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**TRANSIENT MULTIPHASE MULTICOMPONENT  
FLOW IN POROUS MEDIA**

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## Contents

Abstract . . . . .	1
Introduction . . . . .	1
Formulation of the Problem . . . . .	1
Numerical Solution . . . . .	6
Results of Sample Calculation . . . . .	8
Extent of Penetration . . . . .	11
Conclusions . . . . .	12
References . . . . .	13
Appendix A—Nomenclature . . . . .	14
Appendix B—Program Listing for PORE . . . . .	15
Appendix C—Properties Used in Calculations . . . . .	26

# TRANSIENT MULTIPHASE MULTICOMPONENT FLOW IN POROUS MEDIA

## Abstract

One-dimensional transient two-phase, two-component flow in a porous medium is investigated analytically. The flow is described by the use of a continuity equation for each component, a momentum equation for each phase, the first law of thermodynamics, state equations, and relations for the transport properties. Local thermodynamic equilibrium is assumed. The effects of condensation and evaporation are included. The analysis shows that the flow resulting from a

step change in the state at the surface of a uniform semi-infinite porous medium may be described by a single variable which is a simple function of position and time. Under other conditions, position and time must be separately specified to determine the state of the mixture. A computer program to describe the flow of an air-water mixture is presented. Results of calculations relevant to the flow of cavity gas following an underground nuclear explosion are given.

## Introduction

The cavity formed by an underground nuclear explosion ceases growing about a tenth of a second after the explosion. The vapor in the cavity consists predominantly of a mixture of rock gas and steam at a temperature of a few thousand degrees Kelvin and a pressure near the overburden pressure. The cavity gas ordinarily remains at temperatures and pressures of this magnitude until collapse of the cavity, which usually occurs after

several minutes. During this period, should the cavity gas have access to either the porous medium surrounding the cavity or the stemming column, the gas will flow into the porous material. The purpose of this report is to present the results of analysis of such a transient multiphase multicomponent flow. This work is part of a continuing program of the Containment Group of the Nuclear Test Engineering Division.

## Formulation of the Problem

In recent years, the analysis of transient multiphase flow in porous media

has received considerable attention. The subject has been of particular interest to

the petroleum industry because of its importance to the secondary recovery of petroleum. As a result, most of the available literature on such flows<sup>1-12</sup> deals with conditions that may be found in petroleum reservoirs. Multiphase flow resulting from an underground nuclear explosion, however, passes through considerably larger ranges of temperature and pressure. Consequently, assumptions in these studies usually prohibit their application to the problem of interest here. For example, the fluids are ordinarily considered incompressible<sup>1-4</sup> or barotropic.<sup>5-10</sup> Although such restrictive assumptions are not made in Refs. 11 and 12, the formulation of the first law of thermodynamics is in error in both cases.

The flow will be assumed to be a two-phase, two-component flow. When applying the analysis to flow of the cavity gas, the two components are taken to be air and water. Air is initially present in the porous medium. The gas flowing from the cavity is predominantly steam. Because the temperature of the gas entering the porous medium is considerably higher than the ambient temperature of the medium, condensation will occur and a two-phase flow results.

The equations governing this flow may be formulated in a straightforward manner. The equations will be written for a one-dimensional flow; the extension to more dimensions being obvious.

Conservation of mass is expressed by a continuity equation for each component. For air, this equation is

$$\frac{\partial}{\partial x} (\rho_a u_m) + \epsilon \frac{\partial}{\partial t} (\rho_a S_m) = 0. \quad (1)$$

Here, air is assumed to be present only in the vapor phase. Any air dissolved in the liquid phase is negligible;  $x$  is the distance in the direction of flow from the surface of the porous medium;  $t$  is the time;  $\rho_a$  is the mass density of the air;  $u_m$  is the apparent velocity, i.e., volume flow rate/unit area normal to flow, of the mixture of air and water vapor. The two phases will ordinarily have different apparent velocities.  $S_m$  is the saturation of the vapor phase. The saturation of a phase is the void volume fraction occupied by that phase.

The corresponding equation for water is

$$\frac{\partial}{\partial x} (\rho_l u_l + \rho_v u_m) + \epsilon \frac{\partial}{\partial t} (\rho_l S_l + \rho_v S_m) = 0. \quad (2)$$

The additional terms result from the presence of water in both phases. The subscript  $l$  refers to the liquid and the subscript  $v$  refers to water vapor. The air and water vapor have the same apparent velocity, diffusion being neglected.

The flow results from the pressure gradient in the medium. Flow due to gravity is not considered. For sufficiently small Reynolds numbers, the apparent velocity of a phase will be given by Darcy's law with the permeability reduced by the presence of the other phase:

$$u_l = - \frac{k_{rl} k}{\mu_l} \frac{\partial p}{\partial x}, \quad (3)$$

$$u_m = - \frac{k_{rm} k}{\mu_m} \frac{\partial p}{\partial x}. \quad (4)$$

The reduction in permeability is expressed by use of the relative permeability;  $k_{r\ell}$  and  $k_{rm}$  are the relative permeabilities of the liquid and the vapor mixture, respectively. The relative permeability of a phase is the ratio of the effective permeability for the phase to the absolute permeability  $k$  of the medium. Although not strictly functions of saturation only, the relative permeabilities depend most strongly on saturation and are ordinarily taken to depend only on saturation. The functional form of the dependence is different for the two phases because of the wetting of the liquid phase.  $\mu$  and  $p$  are viscosity and pressure respectively.

