stokes terms, at least in the circumferential direction. The equations remain, presumably, properly elliptic in the circumferential direction and an elliptic simultaneous numerical solution procedure can be used for each crossflow plane. Marching is then restricted to the longitudinal direction. If second derivative terms in the longitudinal tangential direction are also included, there is no marching at all and with the addition of the normal momentum equation the problem becomes one of solution of the complete Navier-Stokes equations. Clark's [7] solutions were obtained by solving the complete Navier-Stokes equations. The partial as well as the complete Navier-Stokes formulations are susceptible to all the pitfalls one encounters in numerical solution of the Navier-Stokes equations such as stability difficulties, low Reynolds number limitations, and excessive computer time and storage requirements.

It was decided to attempt the solution to the problem via the first (initial value) approach because of its inherent advantages. This approach was well formulated by Dwyer.[6] The anomalies in Dwyer's solutions arise from marching circumferentially in a direction opposite that of the surface crossflow thus violating the zones of influence of the flow. Because there is no plane of symmetry in the three-dimensional boundary layer on a spinning body, it is necessary

to march completely around the body. The surface and inviscid crossflow over some portions of the body are going to be in opposing directions; it would therefore seem impossible to avoid violating zones of influence by marching against the direction of either the surface crossflow or the essentially inviscid crossflow at the boundary layer edge. This apparently dilemmatic state of affairs is illustrated in Fig. 1 which is a typical streamline pattern over a spinning body at angle of attack (See Fig. 2). Figure 1 shows that marching to the right through 360° will result in eventual marching in a direction opposite the inviscid crossflow velocity component, while marching to the left leads to marching in a direction opposite to the surface velocity. Fortunately, proper consideration of the three-dimensional zones of influence does not lead one to despair of obtaining a solution by a suitable difference scheme. For this investigation a three-dimensional difference scheme which will allow some relaxation of the requirement for marching with the flow direction in a crossflow plane was adopted. This second-order, zig-zag scheme was first proposed by Krause.[10] The computational molecule for the method is shown in Fig. 3. The unusual stability property of Krause's method is that, by making use of the upstream crossflow history, it permits some marching against the crossflow without marching opposite the flow direction in the three-dimensional sense. Zones of influence are not violated provided the stability criterion of the method is obeyed.

The zones of influence and dependence of the three-dimensional boundary layer equations at a point on a body are determined by the

limiting streamlines passing through that point.[8] These streamlines are subcharacteristics of the equations.[11] The Raetz influence principle[8] interpreted as the Courant-Frederichs-Lewy (CFL) stability condition restricts the zone of dependence of the differential equations to lie within that of the difference equations. Since the streamlines have the direction of the velocity vector at each point, the CFL condition restricts the orientation of the velocity vector. Figure 4 shows that the velocity vector must lie within a 135° range for stability of Krause's method with equal mesh intervals in planes parallel to the surface. This CFL stability criterion can be obtained more directly from a Von Neumann stability analysis.[10] By adjusting the mesh aspect ratio and marching in the direction of the surface velocity one can, in principle, march against almost any amount of boundary layer crossflow velocity.

The use of Krause's method should be advantageous even on nonspinning bodies since it admits solutions containing crossflow reversal. This type of boundary layer solution is obtained in the vicinity of the leeward plane of a body at angle of attack. The appearance of the vortex structure indicated by this reverse crossflow is associated with the phenomenon of three-dimensional separation.[12]

II. Basic Equations

The nonrotating body-oriented coordinate system shown in Figure 2 was used in the analysis. In this coordinate system the x coordinate is measured along the body surface from the tip, y is the distance along the normal to the surface and, ϕ is the meridional angle which specifies circumferential location. The velocity components in the x, y , and ϕ directions are u, v , and w . The use of this type of simple coordinate system is suitable only for sharp bodies. The present analysis is restricted to sharp bodies because of the complexity of the boundary layer flow on the nose of a blunt spinning body at angle of attack. The three-dimensional boundary layer equations for a perfect gas in the body-oriented coordinate system are given in [13]. These equations are transformed into similarity form to facilitate their numerical solution. The form of the final equations is the three-dimensional generalization of the form used by Blottner[14] in axisymmetric flow computations.

