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TWO PHOTON PRODUCTION OF LEPTON PAIRS
IN HADRON-HADRON COLLISIONS

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ABSTRACT

The cross-section for the two photon exchange production of lepton pairs in hadron-hadron collisions is evaluated here up to the leading order in $\log S$ for $S \rightarrow \infty$ holding either M^2 or M^2/S fixed. To take into account elastic as well as inelastic emission of photons from hadrons the general spectrum of equivalent photons is derived in terms of the structure functions of the hadrons and this spectrum is then used for evaluating the leading $\log S$ terms in the two photon cross section. The lepton mass dependence and the angular distribution of the two photon process are also discussed.

INTRODUCTION

We use the equivalent photon approximation¹ to evaluate the cross-section, up to leading order in $\log S$ as $S \rightarrow \infty$, for the two photon exchange production of lepton pairs in hadron-hadron collisions in the reaction² $p + p \rightarrow l^+ + l^- + \text{"anything"}$. We can take the limit $S \rightarrow \infty$ by holding either (1) M^2 fixed (M is the invariant mass of the lepton pairs) or (2) M' fixed. Each of these cases is tackled separately.

We first derive the relation for the spectrum of equivalent photons¹ in terms of the structure functions W_1 and W_2 of the hadrons³. This enables us to incorporate both elastic as well as inelastic emission of photons from each hadron. The integral over q^2 (the mass of the photon) in the spectrum is split up into two regions. For $q_{\min}^2 \lesssim q^2 \lesssim m_c^2$ (where $m_c^2 = 2 \text{ GeV}^2$ ⁵) we have both the elastic and inelastic ("shallow-inelastic") emission of photon. For $q^2 \gtrsim m_c^2$ we are in the deep inelastic scaling region. The inelastic electron scattering data from the SLAC-MIT experiments^{4,5} is used for evaluating the cross-section.

For case (1) we find that in the leading log approximation the two photon cross-section grows as $\log^3 S$. For case (2) it grows as $\log^3 M^2$ and in this case even though the approximation is well defined so long as $M^2 \gg m_c^2$, the correction to the leading log term are quite significant unless M^2 is very very large ($M \gtrsim 100 \text{ GeV}$). So that for values of S that are presently available the two approximations give results which are not in very close agreement.

The two photon cross section thus obtained is compared to the experimentally observed cross section (at BNL² for $S = 60 \text{ GeV}^2$) and to the one photon exchange contribution evaluated by Drell et al⁶ using scaling arguments. The two photon contribution is found to be small in general except when $M^2/S \ll 1$.

Small as the two photon cross section, for massive lepton pairs, is compared to the one photon contribution, the two processes are shown to have characteristically different dependence on the lepton mass and also have very different angular distribution for the

emerging leptons. The one photon cross section is independent of the lepton mass μ , up to order μ^2/M^2 (where M is the invariant mass of the lepton pairs) whereas the two photon cross section has a $\ln M/\mu$ dependence, (for $(m^4/S)(M^2/S) \ll \mu^2$) so that the number of electron pairs produced via an exchange of two photons far exceeds the number of muon pairs. However, the difference in their total cross sections (that is, the sum of the one photon and two photon exchange cross sections) is, in general small. This calculation enables one to isolate the difference, based purely on electromagnetic interactions, in the cross section for production of electron and muon pairs in hadron-hadron collisions. An experimental deviation from this difference could, therefore, yield important information about μ - e ^{8,10} symmetry.

Regarding the angular distributions, the two photon process exhibits a strong peaking of the leptons along the collision axis. The one photon process, on the other hand has an angular distribution of the form $(a + b \cos^2 \theta)$. These characteristic differences in the two processes may enable an experimental differentiation between the two contributions.

THE EQUIVALENT PHOTON SPECTRUM

To evaluate the cross section for the two photon exchange (see Figure 1) production of lepton pairs² in hadron-hadron collisions we use the equivalent photon approximation¹. This asserts that the electromagnetic interactions of a relativistic charged particle can be represented by those of a bunch of photons which are essentially forward moving, close to the mass shell and transverse. We derive the general spectrum of the equivalent photons emitted by the hadrons in terms of the structure functions W_1 and W_2 of the colliding hadrons. Introducing³,

$$\begin{aligned}
W_{\mu\nu} &= \frac{(2\pi)^3}{e^2} V \bar{\Sigma} \Sigma \langle p | J_\mu(0) | p^1 \rangle \langle p^1 | J_\nu(0) | p \rangle \delta^4(p - p^1 - q)(E) \\
&= W_1(q^2, q \cdot p) \left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m^2} \right] + \frac{W_2(q^2, q \cdot p)}{m^2} \left[p_\mu - p \cdot q \frac{q_\mu}{q^2} \right] \\
&\quad \left[p_\nu - p \cdot q \frac{q_\nu}{q^2} \right]
\end{aligned} \tag{1}$$