For an isothermal flow these equations, together with state and transport property relations and the initial and boundary conditions, would be sufficient to describe the flow. Gas flow, under the conditions considered here, can be treated as isothermal with very little loss of accuracy. The temperature of the incoming gas from the cavity is very much larger than the initial temperature of the porous medium. As the gas flows, the temperature at any location in the medium will rise. However, before this change in temperature can become appreciable at a position, the mass of gas passing that position must be of a magnitude near to the mass of the solid lying between that position and the inlet. Consequently, for gas flow, the time required to produce significant changes in the temperature is extremely long compared to the time in which changes in pressure become significant. This gas flow is, for all practical purposes, isothermal. Morrison<sup>13</sup> has analyzed transient ideal gas flow under these conditions.

In the flow of interest here, however, condensation will occur. The temperature in the porous medium will rise more rapidly as a result. Moreover, the partial pressure of the water vapor cannot exceed the saturation pressure at the local temperature. Until the temperature at a position begins to change significantly, the steam will exert little pressure there. We conclude that, in this case, the local temperature and fluid pressure will begin to change rapidly at the same time and that this change will result from local condensation of the steam.

To describe this flow, it is necessary to apply the first law of thermodynamics. Because the response time for local heat transfer from the fluid to the solid grains is several orders of magnitude smaller than the times of interest, this response time may be neglected. Thus, the fluid and solid temperatures are assumed equal. This condition of local thermodynamic equilibrium simplifies the formulation of first law, avoids ambiguities in the designation of local temperatures, and makes a model for heat transfer between fluid and solid superfluous. In analysis of flows through beds of small particles, the assumption of temperature equality is common, e.g., see Refs. 11-15.

For an elemental slab with surfaces normal to the direction of flow, the first law is written

$$\begin{aligned} \frac{\partial}{\partial x} (\rho_{\ell} u_{\ell} h_{\ell} + \rho_v u_m h_v + \rho_a u_m h_a) \\ + \epsilon \frac{\partial}{\partial t} (\rho_{\ell} S_{\ell} e_{\ell} + \rho_v S_m e_v + \rho_a S_m e_a) \\ + (1 - \epsilon) \rho_s \frac{\partial e_s}{\partial t} = 0. \quad (5) \end{aligned}$$

Here  $e$  and  $h$  are the internal energy and enthalpy per unit mass, respectively. The subscript  $s$  refers to the solid. Changes in kinetic energy are neglected in agreement with the low Reynolds number restriction implicit in the use of Darcy's law. Conduction of heat through the bed is neglected in comparison to the heat transfer by the motion of the fluid. This, too, is a frequently used assumption in the analysis of flow through porous media.<sup>16,17</sup> The thermal conductivity of unconsolidated sand<sup>18</sup> is sufficiently small that this mechanism of heat transfer becomes important only after long times as the rate of fluid flow decreases.

Defining a few dimensionless quantities makes easier the numerical solution of these equations and permits easy comparison of the results with results of corresponding ideal gas flows. Accordingly, we define a dimensionless position,

$$X = x/L, \quad (6)$$

a dimensionless pressure,

$$P = p/p^*, \quad (7)$$

and a dimensionless time,

$$\tau = kp^*t/\epsilon\mu^*L^2, \quad (8)$$

where  $L$  is the length of the bed,  $p^*$  is the highest pressure at the inlet (simply the inlet pressure for the step change boundary condition employed here) and  $\mu^*$  is the viscosity of the air in the medium at ambient conditions. If the permeability and porosity of the medium are not uniform and constant, reference values of these properties should also be used. We will not consider such variations here. The difference between the definition of  $\tau$

used here and that used in Ref. 13 is small when the inlet pressure is large. Results of the two analyses may be readily compared.

Elimination of the apparent velocities from the continuity equations and the first law is accomplished by substitution of the two Darcy's law expressions into these equations. With these substitutions, we obtain relations that contain only thermodynamic and transport properties as dependent variables. Making these substitutions and introducing the dimensionless variables, we obtain for air continuity

$$\frac{\partial}{\partial X} \left( \frac{\rho_a k_{rm} \mu^*}{\mu_m} \frac{\partial P}{\partial X} \right) = \frac{\partial}{\partial \tau} (\rho_a S_m), \quad (9)$$

for water continuity

$$\frac{\partial}{\partial X} \left[ \left( \frac{\rho_l k_{rl} \mu^*}{\mu_l} + \frac{\rho_v k_{rm} \mu^*}{\mu_m} \right) \frac{\partial P}{\partial X} \right] = \frac{\partial}{\partial \tau} (\rho_l S_l + \rho_v S_m), \quad (10)$$

and for the first law

$$\begin{aligned} \frac{\partial}{\partial X} \left\{ \left[ \frac{\rho_l h_l k_{rl} \mu^*}{\mu_l} + (\rho_v h_v \right. \right. \\ \left. \left. + \rho_a h_a) \frac{k_{rm} \mu^*}{\mu_m} \right] \frac{\partial P}{\partial X} \right\} \\ = \frac{\partial}{\partial \tau} \left( \frac{1-\epsilon}{\epsilon} \rho_s e_s + \rho_l S_l e_l \right. \\ \left. + \rho_v S_m e_v + \rho_a S_m e_a \right). \quad (11) \end{aligned}$$

Equations (9), (10) and (11) govern the one-dimensional transient two-phase two-component flow through a finite porous bed with uniform permeability and porosity.

Initial conditions, boundary conditions, state and transport properties remain to be specified.

Boundary conditions at the inlet to the porous medium are given by the state of the gas within the cavity. In an underground nuclear experiment, the cavity pressure attains a value near the overburden pressure after cessation of cavity growth. The temperature in the cavity is of the order of a few thousand degrees Kelvin. Cavity conditions then ordinarily change slowly until the collapse of the cavity. Cavity collapse may occur several minutes after the detonation. Olsen<sup>19</sup> describes the cavity pressure and temperature history resulting from an underground nuclear experiment in alluvium.

