LASER FUSION AND PARAMETRIC INSTABILITIES

by

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Recently there has been great interest in the possibility of producing a thermonuclear reactor by means of irradiating a DT pellet with an intense pulsed laser. The laser light produces strong heating of electrons through possibly rather complex interactions with the plasma. It is our purpose in this Comment to discuss some plasma physics problems which determine the effective coupling between laser and plasma. To show the importance of such effects we first give a brief review of the basic compression physics.

The absorbed laser energy creates very high temperatures and pressure in the outer plasma regions. This leads to a blow-off of hot plasma whose pressure compresses the cold interior. Extensive numerical hydrodynamic [1,2] calculations indicate that if the profile of laser power versus time is chosen properly, then the core may be compressed to extremely high densities limited only by the small electron degeneracy pressure. Densities of the order of 10^4 times normal density have been

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"observed" in such computations. Fig. 1 shows a schematic density profile. Note that the light absorption takes place at very low densities $\omega \gtrsim \omega_p$. For a CO_2 laser $n = 10^{19}$ for $\omega = \omega_p$, where ω_p is the plasma frequency $(\frac{4\pi n e^2}{m})^{\frac{1}{2}}$. The crucial trick in obtaining high compression is to maintain the compressed material at low entropy so the thermal pressure is not too high compared to the zero-point pressure. This requires a carefully shaped rising profile of laser power versus time. A constant laser power would send in a strong shock which, with spherical convergence and reflection, could only yield compressions of about 50. A high degree of spherical symmetry must also be maintained to achieve high compression.

Now the $n\tau$ value which governs thermonuclear reaction production is easily computed, where the confinement time τ is given roughly by the compressed radius divided by sound speed. Suppose that we have a laser producing energy E , which is able to couple with efficiency ϵ into compressing and heating the high density core, and that we wish to produce thermonuclear energy, ZE . Energetic balance [2] then leads to the conclusion that if we optimize all factors, the laser output E must exceed a critical value:

$$E_c = \left(\frac{n}{n_0}\right)^{-2} \frac{Z^3}{\epsilon^4} \text{ megajoules .}$$

The factor Z must allow for laser efficiency and electrical conversion efficiency and is probably at least 10. Numerical calculations

indicate that purely hydrodynamic efficiencies of coupling the hot blow-off plasma to the cold interior yield $\varepsilon \approx .1$. If the laser-plasma interaction is imperfect, a further decrease in ε will occur, leading to unreasonably large laser energy requirements. Thus we must turn our attention to the mechanism of light absorption by the plasma.

The dispersion relation for an electromagnetic wave in a plasma is given by $\omega^2 = k^2 c^2 + \omega_p^2 (1 + i \frac{\nu}{\omega})$, where ν is the electron-ion collision frequency. This tells us 1) that the wave as it enters the plasma will be reflected when it reaches a density $\omega = \omega_p$, and 2) that under conditions of interest for laser fusion (electron temperatures of many Kev) the collision frequency in hydrogen is such that absorption will be negligibly small. Note that the radiation pressure is far too small for economical implosion--the energy must be absorbed by the plasma.

Fortunately collective processes, in particular the three-wave decay [3] process provide strong laser-plasma interaction and the possibility of absorption, by decay of the laser photon into a plasmon and a phonon [4]. The conditions for decay of a wave of frequency ω_0 and wave number k_0 into two other waves in an infinite homogeneous plasma are the familiar sum rules for energy and momentum of the wave "quanta" $\omega_0 = \omega_1 + \omega_2$ and $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$. Further, conservation of action (number of quanta) requires $|\omega_0| > |\omega_1|, |\omega_2|$. A detailed example will be given in the Appendix. Here we note however that the possibility of decay of the laser photon into a Langmuir plasma oscillation and an ion acoustic

mode (occurring when the photon has penetrated to plasma density such that ω_p is slightly less than ω) provides a very efficient mechanism for anomalous absorption, as has been discussed by many authors, and verified experimentally in microwave and laser experiments as well as in ionospheric heating. Moreover at 4 times smaller densities the decay of laser photons into two plasmons can occur.

