FEB 18 1964

MASTER

University of California

Ernest O. Lawrence
Radiation Laboratory

MEASUREMENTS OF
THERMALLY INDUCED STRESS WAVES IN
A THIN ROD USING BIREFRINGENT COATINGS

This paper was submitted for publication in the open literature at least 6 months prior to the issuance date of this Microcard. Since the U.S.A.E.C. has no evidence that it has been published, the paper is being distributed in Microcard form as a preprint.

Livermore, California

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Livermore, California

Contract No. W-7405-eng-48

MEASUREMENTS OF THERMALLY INDUCED STRESS WAVES IN
A THIN ROD USING BIREFRINGENT COATINGS

A. L. Austin

January 7, 1964

Measurements of Thermally Induced Stress Waves in a Thin
Rod Using Birefringent Coatings

A. L. Austin

Lawrence Radiation Laboratory, University of California
Livermore, California

January 7, 1964

ABSTRACT — When an elastic body is heated rapidly, significant inertial stresses are developed if the imposed heating rates cause a substantial temperature change in times which are short relative to the mechanical response time of the body. This work describes a method for inducing and measuring the thermally induced elastic stress waves in an unrestrained thin rod. Rapid heating was accomplished electrically by discharging a low-inductance capacitor bank ($0.1 \mu\text{H}$, 2800 J) through the rod. Utilizing the frozen-stress technique, an initial fringe pattern was introduced into thin strips of a birefringent material securely bonded to each side of the rod. The longitudinal strain oscillations were measured by direct observation of the movement of the fringe patterns with a high speed framing camera. Interpretation of these measurements required a dynamic calibration and application of a dynamic correction factor for the reinforcing effect of the coating. Oscillations with periods from 35 to 250 μsec and stress amplitudes up to 9000 psi were measured and compared with the uncoupled thermoelastic theory. Good agreement was obtained, and it was concluded that this technique is suitable for transient measurements in the presence of large magnetic fields which normally restrict the use of electronic methods.

Nomenclature

- A = cross-sectional area, in^2
a = sonic velocity, in./sec
c = specific heat, $\text{Btu/lb. } ^\circ\text{F}$
C = capacitance, μF
E = elastic modulus, lb/in^2
 f_ϵ = strain-optic coefficient, $\text{in./in./fringe/in.}$
 f_σ = stress-optic coefficient, $\text{lb/in}^2/\text{fringe/in.}$
h = thickness of birefringent coating, in.
i = current, amperes
l = half-length of "free-free" rod, in.
L = inductance, henries
N = fringe order
q = heating rate, Btu/sec/in^3
R = electrical resistance, ohms
t = time, sec
T = temperature, $^\circ\text{F}$
V = voltage, volts
W = total weight of heated bar, lb
x,y,z = rectangular Cartesian coordinates, in.
 α = coefficient of thermal expansion, $\text{in./in./}^\circ\text{F}$
 γ = specific weight, lb/in^3
 ϵ = strain, in./in.
 ν = Poisson's ratio
 ρ = mass density, $\text{lb-sec}^2/\text{in}^4$
 σ = stress, lb/in^2
 τ = fundamental period of longitudinal oscillation, sec

Subscripts

x,y,z = x,y,z directions

0 = initial

ST = static

Dyn = dynamic

s = specimen

c = coating

m = maximum

Measurements of Thermally Induced Stress Waves in a Thin
Rod Using Birefringent Coatings

A. L. Austin

Lawrence Radiation Laboratory, University of California
Livermore, California

January 7, 1964

Introduction

The effect of high heating rates on the behavior of mechanical systems is becoming a problem of greater and greater engineering importance in nuclear technology. In addition to problems of temperature gradients and the resulting quasi-static thermal and contact stresses, the designer must also consider the initial inertia effects which govern the response during the rapid temperature change. In order for significant inertial stresses to be developed, it has been shown¹⁻⁴ that the imposed heating rates must cause a substantial temperature change in a structure in times short compared to the mechanical response time. Hence, the ratio of a characteristic heating time to a characteristic mechanical time will govern the amplitude of the dynamic overstress caused by rapid heating. Since the mechanical response times, or periods of free vibration, for structural members such as beams, plates, shells, and rods usually vary anywhere from 10^{-4} to 10^{-1} sec, inertial effects become important for heating times which may vary from less than 10^{-6} sec to as high as 10^{-2} sec. It is evident, then, that even though heating times may be long in an absolute sense, serious design problems may still exist.