It is convenient to consider idealized boundary conditions corresponding to step changes of cavity temperature and pressure to constant values. Results of flow calculations, using step change boundary conditions, can be presented in a particularly simple and useful form. The advantage extends until such time as significant property changes begin to occur near the other end of the porous medium.

If we restrict our attention to those times when the effects of flow from the cavity cause only insignificant changes at the far boundary, the far boundary may be removed and the porous medium considered to be semi-infinite. The time during which the effect of a distant boundary is negligible may be easily estimated from the resulting calculations. A numerical example is presented later.

The advantage of considering flow in a semi-infinite medium resulting from a step change in the inlet conditions is due to the existence of a

similar solution. Defining a dimensionless variable,

$$\theta = \frac{X}{2\sqrt{\tau}} = \frac{x}{2} \sqrt{\frac{\epsilon \mu^*}{k p^* t}} \quad (12)$$

we find that the partial differential equations (9), (10) and (11) describing the flow may be written as ordinary differential equations in the independent variable,  $\theta$ . The only restriction is that all dependent variables must be functions solely of the local thermodynamic state. No property may be taken, a priori, to have an explicit dependence on either position or time. In terms of the similarity variable, equations (9), (10) and (11) then become

$$\frac{d}{d\theta} \left( \frac{\rho_a k_{rm} \mu^*}{\mu_m} \frac{dP}{d\theta} \right) + 2\theta \frac{d}{d\theta} (\rho_a S_m) = 0, \quad (13)$$

$$\frac{d}{d\theta} \left[ \left( \frac{\rho_l k_{rl} \mu^*}{\mu_l} + \frac{\rho_v k_{rm} \mu^*}{\mu_m} \right) \frac{dP}{d\theta} \right] + 2\theta \frac{d}{d\theta} (\rho_l S_l + \rho_v S_m) = 0, \quad (14)$$

and

$$\frac{d}{d\theta} \left\{ \left[ \frac{\rho_l h_l k_{rl} \mu^*}{\mu_l} + (\rho_v h_v + \rho_a h_a) \frac{k_{rm} \mu^*}{\mu_m} \right] \frac{dP}{d\theta} \right\} + 2\theta \frac{d}{d\theta} \left( \frac{1-\epsilon}{\epsilon} \rho_s e_s + \rho_l S_l e_l + \rho_v S_m e_v + \rho_a S_m e_a \right) = 0, \quad (15)$$

respectively.

Boundary conditions at the cavity-medium interface correspond to conditions at  $\theta$  equal to zero. Only step change boundary conditions may be expressed in this manner. A similar solution does not exist for less idealized cavity conditions. In those cases, the local thermodynamic state in the medium is not a function of the single variable  $\theta$ , but depends on position and time separately.

Initial conditions in the medium correspond to conditions as  $\theta$  goes to infinity. Only uniform initial conditions in a semi-infinite medium may be so expressed. Note that the length of the bed, present in the definitions (6) and (8) of dimensionless position and time, does not appear in the definition (12) of  $\theta$ . Because the changes of state become quite small for sufficiently large finite values of  $\theta$ , the results may be applied to flow in finite beds for significant periods of time.

The solution of the governing set of equations, (9)-(11) or (13)-(15), is best accomplished by the use of numerical techniques. One may readily demonstrate

that a nontrivial, closed-form analytical solution in terms of well-tabulated functions does not exist. If we consider only the relatively simple special case of isothermal ideal gas, then the flow is governed by Eq. (13) alone since no water is present and the temperature is constant. In this limiting case, Eq. (13) can be considerably simplified. With only a single phase present, the relative permeability and saturation are each unity. When the temperature is constant and no water vapor is present,  $\mu_m$  is everywhere equal to  $\mu^*$ . Finally, the air density is proportional to the pressure and Eq. (13) reduces to

$$\frac{d}{d\theta} \left( P \frac{dP}{d\theta} \right) + 2\theta \frac{dP}{d\theta} = 0. \quad (16)$$

Using the integral transformation of von Mises,<sup>20</sup> this equation may be transformed to Blasius' equation. Because we obtain Blasius' equation in this limiting case, we verify that closed form solutions may not be obtained for the flows governed by Eq. (13)-(15).

## Numerical Solution

The existence of a similar solution does not imply that the ordinary differential equations in the similarity variable are easier to solve numerically than the partial differential equations which have position and time as independent variables. Because any advantage in numerically solving the ordinary differential equations is not apparent and because a program to solve the partial differential equations can be used under less restrictive conditions, a program was written to solve the

partial differential equations (9)-(11). Results of calculations using this program may be presented in the similar form simply by using the value of  $\theta$  corresponding to the values of  $X$  and  $\tau$ . The program is explicit and self-starting. The finite difference expressions employed have truncation errors of the order of the step size. Use of higher order expressions results in the appearance of a Gibbs phenomenon because of shocklike features of the flow.



The solution of equations (9)-(11) is accomplished by first calculating the finite difference approximations of the derivatives on the left-hand sides of these equations. The equations are then numerically integrated in time for one time step. This yields values of the three quantities in parentheses on the right-hand sides of Eqs. (9), (10), and (11) at the next discrete value of time. In accordance with Gibbs' phase rule, three such quantities are necessary and sufficient information to determine the state of the mixture, including the amounts of water present as liquid and as vapor.