New independent variables are

$$\xi = \int_0^x (\rho \mu)_r (x^*) r^2(x^*) dx^* \quad (1a)$$

$$\eta = \left(\frac{u_e}{2\xi} \right)^{\frac{1}{2}} r \int_0^y \rho dy^* \quad (1b)$$

with the new dependent variables:

$$f' = \frac{u}{u_e} \quad \theta = \frac{T}{T_e} \quad g' = \frac{w}{u_e} \quad (2a,b,c)$$

$$V = \frac{\rho v}{r(\rho \mu)_r} \left(\frac{2\xi}{u_e} \right)^{\frac{1}{2}} + f' \delta r \frac{\partial \eta}{\partial x} + g' \delta \frac{\partial \eta}{\partial \phi} \quad (2d)$$

The transformed equations are as follows:

Continuity:

$$\frac{\partial V}{\partial \eta} + 2\xi \frac{\partial f'}{\partial \xi} + f' \left(1 + \frac{\beta_1}{2} \right) + \delta \frac{\partial g'}{\partial \phi} - \frac{\gamma_1}{2} g' = 0 \quad (3a)$$

x-momentum (tangential):

$$2\xi f' \frac{\partial f'}{\partial \xi} + \beta_1 (f'^2 - \theta) + (V - \bar{\ell}') \frac{\partial f'}{\partial \eta} + \delta \left(g' \frac{\partial f'}{\partial \phi} - \alpha \gamma_1 \theta \right. \\ \left. + \gamma_1 f' g' \right) - \epsilon (g'^2 - \alpha^2 \theta) - \bar{\ell} \frac{\partial^2 f'}{\partial \eta^2} = 0 \quad (3b)$$

ϕ -momentum (crossflow):

$$2\xi f' \frac{\partial g'}{\partial \xi} + \beta_1 f' g' + \beta_2 \theta + (V - \bar{\ell}') \frac{\partial g'}{\partial \eta} + \delta \left(g' \frac{\partial g}{\partial \phi} + \gamma_1 g'^2 - \alpha \gamma_2 \theta \right) \\ + \epsilon (f' g' - \alpha \theta) - \bar{\ell} \frac{\partial^2 g'}{\partial \eta^2} = 0 \quad (3c)$$

Energy:

$$\begin{aligned}
 & 2\xi f' \frac{\partial \theta}{\partial \xi} + \bar{e}_1 f' \theta + \left[v - \frac{1}{c_p} \left(\frac{c_p}{Pr} \bar{l} \right)' \right] \frac{\partial \theta}{\partial \eta} + \delta (g' \frac{\partial \theta}{\partial \phi} + \bar{e}_2 g' \theta) \\
 & - \frac{\theta}{c_p} \frac{u_e^2}{T_e} \left[\beta_1 f' + \beta_2 g' + \alpha \epsilon (g' - \alpha f') + \delta \alpha \gamma_1 f' + \delta \alpha \gamma_2 g' \right] \\
 & - \frac{\bar{l} u_e^2}{c_p T_e} \left[\left(\frac{\partial f'}{\partial \eta} \right)^2 + \left(\frac{\partial g'}{\partial \eta} \right)^2 \right] - \frac{\bar{l}}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} = 0 \quad (3d)
 \end{aligned}$$