and neglecting terms of order m^2/E^2 , q^2/ω^2 , as compared to unity, the spectrum $F(\omega)$, of transverse photons is given by

$$F(\omega) d\omega \simeq \frac{\alpha}{\pi} \frac{\omega}{E^2} \int \frac{dq^2 dq \cdot p}{q^4} \left\{ 2W_1(q^2, q \cdot p) + \frac{W_2(q^2, q \cdot p)}{m^2} \frac{EE^1}{\omega^2} q^2 \right\} d\omega \tag{2}$$

where E and m are the energy and mass of the hadron emitting a photon of energy ω and mass q^2 and E^1 , m_f are the energy and mass of the hadron after emission of the photon. To make the problem completely symmetrical with respect to the two protons we are using their $c \cdot m$ system throughout this article.

The mass, q^2 , of the exchanged photons (of Figure 1), though small in general, grows with its energy ω and is given, in the small angle approximation by

$$q^2 \simeq \frac{m^2 \omega^2}{EE^1} + EE^1 \theta^2 \tag{3}$$

for the elastically emitted photons, and by

$$q^2 \simeq \left(\frac{m_f^2}{E^1} - \frac{m^2}{E} \right) \omega + EE^1 \theta^2 \tag{4}$$

for the inelastically emitted photons, where θ is the angle between the vector p of the initial hadron and p^1 of the final hadron. The integral over q^2 in (2) is split up into two

regions: for $q^2 \gtrsim m_c^2 \simeq 2 \text{ GeV}^2$ the emitted photon is in the deep inelastic region⁵ where scaling holds and for $q^2 < m_c^2$ the photon can be emitted either inelastically (we call it the shallow inelastic region) or elastically.

The elastic spectrum is obtained by using

$$W_2(q^2, q \cdot p) = m^2 \delta \left[q \cdot p - q^2/2 \right]$$

$$W_1(q^2, q \cdot p) = q^2/4m^2 W_2(q^2, q \cdot p). \quad (5)$$

So that we get from (2)

$$F^{\text{el}}(\omega) d\omega = \frac{\alpha}{\pi} \frac{E^2 + (E - \omega)^2}{E^2} \frac{d\omega}{\omega} \ln \frac{\epsilon_1^2 m^2}{q_{\min}^2} \quad (6)$$

where $q_{\min}^2 = m^2 \omega^2 / E^2$.

The photon mass is cut-off here at $q^2 = \epsilon_1^2 m^2$ where $\epsilon_1^2 m^2 \ll m_p^2$ because in the region $q^2 \gtrsim m_p^2$ the elastic form factors⁹ of the proton become very important and the equivalent photon approximation, in its simplest form, fails. However, the contribution to the leading logarithm in E as $E \rightarrow \infty$ comes from the lower limit in q^2 so that the region $\epsilon_1^2 m^2 \leq q^2 \leq m_c^2$, being bounded both from above and below, does not make any contribution to that logarithm and can therefore be safely discarded. Hence, using (3) we have

$$F^{\text{el}}(\omega) d\omega = \frac{\alpha}{\pi} \frac{E^2 + (E - \omega)^2}{E^2} \frac{d\omega}{\omega} \ln \frac{\epsilon_1 E}{\omega} \quad (7)$$

For the shallow inelastic region where the proton inelastically emits a photon of mass less than m_c^2 , using the relation

$$W_1(q^2, q \cdot p) = (1 + v^2/q^2) W_2(q^2, q \cdot p) \frac{1}{1 + \sigma_l/\sigma_t} \quad (8)$$

where $v = (q \cdot p)/m$, the spectrum of equivalent photons becomes

$$F^S \cdot I(\omega) d\omega = \frac{\alpha}{\pi} \frac{\omega}{E^2} d\omega \int \frac{dq^2 d(q \cdot p)}{q^4} \left\{ 2 \frac{1 + v^2/q^2}{1 + \sigma_\ell/\sigma_t} + \frac{EE^1}{\omega^2} \frac{q^2}{m^2} \right\} W_2(q^2, q \cdot p) \quad (9)$$

To extract the leading log E term from this relation we assume that

$$W_2(q^2, q \cdot p) \xrightarrow[v \rightarrow \infty]{} \frac{m}{v} q^2 F(q^2) \quad (10)$$

an assumption⁴ supported very well by experimental data. Now in the limit of $E \rightarrow \infty$ holding ω fixed we get

$$F^S \cdot I(\omega) d\omega \xrightarrow[E \rightarrow \infty]{} \frac{\alpha}{\pi} \frac{E^1}{E} \frac{d\omega}{\omega} \ln \frac{E}{\omega} \cdot I \quad (11)$$

where $I = \int_{q_{\min}^2}^{m_c^2} F(q^2) dq^2$.