If the water present at a location is a superheated vapor, then the expressions for the three calculated quantities are significantly simplified by setting the water saturation and vapor mixture saturation equal to zero and one, respectively. Determination of the state is then straightforward. If the calculated values of the three quantities are inconsistent with a superheated water state, the system contains a mixture of saturated liquid and saturated vapor. The state is then determined iteratively by using successive linear interpolation to find the temperature. Saturation properties, the relations for air and solid internal energies, and the quantity determined from Eq. (9) are substituted into the expressions for the quantities that were found by integration of Eqs. (10) and (11). Two estimates of the water saturation result. The iteration proceeds toward the temperature at which the water saturation estimates agree, yielding the state of the system.

Subsidiary equations to calculate state and transport properties must also be

used. Air is considered to be an ideal gas with a variable specific heat. The viscosity of air is considered to be a function only of temperature, and is calculated using an empirical correlation given by Hilsenrath and Touloukian.<sup>21</sup>

Because of limitations on computation time, the model for water properties is simpler, but less accurate, than other formulations, e. g. Ref. 23, generally available. The pressure of saturated vapor is calculated using the relation of Keenan *et al.*<sup>23</sup> The vapor is assumed to obey the Clausius equation of state. The constant in the Clausius equation of state is chosen so that calculated values of the density of saturated vapor agree well with tabulated values. The agreement, of course, is poor in the neighborhood of the critical point. A relation given by Keenan and Keyes<sup>24</sup> is used to determine the density of saturated liquid. Variable specific heats are used for liquid and vapor. The viscosities of liquid water and steam are taken to be functions only of temperature. The viscosity of the liquid is very accurately represented in the temperature range of interest by taking this viscosity to be proportional to the inverse Celsius temperature. The empirical correlation of Hilsenrath and Touloukian<sup>21</sup> for steam viscosity is used. Note that the value 33.15 appearing in the correlation in Ref. 21 is a misprint and should read 3315.0.

The porous medium is considered incompressible with constant specific heat, permeability and porosity. The viscosity of the air-water vapor mixture is calculated from the viscosities,

molecular weights and mole fractions of the two species. The relation for the mixture viscosity is due to Wilke,<sup>25</sup>

The relative permeabilities of the liquid and vapor phases are calculated using relations proposed by Wyllie and Gardner<sup>26</sup> for liquid and gas flow in porous materials. The relations of Wyllie and Gardner have been simplified by assuming constant capillary pressure. The relative permeabilities are taken to

be functions only of saturation. When, because of evaporation, the water saturation becomes less than the "irreducible" water saturation, the relative permeabilities of the liquid and vapor phases are taken to be zero and one, respectively. The resulting relative permeability curves lie within the broad band of experimental results given by Wyckoff and Botset<sup>27</sup> for flow of gas-liquid mixtures through unconsolidated sand.

## Results of Sample Calculation

As an example of the type of flow that would result from an underground nuclear detonation, we may consider idealized cavity conditions based on the measurements of Olson. The cavity pressure is taken to be a suddenly applied constant pressure of 45 bars. The cavity temperature is taken to be 1710°K. These values are in good agreement with values deduced for the Rainier Event from examination of core samples.<sup>28</sup> Water in the cavity is superheated under these conditions. Since any rock vapor will rapidly condense, the gas in the cavity may be considered to be predominantly steam. In the example, the assumed cavity gas composition is 90% steam and 10% air by mass.

The initial temperature of the porous medium is chosen to be 310°K. The porosity of the medium is 0.35 and the medium is assumed to be initially dry.

The results of numerical calculations of flow under these conditions are shown in Figs. 1-4. The abscissae of Figs. 1-3 have identical scales. These figures show the nature of the flow in the region

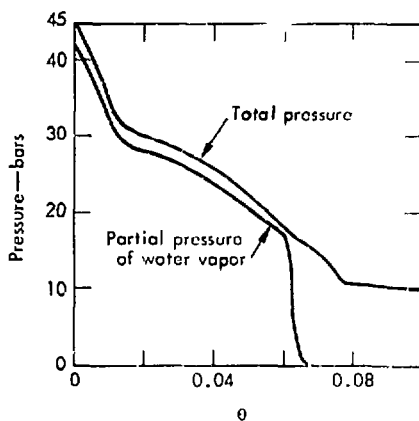


Fig. 1. Pressure distribution in an infinite medium.

where effects of condensation, evaporation and multiphase flow are important.

Figure 4 shows the pressure distribution over a larger range and, for comparison, shows results of calculations for isothermal ideal gas flows. Further discussion of Fig. 4 is deferred until consideration of Figs. 1-3 is completed.

The nature of the flow in the region where the presence of water is important

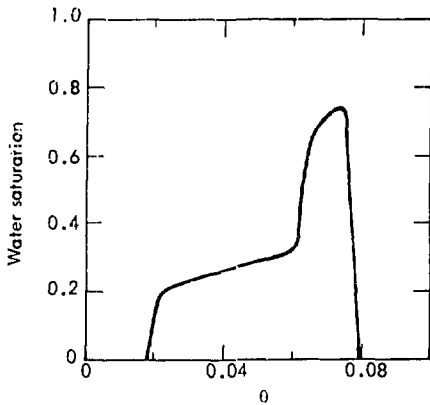


Fig. 2. Water saturation distribution in an infinite medium.

