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**Measurement of Helium and Hydrogen Viscosities
to 2340°K**

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**LOS ALAMOS SCIENTIFIC LABORATORY
of the
University of California**

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**Measurement of Helium and Hydrogen Viscosities
to 2340°K**

by

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ABSTRACT

This paper describes an apparatus used to measure the viscosities of helium and hydrogen in the temperature range from room temperature to 2340°K. A capillary viscometer with a relatively short channel was used to make the measurements. A series of corrections is detailed, and the experimental technique of eliminating the effects of some of them is given. The results are presented as viscosity ratios relative to the viscosities at 283°K.

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I - INTRODUCTION

The scientist or engineer who needs the value of the viscosity of some gas at a high temperature has recourse only to extrapolated values calculated from lower temperature data using any one of several models. Viscosity data available heretofore are given at maximum temperatures ranging from 1100°K for helium to about 1800°K for nitrogen and air. If experimental data with reasonable accuracy at higher temperatures were available, errors due to extrapolation could be reduced.

Although some work has been done at intermediate temperatures with other types of viscometers, ^(1, 2, 3, 4) the instrument that seems to be the best choice at the present time for the measurement of viscosity at high temperatures is the capillary viscometer. ^(5, 6) It is easily operated in a stable, near-equilibrium regime, and the elementary theory of such a device is well-known. If measurements of sufficient sensitivity can be made, the main problem is a good evaluation of necessary corrections. ^(1, 7, 8, 9, 10)

Corrections may be minimized by using a long, straight capillary; but it is difficult to maintain a uniform temperature over the entire length of such capillaries. Coiling the capillary would make it easier to obtain a uniform temperature, while the additional corrections because of a circu-

latory motion, (7, 8, 11, 12) due to centrifugal forces on the molecules, can be minimized by using a large ratio of spiral-radius to channel-radius. When these experiments were undertaken neither of these types of capillaries was readily available in tungsten or molybdenum, but they show promise for improved measurements in the future.

A compromise is to use a short channel and to apply appropriate corrections. This compromise is the basis for the high-temperature viscosity measurements presented in this report. The effects of some of the corrections are eliminated by doing a series of measurements along an isotherm at different flows and extrapolating to the condition of zero flow. A precise knowledge of the radius and length of the capillary is not necessary if the experiment is used for relative measurements, in which case the "geometry factor" may be calibrated against known viscosities. A measurable pressure drop through the short channel may be ensured by using a small-bore capillary.

II - APPARATUS

The capillary viscometer used to make the measurements reported in this paper consists of a resistance-heated molybdenum tube that is 14.0 cm long, has an inside diameter of 0.95 cm, and a wall thickness of 0.25 cm. The capillary channel itself is a hole of approximately 0.040 cm diameter which is drilled through the wall of the tube midway between the ends. See Fig. 1.

Flanges at each end of the heater tube furnish a low-resistance contact with water-cooled copper electrodes. The top end of the tube is left undrilled to assure leak tightness. A 10-cm disk is hard-soldered to the lower flange so that an O-ring seal may be kept away from the main heat-generating region. A circular stainless steel clamp bolted to the lower electrode holds the tube assembly in position.

The upper electrode is part of a piston-cylinder arrangement that allows thermal expansion of the tube while maintaining a constant force downward which is adequate to keep good contact between the electrode and the tube flange.

A water-cooled brass jacket with two observation ports serves as a furnace wall to isolate the viscometer tube from the atmosphere. The

Fig. 1. Viscometer tube and base

electrodes themselves are the top and bottom of the furnace. Joints between parts of the furnace are equipped with lightly greased polyethylene gaskets and are leak-tight within the sensitivity of a 3×10^{-11} cc/sec helium leak detector. An assembly on the bottom electrode serves as the inlet for the gas and permits the insertion of various devices (e.g. thermocouples) into the tube.

Electronically regulated power is supplied to the viscometer by a 20-kVA, 5000-amp transformer. The power level is set by a variable voltage autotransformer in the transformer primary. Figure 2 is a schematic representation of the circuitry.