Finally, for $m_c^2 \leq q^2 \leq q_{\max}^2$ (with $q_{\max}^2 \ll \omega^2$) we are in the deep inelastic scaling region where the structure functions become functions of a single variable⁵

$x = \frac{q^2}{2q \cdot p}$. The equivalent photon spectrum in this region then takes the form, from (6), as

$$F^D \cdot I(\omega) d\omega = \frac{\alpha}{2\pi} \frac{d\omega}{\omega} \int \frac{1}{E} \frac{F(x) dx}{x} \frac{g^2 + (g - \omega)^2}{g^2} \ln \frac{q_{\max}^2}{m^2} \quad (12)$$

where $g = xE$, $F(x) = \frac{\sqrt{W_2(x)}}{m}$. Using the fact that $\sqrt{W_2(x)}$ is a constant for $x \lesssim 0.25$ ¹¹

$$F^D \cdot I(\omega) d\omega \xrightarrow[E \rightarrow \infty]{} \frac{\alpha}{\pi} \frac{d\omega}{\omega} F(0) \ln \frac{E}{2\omega} \ln \frac{q_{\max}^2}{m_c^2} \quad (13)$$

So that as $E \rightarrow \infty$ with ω fixed the leading log E contribution to the spectrum of photons is given by

$$F(\omega) d\omega = \left[F^{el}(\omega) + F^S \cdot I(\omega) + F^D \cdot I(\omega) \right] d\omega \quad (14)$$

where the spectrums for elastic, shallow inelastic and deep inelastic photons are given

respectively by (7), (11) and (13). While each of these grows as $\ln \frac{E}{\omega}$ as $E \rightarrow \infty$, holding ω fixed, the logarithms originate differently in each case. The elastic spectrum yields the logarithm upon integration over q^2 from $q^2 = q_{\min}^2$ limit. The shallow inelastic region gives the logarithm upon integration over $\nu = (q \cdot p)/m$ for any fixed $q^2 < 2 \text{ GeV}^2$ and finally the deep inelastic spectrum has its leading logarithm coming from integration over the scaling variable x .

THE TWO PHOTON CROSS-SECTION

The two photon cross section, $(d\sigma/dM^2)_{2\gamma}$, in the equivalent photon approximation, can be written as

$$\left(\frac{d\sigma}{dM^2}\right)_{2\gamma} \simeq \iint F(\omega_1) F(\omega_2) d\omega_1 d\omega_2 \delta[M^2 - 4\omega_1\omega_2] d\sigma_{\ell^+\ell^-}(M^2, q_1^2, q_2^2) \quad (15)$$

where $F(\omega)$ is the equivalent photon spectrum given by equation (2) and $d\sigma_{\ell^+\ell^-}(M^2, q_1^2, q_2^2)$, the cross section for the production of a lepton pair of mass M^2 by collision of two photons of energy ω_1, ω_2 (see Figure 2) and mass q_1^2, q_2^2 respectively, is (with $q_1^2, q_2^2 \ll M^2$)

$$d\sigma_{\ell^+\ell^-}(M^2, q_1^2, q_2^2) \simeq \frac{4\pi\alpha^2}{M^2} \left[-\ln\left(\frac{\mu^2}{M^2} + \frac{q_1^2 q_2^2}{M^4}\right) - 1 \right] \quad (16)$$

where μ is the lepton mass. From (3) we see that for $S \rightarrow \infty$ (with M^2 , or M^2/S fixed)

$$\frac{q_1^2 q_2^2}{M^2} \ll \mu^2.$$

So that,

$$d\sigma_{\ell^+\ell^-}(M^2, q_1^2, q_2^2) \xrightarrow{S \rightarrow \infty} d\sigma_{\ell^+\ell^-}(M^2, 0, 0) = \frac{4\pi\alpha^2}{M^2} \left[\ln \frac{M^2}{\mu^2} - 1 \right]. \quad (17)$$

From (14) and (15) we see that

$$\begin{aligned} \left(\frac{d\sigma}{dM^2} \right)_{2\gamma} = & \left(d\sigma/dM^2 \right)^{\text{el-el}} + \left(d\sigma/dM^2 \right)^{\text{el-S} \cdot \mathbf{I}} + \left(d\sigma/dM^2 \right)^{\text{el-D} \cdot \mathbf{I}} + \left(d\sigma/dM^2 \right)^{\text{S} \cdot \mathbf{I}-\text{D} \cdot \mathbf{I}} \\ & + \left(d\sigma/dM^2 \right)^{\text{S} \cdot \mathbf{I}-\text{S} \cdot \mathbf{I}} + \left(d\sigma/dM^2 \right)^{\text{D} \cdot \mathbf{I}-\text{D} \cdot \mathbf{I}} \end{aligned} \quad (18)$$

thus allowing for each of the photons to be in the elastic, shallow inelastic or deep inelastic region. The contribution, from each of these terms, up to the leading order in $\log S$ as $S \rightarrow \infty$ can be obtained by using the expressions for the spectrums derived in the previous section.