What is the result of such absorption? From the point of view of linear theory the plasmons can propagate to a region of lower density where Landau damping converts their energy into electron thermal energy. However, in fact for the laser intensities characteristic for laser fusion strong nonlinear effects occur. These have been studied both analytically [5] and by numerical simulation [6]. Qualitatively what is observed is that the plasma oscillation grows to a large amplitude leading to electron trapping at the phase velocity of the fastest growing mode, about $10 v_{the}$. Thus the parametric instability tends to convert the laser light into very high energy electrons, which raises further difficulties since the very long mean free path of such electrons makes them couple only very weakly with the main thermal plasma, a problem discussed by Kidder [7].

Additionally they may penetrate into and preheat the core raising its entropy and making further compression difficult. Thus one interesting area on which plasma physicists might speculate is possible nonlinear mechanisms for thermalization of these electrons. Some preliminary estimates indicate that the simple 2-stream instability should probably

not occur [8], as the non-thermal distribution tends to be monotonically decreasing in energy.

There is another important reason that the parametric instability may not provide the desired efficient coupling between laser and plasma--namely that other forms of parametric decay instability may occur at lower densities--i.e. before the photon reaches the density $\omega \sim \omega_p$ at which conversion to plasmons and acoustic waves occurs. In Table I we list the various types of possible parametric conversion of photons (numerical factors omitted):

TABLE I

Decay Products	Plasma Density	Approximate Growth Rate	Approximate Inhomogeneity Threshold
Plasmon, Acoustic	$\omega_p = \omega$	$\alpha \frac{c}{v_e} \sqrt{\omega \omega_a}$	$\alpha^2 \frac{c^2}{v_e^2} k_a L > 1$
2 Plasmon	$\omega_p = \frac{\omega}{2}$	$\alpha \omega_p$	$\alpha^2 \frac{c^2}{v_e^2} kL > 1$
Plasmon, Photon	$\omega_p \geq \frac{T_e}{mc^2} \omega$	$\alpha \sqrt{\omega \omega_p}$	$\alpha^2 kL > 1$
Acoustic, Photon	$\omega_p \geq \frac{T_i}{mc^2} \omega$	$\alpha \left[\frac{\omega_p}{\omega} \frac{c}{v_e} \right] \sqrt{\omega \omega_a}$	$\alpha^2 \frac{c^2}{v_e^2} kL > 1$

Here v_e is electron thermal velocity, ω and k the photon frequency and wave number, ω_a and k_a the acoustic wave frequency and wave number, $\alpha = \frac{e \mathcal{E}}{m \omega c}$ with $\mathcal{E}$ the laser electric field and $L = \left(\frac{1}{n_0} \frac{dn_0}{dx} \right)^{-1}$.
 Note
 that the last two entries in the table represent parametric production of another photon which is in fact primarily backscattered, i. e. reflected and lost. Note further that these backscatter reactions occur at lower density than the others, even though the growth rate is somewhat smaller, so that if the coupling is efficient the photons will be primarily reflected rather than absorbed [9]. The density limitation for backscatter occurs since from the sum rule $k_a \approx 2k < \lambda_D^{-1}$.

These growth rates of course refer to the usual idealized infinite medium linear theory, and it is clear that we have to consider the limitations of such theory to determine if backscatter in fact occurs and how it may saturate. On the level of linear theory we may consider three stabilizing effects: damping of the product waves, finite laser bandwidth, and plasma inhomogeneity. The effect of damping of decay waves is to stabilize the decay if $\nu_1 \nu_2 > \gamma_0^2$ [10] where γ_0 is the growth rate listed in the table. Insofar as the damping is classical (collisional or Landau) this does not appear an important mechanism of stabilization for laser fusion conditions. Bodner [11] has considered the effect of finite laser bandwidth and finds roughly that for $\gamma_0 > \delta \omega$ the instability persists, again usually the case of interest.

