FIXED-PARITY SUM RULES FOR INELASTIC FORM FACTORS[†]

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If we consider the quantity

$$\rho(\vec{q}) = \sum_1 \epsilon_1 e^{i\vec{q} \cdot \vec{r}_1} \quad (1)$$

and apply the standard double-commutator technique we obtain the sum rule

$$\begin{aligned} \sum_{n \neq 0} (E_n + \frac{\hbar^2}{2mA} \vec{q} \cdot \vec{q}') \rho_{no}^*(\vec{q}') \rho_{no}(\vec{q}) \\ = \frac{\hbar^2}{2m} \vec{q} \cdot \vec{q}' (\rho_{oo}(\vec{q} - \vec{q}') - \frac{1}{A} \rho_{oo}^*(\vec{q}') \rho_{oo}(\vec{q})) \end{aligned} \quad (2)$$

The coordinates $\vec{r}_1$ in Eq. (1) are measured with respect to the center of mass of the system and ϵ_1 is a parameter assumed to satisfy $\epsilon_1^2 = \epsilon_1$ e.g. $\epsilon_1 = 1$ or $\epsilon_1 = 1/2 \pm t_3(i)$. The quantities $\rho_{no}(\vec{q})$ are (apart from constant factors) form factors for elastic and inelastic scattering. The result of Eq. (2) differs from more traditional expressions because of the second terms in both the left and the right-hand sides. These are essentially center-of-mass corrections and appear because relative coordinates were used in Eq. (1). If π_n denotes the parity of the n th state we can insert the relation $\rho_{no}(-\vec{q}) = \pi_o \pi_n \rho_{no}(\vec{q})$ into Eq. (2) and obtain

$$\begin{aligned} \sum_{n \neq 0} (E_n - \frac{\hbar^2}{2mA} \vec{q} \cdot \vec{q}') \pi_o \pi_n \rho_{no}^*(\vec{q}') \rho_{no}(\vec{q}) \\ = - \frac{\hbar^2}{2m} \vec{q} \cdot \vec{q}' (\rho_{oo}(\vec{q} + \vec{q}') - \frac{1}{A} \rho_{oo}^*(\vec{q}') \rho_{oo}(\vec{q})) \end{aligned} \quad (3)$$

It is clear that Eqs. (2) and (3) cannot in general be combined to give sum rules involving only states of a given parity. If the center of mass correction on the left hand side is neglected, however, the separation is easy and yields (for the special case $\vec{q}' = \vec{q}$)

$$\sum_n E_n \frac{1 + \pi_o \pi_n}{2} |\rho_{no}(\vec{q})|^2 = \frac{\hbar^2 q^2}{4m} [\rho_{oo}(0) - \rho_{oo}(2\vec{q})] \quad (4a)$$

$$\sum_n E_n \frac{1 - \pi_o \pi_n}{2} |\rho_{no}(\vec{q})|^2 = \frac{\hbar^2 q^2}{4m} [\rho_{oo}(0) + \rho_{oo}(2\vec{q}) - \frac{2}{A} |\rho_{oo}(\vec{q})|^2] \quad (4b)$$

We note that for the isoscalar case $\epsilon_1 = 1$ the term in the bracket of Eq. (4b) vanishes for $q = 0$ i.e. the equation includes the selection rule for isoscalar dipole transitions. The usefulness of expressions such as Eqs. (4a) and (4b) is related to the fact that the distribution of strength is often dominated by collective giant doorway excitations. The location of these modes depends on their parity (and isospin) much more than on their angular momentum.

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