CASE I: $S \rightarrow \infty$ holding M^2 fixed

Elastic-Elastic

From (15) and (7) we get the expression for the elastic-elastic cross section, a process in which each of the photons is emitted elastically with a mass $< \epsilon_1^2 m^2 \ll m^2$, as

$$\begin{aligned} \left(\frac{d\sigma}{dM^2} \right)^{\text{el-el}} = & \frac{\alpha^2}{4\pi^2} \int \frac{d\omega}{\omega} \frac{E^2 \epsilon(E-\omega)^2}{E^2} \frac{E^2 + (E - M^2/4\omega)^2}{E^2} \ln \frac{\epsilon_1^2 E^2}{\omega^2} \\ & \ln \frac{16\epsilon_1^2 E^2 \omega^2}{M^4} \frac{d\sigma_{\ell+\ell-}(M^2)}{M^2} \\ \rightarrow & \frac{\alpha^2}{6\pi^2} \frac{d\sigma_{\ell+\ell-}(M^2)}{M^2} (2 + M^2/S)^2 \left[\ln(\epsilon_1^2 S/M^2) \right]^3 \end{aligned} \quad (19)$$

where

$$d\sigma_{\ell+\ell-}(M^2) = d\sigma_{\ell+\ell-}(M^2, 0, 0) .$$

Since $q_{\text{max}}^2 \sim \frac{m^2 \omega^2}{E^2} < \epsilon_1^2 m^2 \Rightarrow \omega_{\text{max}} = \epsilon_1 E$ so that

$$M_{\text{max}}^{\text{el-el}} = \epsilon_1^2 S .$$

ELASTIC-SHALLOW INELASTIC

The cross section for this process is given (from eqs. 7, 11 and 15) by

$$\begin{aligned} \left(\frac{d\sigma}{dM^2}\right)^{\text{el-S} \cdot \mathbf{I}} &= \frac{\alpha^2}{\pi^2} \frac{I^2}{M^2} \int \frac{d\sigma_{\ell+\ell^-(M^2)}}{\omega} d\omega \frac{E^2 + (E - \omega)^2}{E^2} \left(1 - \frac{M^2}{4\omega E}\right) \ln \frac{\epsilon_1 E}{\omega} \ln \frac{4E\omega}{M^2} \\ &\rightarrow \frac{\alpha^2}{3\pi^2} \frac{I^2}{M^2} \frac{d\sigma_{\ell+\ell^-(M^2)}}{M^2} \left[\ln \frac{\epsilon_1 \epsilon_2 S}{M^2} \right]^3 (1 + 2M^2/S). \end{aligned} \quad (20)$$

Since the maximum mass of the elastic photon is $\epsilon_1^2 m^2$ and that of the shallow inelastic is m_c^2 equation (3) implies that $(M^2)_{\text{max}}^{\text{el-S} \cdot \mathbf{I}} = \epsilon_1 \epsilon_2 S$.

SHALLOW INELASTIC - SHALLOW INELASTIC

From (11) and (15) we get

$$\begin{aligned} \left(\frac{d\sigma}{dM^2}\right)^{\text{S} \cdot \mathbf{I-S} \cdot \mathbf{I}} &= \frac{\alpha^2}{\pi^2} \frac{I^2}{M^2} \int \frac{d\omega}{\omega} \left(1 - \frac{\omega}{E}\right) \left(1 - \frac{M^2}{4\omega E}\right) \ln \frac{E}{\omega} \ln \frac{4\omega E}{M^2} d\sigma_{\ell+\ell^-(M^2)} \\ &= \frac{\alpha^2}{6\pi^2} \frac{I^2}{M^2} d\sigma_{\ell+\ell^-(M^2)} \left[\ln \frac{\epsilon_2^2 S}{M^2} \right]^3 (1 + M^2/S) \end{aligned} \quad (21)$$

with

$$(M^2)_{\text{max}}^{\text{S} \cdot \mathbf{I-S} \cdot \mathbf{I}} = \epsilon_2^2 S.$$

DEEP INELASTIC CONTRIBUTIONS

In the deep inelastic spectrum (equation 13), as pointed out earlier, the leading $\log S$ factor comes from the integration over the scaling variable $x = q^2/2q \cdot p$ of the structure functions. The coefficient of this leading logarithm involves the integral

$$\int_{m_c^2}^{q_{\max}^2} \frac{dq^2}{q^2}$$

where the equivalent photon approximation restricts $q_{\max}^2 < \omega^2$. In general there will be a leading logarithmic contribution to $(d\sigma/dM^2)_{2\gamma}$ coming from photons with $q^2 > \omega^2$.