Thus the principal linear stabilization mechanism appears to

result from the fact that the decay instability is not taking place in an infinite uniform plasma, but in the steeply varying density of the blowing-off plasma. For reasonable pellet sizes $L = (\frac{1}{n} \frac{dn}{dx})^{-1}$ is of the order of a few mm, and these considerations lead to the last column of the table.

We may understand the effect qualitatively in the WKB approximation by noting that for $\omega_0 = \omega_1 + \omega_2$, the condition $k_0 = k_1 + k_2$ depends on plasma density through the dispersion relation of the waves. Hence the sum rules are only satisfied at a single density value. We may define therefore a region of effective wave interaction X by the phase condition $\int_0^X K(x)dx = 1$ with $K = k_0 - k_1 - k_2$. The decay must now occur before the product waves have time to convect out of the interaction region. A detailed consideration shows that this results in a growth of fluctuations by a factor $\exp(2\pi y_0^2 / V_{g1} V_{g2} dK/dx)$ -- (V_g is the group velocity) which we have used to derive our threshold estimates [12]. Inserting numerical values it appears that most present experiments are below the threshold for the backscatter reactions, while the higher powers required for laser fusion should exceed such threshold. It thus becomes necessary to discuss possible nonlinear saturation of backscatter.

A highly idealized nonlinear problem which is soluble is the case where the spectrum is limited to only three modes, which reach an equilibrium with equipartition between numbers of waves. However in the laser case it is obviously a crucial feature that the pump will not stimulate only a single pair of decay waves, but a whole spectrum which may then interact with each other.

In the weak turbulence limit this may then be studied by the wave-kinetic equation. The wave kinetic equation for photon decay into plasma oscillations and heavily damped acoustic waves has been solved [13] in connection with the ionospheric heating problem. Here the plasma oscillations produced by the parametric instability, are transferred towards longer wavelength by induced scattering (nonlinear ion Landau damping) being eventually removed by collisional damping. The predicted spectrum agrees well with experiment. Whether such weak-turbulence methods are applicable to the collisionless laser fusion problem remains doubtful.

A rough estimate for nonlinear absorption by the parametric decay instability into plasmon and acoustic wave may be made as follows.

We have seen earlier that the collisional threshold for such decay is determined by $\gamma_0^2 > \nu_p \nu_a$, where the collisional (or Landau) damping (ν_a) of ion waves is fairly large but ν_p is small. We may suppose that some nonlinear interaction process leads to an effective electron scattering $\nu_{\text{eff}} \gg \nu_p$ such that the effective plasmon damping becomes sufficient to stabilize the parametric mode, i. e. $\gamma_0^2 = \nu_{\text{eff}} \nu_a$, giving an estimate for:

$$\nu_{\text{eff}} = \frac{\omega_p \omega_a}{2\nu_a} \frac{|\xi_0|^2}{4\pi n T e}$$

in terms of the laser power. It is reasonable to suppose that the same effective electron scattering ν_{eff} will also lead to absorption of the laser light according to the usual classical collisional absorption discussed earlier. Such estimates tend to predict very strong absorption under

laser fusion conditions, but it is not obvious how to make such an estimate for the efficiency of backscatter.

three-wave

As we have noted, there are two possible backscatter decay modes-- Raman scattering off plasmons and Brillouin scattering off ion acoustic modes. One simple mechanism for saturating Raman scattering is electron heating by nonlinear processes. Since $k_p \sim 2k_0$ and $k_p \lambda_{De} \lesssim \frac{1}{3}$ to avoid Landau damping it follows that rather modest electron temperatures $T_e \sim .05 mc^2 \left(\frac{\omega_p}{\omega} \right)^2$ will stabilize the backscatter. Note, however, that ion acoustic waves persist down to much shorter wavelengths (λ_{Di}) and are relatively insensitive to electron heating. Moreover, the Brillouin backscatter deposits very little energy into the plasma, another reason for expecting it to be a more dangerous process than Raman.