A literature review previously performed by the author⁴ indicates that only within the last ten years has considerable attention been given to these problems. Most of this work has been restricted to theoretical developments in the dynamical theory of thermoelasticity and to solutions of problems which illustrate the nature of the theory. Specifically, rigorous investigations have been performed which include the thermomechanical coupling between the elastic and thermal field equations, i. e., where the state of deformation of a body affects the temperature field such that the heat conduction equation is coupled to the equations of equilibrium. However, for most engineering design purposes it has been demonstrated that coupling can be neglected.^{4,5-7} This allows considerable simplification of solutions to dynamic problems in thermoelasticity since the temperature field can be determined independently of the state of stress, or strain. However, even within the scope of this simplification, experimental verifications of theoretical results have been noticeably absent.

To this author's knowledge, there have been only two experimental investigations reported. In 1961, Michaels⁸ investigated the effect of high intensity thermal radiation pulses applied to the end of a cylindrical rod. He successfully demonstrated the existence of the resulting elastic wave and obtained good agreement with the uncoupled theory. Lindholm,⁹ in 1962, designed an arc-image furnace which was used to produce an extremely high, short-duration thermal flux on the surface of a thin spherical cap. His results consisted primarily of subjective observations of the gross effects of the rapid heating, but were generally inconclusive. In view of this situation, it is evident that additional experimentation in the field of thermoelastic dynamics is highly desirable. Hence, the purpose of the present study is to develop and utilize a technique for measuring the inertial stresses induced by rapid, but

uniform, heating of an unrestrained elastic solid. This work consists of observations of the behavior of a long, thin rod subjected to high internal heat generation rates accomplished by discharge of a low-inductance capacitor bank through the rod. The transient axial stress is measured by observation of the movement of the isochromatic fringe pattern in a birefringence coating on the rod. A suitable calibration technique was developed and used, and a one-dimensional analysis provided a theoretical comparison for the experimental measurements. The results of these activities are shown and discussed in the following.

Theoretical Results

The transient stress distribution in a long, unrestrained elastic rod subjected to uniform rapid internal heat generation has been worked out for several types of heat inputs.⁴ Of interest here is the solution for a rod subjected to a very high heating rate such that the temperature rise is uniform along the rod and has the time dependence given by

$$T - T_0 = T_m \left(1 - e^{-t/t_1} \right). \quad (1)$$

The one-dimensional axial stress distribution in the rod of length $2l$ with origin of coordinates at the center with the x -axis coincident with the axis of the bar, was found to be

$$\frac{\sigma(x,t)}{\sigma_{ST}} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \cos\left(n - \frac{1}{2}\right) \frac{\pi x}{l}}{\left(n - \frac{1}{2}\right) \left[1 + 16\pi^2 \left(n - \frac{1}{2}\right)^2 \left(\frac{t_1}{\tau}\right)^2 \right]} \left[\cos \left[4\pi \left(n - \frac{1}{2}\right) \frac{t}{\tau} \right] + 4\pi \left(n - \frac{1}{2}\right) \left(\frac{t_1}{\tau}\right) \sin \left[4\pi \left(n - \frac{1}{2}\right) \frac{t}{\tau} \right] - e^{-t/t_1} \right]. \quad (2)$$

where

$\sigma_{ST} = E\alpha T_m$ = "static" stress if bar were restrained axially and heated slowly to temperature T_m ,

$\tau = 4l/a$ = fundamental period of free longitudinal vibration,

t_1 = heating time constant.

In order to illustrate the physical nature of this solution, the series in Eq. (2) was evaluated with the IBM 650 digital computer at the Lawrence Radiation Laboratory. The results of these calculations are shown in Figs. 1, 2, and 3 where, respectively, the transient axial stress distribution as a function of time, the stress histories at points $x = l/2$ and $x = 0$, and the amplitude dependence upon t_1/τ are shown. The most significant result is the dependence of stress amplitude upon the ratio of a characteristic heating time to the mechanical response time as shown in Fig. 3. If the heating is instantaneous, i. e., $t_1 = 0$, the stress amplitude is a maximum and is given by $E\alpha T_m$. This condition is identical to restraining the rod, heating it slowly to T_m , with free oscillation resulting from sudden release of the ends. As t_1/τ increases, the stress decreases and approaches zero. Hence, the meaning of "rapid heating" is made clear since inertial effects become important only when the heating time is significantly less than the mechanical response time.

It should be pointed out that the theoretical result shown here was obtained by neglecting the thermomechanical coupling as well as damping in the rod. Further, it was assumed that material properties are constant and the motion remained entirely elastic. This was done in order to provide an accurate theoretical comparison for verification of the experimental technique and results. Hence, the temperature changes in the experiment are purposely kept low to insure validity of these assumptions.