Fig. 2. Power circuitry

A disappearing filament optical pyrometer is used to measure the high temperatures through one of the observation ports as shown in Fig. 3. The capillary channel is positioned in such a way that the pyrometer "looks" straight into the channel, giving nearly blackbody conditions for the pyrometer. The optical system gives a magnification such that the channel has an apparent diameter approximately six times the width of the pyrometer filament. The pyrometer has been compared with a standard tungsten lamp and a NBS-certified pyrometer.

Fig. 3. Apparatus

A Pt-Rh 10 percent vs Pt thermocouple may be inserted through an access hole in the bottom electrode to measure temperatures from room temperature up to 1100°K. The thermocouple touches the wall of the tube near

the capillary so that effects of radiation losses to the cold ends of the tube are minimized. Agreement between the thermocouple and window-corrected pyrometer readings is within one degree in the range of overlap.

The other observation port is used for a radiation sensor, the output of which is monitored on a variable-span, variable-sensitivity recorder. This system is capable of detecting viscometer tube temperature changes as small as 3°K or less at any temperature above 1200°K.

The pressure drop through the viscometer capillary is measured by a Consolidated Electrodynamics Corp. Model 1-126 electromanometer equipped with a Type 4-332, 0 ± 1.5 -psid precision pressure balance. The electromanometer output is fed to a potentiometer-recorder combination that makes it possible to detect pressure changes of 10^{-5} psi. The extreme sensitivity of the precision pressure balance is used to good advantage in that it makes it possible to work at very low Reynolds numbers through the capillary. The same pressure measuring system is used to measure the pressure drop across a second capillary that serves as a flowmeter. A system of bellows valves allows a choice of measuring the pressure drop across the flowmeter, across the viscometer, or across both. Figure 4 is a schematic representation of the flow.

Fig. 4. Flow diagram

High-purity gas^{*} is supplied to the system from a high-pressure cylinder through a two-stage pressure regulating valve. Two sensitive needle valves in series with very little volume between them serve as coarse and fine controls for setting flow rates. The needle valves, flow-meter capillary, bellows valves, and electromanometer pressure balance are all submerged in a thermostated 40-liter tank of transformer oil held at 295°K. The temperature of the oil is uniform to 0.2°K, and drift is negligible.

^{*}Purity of the helium was greater than 99.99% and that of the hydrogen was greater than 99.995% as commercially supplied.

From the flowmeter the gas goes into the viscometer tube where it is heated to the wall temperature before entering the capillary itself. After leaving the viscometer the gas passes to a 100-liter ballast tank and then through an exit valve to the atmosphere. The tank furnishes good isolation from microbarometric effects with the exit valve open. Closing the valve permits operation of the viscometer at elevated pressures without the need for the tedious matching of flows into and out of the system.

III - THEORY

If it is assumed that the viscosity of a gas is independent of pressure, that the density of this gas is proportional to the pressure, and that the change in density of the gas over the length of the capillary is small, the mass flow rate through the capillary may be written as

$$Q = \frac{\pi \rho a^4}{8 \eta \ell} \Delta P. \quad (1)$$

In this expression, Q is the mass flow in gm/sec, ρ is the density of the gas in gm/cc, η is its viscosity in poises, ℓ and a are the length and radius of the capillary channel in cm, and ΔP is the pressure drop through the channel in dynes/cm². This is the well-known Poiseuille equation for laminar flow that holds for a well-established parabolic velocity profile with no slip at the walls.

III A - CORRECTIONS

Several corrections must be applied to this formula. End corrections are necessary even when considering flow through a long tube; and, of

course, they become extremely important as the tube is shortened. Two types of end corrections are usually considered, one due to viscous effects⁽¹⁰⁾ and the other due to inertial effects.^(13, 14, 15) Viscous end effects arise because a parabolic velocity profile is not attained immediately at the entrance to the channel. Acceleration of the gas from essentially zero velocity in the entrance plenum to a finite velocity through the channel gives rise to the inertial effects. Finally, a correction for slippage of the gas at the walls must be applied.