In the above equations

$$\beta_1 = \frac{2\xi}{u_e} \frac{\partial u_e}{\partial \xi},$$

$$\beta_2 = \frac{2\xi}{u_e} \frac{\partial w_e}{\partial \xi}$$

$$\bar{e}_1 = \frac{2\xi}{T_e} \frac{\partial T_e}{\partial \xi},$$

$$\bar{e}_2 = \frac{1}{T_e} \frac{\partial T_e}{\partial \phi}$$

$$\gamma_1 = \frac{1}{u_e} \frac{\partial u_e}{\partial \phi},$$

$$\gamma_2 = \frac{1}{u_e} \frac{\partial w_e}{\partial \phi}$$

$$\delta = \frac{2\xi}{r^3(\rho\mu)_r},$$

$$\epsilon = \frac{2\xi}{r} \frac{dr}{d\xi}$$

$$\alpha = \frac{w_e}{u_e}$$

The boundary conditions on equations (3) are at the surface of the body

$$V(\xi, 0, \phi) = 0 \quad (4a)$$

$$f'(\xi, 0, \phi) = 0 \quad (4b)$$

$$g'(\xi, 0, \phi) = \Omega r(\xi)/u_e(\xi, \phi) \quad (4c)$$

$$\theta(\xi, 0) = T_w/T_e(\xi, \phi) \quad (4d)$$

At the outer edge

$$f'(\xi, \infty, \phi) = 1 \quad (5a)$$

$$g'(\xi, \infty, \phi) = w_e(\xi, \phi)/u_e(\xi, \phi) \quad (5b)$$

$$\theta(\xi, \infty, \phi) = 1 \quad (5c)$$

Initial boundary layer profiles must be specified at the tip and at the inviscid-flow plane of symmetry $\phi = 0$. The transformed space domain can be represented as a rectangular box as in Fig. 5. [6] Initial profiles are specified on the front face ($\xi = 0$) and on the vertical cross section perpendicular to the front face ($\phi = 0$). Boundary conditions, equations (4) and (5), are specified at the top ($\eta = \infty$) and the bottom ($\eta = 0$) faces.

A set of two-dimensional equations in η and ϕ which apply at the tip can be derived by taking the limit of equations (3) as $\xi \rightarrow 0$. [6] These equations are the similar boundary layer equations for a nonspinning sharp cone tangent to the body at the tip. Similarly, a set of quasi-two-dimensional equations in ξ and η can be obtained from equations (3) which apply in the plane of symmetry of a nonspinning body. [13] On the spinning body these equations are no longer quasi-two-dimensional.

However, for small spin rates a perturbation technique was used by Watkins [9] to derive sets of quasi-two-dimensional equations for the inviscid-flow plane of symmetry which include effects due to spin.

It should be noted that the present formulation contains the tacit assumption that spin rates are not sufficiently large to induce a significant normal pressure gradient. The usual first-order boundary layer approximation is assumed to be valid. This does not preclude the use of the normal momentum equation to compute the centrifugal contribution to Magnus force.

Krause's method as illustrated in Fig. 3 was applied to the boundary layer equations, equations (3). The differential equations were linearized by Newton iteration[15] For example at the $j + 1$ st iteration the term $f' \partial f' / \partial \xi$ is linearized according to

$$f' \frac{\partial f'}{\partial \xi} = f'^{j+1} \frac{\partial f'^j}{\partial \xi} + f'^j \frac{\partial f'^{j+1}}{\partial \xi} - f'^j \frac{\partial f'^j}{\partial \xi} \quad (6)$$

In using this procedure, an iteration consists of the sequential solution of the difference approximations to equations (3) for all the discrete values of η at fixed points along the body. At each point two iterations were performed. One iteration is usually sufficient in calculations of this type because it is equivalent in accuracy to iteration to completion. The extra iteration here is justified on the basis that iteration is not being performed on a coupled set of difference equations. Instead, the difference equations for each dependent variable are solved sequentially using the most recently computed values of the other variables. A tridiagonal matrix is generated from the difference equations for f' , g' , and θ at a point due to the implicit nature of the Krause method. In the present case, these three equations are only moderately coupled and the sequential procedure was taken to avoid additional matrix operations. Two iterations actually consist of three calculations of the new variables; so only a one-third reduction in time could be achieved by going to one iteration per point.

Krause's method is subject to the following stability criterion[10]

$$-1 \leq \frac{w \Delta x}{u r \Delta \phi} \leq \infty \quad (7)$$

When marching opposite the inviscid crossflow direction, the ratio w/u may be negative; then considering the left-hand limit, the inequality of equation (7) may be rewritten as

$$\Delta x \leq |u/w|_{\min} r \Delta \phi \quad (8)$$

where $|u/w|_{\min}$ is the minimum absolute value of the ratio at the axial location where equation (8) is applied. A problem exists with the stability criterion, equation (8), near the tip since as r tends to zero so does Δx . This difficulty is only encountered when marching against the inviscid crossflow, which is necessary on the spinning body for $\phi > 180^\circ$ as shown in Fig. 1. On the nonspinning body it is not necessary to continue the computation into this region because symmetry dictates that the boundary layer for $180^\circ \leq \phi \leq 360^\circ$ is the mirror image of the boundary layer for $0^\circ \leq \phi \leq 180^\circ$. Therefore, it was decided to approximate the solution in the vicinity of the tip with the nonspinning solution and simply initiate spin at some small distance back from the tip where equation (8) is not as restrictive. This approximation should distort the solution only slightly and only in the immediate vicinity of the tip.