Experimental Equipment and Procedures

In experimental studies of the inertial effects of rapid heating the primary difficulty is obtaining very high heat rates which can be applied in short times. Heat sources such as solar energy, plasma jets, and propellents produce high thermal fluxes, but the heating times are necessarily long (about 10^{-2} sec), demanding excessive natural periods of the heated test specimens. It was decided, therefore, to make use of Joule heating by the oscillatory discharge of a capacitor bank through the rod. If the rod cross-sectional dimensions are less than twice the current skin depth, the temperature is nearly uniform until heat transfer away from the bar takes place. In order to obtain short heating times, the electrical system must have very low inductance for minimum energy deposition time and also the capability of high charge voltage to allow high energy storage while maintaining a reasonably low capacitance. In addition, for maximum energy transfer to the test specimen, the circuit resistance must be low. At the Lawrence Radiation Laboratory researchers have designed and built several types of low-inductance, high energy, mobile capacitor banks for use in other activities involving similar requirements. These units have inductances on the order of 10^{-7} H, resistances as low as 10^{-2} Ω , and provide reproducible discharge times. Hence, one of these units was selected for use in this work.

Specimen Geometry and Heating Method

Due to the spatial symmetry exhibited by the theoretical solution (Eq. (2)), the results for a "fixed-free" rod of length l are identical for a "free-free" rod of length $2l$. Hence, the test specimen was chosen to be a rod clamped at one end and submerged at the other in a shallow pool of mercury for the necessary electrical continuity. Heating was accomplished with a Lawrence Radiation Laboratory "Mosquitoette" capacitor bank having a

capacity of 14 μF , inductance of about 0.1 μH , internal resistance of about 0.02 Ω , and a voltage control system for charging to any voltage up to 20 kV and giving a maximum energy storage of 2800 J.

The heating jig is shown schematically in Fig. 4. This arrangement was chosen in order to equalize the magnetic forces on the specimen during discharge, and to allow control of circuit inductance, if necessary, by varying the distance between the return conductor rods. The specimens were fabricated from Type 304 stainless steel, rectangular in cross section with dimensions of 0.05×0.250 and 0.125×0.250 in., with lengths varying from 1.50 in. to 12 in. In order to maximize the $i^2 R$ heating and the time for heat conduction out the ends, stainless steel was chosen for its high electrical resistivity and low thermal conductivity. Also, all other physical and mechanical properties of stainless steel are well known.

Electrical Heating Measurements

Since the system is simply an RLC circuit, the current in the bar is

$$i = \frac{CV_0}{b} (a^2 + b^2) e^{-at} \sin bt \quad (3)$$

where

$$a = \frac{R}{2L}; \quad b = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad (4)$$

In order to calculate the heating time, it is assumed that the current is uniform throughout the specimen, whose dimensions were chosen to be approximately twice the skin depth. Using standard formulae, the skin effect was evaluated, and it was found that the high-frequency resistance was only about 5% higher than the dc value. Hence, the temperature history can be obtained directly from

$$q = i^2 R_s = \gamma C \frac{\partial T}{\partial t}$$

Substituting (3) and integrating with the assumption of constant properties, i. e., small temperature changes, the temperature is

$$T - T_0 = \frac{CV_0^2}{2} \left(\frac{1}{Wc} \right) \frac{R_s}{R} \left[1 - \left(\frac{2a^2}{b^2} \sin^2 bt + \frac{a}{b} \sin 2bt + 1 \right) e^{-2at} \right] \quad (5)$$

For the particular system constants used in this work, it can be shown that the terms in parentheses total nearly unity, giving the close approximation

$$T - T_0 = T_m \left(1 - e^{-t/t_1} \right) \quad (6)$$

where

$$T_m = \frac{CV_0^2}{2} \left(\frac{1}{Wc} \right) \frac{R_s}{R} \quad (7)$$

and

$$t_1 = L/R. \quad (8)$$

Hence, the characteristic heating time can be controlled by the inductance and total circuit resistance of the system, and since the mechanical response time varies as the length of the bar, experimentation at any desired ratio t_1/τ is allowed. Also, within the energy capability of the bank, the heat capacity of the specimen, and the ratio of bar to circuit resistance, the desired test temperature can be selected. With the capacitance, bar resistance, and charge voltage known, only the inductance and total resistance require measurement. This was done with an inductive probe consisting of a small coil wound around a glass tube placed next to the specimen. The current induced in the coil was measured with a dual-beam Tektronix 551 oscilloscope fitted with Type L preamplifiers. The coil output is proportional to the first time

derivative of the current in the bar. Hence, a measurement of the period of oscillation $2\pi/b$ and the logarithmic decrement $-a$ of the probe signal allows a calculation of L and R from Eq. (4). The circuitry is shown in Fig. 5, and sample oscilloscope traces are shown in Fig. 6. The calculated temperature was checked with a thermocouple mounted on the bar. The temperature cooling curve was plotted on semi-log paper, and extrapolation back to zero time came within 5% of the calculated value.