An empirical approach to the flow law has been taken for this experiment because these corrections, which are straightforward when applied to a long capillary, may be subject to errors which make it impractical to consider using them for flow through a short capillary. An empirical equation that adequately describes flow through a short capillary is

$$\Delta P/Q = A + BQ. \quad (2)$$

Q and ΔP have the same meaning as before; and A and B are parameters which depend on pressure, temperature, the gas being used, and the capillary channel geometry and dimensions. Values of A and B may be found from a plot of the experimental ratio of ΔP to Q , as shown in Fig. 5.

Fig. 5. Empirical flow law

The inertial effects become smaller as the flow is decreased, and they disappear altogether in the limit as the flow approaches zero. Thus, the parameter B is a measure of these inertial effects. Goldstein⁽¹⁵⁾ and others show that for a long channel

$$B = \frac{k}{2\pi^2 \rho a^4}, \quad (3)$$

where k is a dimensionless constant known as the kinetic energy correction factor. The parameter A may now be evaluated from Eq. (1) as

$$A = \frac{8\eta\ell}{\pi\rho a^4}. \quad (4)$$

If the dimensions of the capillary and the density of the gas are known, the viscosity, η , may be determined from the measured value of A .

$$\eta = \frac{\pi \rho a^4 A}{8 \ell} \quad (5)$$

Conventionally, viscous end effects are explicitly taken into account by the addition of a fictitious length to the actual tube length.⁽¹⁰⁾ These viscous end effects may be considered to be implicitly included in the parameter A of this empirical approach.

A correction for slip modifies the Poiseuille formula, Eq. (1), by a multiplicative factor⁽¹⁶⁾

$$f_s = (1 + 4\beta\lambda/a), \quad (6)$$

in which λ is the mean free path of the gas in cm and β is a numerical constant of order unity which has been calculated to be $\beta = 1.147$.⁽¹⁷⁾ The mean free path depends on the pressure and temperature of the gas and, unfortunately, on its viscosity.⁽¹⁸⁾

$$\lambda = 8.589 \frac{\eta}{P} \sqrt{\frac{T}{M}} \quad (7)$$

In the above expression M is the molecular weight of the gas, P is the operating pressure in mm Hg, and T is the temperature of the gas in the viscometer in °K. Inasmuch as the slip correction is small, it is permissi-

ble to use viscosity values in the formula which are calculated from a modified Buckingham potential.

Some of the above corrections are discussed more fully later in this report, along with other corrections such as those for thermal expansion of the channel and temperature variations of the viscometer, which are more properly considered as being applied to the experiment itself rather than to the theory.

III B - RELATIVE MEASUREMENTS

Several simplifications in the experiment and in the data analysis result if relative, rather than absolute, measurements of the viscosity are acceptable. Viscosity ratios as a function of temperature are calculated from Eq. (5) as

$$R(\eta) = \frac{\eta(T_2)}{\eta(T_1)} = \frac{A(T_2)\rho(T_2)}{A(T_1)\rho(T_1)} = \frac{T_1 A(T_2)}{T_2 A(T_1)}, \quad (8)$$

where appropriate corrections remain to be made. T_1 may be chosen to be some easily reproducible reference temperature at which the viscosity is well-known. For this experiment T_1 is the temperature of the furnace cooling water, 283°K. Note that it is no longer necessary to have a precise knowledge of the dimensions of the channel because, except for a thermal expansion correction factor, they cancel out when the ratio is taken.

An important assumption made here is that the connections for viscous end effects also cancel in this approach. The validity of this assumption is inferred from the literature,^(9, 10) but it has not been vigorously justified.

This relative approach makes it possible to use arbitrary units for the measured values of ΔP and Q . The ratio of the high-temperature intercept, $A(T_2)$, to the reference-temperature intercept, $A(T_1)$, is dimensionless and, therefore, independent of the units used. If the pressure drops in arbitrary units across the viscometer and flowmeter are denoted by V and F , respectively, then at any fixed viscometer temperature a plot of V/F vs F yields an extrapolated intercept, A_m . The ratio $A(T_2)/A(T_1)$ required in Eq. (8) may be replaced by the equivalent ratio, $A_m(T_2)/A_m(T_1)$.