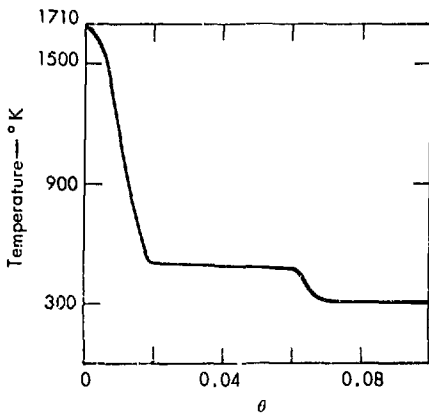


Fig. 3. Temperature distribution in an infinite medium.

is perhaps best perceived by simultaneously considering Figs. 1-3 which show pressure and partial pressure, water saturation, and temperature distributions, respectively. Since the properties are shown as functions of the similarity variable  $\theta$ , these curves represent the spatial distributions of these properties at any

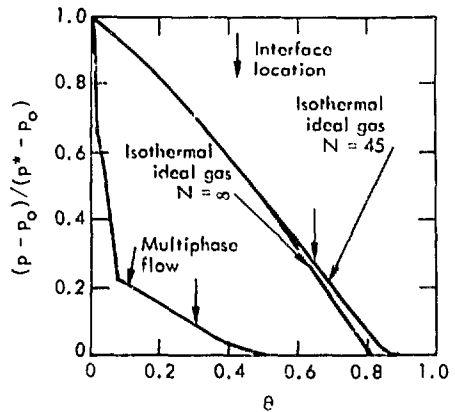


Fig. 4. Pressure distribution in an infinite medium.

time. Given a time, the scale of the abscissae may be converted to the corresponding linear scale showing distance from the inlet. Values of position are determined by reference to the definition of  $\theta$ , Eq. (12). Conversely, these figures may be used to show the temporal variation of state at any given location. This transformation is nonlinear, as Eq. (12) shows.

The flow may be characterized as having several distinct regimes. Nearest the inlet to the porous bed, the water exists as a superheated vapor. In the example shown, this regime extends from the inlet, where  $\theta$  is equal to zero, to a value of  $\theta$  roughly equal to 0.02. Because no liquid is present, the water saturation, shown in Fig. 2, is zero here. The decrease in total pressure and partial pressure, as seen in Fig. 1, is accompanied by the decrease in temperature shown in Fig. 3.

The flow then enters a second regime where a mixture of liquid and water vapor

is present. The temperature is relatively constant through most of this regime, changing slowly in the region bounded by  $\theta$  equal to 0.02 and 0.06. Changes in the partial pressure of the water vapor here correspond to changes of the saturation pressure with temperature. The change in total pressure is largely determined by the change of the water vapor pressure. The water saturation changes relatively slowly through most of this region. The positive slope of the water saturation curve indicates that the fractional pore volume occupied by liquid at a bed position in this region decreases with time due to evaporation and the flow of liquid toward regions of lower pressure.

From  $\theta$  approximately equal to 0.06 to  $\theta$  nearly equal to 0.08, the temperature of the bed again decreases rapidly with distance from the inlet. At  $\theta$  near 0.08, the temperature is essentially the initial bed temperature. The increase in temperature with time at a bed location in this region is due to the motion of liquid and condensation of the vapor. The vapor condenses rapidly as it enters this cooler region. The appearance of a sharp condensation front or water saturation front, as in Fig. 2, is commonly encountered in the steam or water flooding of petroleum reservoirs, e.g., see Ref. 29. After this sudden appearance of liquid at a location producing a high water saturation, the water saturation then decreases with increasing time (or decreasing  $t$ ). The rate of decrease is more rapid here than in the region where  $\theta$  ranges from 0.02 to 0.06 because of the large relative permeability of the liquid in this range of water saturation. The rapid change in the vapor pressure, shown in Fig. 1, results from

the temperature change. The vapor pressure does not vanish near the front of this region, but has values corresponding to the saturation pressures at temperatures near ambient.

Beyond the water saturation front,  $\theta$  greater than about 0.08, no liquid exists. Water, in small amounts, exists beyond this point as superheated vapor. The flow is almost entirely a flow of air. The heat capacity of the air is not enough for air flow to cause significant changes in the local temperature. Consequently, the flow is, in essence, an isothermal ideal gas flow of air at the initial temperature of the porous medium. Referring now to Fig. 4, the pressure distribution of the multiphase flow described here is shown in dimensionless form. The saturation front is located at the near discontinuity in the pressure gradient. The pressure drop in the regime of isothermal gas flow is very gradual compared to that in the two-phase regime preceding it.

For ready comparison of the multiphase flow calculations with calculations based on an isothermal ideal gas model, the dimensionless pressure distributions for two isothermal ideal gas flows are also shown in Fig. 4. These flows correspond to ratios of inlet pressure to ambient pressure  $N$  of infinity and 45. Transient isothermal gas flow under these conditions has been analyzed<sup>13</sup> where a slightly different definition of  $\theta$  is used. Because of the absence of condensation, these gas flows propagate more rapidly than the multiphase flow. Because the isothermal ideal gas calculations are much faster than the multiphase calculations, the ideal gas model provides a convenient means

of rapidly determining a bound for the flow into the porous medium.

The "interface location" whose position is noted on Fig. 4 is the position of the interface between gas from the cavity and gas originally in the bed when a piston-like displacement is posited. The location is determined by a mass balance. The analysis providing a simple means of finding the interface location is given in the following section. The interface for the infinite pressure ratio isothermal ideal gas flow is located at  $\theta$  equal to 0.81 where the dimensionless pressure goes abruptly to zero.

As an example of the application of these analyses, let us predict the actual times and distances associated with certain occurrences. The infinite pressure

ratio ideal gas analysis predicts the effects of the flow to extend up to  $\theta$  equal to 0.81. The multiphase flow considered here has the water saturation front near  $\theta$  equal to 0.98. Referring to the definition (12) of  $\theta$ , choose  $\epsilon$  to be 0.35,  $p^0$  to be 45 bars, and  $\mu^0$  to be 189 micropoise as in the multiphase flow computations. Further select a permeability  $k$  of 1 darcy (1 darcy =  $9.8 \times 10^{-9}$  cm<sup>2</sup>), typical of stemming materials used in these tests. Substitution into Eq. (12) then gives the distance of the water saturation front from the cavity as 1, 3.2, and 25 meters at times of 1 min, 10 min, and 10 hr, respectively. At these same times,  $\theta$  equal to 0.81 corresponds to distances of roughly 10, 32 and 250 meters.

