A MODEL CALCULATION
OF CORRELATIONS AT HIGH ENERGY

J. Finkelstein

Columbia University, New York, New York 10027

2154000

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

THIS
1850000

This research was supported in part by the U. S. Atomic Energy Commission.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

leg

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

1

ABSTRACT

The inclusive two-body correlation function for particles produced in high-energy hadronic collisions is calculated in a multiperipheral-like model, and compared with experiment.

It has been recognized lately that different models of particle production processes at high energy that may agree with each other and with the data on single-particle inclusive spectra may greatly differ in their predictions for two-particle spectra, and that studies of two-particle spectra could therefore be useful in distinguishing between these models.¹ One class of model which is often considered is the class of multiperipheral-like or short-range-correlation-models. It is the purpose of this note to report a calculation of the two-particle spectrum at high energy in a simple yet hopefully representative model of this class which has been proposed earlier by Peccei and myself² and by Krzywicki and Petersson.³

In this model, particle production is described by distinguishing one particle (called the 'leading particle') from the rest (the 'fireball'); the distribution of particles in the fireball is assumed to be, in the proper Lorentz frame, the same as the distribution of all the particles in a typical event. Let $f_1(x)$ be the scaling limit of the normalized inclusive single-particle distribution:

$$f_1(x) \equiv \lim_{s \rightarrow \infty} \frac{x}{\sigma_{\text{tot}}(s)} \frac{d\sigma(x,s)}{dx}. \quad (1)$$

In the model described in references 2 and 3, $f_1(x)$ is related to $g(x)$, which is the distribution function for leading particles, by the integral equation

$$f_1(x) = g(x) + \int \frac{dy}{y} g(y) f_1\left(\frac{x}{1-y}\right). \quad (2)$$

Once the function $g(x)$ is known, then the two- (and in fact the many-) particle distribution can be calculated. In particular, the two-particle distribution with both particles in the central ($x \approx 0$) region can be expressed rather simply in terms of g and f_1 . Let $f_2(x_1, x_2)$ be the scaling limit of the two-particle

inclusive distribution, defined analogously with Equation (1), and let

$$f_2(R) \equiv \lim_{x \rightarrow 0} f_2(x, Rx). \quad (3)$$

Then, according to the model,

$$f_2(R) = f_1(0) \int \frac{dx}{x} \left[g(x) f\left(\frac{Rx}{1-x}\right) + g(Rx) f\left(\frac{x}{1-Rx}\right) \right]. \quad (4)$$

The reader is referred to reference 2 for a derivation and discussion of this equation. The variable R is closely related to the relative rapidity of the two particles:

$$\ln R \approx y_2 - y_1, \quad (|y_2 - y_1| \text{ large}). \quad (5)$$

In the central region, the two-particle distribution is independent of the sum of the two rapidities.

We will adopt the following strategy to calculate $f_2(R)$: first, we use data on the single-particle distribution to guess at the form of $f_1(x)$; second, we make use of this $f_1(x)$ to obtain $g(x)$ through Equation (2); finally, we calculate $f_2(R)$ through Equation (4). The model as originally formulated makes no reference to any internal quantum numbers; thus the distributions f_1 and f_2 should be understood to be the distribution of particles of all types, neutral as well as charged. The model can be generalized to recognize the existence of several types of particles;⁴ however, for the present purpose of illustrating the kind of distribution one obtains in multiperipheral-like models, this extra complication is probably not warranted, and so will not be considered here.

A more serious limitation of this type of model stems from the fact that, as pointed out by Le Bellac,⁵ no model with only short-range correlations can be entirely correct if total cross sections are asymptotically constant, since such models can never describe diffraction. We thus have to be able to ignore diffraction; for example, if individual events can be classified as diffractive or non-diffractive, then the quantities f_1 and f_2 must be understood to represent the distributions of particles from non-diffractive events, and the quantity σ_{tot} in Equation (1) should be understood to be the total non-diffractive cross section, which is presumably somewhat less than the inelastic cross section.

