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On the Equivalence of Regge and Optical Models

Archibald W. Hendry*

Physics Department, Indiana University

Bloomington, Indiana 47401

Abstract:

We suggest that the Regge model is equivalent to an optical model. We show that the effect of combining a Regge pole and destructive cut is to imitate a direct channel Regge pole; the rotating Regge part is "dual" to scattering from the edge of the interaction region. We interpret Schmid loops. Regge energy dependence is related to the energy independence of the interaction radius.

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In low energy collisions, the most characteristic phenomenon is the excitation of the interacting particles from their ground states to higher excited levels.¹ Above these energies, say $p_{lab} \geq 3\text{GeV}/c$, what is the appropriate description for the scattering process?

So far, two apparently different types of model have been shown, by explicitly fitting a large amount of data, to work. The first and more familiar type is based on exchanges in the crossed t , u channels. Data fitting requires both poles and cuts. The second type is based on a direct channel picture, its original appeal being its ability to account simultaneously² for the three well-known structures, at $t = 0$, $-.6$ and $u = -.2$ $(\text{GeV}/c)^2$, in the π^+p CEX differential cross section. The usefulness of such a model has subsequently been demonstrated³ by fits to data at all angles for π^+p elastic and CEX in energy range $p_{lab} = 2.74 - 5.0$ GeV/c . It was shown that there were two dominant parts to the scattering: (i) diffraction, arising from the partial absorption of all partial waves with impact parameter $b \leq 0.8F$, and (ii) scattering of those partial waves with angular momenta centered around $(j_c + \frac{1}{2}) = kb_c$ where b_c was determined by the fits to be about $0.8F$ at all energies. The fact that b_c occurs at just the place where the absorption suddenly falls off clearly indicates the overall consistency of the picture.

The question therefore arises, are the two apparently different models somehow equivalent? In the present paper, we try to indicate that this is, at least partly true, by showing that the rotating Regge term arising from pole plus cut corresponds to the scattering from the $0.8F$ edge of the interaction region in the s -channel picture. (The partial waves for $b \leq 0.8F$ which, for example, give rise to diffraction in elastic scattering are different in the two models; see Ref. 3 for further discussion.)

However, before proceeding with the detailed analysis, we comment first on the appropriate parametrisation of the s-channel partial waves which get scattered at the edge of the interaction region. In Ref. 3, their contribution was taken to have the form (ignoring spin)

$$(r/2)/(l-l_c - i r/2) \quad (1)$$

where l_c was shown to have the energy dependence $l_c = kb_c$, with b_c energy independent and having the value 0.8f. This Breit-Wigner form will clearly give the required enhancement of the peripheral band of partial waves. However, a more important property is that it is a pole in the angular momentum plane for the s-channel process. In all classical descriptions of scattering of grazing rays (e.g. surface or creep waves, or the glory effect in optics) such a form is the appropriate one.⁴

In order to make a comparison between the two types of model, we shall partial wave analyse typical Regge terms and see if they in any way reproduce the characteristics of the s-channel approach. For a linear trajectory $\alpha(t) = \alpha_0 + \alpha' t$, we have that

$$-s^{\alpha(t)} = \sum_{\ell} (2\ell+1) a_{\ell} P_{\ell}(\cos \theta_s) \quad (2)$$

$$-e^{-i\pi\alpha(t)} s^{\alpha(t)} = \sum_{\ell} (2\ell+1) c_{\ell} P_{\ell}(\cos \theta_s) \quad (3)$$

where a_{ℓ} , c_{ℓ} are spherical Bessel functions with complex argument; they depend upon the energy s as well as on the trajectory parameters α_0 , α' .

We now plot a_{ℓ} , c_{ℓ} as functions of the impact parameter $b = \ell/k$ (in fermis) for various energies. Figs. 1a, 2a, 3a show a_{ℓ} , $\text{Re } c_{\ell}$, $\text{Im } c_{\ell}$ respectively with one exchanged pole, the trajectory being taken as $\alpha(t) = \frac{1}{2} + t$; Figs. 1b, 2b, 3b show the more realistic case with a pole plus cut.⁵

From Figs. 2b, 2c, it is easy to interpret the rotating Regge term: its Breit-Wigner characteristics clearly reflect a (Regge) pole in the s-channel angular momentum plane. Moreover, we see that at all energies this

pole remains at a fixed position $b \approx 0.8F$ in the impact parameter space. This distance $0.8F$ is determined by the slope α' of the crossed pole trajectory and corresponds to $\alpha' = 1$; the property of "universality of slopes" for leading meson and baryon exchanges is therefore just another way of saying that the interaction region has constant radius $b = 0.8F$.