## Extent of Penetration

The extent of cavity gas penetration into the porous medium is a matter of considerable interest. Fluid from the cavity clearly extends beyond the saturation front since all of the water was initially in the cavity. The extent to which air from the cavity has traveled is less obvious. The air in the medium consists of air initially in the medium as well as air that entered from the cavity. The flow will result in some mixing of these gases because of such things as dead spaces, non-uniform velocity distribution across open areas, or the distribution of path lengths. Danckwerts<sup>30</sup> has examined incompressible flows with a distribution of residence times in considerable detail.

If mixing may be neglected, the gas from the cavity will entirely displace the

gas originally in the bed. This model corresponds to a piston-like displacement and is commonly used in the analysis of flows through packed beds. Air from the cavity would be separated from air initially in the porous medium by an interface which moves through the bed. The location of this interface may be determined by a simple extension of the method employed by the author.<sup>13</sup> Because of the presence of some mixing, the interface location corresponds to a lower bound for the distance from the inlet of the leading gas originating in the cavity.

In the similarity analysis of Ref. 13, the interface location is readily determined from the pressure distribution because an isothermal flow is barotropic. The local gas density and the accumulated

mass of gas in any region are obtained directly from the pressure and its spatial integral in that region. Although the multiphase flow of interest here is not barotropic throughout, it becomes barotropic in the region that is considered in finding the interface. This region is that portion of the bed that lies ahead of the water saturation front. As we have seen, the interface location must lie in this region and the flow there is, with considerable accuracy, an isothermal flow of air.

As in Ref. 13, the interface location corresponds to that bed position where the mass of air originally lying closer to the inlet has been added to the gas lying further from the inlet:

$$\int_0^{\infty} (\rho - \rho_0) dx = \rho_0 cx. \quad (17)$$

The subscript 0 denotes air at ambient conditions.

Considering air as an ideal gas and recognizing that the temperature beyond the interface is constant and equal to the initial bed temperature, this relation may be written

$$\int_0^{\infty} (P - P_0) d\theta = P_0 \theta. \quad (18)$$

The definitions of dimensionless variables are different from those used in Ref. 13.

Although Eqs. (13)-(15) are needed for a complete description of the multiphase flow, Eq. (13) reduces to Eq. (16) and Eqs. (14) and (15) are identically satisfied in the region beyond the interface. This simplification results immediately from the constant temperature and absence of water in that region.

Equation (16) is used to evaluate the integral in Eq. (18) using substitutions analogous to those of Ref. 13. The interface location is again found to be the position where

$$\frac{dP}{d\theta} + 2\theta = 0 \quad (19)$$

or

$$\frac{d^2P}{d\theta^2} = 0. \quad (20)$$

The interface is located at the inflection point of the pressure distribution curve as in Fig. 4.

## Conclusions

Multiphase multicomponent flow issuing from a cavity following an underground nuclear experiment has been investigated analytically. A program was developed and results obtained for the one-dimensional transient flow of air, steam, and water through a uniform unfractured

porous medium. A numerical example, using idealized but meaningful cavity conditions, was presented.

These results were compared with those obtained from an isothermal ideal gas model, and reveal the extent to which such an isothermal ideal gas model is conservative in containment calculations.

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## Appendix A – Nomenclature

e	internal energy per unit mass;
h	enthalpy per unit mass;
k	permeability;
$k_r$	relative permeability;
L	length of porous bed;
p	pressure;
$p^*$	inlet pressure;
S	saturation;
t	time;
u	apparent speed;
x	position coordinate in direction of flow;
$\epsilon$	porosity;
$\mu$	viscosity;
$\mu^*$	viscosity of air at ambient temperature;
$\rho$	density.

### Dimensionless quantities

P	dimensionless pressure, $p/p^*$ ;
X	dimensionless position, $x/L$ ;
$\theta$	similarity variable, $X/2\sqrt{\tau}$ ;
$\tau$	dimensionless time, $kp^*t/\epsilon\mu^*L^2$ .

### Subscripts

a	air;
l	liquid;
m	air-water vapor mixture;
s	solid;
v	water vapor;
o	air at ambient conditions.