III C - HEAT TRANSFER

The assumption that the gas reaches the temperature of the heater wall before entering the capillary must be justified. By using an analysis from Goldstein,⁽¹⁹⁾ a characteristic length for heat transfer is determined for this heater tube. The leading term from the expression for the mean temperature, weighted with respect to the fluid velocity for the case of a Poiseuillian flow through a pipe with constant wall temperature, gives the characteristic length as

$$L = \frac{4u_m a^2}{14.6 \kappa}, \quad (9)$$

where u_m is the mean fluid velocity, and κ is the thermal diffusivity.

The wall temperature profile with the ends of the heater tube held at 283°K is such that the temperature is constant for at least three centimeters. For the worst possible values of the heat-transfer parameters in this experiment, L is evaluated as 0.06 cm. Thus, it is concluded that the mean gas temperature is essentially at the wall temperature when the gas enters the capillary channel.

IV - DISCUSSION

The viscometer described in this report is unique in that it consists of a relatively short capillary. The design factors and the operating characteristics of this viscometer introduce several interesting effects that require further discussion.

IV A - CAPILLARY SIZE

The design of this viscometer entailed a compromise in the choice of the length of the capillary. It is desirable to use as long a channel as possible, but the difficulties associated with molybdenum fabrication severely limit the capillary length. The dimensions chosen result from several considerations.

To reduce end effects to the point where they may be considered negligible, it is necessary to make the channel such that the length-to-radius ratio is well over 100. This has been a usual design criterion in the past, but it was abandoned in this experiment. Power demands for this type of electrically heated tube make it impractical to have a wall thickness much over 0.25 cm, and the above criterion would require a channel diameter of less than 0.005 cm. It is clearly a difficult task to drill such a tiny hole

through materials like molybdenum and tungsten. Furthermore, slip effects would be very large in such a narrow channel. A hole having a diameter of 0.040 cm was the smallest that could be drilled by conventional methods through the 0.25-cm molybdenum wall. This results in a length-to-radius ratio of only about twelve.

Once the size of the channel is fixed, it is necessary to be sure that flow through the viscometer is in the viscous range rather than in the range of molecular flow or somewhere in the transition region. A criterion that the flow be viscous is that⁽²⁰⁾ $ap > 500$, in which a is the channel radius in cm, and p is the operating pressure in microns of mercury. For this viscometer $a = 0.020$ cm and $p = 5.9 \times 10^5$ microns,* so $ap = 11,800$; and the flow is viscous.

IV B - CORRECTIONS

It has been pointed out that temperature variations of the oil bath are insignificant. Small variations in the cooling water temperature lasting for less than two minutes do not affect the viscometer because of the relatively large thermal mass involved. Longer-term cooling water temperature variations, which seldom occur, are felt in the viscometer and are corrected for in the following manner. In the neighborhood of room temperature $\eta \propto T^{\frac{3}{3}}$ and

* Typical Los Alamos atmospheric pressure.

$\rho \propto T^{-1}$, thus

$$\frac{\Delta(\Delta P_v/Q)}{(\Delta P_v/Q)} = \frac{\Delta(\eta/\rho)}{(\eta/\rho)} = \frac{5}{3} \frac{\Delta T}{T} \quad (10)$$

This correction is applied to individual measurements or to an entire set of data, as necessary. Comparison of reference temperature data intercepts, generated from isotherms differing by as much as 3°K, confirms the validity of this correction. This same correction, also experimentally verified, is applied for temperature variations less than 10°K at high temperatures. When a temperature change of more than 10°K occurs, the power to the viscometer is adjusted to bring the temperature back more closely to its original level.

Temperatures measured by sighting through an observation port with an optical pyrometer must be corrected for the absorption of radiation by the pyrex window. A correction curve for the temperature range of interest was obtained by reading the temperature of a standard strip lamp with and without the window interposed at a number of lamp temperatures throughout the range. The fact that no significant changes occurred in the absorption characteristics of the window was verified by making transmissivity measurements of the glass with a spectrophotometer both before and after use. The spectrophotometer measurement also served to quantitatively verify the correction curve. The values of the corrections added range from 5°K at 1100°K

to 44°K at 2400°K.