Measurement of Axial Strain in the Heated Rod

Since high voltage electrical heating provided the only convenient method of rapid heating, any electronic measurement of axial strain was complicated by shielding difficulties. The use of strain gages mounted on the bar was attempted, but with bank voltages above 5 kV, arcing occurred and destroyed gage continuity. Hence, some optical means was sought to record the small elastic strains in the bar. Normal "Photostress"¹⁰⁻¹² techniques proved to be too insensitive since the birefringent coating must be necessarily thin, which limits the fringe order and the resultant accuracy. Hence, this technique was modified in the following manner.

A 0.250-in. -thick by 1.00-in. -deep beam of birefringent material (Bakelite ERL 2774) was loaded in pure bending. The resulting fringe pattern was frozen into the loaded beam by holding it at a temperature of 165°F for four hours, and then allowing slow cooling. The stressed beam as shown in Fig. 7 was polished and then carefully cut by hand into 0.050-in. strips which were polished in successive stages by using emery cloth of decreasing grit size mounted on plate glass. As shown in Fig. 8, a strip was bonded to each side of the rod with an epoxy-resin glue. If careful attention was given to the cutting and polishing, the fringe pattern remained unaltered.

With the birefringent coating mounted in this manner it was possible to use incident rather than reflected light, and the initial fringe pattern allowed accurate determination of fractional fringe order changes. Also, by observing the entire fringe pattern, a complete stress distribution over the coating length could be obtained, if desired. This technique is discussed in some detail by Zandman¹³ who, in addition, discusses other interesting variations and uses of the photoelastic stress gage utilizing incident light. In order to observe the fringe pattern motion during, and after, the rapid heating, a Model 601 high-speed framing camera¹⁴ was used with a crossed circular polariscope and white light. The light source consisted of an Edgerton, Germeshausen, and Grier FX-1 Xenon flash tube mounted at the focus of a cylindrical parabolic reflector and powered by the discharge of a bank of nine 1.0- μ F, 4-kV capacitors connected in parallel through 1.0-mH inductances. This system provided approximately 250 μ sec of sufficient light with a 2- μ sec rise time to expose 35-mm Kodak high-speed Ektachrome film with camera speeds of about 350 000 frames/sec at f:23.

Operation Procedure

The experimental apparatus is shown schematically in Fig. 9 with the corresponding block diagram shown in Fig. 10. With the room darkened, the camera was brought up to the desired speed with the mechanical shutter held open manually. Closure of the FIRE switch on the control unit dispatched a 500-V synchronization pulse which triggered the oscilloscopes, discharged a capacitor unit to explode a bridge-wire next to the specimen, discharged the main capacitor bank through the specimen, and triggered the flash-lamp unit. Time delay units were available, but were not needed since the inherent delays in the system proved satisfactory. The oscilloscopes triggered first, at 5 μ sec the bridge-wire and flash lamp discharged, and at 12 to 20 μ sec the

main capacitor bank discharged. Since both the light source and bridge-wire light outputs had rise-times of about 2 μ sec, both appeared on the "zero" frame of the film strip. The oscilloscope traces were recorded with Dumont oscillograph cameras loaded with Type 400 Polaroid film. The inductive probe signal is shown in Fig. 6, and the camera rotor speed was measured with a 535 Tektronix oscilloscope fitted with a Type H preamplifier to record the output of a reluctance pickup mounted on the camera rotor. Several tests were run in this manner; however, some difficulty was experienced with the clamping at the fixed end of the specimen. Hence, separate tests were performed on bars with both ends immersed in shallow mercury pools which closely simulated the condition of "free-free" end conditions. The complete data reduction procedure for two sample runs is given in the following.

Data Reduction Procedure

Since white light was used, the positions of the zero and two first order fringes were always clearly visible. The zero order is always black, and the two first orders are always marked by the passage from red to blue. With 5461-Å green light and black and white film, the entire fringe pattern could have been observed to obtain the stress distribution over the coating length. However, difficulty was experienced in following the zero order from frame to frame on the film strip. Hence, white light and color film were chosen since it was considered sufficient to obtain only the stress at a point to obtain verification of the experimental procedure.

The color film was scanned with a 10X Bausch and Lomb binocular microscope fitted with an "x-y" travelling stage graduated to 0.01 mm. The positions of the zero- and two first-order fringes in the coatings were measured in each frame with respect to the fixed calibration scribe marks on a piece of Polaroid mounted next to the specimen which allowed light to pass

through the crossed polariscope. Also, the known calibration length between scribe marks provided a correction for film shrinkage. Hence, the measurements per frame consisted of six fringe positions and a calibration length, giving a total of seven separate measurements per frame. In order to eliminate lateral bending effects, the measurements of fringe movements in the coatings glued on each side of the specimen were averaged and plotted as a function of time as shown in Fig. 11. Hence, only measurement of longitudinal motion was insured. With these plots, the change in fringe order at the initial position of the zero-order fringe was calculated graphically for as many points in time as desired, and the stress-optic law used to calculate the stress as a function of time. The derivation of the relationships are given in the following section.