Another strong saturation mechanism for the Raman process is the so-called "modulational instability" [14] which arises when the energy in plasmons becomes large. The instability arises because of the effective "negative pressure" of plasmons. From the dispersion relation $\omega^2 = \omega_p^2 + k^2 v_e^2$ we see that plasmons are reflected away from high density regions. Hence if a low-frequency density perturbation arises, the plasmons will concentrate in the low density region thereby increasing the energy density, and destabilizing the perturbation by causing the low density region to expand further. When $W_p/nT > k^2 \lambda_D^2$ with k a typical plasmon wave number, this negative plasmon pressure will exceed the stabilizing thermal pressure and the modulational instability will occur.

Hence we expect that the plasmon energy density will tend to saturate at $W_p \sim k^2 \lambda_D^2 n T_e$. What will now limit the amplitude of the modulational instability is that when the density fluctuations become too large, they will in turn stabilize the parametric Raman scatter in much the same way as a regular density gradient by destroying the three-wave resonance condition since k_p will now be a fluctuating function of position. Thus the matrix element for the parametric process will contain factors

$$\int_0^X \exp i \left\{ \frac{k}{(k\lambda_D)^2} \int_0^{X'} \frac{\delta n(x')}{n} dx' \right\} dx$$

which vanish rapidly for $\frac{\delta n}{n} > (k\lambda_D)^2$ assuming that the spectrum of the modulational density fluctuations δn , is also centered around the driving wave numbers characteristic of the Raman backscatter. Finally, with such a level of density fluctuations we may crudely estimate the mean free path for backscatter as:

$$\lambda_{b.s.} = \lambda \left(\frac{\varepsilon}{\delta \varepsilon} \right)^2 \approx \lambda \frac{\omega^2}{\omega_p^2} \left(\frac{n}{\delta n} \right)^2 = \lambda \frac{\omega^2}{\omega_p^2} (k\lambda_D)^{-4} \approx \lambda \frac{\omega^2}{\omega_p^2} \left(\frac{c}{v_e} \right)^4$$

with λ the photon wavelength and ε the dielectric constant. This would indicate a rather low-level saturation of Raman-backscatter.

On the other hand no such instability seems to limit the buildup of ion acoustic waves, i.e. the Brillouin backscatter, and as of this writing it remains unclear whether saturation of low frequency density fluctuations much below the level $\frac{\delta n}{n} \sim \mathcal{O}(1)$ with consequent enormous

backscatter is to be expected under the intense laser fusion conditions. Going to shorter wavelength lasers and the use of high Z materials to enhance collisional damping of the backscatter modes represent possible directions of investigation for lessening the importance of collective phenomena.

In conclusion we may note that while the principal problems involved with the possibility of laser fusion are probably the extremely severe technological and economic ones, nonetheless important questions remain unanswered in the physics of laser absorption and/or reflection by collective plasma processes. One such set of problems has to do with the thermalization of the high-energy electron tail produced in parametric absorption; the other concerns the backscatter instabilities, which should occur at the intensities necessary for laser fusion but not in most present experiments with weaker lasers. We have presented in this Comment a possible saturation mechanism for the backscatter into plasmons which may occur at not too dangerous a level. However, since acoustic waves are not so sensitive to trapping and scattering, we have not yet discovered an appropriate saturation mechanism for $\frac{\delta n}{n} \ll 1$ for the acoustic backscatter. Evidently further turbulent analysis, extensive numerical simulations and, of course, experiments are required.

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APPENDIX

Simplified Derivation of Decay Instabilities

When the damping of the decay product waves may be neglected, the problem consists of the calculation of the so-called 3-wave interaction matrix element, V_{123} . The general symmetry properties of V are well known and the growth rate for the decay instability $\omega_1 \rightarrow \omega_2 + \omega_3$ is given by $\gamma = |V| |c_1|$ where c_1 is the amplitude of the pump wave normalized such that the energy density of the pump wave $W_1 = |c_1|^2 \omega_1$.