Another correction must be applied to take into account thermal expansion of the channel. The dimensions of the channel enter into the viscosity determination as a^4/ℓ . At the high temperatures, $a = a_0 (1 + \alpha \Delta T)$ and $\ell = \ell_0 (1 + \alpha \Delta T)$. Here α is the linear coefficient of thermal expansion⁽²¹⁾ of the viscometer heater at the high temperature, and ΔT is the difference between the high and low temperatures. Thus

$$\frac{a^4}{\ell} = \frac{a_0^4}{\ell_0} (1 + \alpha \Delta T)^3 \approx \frac{a_0^4}{\ell_0} (1 + 3\alpha \Delta T) = \frac{a_0^4}{\ell_0} f_e, \quad (11)$$

where f_e is the thermal expansion correction factor that must be applied to Eq. (8).

The correction for slip in the flowmeter cancels when relative measurements are made because the temperature and pressure in the flowmeter are constant. This is not the case for the viscometer slip correction, however; and Eq. (8) needs to be suitably modified to take slip into account. The ratio of the slip corrections becomes

$$f'_S = \frac{f_S(T_2)}{f_S(T_1)} = \frac{1 + 4\beta\lambda(T_2)/a}{1 + 4\beta\lambda(T_1)/a} \approx 1 + \frac{4\beta\lambda(T_2)}{a} - \frac{4\beta\lambda(T_1)}{a}. \quad (12)$$

It is seen that inasmuch as the slip correction at the reference temperature is relatively small, the ratio of the corrections may be approximated by simply

subtracting the correction for the reference temperature from the correction for the high temperature.

Following a suggestion by Present,⁽²²⁾ an attempt was made to verify the calculated slip correction experimentally, but the attempt failed. A series of runs was made at various temperatures and at operating pressures ranging from 5 to 60 psia. A plot of the values of A (see Eq. 2) versus reciprocal pressure should result in a straight line with the infinite pressure intercept giving the value of A in the absence of slip.

The infinite pressure values are consistently higher than the values calculated from the theoretical slip correction, and the use of these infinite pressure values results in helium viscosities that are higher than those of other workers by five percent at 1100°K. It is possible that there are some effects other than slip which are pressure-dependent, but there is no really satisfactory explanation of this discrepancy. Because the slip correction calculated from theory results in better agreement with other workers in the region of overlap of data, it was decided to apply the theoretical correction to the viscosity ratio measurement.

IV C - EXPERIMENTAL OBSERVATIONS

Data for hydrogen and helium at 283°K plotted in Fig. 6 show a deviation of the experimental data away from the high-flow, straight-line behavior of the empirical V/F vs. F plot. Both sets of data are asymptotic to a straight line for flows through the viscometer corresponding to Reynolds numbers above forty. This behavior was observed throughout the range of temperatures and may be explained as a variation in the orifice coefficient of a short tube. ⁽²³⁾

The same value for a viscosity ratio is obtained whether the "observed" intercept or the extrapolated, straight-line intercept is used as long as they are not intermixed. For convenience in data analysis only the straight-line portion of the curve is used in obtaining the results reported here. Consequently, flows are always set above a Reynolds number of 40 in order to obtain points generating the straight line of Fig. 5. The slope and intercept of this straight line are numerically determined by using a method of averages.

The detailed reference temperature lines shown in Fig. 6 display the same intercepts for helium and hydrogen. The following simple analysis shows that these intercepts should be the same for all gases to a first order

Fig. 6. Reference temperature data plot

approximation. With A as defined in Eqs. (2) and (4) and A_m as defined on p. 21, it is seen that

$$A = \frac{8\eta_v \ell_v}{\pi \rho_v a_v^4} = \lim_{Q \rightarrow 0} \frac{\Delta P_v}{Q} = \lim_{\Delta P_f \rightarrow 0} \frac{\Delta P_v}{(\pi \rho_f a_f^4 / 8 \eta_f \ell_f) \Delta P_f}$$