Calculation of Stress from Experimental Data

Following the result given previously, the heated rod is assumed to be in a state of uniaxial stress σ_x , with lateral inertial effects neglected. The axial strain at any time in the specimen is

$$\epsilon_{x_s} = \frac{\sigma_{x_s}}{E_s} + \alpha T(t).$$

If the coating, of thickness h , is securely bonded to the specimen, then the strain in the birefringent coating is equal to the strain in the specimen.

Hence,

$$\epsilon_{x_s} = \epsilon_{x_c}$$

and the stress in the coating is

$$\sigma_{x_c} = E_c \epsilon_{x_c} = \frac{E_c}{E_s} \sigma_{x_s} + E_c \alpha T(t)$$

which gives

$$\sigma_{x_s} = \frac{E_s}{E_c} \sigma_{x_c} - E_s \alpha T(t) \quad (9)$$

where the subscripts s and c refer to the specimen and coating, respectively. The observed birefringence N_x in the coating is related to the stress by the stress-optic law

$$\sigma_1 - \sigma_2 = \frac{f_\sigma}{h} N_x$$

where σ_1 and σ_2 are the largest and smallest principal stresses acting in the plane of observation. The stress σ_2 results from inertial loading of the coating by the lateral displacement. A simple calculation using Newton's second law and the lateral acceleration obtained from the theoretical solution showed that σ_2 is very small and can be neglected. Hence,

$$\sigma_1 = \sigma_x = \frac{f_\sigma}{h} N_x$$

and

$$\sigma_{x_s} = \frac{E_s}{h} \left(\frac{f_\sigma}{E_c} \right) N_x - E_s \alpha T(t).$$

Dividing through by $\sigma_{ST} = E_s \alpha T_m$, and dropping the subscripts,

$$\frac{\sigma}{\sigma_{ST}} = \frac{1}{\alpha T_m} \left(\frac{f_\sigma}{E_c} \frac{N}{h} \right) - \frac{T(t)}{T_m}. \quad (10)$$

The average axial strain would result if the bar were heated slowly to T_m , and would be indicated by the birefringence N_{avg} , i.e.,

$$\epsilon_{avg} = \alpha T_m = \left(\frac{f_\sigma}{E_c} \right)_{ST} \left(\frac{N_{avg}}{h} \right). \quad (11)$$

Substituting (11) into (10) gives

$$\frac{\sigma}{\sigma_{ST}} = \frac{\left(f_{\sigma/E_c}\right)_{Dyn}}{\left(f_{\sigma/E_c}\right)_{ST}} \left[\frac{N}{N_{avg}} \right] - \frac{T(t)}{T_m} \quad (12)$$

The relation between the stress-optic coefficient and the strain-optic coefficient is

$$\frac{f_{\sigma}}{E_c} = \frac{f_{\epsilon}}{1 + \nu_c} \quad (13)$$

and since Poisson's ratio is assumed to be rate independent, substitution of (13) into (12) gives*

$$\frac{\sigma}{\sigma_{ST}} = \frac{\left(f_{\epsilon}\right)_{Dyn}}{\left(f_{\epsilon}\right)_{ST}} \left[\frac{N}{N_{avg}} \right] - \frac{T(t)}{T_m} \quad (14)$$

If $T(t)$ is given by (6), the stress in the bar can be calculated from the fringe movements shown in Fig. 11.

As pointed out by Goldsmith,¹⁵ the strain-optic coefficient may vary appreciably with the rate of loading, and any interpretation of photoelastically obtained data must rely upon a calibration at substantially the same rates.

*In order to account for shear stresses in the coating, Zandman¹³ has provided the correction factor

$$N^2 / \sqrt{N^2 + 4h^2(\Delta N/\Delta x)^2}$$

However, observations of the fringe patterns have shown that $4h^2(\Delta N/\Delta x)^2$ is small relative to N^2 . Hence, (12) was used directly to calculate the stress from measured values of birefringence.

Hence, in order to evaluate the ratio of the dynamic to the static strain-optic coefficient in (14), a calibration was performed on the test specimens. This was done by clamping the rod at its upper end and loading it with a weight suspended by a thin wire fastened at the lower end as shown in Fig. 12. Sudden release of the weight was accomplished by using the capacitor bank to explode the lower portion of the wire. The resulting longitudinal oscillations of the rod were recorded photoelastically in the same manner as previously described. An independent measurement was obtained simultaneously from two 350- Ω semiconductor strain gages mounted on the uncoated sides of the bar next to the birefringent coating. These gages were series connected, to eliminate lateral bending strains, in a simple voltage divider circuit with the output recorded on a Tektronix 535 oscilloscope fitted with a Type H preamplifier and a Dumont oscillograph camera loaded with Type 400 Polaroid film. A sample of the data obtained in this manner is shown in Fig. 13. Since good agreement was obtained in these calibrations, it was concluded that the strain-optic coefficient was essentially constant for this frequency range and that the ratio of the dynamic to static coefficients in (14) could be taken as unity. Hence, all data were reduced by the relationship

$$\frac{\sigma}{\sigma_{ST}} = \frac{N}{N_{avg}} \left(1 - e^{-Rt/L} \right), \quad (15)$$

where (6) has been substituted for the temperature function $T(t)$, since no experimental method for measuring temperature rise during the rapid heating was available.