We are therefore unable to evaluate this cross section in the leading log approximation in the limit of $S \rightarrow \infty$ holding M^2 fixed. However, we notice from (16) that

$$d\sigma_{\ell+\ell-}(M^2, q_1^2, q_2^2) \xrightarrow{S \rightarrow \infty} d\sigma_{\ell+\ell-}(M^2, 0, 0) = \frac{4\pi\alpha^2}{M^2} \left[\ln\left(\frac{M^2}{\mu^2}\right) - 1 \right]$$

for $q_1^2/M^2, q_2^2/M^2 \ll 1$. So that by taking $q_{\max}^2 = \epsilon M^2$ (with fixed small ϵ) we will be able to evaluate the term containing $\ln M^2/\mu^2$ in the leading log S contribution to $(d\sigma/dM^2)_{2\gamma}$ from the deep inelastic photons for $S \rightarrow \infty$ with fixed M^2 .

We then have from (13)

$$F^{D \cdot I}(\omega) d\omega = \frac{\alpha}{2\pi} \int \frac{d\omega}{\omega} \frac{F(x)}{x} \frac{g^2(g-\omega)^2}{g^2} \ln \frac{M^2}{m_c^2}$$

$$\xrightarrow[E \rightarrow \infty]{\omega \text{ fixed}} \frac{\alpha}{\pi} \frac{d\omega}{\omega} F(0) \ln E/2\omega \ln \frac{M^2}{m_c^2} \quad (22)$$

ELASTIC - DEEP INELASTIC

From (7) and (10) we have

$$\left(\frac{d\sigma}{dM^2}\right)^{\text{el-D} \cdot I} = \frac{\alpha^2}{\pi^2} \frac{F(0)}{M^2} \int \frac{d\omega}{\omega} \frac{E^2 + (E-\omega)^2}{E^2} \ln \frac{\epsilon^2 E^2}{\omega^2} \ln \frac{4\omega E}{M^2} d\sigma_{\ell+\ell-}(M^2) \ln \frac{M^2}{m_c^2}$$

$$\rightarrow \frac{\alpha^2}{3\pi^2} \frac{F(0)}{M^2} \ln\left(\frac{M^2}{m_c^2}\right) d\sigma_{\ell+\ell-}(M^2) \left(\ln \frac{\epsilon^2 S}{2M^2}\right)^3 \quad (23)$$

SHALLOW INELASTIC - DEEP INELASTIC

From (9) and (12) we get

$$\begin{aligned} \left(\frac{d\sigma}{dM^2} \right)^{S \cdot I-D \cdot I} &= \frac{\alpha^2}{\pi^2} \frac{F(0) I(q^2)}{M^2} \int \frac{d\omega}{\omega} \left(1 - \frac{M^2}{4\omega E} \right) \ln \frac{E}{\omega} \ln \frac{4\omega E}{M^2} \ln M^2/m_c^2 d\sigma_{\ell+\ell}-(M^2) \\ &\rightarrow \frac{\alpha^2}{6\pi^2} \frac{F(0) I(q^2)}{M_c^2} \ln \left(\frac{M^2}{m_c^2} \right) d\sigma_{\ell+\ell}-(M^2) \left(\ln \frac{E}{2M^2} \right)^3. \end{aligned} \quad (24)$$

DEEP INELASTIC - DEEP INELASTIC

From (12) we have

$$\begin{aligned} \left(\frac{d\sigma}{dM^2} \right)^{D \cdot I-D \cdot I} &= \frac{\alpha^2}{\pi^2} \frac{F^2(0)}{M^2} \int \frac{d\omega}{\omega} \ln \frac{E}{\omega} \ln \frac{4\omega E}{M^2} \left[\ln (M^2/m_c^2) \right]^2 d\sigma_{\ell+\ell}-(M^2) \\ &\rightarrow \frac{\alpha^2}{6\pi^2} \frac{F^2(0)}{M^2} \left[\ln M^2/m_c^2 \right]^2 d\sigma_{\ell+\ell}-(M^2) \left(\ln \frac{S}{4M^2} \right)^3. \end{aligned} \quad (25)$$

To get an idea of their relative importance we plot each of these contributions along with their sum for $S = 3200 \text{ GeV}^2$. We notice that for $M^2/S \ll 1$ where all of them contribute their leading logarithmic contributions are roughly comparable to each other. (See figures 3a and 8b)¹²

CASE II: $S \rightarrow \infty$ with M^2/S Held Fixed.