## Appendix B - Program Listing for PORE

A COMPLETE CODE FOR TRANSIENT MULTIPHASE  
MULTICOMPONENT FLOW IN POROUS MEDIA

```
PROGRAM FAM(TAPE59,SEE,TAPE3=SEE,IN,TAPE2=IN)
CALL CHANGE(4R+FAM)
DIMENSION AGAS(201),AH2O(201),
1 C(201),ENERGY(201),GASMAS(201),HUMID(201),
2 H2OMAS(201),NRGGAS(201),NRGLIQ(201),NRGVAP(201),
3 P(201),PPSI(201),PPSIG(201),PPSIWV(201),
4 RHOGAS(201),RHOLIQ(201),RHOVAP(201),SW(201),
5 TEMPK(201),AWDP(201),AGDP(201),CDP(201),BLOK(5)
REAL MOLWTG,MOLWTW,MUAIR,MULIQ,MUMIX,MUSTAR
1 MUVAP,NRGGAS,NRGLIQ,NRGSOL,NRGVAP
CALL DEVICE(6HCREATE,3HSEE,500000)
NIN=2 $ NOUT=3
1 CALL DATA(BLOK,4.,NIN,NOUT)
PPSI(1)=BLOK(1) $ PPSIWV(1)=BLOK(2)
TEMPK(1)=BLOK(3) $ SW(1)=BLOK(4)
C
C   ...SPECIFY VALUES OF CONSTANTS THAT ARE NOT
C   FREQUENTLY CHANGED...
C   ..INITIAL COLUMN TEMPERATURE IN DEGREES KELVIN..
TEMPKO=310.0
C   ..INITIAL COLUMN PRESSURE IN LBF./SQ.IN. ABS..
PPSIO=14.6959
C   ..INITIAL WATER SATURATION IN COLUMN..
SWO=0.0
C   ..INITIAL RELATIVE HUMIDITY IN COLUMN..
RELHUM=0.0
C   ..SOLID DENSITY IN G./CC...
RHOSOL=2.4
C   ..POROSITY..
PROSTY=.35
C   ..INITIAL TIME..
TAU=0.0
C   ..NUMBER OF ITERATIONS BEFORE PRINTING VALUES..
NSTEP=200
C   ..NUMBER OF TIMES PROPERTIES PRINTED..
NPRINT=500
C   ..NUMBER OF COLUMN SECTIONS..
N=200
REALN=N
NPLUS1=N+1
NMINUS=N-1
NMIN2=N-2
C   ..DISTANCE BETWEEN PRINTOUT POSITIONS..
M=1
C   ..SPATIAL STEP SIZE..
DELTAZ=1./REALN
C   ..TEMPORAL STEP SIZE..
DELTAU=DELTAZ*DELTAZ/5.
C   ..MOLECULAR WEIGHT OF GAS (AIR)..
MOLWTG=28.969
C   ..MOLECULAR WEIGHT OF WATER..
MOLWTW=18.016
C   ..VISCOSITY OF AIR IN MICROPOISE AT INITIAL
C   COLUMN TEMPERATURE..
MUSTAR=VISAIR(TEMPKO)
C   ..HIGH PRESSURE..
```

```

PSTAR=PSSI(1)
C ..INCREMENT FOR TEMPERATURE ESTIMATE..
DELTEM=0.1
C ..CRITICAL TEMPERATURE OF WATER IN DEGREES KELVIN..
TCRIT=647.286
DO 100 I=2,NPLUS1
C
C ....SET INITIAL COLUMN TEMPERATURE, PRESSURE,
C WATER SATURATION, AND RELATIVE HUMIDITY....
TEMPK(I)=TEMPKO $ PSSI(1)=PSSIO
SW(I)=SWO $ HUMID(I)=RELHUM
C
C ....CALCULATE SATURATION PRESSURE AT MIXTURE
C TEMPERATURE AND PARTIAL PRESSURES IN GAS
C PHASE IN LBF./SQ.IN.....
CALL VAPORP(TEMPK(I),PSAT,K)
IF (K.EQ.1) GO TO 999
PPSIWV(I)=HUMID(I)*PSAT
PPSIG(I)=PSSI(1)-PPSIWV(I)
100 CONTINUE
PPSIG(I)=PSSI(1)-PPSIWV(I)
DO 125 I=1,NPLUS1
P(I)=PSSI(1)/PSTAR
C
C ....CALCULATE DENSITIES IN GAS AND LIQUID PHASES
C IN G./CC.....
RHOGAS(I)=PSSI(I)/(41.6317*TEMPK(I))
RHOVAP(I)=SUPERD(TEMPK(I),PPSIWV(I))
RHOLIQ(I)=1.
CALL WATERD(TEMPK(I),RHOLIQ(I),K)
C
C ....CALCULATE APPARENT DENSITIES OF SPECIES
C IN PORES IN G./CC....
H2OMAS(I)=RHOLIQ(I)*SW(I)+RHOVAP(I)*(1.-SW(I))
GASMAS(I)=RHOGAS(I)*(1.-SW(I))
C
C ....CALCULATE INTERNAL ENERGIES IN GAS, LIQUID,
C AND SOLID PHASES IN CAL./G....
NRGGAS(I)=AIRNRG(TEMPK(I))
NRGVAP(I)=SUFERE(TEMPK(I))
NRGLIQ(I)=0.0
CALL WATERE(TEMPK(I),NRGLIQ(I),K)
NRGSOL=SOLIDE(TEMPK(I))
C
C ....CALCULATE (TOTAL INTERNAL ENERGY/POROSITY)
C IN CAL./CC....
ENERGY(I)=(1.-PROSTY)*RHOSOL*NRGSOL/PROSTY
1 +RHOLIQ(I)*NRGLIQ(I)*SW(I)+(RHOVAP(I)*NRGVAP(I)
2 +RHOGAS(I)*NRGGAS(I))*(1.-SW(I))
ISAVE=5
125 CONTINUE
DO 400 III=1,NPRINT
DO 300 II=1,NSTEP
DO 150 I=1,ISAVE
C
C ....CALCULATE VISCOSITIES OF GAS AND LIQUID
C PHASES IN MICROPOISE....
MUAIR=VISAIR(TEMPK(I))