An alternative way of looking at this is to examine each c_l as a function of energy. Fig. 4a shows the Argand diagrams for the first 10 partial waves for $p_{lab} = 3-10$ GeV/c with only the pole term present. The Schmid loops⁶ are clearly visible; these have led to speculations about resonances at high energies and infinitely rising trajectories. The situation becomes much clearer in Fig. 4b with a pole plus cut combination (the same as in Figs. 2b, 3b). It is easy to check that for $l \geq 5$, each partial wave resonates (i.e. crosses the imaginary axis) at just the energy where $l = kb_c$ is satisfied, with b_c constant at about $0.8F$. Thus Schmid loops (from pole plus cut) do not correspond to the same type of liquid drop resonances as occur at low energies, but are just the reflection of resonant scattering from the $0.8F$ edge of the interaction region.

It is also of interest to examine the origins of the $\frac{d\sigma}{dt}$ dips in the two models, e.g. at $t = -.6$ (GeV/c)². With a dominant band of s-channel angular momentum states at the surface, one immediately gets² dips in the "right" positions from the oscillations of the d-functions; there is no need for extra nonsense factors. In the crossed channel model, one has a zero arising in the odd-signature combination of (2), (3) when $\alpha(t) = 0$, but not in the even signature one. However, we see from Figs. 1a, 2a that in the odd-signature combination, the low b components are of opposite sign and more or less cancel, thus giving rise to something like Fig. 2b. One therefore sees that the odd signature combination tends to be more like the s-channel picture which picks out a band around $0.8F$, whereas in the even

signature combination no such cancellation takes place and the combination is swamped by low b components.

We conclude therefore that the present treatment of particle scattering with exchanges in the crossed channels (t, u channel Regge poles, modified by cuts) is substantially equivalent⁷ to the simpler s -channel picture of having an interaction region of definite radius, inside of which there is substantial absorption and at the edge of which there are strong surface effects. We have shown that the rotating Regge part corresponds to surface scattering, that scattering taking place at a distance of about $0.8F$ which remains the same at all intermediate energies. We interpret Schmid loops as reflections of these surface waves. Moreover the dips in $\frac{d\sigma}{dt}$ come about in a somewhat similar way; the crucial feature of all the crossed-channel models is that through the parametrisation of their exchanges they essentially reduce the low b component and pick out a band centered around $b = 0.8F$, which indeed is the s -channel picture.

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5. We have taken pole + c *cut, with the cut of the form (2), (3) and $\alpha_c(t) = \frac{1}{2} + \frac{1}{2}t$. The coefficient $c = -0.4$ is taken sufficiently negative to make the interpretation clearer. The effect of a cut is to reduce the lower partial waves.
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Figure Captions

- Fig. 1. Partial wave analysis of $s^{\alpha(t)}$ for (a) pole alone (b) pole plus cut, against impact parameter $b(F)$.
- Fig. 2. Partial wave analysis of $\text{Re}(e^{-i\pi\alpha(t)} s^{\alpha(t)})$ for (a) pole alone (b) pole plus cut.
- Fig. 3. Partial wave analysis of $\text{Im}(e^{-i\pi\alpha(t)} s^{\alpha(t)})$ for (a) pole alone (b) pole plus cut.
- Fig. 4. Argand diagrams for partial waves $l = 0, 1, \dots, 9$ of $e^{-i\pi\alpha(t)} s^{\alpha(t)}$ for (a) pole alone, (b) pole plus cut. The cross-slashes correspond to $P_{\text{lab}} = 3.0$ (1.0) 10.0 GeV/c.

Fig. 1

Fig. 2

Fig. 3

Fig. 4a

Fig. 4b