While the calculation of processes of interest to laser fusion have been given elsewhere, we show here how to obtain γ in a very simple way. Consider for example the Raman backscatter process $\text{photon}_1 \rightarrow \text{photon}_2 + \text{plasmon}_3$ in the one-dimensional case.

The equation for the electric field of photon 2 is

$$(\omega_2^2 - \omega_p^2 - k_2^2 c^2) E_2 = 0 \quad . \quad (\text{A.1})$$

In the presence of the density fluctuations of the plasmon, and the electric field of the pump wave, a nonlinear term of the proper frequency and wave number arises and our equation is modified to be:

$$(\omega_2^2 - \omega_p^2 - k_2^2 c^2) E_2 = \omega_p^2 \frac{\delta n_3}{n_0} E_1 \quad . \quad (\text{A.2})$$

The linear equation for the plasmon is

$$(\omega_3^2 - \omega_p^2 - 3k_3^2 v_e^2) \delta n_3 = 0 \quad . \quad (\text{A.3})$$

The final term comes as the gradient of the perturbed plasma pressure.

To it we must add the nonlinear coupling--the Lorentz force $j_1 B_2 + j_2 B_1$.

Here we note $j_1 \approx \frac{n_0 e^2}{m \omega_1} E_1$, $B_2 = \frac{k_2 E_2}{\omega_2}$ obtaining,

$$(\omega_3^2 - \omega_p^2 - 3k_3^2 v_e^2) \delta n_3 = 2k_3^2 \frac{e^2 E_1 E_2}{m \omega_1^2} n_0 \quad (A.4)$$

Equations (A.2) and (A.4) together with the sum rule $\omega_3 = \omega_1 - \omega_2$ give the solubility determinant

$$\begin{vmatrix} \omega_2^2 - \omega_p^2 - k_2^2 c^2 & \omega_p^2 \frac{E_1}{n_0} \\ 2k_3^2 \frac{e^2 E_1}{m \omega_1^2} n_0 & (\omega_1 - \omega_2)^2 - \omega_p^2 - 3k_3^2 v_e^2 \end{vmatrix} = 0$$

Looking for a solution $\omega_2 = \sqrt{\omega_p^2 + k_2^2 c^2} + i\gamma$ with $\gamma \ll \omega_2$ we find:

$$\gamma = \sqrt{\frac{\omega_p^2}{\omega_2 \omega_3}} \frac{k_3}{\sqrt{2}} \frac{e E_1}{m \omega_1} \approx \frac{e E_1}{m \omega_1 c} \sqrt{\omega \omega p} \quad (A.5)$$

Similarly for the case of Brillouin scattering: photon₁ → photon₂ + acoustic₃ we have

$$\begin{vmatrix} \omega_2^2 - \omega_p^2 - k_2^2 c^2 & \omega_p^2 \frac{E_1}{n_0} \\ 2k_3^2 \frac{e^2 E_1 n_0}{m \omega_1^2} \frac{m}{M} & (\omega_1 - \omega_2)^2 - k_3^2 c_s^2 \end{vmatrix} = 0 \quad (A.6)$$

with

$$\gamma_0 \approx \frac{\omega_p}{\omega_1} \frac{C}{v_e} \sqrt{\omega_1 \omega_a} \frac{e E_1}{m \omega_1 c}$$

Note that in the case of Brillouin scattering a rather modest laser intensity will suffice to give $\gamma > k_3 c_s$. In this case the determinant (A.6)

is no longer adequately represented as a quadratic for ω_2 , but for $\omega_2 > \gamma > k_3 c_s$ must be treated as a cubic yielding the growth rate for the modified [15] decay instability $\gamma = (\gamma_0^2 \omega_3)^{1/3}$. Thus when the growth rate exceeds the frequency of the acoustic mode the rate increases less strongly with pump power, but the mode does not saturate.

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