In order to determine the shape of $f_1(x)$, we use data on particles produced in proton-proton collisions. At small values of x , most of the particles produced are pions. The π^+ and π^- distributions are reasonably well determined⁶ at the ISR, where they are seen to scale. Because the Pomeron has isospin zero, the limiting π^0 distribution should be the average of π^+ and π^- ; thus the total pion distribution is three times the average of π^+ and π^- . At large values of x , protons dominate. The proton distribution is seen⁷ to have a peak near $x = 1$; since this peak is presumably a diffractive effect, it should not be included in our non-diffractive distribution function $f_1(x)$. A detailed (and t -dependent) triple-Regge analysis near $x = 1$ would presumably enable us to subtract the diffractive from the measured distribution; otherwise, we can use data on $pp \rightarrow pX$ for x outside the peak ($x < 0.9$) and then extrapolate to a constant value at $x = 1$. In fact, since scaling is seen to be reasonably well obeyed in this region, we can use data at lower energy.⁸ Finally, although not quite all of the produced

particles are protons or pions, we assume in what follows that the distribution of all particles has the same shape as the sum of the proton and the pion distributions.

Some further constraints on $f_1(x)$, which are valid for the type of model we consider, are:

- (i) The energy conservation sum rule, applied to non-diffractive events, reads

$$\int_0^1 dx f_1(x) = 1. \quad (6)$$

Thus the normalization of $f_1(x)$ is determined once its shape is known.

- (ii) If $\langle n \rangle$ is the average number of produced particles of all types, then, as

$$s \rightarrow \infty,$$

$$\langle n \rangle = f(0) \ln s \quad (7)$$

An estimate of $f(0)$ which is consistent with the observed energy dependence of multiplicities is $f(0) \approx 3$.

- (iii) For small x ,

$$f(x) \approx f(0) + Ax^{\frac{1}{2}}, \quad (8)$$

where the parameter A also appears in the expression for the approach to scaling in the central region:⁹

$$\left. \frac{x}{\sigma_{\text{tot}}} \frac{d\sigma}{dx} \right|_{x=0} \rightarrow f(0) + 2A \left(\frac{s}{m^2 + p_{\perp}^2} \right)^{-1/4}, \quad s \rightarrow \infty. \quad (9)$$

If the form given in (9) is valid down to conventional accelerator energies, then the value of A can be seen¹⁰ to be about -3.3.

A parameterization of $f_1(x)$ which respects these three constraints and which has the shape suggested by the sum of the proton and pion spectra is

$$f_1(x) = 3.0 - 3.3x^{\frac{1}{2}} - x + 1.75x^{\frac{3}{2}}. \quad (10)$$

This function is displayed in Figure 1, together with data points from references 6 and 8 representing $\frac{3}{2}(f_1^{\pi^+} + f_1^{\pi^-}) + f_1^p$; the data points have been normalized to make $f_1(x=0) = 3$.

Using $f_1(x)$ as given in (10), it is straightforward to numerically compute $g(x)$ from (2), and then to compute $f_2(R)$ from (4). The two-body correlation function $C_2(R)$ is defined by

$$C_2(R) \equiv f_2(R) - [f_1(0)]^2. \quad (11)$$

The calculated values of $C_2(R)$ are displayed in Figure 2; I have checked that these values are reasonably insensitive to the precise form of $f_1(x)$ assumed. The predicted values for the fractional correlation, which is defined as $(f_2 - f_1^2)/f_1^2$, can be obtained by dividing $C_2(R)$ by nine.

In models which exhibit short-range correlations, the integrated two-particle correlation function $\langle C_2 \rangle$ grows logarithmically with s :

$$\langle C_2 \rangle \rightarrow C_2 \ln s. \quad (12)$$

The constant C_2 is given in terms of $C_2(R)$ by

$$C_2 = 2 \int_1^{\infty} \frac{dR}{R} C_2(R), \quad (13)$$

and, in the calculation reported here, has a numerical value of about 7.6;

thus we predict

$$\langle C_2 \rangle \approx 2.5 \langle n \rangle. \quad (14)$$

At large values of R , $C_2(R)$ behaves as

$$C_2(R) \rightarrow (\text{const.}) \times R^{-\frac{1}{2}}. \quad (15)$$

The form given in (15) corresponds to a correlation length of $\frac{1}{2}$; the calculated value of the constant in (15) is about 2.2. In the approximation suggested by Frazer et al.,¹¹ which is to use the asymptotic form given in (15) for all R in order to calculate C_2 through Equation (13), we would have obtained $C_2 = 8.8$, which is about 16% higher than the actual value of the integral in (13).