Initial values at the windward inviscid-flow plane of symmetry $\phi = 0$, and at the tip $\xi = 0$ are needed to start the three-dimensional solutions. At the tip of the body these initial conditions are the boundary layer profiles obtained from the solution of the two-dimensional problem in ϕ and η that equations (3) reduce to as $\xi \rightarrow 0$. This two-dimensional problem was solved with the Crank-Nicholson implicit

difference method,[14] The techniques used are very similar to those employed by Dwyer in [16]. There are two significant differences between the present schemes and those of [16]. In the present work Newton iteration was used to linearize the differential equations. Also in the present work it was found that, by using the circumferential derivative of the crossflow velocity in the initial conditions at the plane of symmetry, it was possible to avoid the oscillations which forced Dwyer to use the integral form of the continuity equation.

At the plane of symmetry the quasi-two-dimensional equations of [9] were also solved with the Crank-Nicholson method,[14]. The techniques used are described in [17]. The initial values obtained by this procedure are only valid for small spin rates within the range of validity of the perturbation analysis of [9]. For larger spin rates these solutions are used only as tentative initial values to start the computation. The computer code for solving the three-dimensional boundary layer equations was written to, if desired, take advantage of the periodicity of the boundary layer solution in the circular domain of the crossflow plane. This allows one to continue computation to a full 360° , recomputing the windward plane of symmetry profile. The calculation can then be recycled with the new plane of symmetry profile. The convergence of this scheme was not extensively investigated. However, in the test cases run only a few cycles were required for satisfactory convergence. This continuation may eventually diverge because of the buildup of global truncation error. Such a discussion is probably somewhat academic since in practical terms only a few cycles can be completed

because of the excessive computer time required. The success of the cyclic approach suggests that the initial conditions at the plane of symmetry are superfluous and that the problem is properly posed with only a requirement for periodicity of the solution in the circumferential direction.

IV. RESULTS AND DISCUSSION

Solutions were computed for a 11 cm diameter tangent ogive-cylinder[18] at a Mach number of 3.0 and at 4.0° incidence. The two cases computed were a nonspinning body and a body spinning at $\Omega = 22,600$ rpm. These correspond nominally to the experimental conditions of Vaughn and George [18] for spin parameters ($p = \Omega d/V_\infty$) of 0.0 and 0.2, respectively, with a free stream Reynolds number per cm of 3.28×10^4 . The boundary layer assumptions included $Pr = 0.738$, $\bar{\lambda} = 1$, $n_e = 7$ and $T_w = 300^\circ K$. Inviscid flow solutions for the boundary layer edge conditions were obtained from computations performed at Sandia Laboratories (Albuquerque, New Mexico) using a numerical method of characteristics computer code developed by Rakich.[19]

Thirty-four steps were taken along the body in the axial direction to $x = 27.35$ cm. A variable step size was used; controls were the relative change in independent variables between steps and the stability criterion, equation (8). The computational grid divided the transformed domain into 50 equal intervals normal to the surface and into 32 equal sectors of 11.25° each circumferentially. The three-dimensional spinning calculation consumed 2 hours of IBM 360/50 CPU time.

The effects of spin on the boundary layer profiles at one axial location on the cylindrical portion of the body are illustrated in Figs. 6,7, and 8. The most pronounced spin effects on the tangential velocity and temperature profiles of Figs. 6 and 8 occur in the vicinity of $\phi = 180^\circ$ on the leeward side. The asymmetry of the crossflow

velocity profiles caused by the nonzero surface velocity is shown in Fig. 7(c), which compares profiles for $\phi = 90^\circ$ and $\phi = 270^\circ$.