The flowmeter constant may now be moved outside the limit process, and arbitrary units may be used for the pressure drops. Thus

$$\frac{8\eta_v \ell_v}{\pi \rho_v a_v^4} = \frac{8\eta_f \ell_f}{\pi \rho_f a_f^4} \lim_{F \rightarrow 0} \frac{V}{F} = \frac{8\eta_f \ell_f}{\pi \rho_f a_f^4} A_m$$

$$\text{Therefore, } A_m = D \frac{\eta_v \rho_f}{\eta_f \rho_v} \quad (13)$$

where $D = \frac{\ell_v a_f^4}{\ell_f a_v^4}$ and is independent of the gas used. Because the temperatures of the viscometer and flowmeter are only slightly different and because the temperature dependence of the viscosity and density is similar for all gases, it follows that the raw data intercepts at the reference temperature are independent of the specific gas used. This conclusion is not changed when slip is also considered because the difference in this correction for different gases is negligible at this temperature.

A result of particular interest is noted on forming the slope-to-intercept ratio from the data lines. This ratio gives a measure of the dimensionless kinetic energy correction factor, k , without a need for precise

knowledge of the channel radius. From Eqs. (3) and (4),

$$\frac{B}{A} = \frac{k/2\pi^2\rho a^4}{8\eta\ell/\pi\rho a^4} = \frac{k}{16\pi\eta\ell} \quad (14)$$

This observation was made for three gases at room temperature with the results $k = 2.31$ for He, $k = 2.26$ for H₂, and $k = 2.27$ for N₂. Similar values are found at high temperatures even though the slope measurement is subject to greater errors. As these values are the same within experimental error it is concluded $k = 2.28$. This is in the same range as Goldstein's calculation ($k = 2.41$) and Schiller's measurements⁽¹⁵⁾ ($k = 2.32$) for a long tube.

IV D - ACCURACY

The temperature measurement is the limiting accuracy of the quantities measured. The pressure transducer has an accuracy specification of 0.05 percent, and the differential voltmeter/recorder combination is good to 0.02 percent; both values are referred to full scale. These specifications are checked periodically against known standards. Temperature measurements with an optical pyrometer on the other hand are not much better than 0.5 percent. Corrections to the temperature because of absorption of radiation by the glass window are well-known and do not introduce appreciable error. The usual corrections to temperatures measured by the thermocouple are too small to be significant.

The thermal expansion coefficient may be in doubt by as much as three percent but this causes trivial error in the final result. The slip correction should be straightforward and could introduce an error as large as one percent. The failure in obtaining experimental verification of this correction may pessimistically be interpreted as a bias which leaves the final results in doubt by as much as five percent at 1100°K to more than ten percent at 2400°K.

Other factors considered in evaluating accuracy of the experiment include added pressure drops through the plumbing external to the viscometer. These are quite small and vanish completely in the limit of zero flow. The operating pressure is measured with a Bourdon tube to only 0.5 percent, but this is not a significant parameter in the experiment except for the attempt to evaluate slip. Runs were made using different circular channels and a semi-circular channel, and in each case results essentially identical to those reported were obtained. A statistical analysis of the data points on any given high-temperature line indicate a precision of ± 0.1 percent in the value of the intercept, while analysis of the reference temperature data shows that its intercept is determined to $\pm 3/4$ percent. The lower precision at the reference temperature is a consequence of the relatively low pressure drop across the viscometer at this temperature. Finally, periodic checks on the cold line are made to be sure there have been no undetected changes in the capillary.

V - RESULTS

Experimental viscosity ratios referred to 283°K are shown in Table 1 and Fig. 7 for helium and in Table 2 and Fig. 8 for hydrogen. The theoretical curves are calculated from a modified Buckingham (exp-six) potential with parameter values calculated by Mason and Rice.⁽²⁴⁾ No attempt has been made to fit these data using other values of the parameters. Data by other workers⁽⁹⁾ expressed as viscosity ratios are also shown to indicate the general comparison of these measurements with those of others.