In this case the entire contribution to the leading $\log S$ term comes only from the deep inelastic - deep inelastic process and can be evaluated in the equivalent photon approximation by using (15) and (20) as

$$\left(\frac{d\sigma}{dM^2}\right)_{2\gamma} = \frac{\alpha^2}{2\pi^2} \frac{d\sigma_{\ell^+\ell^-}(M^2)}{M^2} \left(\ln \frac{M^2}{m^2}\right)^2 \int \frac{F(x_1)}{x_1} \frac{F(x_2)}{x_2} dx_1 dx_2 \left\{ \frac{1}{2}(2+y)^2 \ln \frac{1}{y} - (1-y)(3+y) \right\} \quad (26)$$

where $y = M^2/(x_1 x_2 S)$ so that with $M^2/S = \delta$

$$\left(\frac{d\sigma}{dM^2}\right)_{2\gamma} \xrightarrow[S \rightarrow \infty]{M^2 \rightarrow \infty, \frac{M^2}{S} = \delta \text{ held fixed}} \frac{2\alpha^4}{M^4 \pi} \left(\ln \frac{M^2}{m^2}\right)^2 \ln \frac{M^2}{\mu^2} F(\delta, x_1, x_2) \quad (27)$$

where

$$F(\delta, x_1, x_2) = \int_{\delta}^1 dx_1 \int_{\delta/x_1}^1 dx_2 \frac{F(x_1)}{x_1} \frac{F(x_2)}{x_2} \left\{ \frac{1}{2} \left(2 + \frac{\delta}{x_1 x_2}\right)^2 \ln \frac{x_1 x_2}{\delta} - \left(1 - \frac{\delta}{x_1 x_2}\right) \left(3 + \frac{\delta}{x_1 x_2}\right) \right\} \quad (28)$$

The results of the calculation of both these cases are plotted in graphs for $S = 60, 1000, 3200$ and 160000 GeV^2 . For comparisons we also plot the one photon cross section⁶ for these values of S and the results of the BNL² experiment. It is quite evident from these graphs that in general the two photon process makes only a small contribution to the lepton pair production processes. (See Figure 4 a-d.)

DEPENDENCE ON THE LEPTON MASS

Even though, as we have seen, the two photon production of lepton pairs in hadron-hadron collisions is just a small correction to the one photon process, it has two important

features which are altogether different from the one photon process and which make a study of the two photon process rather interesting.

The appearance of the lepton mass dependence in the two photon cross section has its origin in the process of Figure 2 where ^{the} cross section for $S \rightarrow \infty$, as in (17), is

$$d\sigma_{\ell+\ell-}(M^2, 0, 0) = d\sigma_{\ell+\ell-}(M^2) \sim \frac{4\pi\alpha^2}{M^2} \left(\ln \frac{M^2}{\mu^2} - 1 \right) \quad (17)$$

resulting in the logarithmic dependence on the lepton mass of the two photon cross section $(d\sigma/dM^2)_{2\gamma}$. The one photon process, on the other hand, is independent of the lepton mass up to order μ^2/M^2 . From (15) and (17) we have, for example, for $S \rightarrow \infty$

$$\frac{\left(\frac{d\sigma}{dM^2}\right)_{2\gamma}^{pp \rightarrow e^+e^- + \text{hadrons}}}{\left(\frac{d\sigma}{dM}\right)_{pp \rightarrow \mu^+\mu^- + \text{hadrons}}} \approx \frac{\ln(M/m_e) - \frac{1}{2}}{\ln(M/m_\mu) - \frac{1}{2}} \approx \begin{cases} 4 & \text{for } M \sim 1 \text{ GeV} \\ 2 & \text{for } M \sim 10 \text{ GeV} \end{cases} \quad (29)$$

thus rendering a characteristic difference in the e, μ production cross sections.

We, therefore, find that the number of electron pairs produced (by the two photon process alone) is much more than the number of muon pairs produced. But as far as the total cross section, that is, the sum of the one photon and two photon cross sections, is concerned the difference in the electron and muon cross sections is rather small. To make the discussion more quantitative let us define a function ϵ as

$$\epsilon = \frac{(d\sigma/dM)_{e^+e^-} - (d\sigma/dM)_{\mu^+\mu^-}}{(d\sigma/dM)_{e^+e^-} + (d\sigma/dM)_{\mu^+\mu^-}} \quad (30)$$

where $(d\sigma/dM)_{\ell+\ell-}$ is the total lepton pair production cross section in hadron-hadron

collisions. Then the contribution to ϵ from the 2γ process of Figure 1, $\epsilon^{2\gamma}$ say,

$$\epsilon^{2\gamma} = \frac{(\frac{d\sigma}{dM})_{e^+e^-}^{2\gamma} - (\frac{d\sigma}{dM})_{\mu^+\mu^-}^{2\gamma}}{(\frac{d\sigma}{dM})_{e^+e^-}^{2\gamma} + (\frac{d\sigma}{dM})_{\mu^+\mu^-}^{2\gamma}} \quad (31)$$

can be obtained from the calculation of the previous sections and those of reference 6.

It is important to realize that for $S \rightarrow \infty$, M^2 fixed the term containing $\ln M^2/\mu^2$ that we have obtained represents the entire leading $\log S$ contribution to $\epsilon^{2\gamma}$ even though it may not represent the entire leading $\log S$ contribution to the two photon cross section. To get a feeling for the numbers involved we plot $\epsilon^{2\gamma}$ (for $S = 3200 \text{ GeV}^2$) in Figure 5. For $M^2/S \lesssim 0.2$ we notice that $\epsilon^{2\gamma}$ is a few percent. One could test for μ - e symmetry by comparing the cross sections for production of electron and muon pairs in hadron-hadron collisions. ϵ is the expected difference of an electromagnetic nature. An experimental difference^{8,10} much larger than ϵ would reveal some hitherto unknown difference in the electron and muon of a non-electromagnetic origin.