Displacement thickness at the same axial location is plotted in Fig. 9.

The displacement thickness based on tangential velocity is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \quad (9)$$

Figure 9 clearly reveals the spin-induced asymmetry of the displacement thickness which can contribute significantly to the Magnus-producing imbalance in pressure forces. The equally important centrifugal contribution to Magnus force can be determined by integration of the normal momentum equation using the computed crossflow velocity profile.

The trends of Figs. 6, 8, and 9 agree with the physical explanation given by Vaughn and George in [18]. According to [18], the boundary layer thickness on the spinning body relative to the boundary layer thickness on the nonspinning body at the same location is governed by the history of a typical boundary layer streamline arriving at that location. The effect of spin is to change the path of this streamline. This causes the boundary layer profile at the point under consideration to tend to take on the characteristics of that portion of the boundary layer through which spin diverted the streamline. For example, a streamline arriving at $\phi = 180^\circ$ where the boundary layer thickness is a maximum would tend to be diverted through the thinner portion of the boundary layer by spin. Hence, the $\phi = 180^\circ$ thickness would be expected to be less on the spinning body.

Figure 10 compares the tangential profiles computed at a slightly different axial location with the experimentally based results of Vaughn and George.[18] The agreement is good considering the assumptions made in the present analysis and the approximations made in [18] to reduce the data. The present results were also compared with the semi-empirical relation developed in [18] for the tangential velocity profiles; agreement was to within a few percent.

In conclusion, the results indicate that unique viscous flow solutions can be obtained for spinning bodies at angle of attack within the framework of conventional boundary layer theory. These solutions are free from the anomalies observed in a previous investigation.

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- INVISCID FLOW STREAMLINES
- - - - - SURFACE STREAMLINES WITH SPIN
- . - . - SURFACE STREAMLINES WITHOUT SPIN

FIGURE I. TYPICAL STREAMLINE PATTERNS

FIGURE 2. SPINNING SHARP BODY AT ANGLE OF ATTACK

STABLE FOR: $-1 \leq \frac{w \Delta x}{u r \Delta \phi} \leq \infty$

GLOBAL TRUNCATION ERROR: $O[(\Delta x)^2] + O[(\Delta y)^2] + O[r^2(\Delta \phi)^2]$

FIGURE 3. KRAUSE'S METHOD

NOTATION FOR COMPUTATIONAL MOLECULE: $\longleftrightarrow$ ϕ DERIVATIVES
 $\updownarrow$ x DERIVATIVES
 $\odot$ y DERIVATIVES

FIGURE 4. CFL CONDITION ON VELOCITY VECTOR

INITIAL CONDITIONS ARE SPECIFIED AT
(BY SOLUTION OF TIP EQUATIONS)

$$\begin{aligned} \xi &= 0 \\ 0 \leq \eta &\leq \infty \equiv \tau \\ 0^\circ \leq \phi &\leq 360^\circ \end{aligned}$$

BOUNDARY CONDITIONS ARE SPECIFIED AT

$$\begin{aligned} \eta &= 0 \text{ AND } \eta = \infty \equiv \tau \\ 0^\circ \leq \phi &\leq 360^\circ \end{aligned}$$

TENTATIVE INITIAL CONDITIONS ARE SPECIFIED AT
(BY SOLUTION OF SYMMETRY-PLANE EQUATIONS)

$$\begin{aligned} \phi &= 0 \\ 0 \leq \eta &\leq \infty \equiv \tau \end{aligned}$$

FIGURE 5. INITIAL AND BOUNDARY VALUE PROBLEM
IN THE TRANSFORMED DOMAIN

FIGURE 6. NONDIMENSIONAL TANGENTIAL VELOCITY PROFILES FOR $x=27.35$ CM

FIGURE 7. NONDIMENSIONAL CROSSFLOW VELOCITY PROFILES FOR $x=27.35$ CM

FIGURE 8. NONDIMENSIONAL TEMPERATURE PROFILES FOR $x = 27.35$ CM

FIGURE 9. DISPLACEMENT THICKNESS AT $x = 27.35 \text{ CM}$

FIGURE 10. COMPARISON OF TANGENTIAL PROFILES WITH EXPERIMENT