Actually, the coating reinforces the steel bar and a correction for this must be applied. Zandman, Redner, and Riegner¹⁶ have worked out correction factors for reinforcing effects, but confined their analysis to quasi-static

stress states. However, for dynamic loading the inertia as well as the stiffness of the coating must be considered. Hence, the analytical procedure of Zandman et al. was modified to give an approximate dynamic correction factor. The results of this analysis¹⁷ showed that only the period of oscillation is affected such that the period τ in the theoretical solution (2) must be replaced with the corrected period τ' , where

$$\tau' = \sqrt{\frac{1 + A_c \rho_c / A_s \rho_s}{1 + A_c E_c / A_s E_s}} \tau. \quad (16)$$

Hence, with N and N_{avg} measured from the film record and R/L obtained from the inductive probe record, (15) was used to determine the inertial stress. This was compared with theory given by (2) modified by (16). The stress σ_{ST} is obtained from a calculation of T_m from (7).

Experimental Results and Discussion

The experimental results were obtained from tests on four bars of different sizes with two of the bars each having two different coating sizes. All bars were fabricated from the same stock of 304 stainless steel, and all coatings cut from the same piece of Bakelite. The bars were rapidly heated to cause temperature changes of approximately 30°F, and bar lengths varied to provide a range of heating ratios t_1/τ from 0.05 to 0.28. The test parameters are shown in Table I and sample data are shown in Figs. 14 and 15. A comparison of test results to the theoretical prediction for the dependence of the initial compressive stress amplitude upon the heating ratio t_1/τ is shown in Fig. 16.

For the "fixed-free" specimens, agreement between the experimental data and theory is good over the first half cycle of response, but gradually decreases with time. Since this subsequent disagreement appears as a deviation in amplitude only, it was apparent that some damping effect was present.

A separate calculation indicated that within the time of measurement no heat was transferred to the coating so that its temperature remained essentially constant at the initial value, and the calibration indicated that accurate measurement of the dynamic strains were obtained. Hence, it was evident that the damping was the result of imperfect clamping of the rod. This was investigated experimentally by repeating the calibration procedure to observe many cycles of oscillation. The strain-gage records so obtained were reproducible, and indicated that damping was present. Hence, a damping factor, or logarithmic decrement, was calculated from these measurements and applied to the theory. The envelope of this exponential decay is shown by the dotted lines superposed over the plots in Fig. 14. The excellent agreement with the experimental amplitudes obtained from the coating indicates that the bar is not behaving as predicted by the analysis, which excluded damping considerations, and that the photoelastic coating provides accurate measurements of the dynamic strains in the bar.

In order to further verify this, a separate test was performed on a bar which was essentially free at the ends. With some difficulty, end supports were designed which did not restrict motion of the bar. This was done by supporting the ends with Styrofoam cushions mounted in suitable copper electrodes. Mercury was used to achieve electrical continuity, and the entire assembly was mounted in the heating jig which had been rotated 90° in the plane of the return conductor rods. The specimen was cut from the 12-in. rod such that the coating was at the center ($x/l = 0$) of a 10-in. rod. The results of this test are shown in Fig. 15, which shows the excellent agreement with theory, and indicates that during the time of measurement very little damping was present. The difference of wave shape is due to the different location of the strain measurement, i. e., $x/l = 0$ instead of $x/l = 0.5$. This is also shown theoretically in Fig. 2.

Since the coating itself might induce some damping, specimens of the same size but with different coating sizes were tested. Also, a separate calibration was performed in which a bar was loaded as previously mentioned, but with and without a coating. The strain-gage records from these tests showed that no significant difference in amplitude occurred. Hence, it was concluded that the coatings used in this work introduced no observable damping. It was found, however, that the coating did affect the frequency of oscillation. The experimental data was first compared with theory given by Eq. (2) and it was found that periods of oscillation differed. Hence, the correction factor given by (16) was derived and applied to (2). The comparison between the data and the corrected theory shown in Figs. 14 and 15 illustrate the excellent agreement.