Data on the two-particle correlation in the central region is now becoming available from the ISR; we consider in particular the preliminary data of the Pisa-Stony Brook collaboration.¹² These data show a fractional correlation¹³ which is consistent with the parameterization $0.6 \times \exp(-|y_2 - y_1|)$. This is somewhat larger than the correlation predicted by our calculation; from Equations (5) and (15), we would say that the coefficient of $\exp(-|y_2 - y_1|)$ should be about $2.2 \div 9 = 0.24$. On the other hand, the fractional correlation as reported in reference 12 is defined with σ_{tot} (e.g. in Equation (1)) replaced by $\sigma_{\text{inelastic}}$, while for the purpose of the present discussion we want it replaced by $\sigma_{\text{non-diffractive}}$; thus the reported correlation is expected to be larger than the one which we predict. In different words, the presence of diffractive inelastic events produces a positive long-range correlation which adds to the short-range correlation we have been discussing.

Although our calculation was done within a specific model, we suspect that the general features of the calculated correlation would be common to almost any multiperipheral-like model; in particular, we think it would be difficult for a model of this type which fits the single-particle spectrum to produce a fractional correlation as large as 60%. If the preliminary data¹² are correct in their suggestion that correlations are in fact this large, we should consider this to indicate a failure of this type of model as applied to all inelastic events. The question of whether or not these models can be reprieved by being applied to a subset of inelastic events is left for future investigation.

Conversations with Roberto Peccei are gratefully acknowledged.

FOOTNOTES AND REFERENCES

1. For a review of models for high-energy production processes, see M. Jacob, rapporteur talk at the XVI International Conference on High Energy Physics (1972).
2. J. Finkelstein and R. D. Peccei, Phys. Rev. D 6, 2606 (1972).
3. A. Krzywicki and B. Petersson, Phys. Rev. D 6, 924 (1972);
B. Petersson, in the Proceedings of the Seventh Rencontre de Moriond, edited by J. Tran Thanh Van (CNRS, Paris, 1972).
4. R. Kronenfeld and R. D. Peccei, Phys. Rev. D 7, 1556 (1973).
5. M. Le Bellac, Phys. Lett. 37B, 413 (1971).
6. M. Banner et al., Phys. Lett. 41B, 547 (1972),
M. G. Albrow et al., Phys. Lett. 42B, 279 (1972),
A. Bertin et al., Phys. Lett. 42B, 493 (1972).
7. M. G. Albrow et al., contribution to the XVI International Conference on High Energy Physics (1972), and discussed in reference 1.
8. J. V. Allaby et al., CERN Report No. CERN 70-12, 1970 (unpublished).
9. R. C. Brower and J. Ellis, Phys. Rev. D 5, 2253 (1972). We take the intercept of secondary Regge trajectories to be $\frac{1}{2}$.
10. Compare the analysis of T. Ferbel, Phys. Rev. Lett. 29, 448 (1972).
11. W. R. Frazer, R. D. Peccei, S. S. Pinsky and C.-I. Tan, Phys. Rev. D (to be published 1 May 1973).
12. Pisa-Stony Brook collaboration, contribution to the XVI International Conference on High Energy Physics (1972), and discussed in reference 1.

13. The data of reference 12 are for charged correlations; however, as long as most of the produced particles are pions, we expect the fractional correlation in the central region for all particles to be the same as that for charged particles, except possibly for very small rapidity separation. This is because the middle of the three Reggeons in the Mueller diagram for the two-particle distribution has isospin zero or one; the isospin one part cancels when one sums π^+ and π^- , while the isospin zero couples equally to π^+ , π^- and π^0 .

FIGURE CAPTIONS

Figure 1. The single-particle distribution from p-p scattering. The data points represent the proton distribution plus three times the average of the π^+ and π^- distribution, normalized to the value 3 at $x=0$. The solid line is the parameterization used in the calculation of the two-particle distribution.

Figure 2. The calculated two-particle correlation in the central region, as a function of $R \approx \exp(-|y_2 - y_1|)$.