For this interesting application of the logarithmic dependence on the lepton mass of the two photon cross section, it may be worthwhile to point out the effect of the correction term $q_1^2 q_2^2 / M^4$ in the argument of the logarithm in equation (16). From (3) we find

$$\frac{q_1^2 q_2^2}{M^2} \gtrsim \frac{m^4}{M^2} \left(\frac{M^2}{S}\right)^2 = \frac{m^4}{S} \left(\frac{M^2}{S}\right) \quad (32)$$

So that the lepton mass dependence, in the simple form $\ln \frac{M}{\mu}$ of the two photon process, is correct only if

$$\frac{m^4}{S} \frac{M^2}{S} \ll \mu^2 \quad (33)$$

Hence for the muon for $S > 100 \text{ GeV}^2$, if M^2/S be sufficiently small we can apply our asymptotic formula (17) for $d\sigma_{\ell+\ell-}(M^2)$ in (15) with some confidence but for the electron the applicability is questionable unless $S > 10^6 \text{ GeV}^2$ except for extremely small values

of M^2/S . However, this is no serious limitation in practice since $(d\sigma/dM^2)^{2\gamma}$ of equation (15), with the spectrum of equation (2) and with $d\sigma_{\ell+\ell-}(M^2, q_1^2, q_2^2)$ of equation (16), can be evaluated numerically with considerable ease. To get a feeling of the effect of the correction term $(q_1^2 q_2^2)/M^4$ such a numerical calculation has been done for M^2/S fixed case. This is shown in Figure 6 for $S = 3200 \text{ GeV}^2$.

ANGULAR DISTRIBUTION OF THE LEPTONS

The second point of interest emerges if one calculates the differential cross-sections in the lepton angle with the collision axis. In the equivalent photon approximation this is given by

$$\frac{\partial^2 \sigma}{\partial M^2 \partial (\cos \theta_\ell)} \simeq \int F(\omega_1) d\omega_1 F(\omega_2) d\omega_2 \delta[M^2 - 4\omega_1 \omega_2] \frac{\partial \sigma_{\ell+\ell-}(M^2)}{\partial (\cos \theta_\ell)} \quad (34)$$

where $F(\omega)$ is the spectrum of the equivalent photons, θ_ℓ is the angle which one of the leptons makes with the collision axes and $(\partial \sigma_{\ell+\ell-})/(\partial (\cos \theta_\ell))$ is the angular distribution of the lepton in the process of Figure (2). The latter has an angular singularity coming from the vanishing of the propagator in the direction of the collision axis of the hadrons (for $q_1^2, q_2^2 \ll M^2$). This singularity persists in (34) giving the two photon exchange process a strong peaking along the collision axis. The predominant contribution to the two photon cross section, thus, comes from leptons angle $\theta_\ell \sim (\mu^2/M^2)^{1/2}$ and therefore the conical spread is expected to be much smaller for electrons than for muons.

No such peaking is exhibited by the one photon process, since it has an angular distribution of the form $a + b \cos^2 \theta_\ell$. This characteristic difference in the angular

distributions of these two processes could prove important in an experimental differentiation of their contributions.

CONCLUSION

In conclusion we would like to discuss the salient features of this work and its prominent limitations.

In evaluating the two photon cross-section in case (1), that is, as $S \rightarrow \infty$ holding M^2 fixed, we find that the leading log approximation yields that the cross-section grows as $\log^3 S$. The leading logarithmic contribution from the deep inelastic photons arises from integration over the scaling variable x for all $q^2 \gtrsim m_c^2$. In general there may therefore be leading $\log S$ contribution from photons of $q^2 > \omega^2$ which we are unable to evaluate in the equivalent photon approach. However, for such photons the leading $\log S$ term will not contain $\log M^2/\mu^2$. Since in general $M^2 \gg \mu^2$ it is not unreasonable to expect that the term containing $\log M^2/\mu^2$ that we are able to calculate will be the dominant leading $\log S$ contribution to the cross-section. Since the simple asymptotic form of the spectrum that we obtain in this limit are valid only for $M^2/S \ll 1$ the resulting leading logarithmic expression for the two photon cross-section is expected to hold only for small invariant masses. As a specific example since νW_2 as a function of the scaling variable x falls rapidly from its constant value for $x \gtrsim .4$, so that the asymptotic expression for the deep inelastic spectrum (equation (13)) fails for $M^2/S \gtrsim 0.2$. In the figures drawn we therefore confine ourselves to $M^2/S \lesssim 0.2$.^{11,12}