Seven different tests were run with ratios of heating time to mechanical response time varying from 0.05 to 0.28. The results of these tests are shown in Fig. 16 where the maximum initial compressive stress in the rod is plotted as a function of t_1/τ . Because of clamp damping, these results should logically lie below the predicted values. The only large disagreement occurs at $t_1/\tau = 0.05$, and is probably due to the inability of the clamp to provide zero displacement at the fixed end of the 12-in. specimen used in this run. Calibration tests on the larger specimens also indicated that erratic behavior occurred for specimen lengths greater than 10 in. A photograph of the specimens is shown in Fig. 17 which includes the calibration bar with the strain gages attached.

Concluding Remarks

In view of these experiments, it is evident that the photoelastic coating technique can provide accurate measurement of the dynamic stress distribution in rapidly heated specimens. In this work only the stress at a point was

measured, but this technique can provide measurements of the stress distribution over the coating length, and could provide measurements over the entire length of the specimen by spacing coatings along the bar. The only major difficulty arises in carrying out the multitude of measurements involved in using more than three fringes in each coating. In addition to the problem of following the zero-order fringe when black and white film is used, observation of a complete fringe pattern would be an extremely tedious task. Visual scanning of only three fringe movements resulted in 7 measurements per frame, or a total of about 500 separate measurements for a complete film record.

The major advantage of the birefringent coating technique is that since it is basically an optical technique, measurements made in short time intervals are independent of environmental conditions, particularly magnetic fields, which normally restrict the use of electronic methods. Hence, this method is useful for studies of the response of bodies subjected to rapid heating. In this work, temperature changes were kept below 30 °F, but additional experimentation was performed in which the coated specimens were rapidly heated to as high a temperature as possible with the 2800-J capacitor bank, and it was found that for temperature changes exceeding 400 °F, the coatings did not remain bonded. No actual photographs were taken to determine the fringe movements at the higher temperatures, but examination of the fringe pattern and glue joint after heating showed no appreciable change for rapid temperature changes up to 400 °F. A heat transfer analysis indicated that the temperature of the coating at a distance of only 0.005 in. from the heated bar remains at the initial ambient value for at least 1 msec. Hence, even for very high rapid temperature changes, accurate transient measurements are possible since significant heat transfer to the coating requires milliseconds. However, for

temperature changes above 400°F, difficulty would be encountered in maintaining a suitable bond.

The primary restriction is that accurate measurement of the transient response during the heating depends upon an independent measurement of the temperature rise. In this work the temperature rise $T(t)$ in Eq. (14) was calculated from an integration of the $i^2 R$ heating resulting from discharge of the capacitor bank since, to this author's knowledge, no successful temperature measurement device with microsecond response time has been developed for use in the presence of large magnetic fields. It is of interest to note that the photoelastic coating technique may have possible use to determine transient temperatures by measuring the fringe order with time and using the strain-optic law for calculation of the thermal strain. However, the temperature dependence of the coefficient of thermal expansion of the heated body and the dynamic strain-optic coefficient of the coating must be known in advance. The latter could easily be determined by a calibration at the desired frequencies in the manner discussed previously.

Acknowledgments

The author wishes to express his appreciation to Professors W. H. Giedt and J. Frisch of the University of California at Berkeley for their encouragement and many stimulating discussions which added significantly to this research. This work was done at the Lawrence Radiation Laboratory, Livermore site, under the auspices of the U. S. Atomic Energy Commission. Among the members of this organization, grateful acknowledgment is extended to M. D. Martin for his support and continued interest in this work. Special thanks are given to J. D. Lawrence who carried out the programming for machine computation of the theoretical results, to J. Stone for his advice concerning the preparation of the birefringent coatings, and to the staff of the Technical Information Division for preparation of this manuscript.

References

1. Boley, B. A., "Thermally Induced Vibrations of Beams," J. Aeron. Sci., 23, (2), 179-181, (1956).
2. Boley, B. A., and Barber, A. D., "Dynamic Response of Beams and Plates to Rapid Heating," J. Appl. Mech., 24, (3), 413-416, (1957).
3. Burgreen, D., "Thermoelastic Dynamics of Rods, Thin Shells, and Solid Spheres," Nucl. Sci. Eng., 12, 203-217, (1962).
4. Austin, A. L., "Transient Stress Distribution in a Rapidly Heated Rod," Univ. of Calif., Inst. of Eng. Res., Tech. Rpt. HE-150-210, Series 128, Issue 11, Feb. 14, 1963. (Dr. Eng. Thesis, Berkeley, Calif.)
5. Boley, B. A., and Weiner, J. H., "Theory of Thermal Stresses," John Wiley & Sons, Inc., New York, (1960). Chapters 1 and 2
6. Chadwick, P., and Sneddon, I. N., "Plane Waves in an Elastic Solid Conducting Heat," J. Mech. Phys. Solids, 6, 223-230, (1958).
7. Lockett, F. J., "Effects of Thermal Properties of a Solid on the Velocity of Rayleigh Waves," J. Mech. Phys. Solids, 7, 71-75, (1958).
8. Michaels, J. E., "Thermal Impact — The Mechanical Response of Solids to Extreme Electromagnetic Radiation," Planetary and Space Sci., 7, 427-433, (1961).
9. Lindholm, U. S., "Effects of Thermal Impact on a Thin Spherical Cap," EXPERIMENTAL MECHANICS, 2, No. 10, pp. 303-306, (Oct., 1962).
10. Mesnager, M., "Sur la Determination Optique des Tensions Interieures dans les Solides à Trois Dimensions," Comptes Rendes, 190, p. 1249, (1930).
11. D'Agostino, J., Drucker, D. C., Liu, C. K., and Mylonas, C., "An Analysis of Plastic Behavior of Metals with Bonded Birefringent Plastic," Proc. of SESA, XII (2), pp. 115-122, (1955).