In Tables 1 and 2 the temperatures shown are corrected for window absorption. The thermal expansion correction factor, f_e , is defined on p. 26 and the correction for slip, f'_s , is derived in Eq. (12) on p. 26. The uncorrected viscosity ratio, $R(\eta)$, discussed on p. 20 must be multiplied by f_e and f'_s to obtain the final results.

Note that the agreement between the theoretical model and this experiment is better for helium than it is for hydrogen, while the data of Trautz and Zink fit the calculated viscosities quite well for both gases. It may be that there is some systematic bias on the hydrogen results from this experiment, but it may also be possible that there are errors in the earlier work.

Table 1. Helium Viscosity Ratios *

T (°K)	R (η)	f_e	f'_s	$\eta(T)/\eta(283^\circ\text{K})$
476	1.410	1.003	1.002	1.42
675	1.796	1.006	1.004	1.81
875	2.124	1.009	1.006	2.16
1135	2.492	1.014	1.018	2.57
1214	2.605	1.016	1.019	2.70
1262	2.665	1.017	1.022	2.77
1311	2.734	1.018	1.023	2.85
1363	2.792	1.019	1.024	2.91
1412	2.829	1.020	1.025	2.96
1462	2.873	1.021	1.027	3.01
1596	3.052	1.024	1.030	3.22
1672	3.092	1.026	1.032	3.27
1789	3.224	1.029	1.035	3.43
1877	3.323	1.031	1.037	3.55
1932	3.349	1.032	1.038	3.59
1991	3.412	1.034	1.039	3.67
2033	3.451	1.035	1.040	3.71
2086	3.503	1.037	1.041	3.78
2139	3.529	1.038	1.042	3.82
2192	3.592	1.040	1.043	3.90
2243	3.599	1.041	1.044	3.91
2290	3.691	1.043	1.045	4.02
2344	3.682	1.044	1.045	4.02

* Experimental ratios R (η) are corrected for thermal expansion (f_e) and slip (f'_s) to yield η/η_0 .

Fig. 7. Helium viscosity ratio

Table 2. Hydrogen Viscosity Ratios^{*}

$T (^{\circ}\text{K})$	$R(\eta)$	f_e	f'_s	$\eta(T)/\eta(283^{\circ}\text{K})$
296	1.022	1.000	1.000	1.02
401	1.267	1.001	1.002	1.27
599	1.670	1.005	1.005	1.69
798	2.049	1.008	1.008	2.08
898	2.232	1.010	1.010	2.28
998	2.390	1.012	1.011	2.44
1092	2.528	1.013	1.013	2.59
1156	2.640	1.015	1.014	2.72
1259	2.799	1.017	1.016	2.89
1311	2.826	1.018	1.017	2.93
1363	2.924	1.019	1.018	3.03
1415	3.002	1.020	1.019	3.12
1519	3.142	1.022	1.021	3.28
1621	3.296	1.025	1.023	3.46
1725	3.430	1.027	1.025	3.61
1827	3.567	1.030	1.026	3.77
1931	3.682	1.033	1.028	3.91
2034	3.815	1.035	1.030	4.07
2142	3.937	1.038	1.033	4.22
2242	4.059	1.041	1.035	4.37
2341	4.206	1.044	1.036	4.55

* Experimental ratios $R(\eta)$ are corrected for thermal expansion (f_e) and slip (f'_s) to yield η/η_0 .

Fig. 8. Hydrogen viscosity ratio

Slow changes in the capillary channel above the temperatures shown caused errors on the order of several percent, so meaningful results were not obtainable. Power limitations also began to cause troubles which were manifested as data scatter caused by short-term temperature variations of the viscometer.

VI - CONCLUSION

It is concluded on examination of the various error sources and of the overall precision that the results should be accurate to ± 1 percent at temperatures below 1100°K, increasing to ± 4 percent at the highest temperature, 2340°K. The question of whether or not there is some systematic bias on the data is not fully answered. It may be that because of the lack of a satisfactory theory of flow through a short tube, the question cannot be answered at this time. In particular, the assumption about viscous end effects (see p. 21), may be a source of difficulty, especially in view of the fact that the correction is relatively large. For this reason these measurements will be repeated when a long capillary version of this viscometer is readied.

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