In case (2) when we let both M^2 and S approach ∞ with M^2/S fixed, the leading logarithmic contribution to the cross-section goes as $\sim \log^3 M^2$. The only contribution to the leading log term in this case comes when both the photons are in

the deep inelastic region. Even though this approximation is well defined so long as $M^2 \gg m_c^2$ the correction terms (that is terms in the cross-section that grow as $\log^2 M^2$ or as $\log^2 S$) arising from elastic or shallow inelastic photons are expected to be quite significant unless M^2 be very very ($M \gtrsim 100$ Gev) large. We therefore witness a considerable difference in the results of the two approximations for energies presently available (see figs.(4) and (5)) .The two approximations however do approach each other as we let S grow sufficiently large. Since this approximation improves as M^2 grows larger, the regions of applicability of the two approximations are complementary to each other.

For $M^2/S \ll 1$ the leading log term containing $\log M^2/\mu^2$ enables us to calculate the function $e^{2\gamma}$ which measures the difference, up to the leading $\log S$ term , in the cross-sections for the $e^+ e^-$ and $\mu^+ \mu^-$ pairs and may therefore be used to test $e\mu$ symmetry.

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8. I take this opportunity to thank Professor L. M. Lederman for suggesting this test of μ -e symmetry.
9. The effect of the elastic form factors of the proton is considered by K. Fujikawa in his two photon calculation (Preprint EFL-71-69), The Enrico Fermi Institute, Chicago.
10. The lepton mass dependence of some other two photon processes is currently under investigation. It will be discussed in a future publication.
11. $\nu W_2(x)$ or $F(x)$ is not a constant for large values of x . Specifically for $x \gtrsim 0.4$, νW_2

drops almost linearly with $1/x$ so that cutting the integral $\int \frac{F(x)}{x} dx$ at $x = \frac{1}{2}$ for $E \rightarrow \infty$ with fixed ω enables it to be represented very accurately by its asymptotic value $F(0) \ln(E/2\omega)$

12. We are using the expressions obtained in this approximation ($S \rightarrow \infty, M^2$ fixed) to plot graphs only for $M^2/S \lesssim 0.2$ because for $M^2/S \gtrsim 0.2$ the contribution from the region where νW_2 is not a constant would become important and our asymptotic spectrum for the deep inelastic case would fail and would start overestimating the cross-section. See also footnote 11.

Figure Captions

Figure 1. The figure shows the process $p + p \rightarrow l^+ + l^- + \text{"anything"}$ via an exchange of two photons. Thick lines represent the unobserved final hadronic states.

Figure 2. The process $\gamma + \gamma \rightarrow l^+ + l^-$

Figure 3a. The leading $\log S$ terms in the elastic-elastic, elastic-shallow inelastic, shallow-shallow and in the total two photon (that is, the sum of six contributions, see equation (18)) cross-sections, obtained in the limit when $S \rightarrow \infty$, holding M^2 fixed, are plotted for $S = 3200 \text{ GeV}^2$. The curves are shown only for $M^2 \geq 0.2 S$ because the asymptotic formulas obtained are not valid beyond that region. See footnotes 11 and 12.

Figure 3b. The leading $\log S$ terms in the elastic-deep, shallow-deep, deep-deep and in the total two photon cross-sections are plotted for $S = 3200 \text{ GeV}^2$. These leading $\log S$ terms are only those that contain $\log M^2 / \mu^2$ in them.

Figure 4a-d. In these figures :

I represents the one photon exchange contribution calculated from Drell and Yan's paper (reference (6))

II (1) is the leading $\log S$ term (containing $\log M^2 / \mu^2$) in the two photon cross-section obtained in the approximation $S \rightarrow \infty$ holding M^2 fixed.

II (2) is the leading $\log S$ term in the two photon cross-section for the case when M^2 and $S \rightarrow \infty$ with M^2/S fixed.

III is the experimental data from reference 2 plotted without taking error bars into account.

Figure 5. $\epsilon^{2\gamma}(1)$ represents the leading $\log S$ term in (equation (31)) obtained in case (1) and $\epsilon^{2\gamma}(2)$ is the one obtained in case (2).

Figure 6. This figure is to show the effect of the correction term $q_1^2 q_2^2 / M^4$ in $\log(\mu^2 / M^2 + q_1^2 q_2^2 / M^4)$ on the two photon cross-section (for production of muon pairs) obtained for case (2). Curve (I) is the two photon cross-section with

$$dG_{l+l-}(M^2) = 4\pi \alpha^2 (\log(M^2/\mu^2) - 1) / M^2$$

and curve II is with

$$dG_{l+l-}(M^2) = -4\pi \alpha^2 (\log(\mu^2/M^2 + q_1^2 q_2^2 / M^4) + 1) / M^2$$

FIG. 1

FIG. 2

Fig 3a.

Fig 3b

Fig 4b

Fig 4d

Fig 5