12. Zandman, F., and Wood, M. R., "Photostress," Prod. Eng., pp. 167-178, (Sept., 1956).
13. Zandman, F., "Concepts of the Photoelastic Stress Gage," EXPERIMENTAL MECHANICS, 2, No. 8, 225-233, (Aug., 1962).
14. Brixner, B., "A High-Speed Rotating-Mirror Frame Camera," J. Soc. Motion Picture Television Engs., 58, pp. 503-511, (Dec., 1952).
15. Goldsmith, W., "Dynamic Photoelasticity," Coll. on Experimental Techniques in Shock and Vibration, Appl. Mech. Div. ASME, Nov. 1962, pp. 25-54.
16. Zandman, F., Redner, S. S., and Riegner, E. I., "Reinforcing Effects of Birefringent Coatings," EXPERIMENTAL MECHANICS, 2, No. 2, 55-64, (Feb., 1962).
17. Austin, A. L., (to be published).

TABLE I — TEST PARAMETERS

Run	Specimen Size ^a	Coating Size ^b	t_1 μsec	τ' μsec	x/l	End Conditions	Capacitor Voltage (kV)	T_m (°F)	σ_{ST} (psi)
1	0.125 × 0.25 × 12.0	0.04 × 1.0	13.2	254	0.50	Fixed-Free	14.1	35	9300
2	0.125 × 0.25 × 5.00	0.03 × 0.56	9.4	105	0.50	Fixed-Free	10.3	23	6100
3	0.050 × 0.25 × 2.47	0.05 × 1.0	7.1	57.7	0.45	Fixed-Free	5.2	35	9450
4	0.050 × 0.25 × 1.46	0.03 × 0.657	7.45	32.6	0.38	Fixed-Free	4.2	27	7400
5	0.050 × 0.25 × 1.46	0.03 × 1.0	9.2	32.6	0.38	Fixed-Free	4.7	25	6800
6	0.125 × 0.25 × 10.0	0.035 × 1.0	11.1	105	0	Free-Free	12.6	22	6300
7	0.125 × 0.25 × 5.00	0.05 × 1.0	11.0	106	0.50	Fixed-Free	10.2	26	7000

^aAll specimens were fabricated from Type 304 stainless steel. Sizes given in inches.

^bAll coating widths were 0.25 in.

 $\tau = 4l/a = \text{NATURAL PERIOD}$

$$\sigma_{ST} = E\alpha T_M$$

LEGEND

--- $t_1/\tau = 0$
 — $t_1/\tau = 0.10$

GLL-6310-2460

Fig. 1. Elastic stress-distribution sequence in a rapidly heated rod.

Fig. 2. Stress histories in a rapidly heated rod.

GLL-6310-2461

Fig. 3. Variation of stress amplitudes with ratio of heating period to fundamental period.

Fig. 4. Heating jig.

Fig. 5. Inductive probe circuitry.

RUN 2

Current

0.5 volt/cm. —

Voltage

2 volts/cm. —

10 μ secs.

RUN 6

Current

0.5 volt/cm. —

Voltage

2 volts/cm. —

Bridge wire Fiducial

GLB-6310-5349

Fig. 6. Output of the inductive probe.

GLB-6310-5348

Fig. 7. Fringe pattern in prestressed beam.

Fig. 8. Specimen preparation.

Fig. 9. Experimental apparatus.

Fig. 10. Block diagram.

Fig. 11. Observed fringe movements.

Fig. 12. Dynamic calibration of photoelastic coating.

Fig. 13. Comparison of photoelastic coating data with strain gage data.

GLL-6310-2432A

Fig. 14. Stress history at a point.

GLL-6310-2431A

Fig. 15. Stress history at a point.

Fig. 16. Variation of initial compression amplitude with the ratio of heating period to fundamental period.

GLB-6310-5347

Fig. 17. Specimens.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission " includes any employee or contractor of the commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.