```

```

MUVAP=VISSTE(TEMPK(I))
XGAS=1./(1.+MOLWTG*RHOVAP(I)/(MOLWTW*RHOGAS(I)))
MUMIX=VISMIX(MUAIR,MUVAP,XGAS)
MULIQ=VISLIQ(TEMPK(I))

C
C
C   ....CALCULATE RELATIVE PERMEABILITIES OF GAS
C   AND LIQUID PHASES....
C   CALL PERM(SW(I),RELIQ,RELMIX)

C
C   ....CALCULATE ENTHALPIES IN GAS AND LIQUID
C   PHASES IN CAL./G.....
HGAS=NRGGAS(I)+.00164679*41.6317*TEMPK(I)
HVAP=SUPERH(TEMPK(I),RHOVAP(I),NRGVAP(I))
HLIQ=NRGLIQ(I)+.00164679*PPSI(I)/RHOLIQ(I)

C
C   ....CALCULATE COEFFICIENTS IN COMBINED MOMENTUM
C   AND CONTINUITY RELATIONS. A IN G./CC.....
AL=RHOLIQ(I)*MUSTAR*RELIQ/MULIQ
DUMMY=MUSTAR*RELMIX/MUMIX
AV=RHOVAP(I)*DUMMY
AH2O(I)=AL+AV
AGAS(I)=RHOGAS(I)*DUMMY

C
C   ....CALCULATE COEFFICIENTS IN COMBINED MOMENTUM
C   AND FIRST LAW. C IN CAL./CC.....
CL=AL*HLIQ
CV=AV*HVAP
CG=AGAS(I)*HGAS
C(I)=CL+CV+CG

C
C   ....CALCULATE SPATIAL DERIVATIVE OF DIMENSIONLESS
C   PRESSURE AND MULTIPLY BY COEFFICIENT....
DP=(P(I+1)-P(I))/DELTAZ
AWDP(I)=AH2O(I)*DP
AGDP(I)=AGAS(I)*DP
CDP(I)=C(I)*DP

C
C   ....CALCULATE FLUX COMPONENTS AT POSITIONS AWAY
C   FROM BOUNDARIES....
C
IF (I.EQ.1) GO TO 150
DAWDP=(AWDP(I)-AWDP(I-1))/DELTAZ
DAGDP=(AGDP(I)-AGDP(I-1))/DELTAZ
DCDP=(CDP(I)-CDP(I-1))/DELTAZ

C
C   ....CALCULATE VALUES OF STORED QUANTITIES AT NEXT
C   TIME TAU....
H2OMAS(I)=H2OMAS(I)+DAWDP*DELTAU
GASMAS(I)=GASMAS(I)+DAGDP*DELTAU
ENERGY(I)=ENERGY(I)+DCDP*DELTAU
150 CONTINUE
ITEST=0
DO 250 I=2,N
IF (H2OMAS(I).LE.0.) H2OMAS(I)=0.

```

```

      IF (GASMAS(I).LE.0.) GASMAS(I)=0.
C
C     ....DETERMINE THE STATE CORRESPONDING TO THE
C     CALCULATED STORAGE TERMS....
C
C     ..FIRST, ASSUME WATER SUPERHEATED AND SOLVE FOR
C     TEMPERATURE IN DEGREES KELVIN..
      CALL SUPERT(TK,H2OMAS(I),GASMAS(I),RHOSOL,PROSTY,
1     ENERGY(I),K)
C     ..IF CALCULATED TEMPERATURE IS COMPLEX OR LESS THAN
C     TRIPLE POINT TEMPERATURE, THEN WATER CANNOT BE
C     SUPERHEATED..
      IF (K.EQ.1) GO TO 210
C     ..CALCULATE THE VAPOR PRESSURE (LBF./SQ.IN.) OF
C     SATURATED STEAM AT THIS TEMPERATURE..
      CALL VAPORP(TK,PTRY,K)
C     ..IF CALCULATED TEMPERATURE EXCEEDS CRITICAL
C     TEMPERATURE, WATER IS SUPERHEATED..
      IF (K.EQ.1) GO TO 205
C     ..CALCULATE THE DENSITY (G./CC.) OF SATURATED
C     VAPOR AT THIS TEMPERATURE..
      RHOSAT=UPERD(TK,PTRY)
C     ..IF H2OMAS(I) EXCEEDS THE DENSITY OF SATURATED
C     VAPOR, WATER IS NOT SUPERHEATED..
      IF (H2OMAS(I).GT.RHOSAT) GO TO 210
205  CONTINUE
C     ..WATER IS SUPERHEATED. SET PROPERTY VALUES..
      RHOVAP(I)=H2OMAS(I) $ RHOGAS(I)=GASMAS(I)
      SW(I)=0.0 $ TEMPK(I)=TK
      NRGAS(I)=AIRNRG(TK) $ NRGVAP(I)=SUPERE(TK)
      PPSIWV(I)=SUPERP(TK,H2OMAS(I))
      PPSIG(I)=RHOGAS(I)*41.6317*TK
      PPSI(I)=PPSIWV(I)+PPSIG(I)
      P(I)=PPSI(I)/PSTAR
      IF (PPSI(I).LT.14.6961) ITEST=ITEST+1
      ISAVE=I+1
      IF (ITEST.EQ.3) GO TO 251
      GO TO 250
210  CONTINUE
C     ..WATER IS NOT SUPERHEATED. TO DETERMINE THE STATE,
C     FIRST GUESS THAT THE TEMPERATURE HAS NOT CHANGED.
C     CALCULATE THE WATER SATURATION FROM WATER
C     CONTINUITY..
      DO 213 IT=1,5
      DELTAT=DELTEM
      CALL WATERD(TEMPK(I),DLTRY,K)
      CALL VAPORP(TEMPK(I),PTRY,K)
      DWVTRY=UPERD(TEMPK(I),PTRY)
      STRYA1=(H2OMAS(I)-DWVTRY)/(DLTRY-DWVTRY)
C     ..NOW, CALCULATE THE WATER SATURATION FROM THE FIRST
C     LAW..
      CALL WATERE(TEMPK(I),ELTRY,K)
      EGTRY=AIRNRG(TEMPK(I))
      EWVTRY=SUPERE(TEMPK(I))
      ESTRY=SOLIDE(TEMPK(I))
      EVAP=EWVTRY-ELTRY

```

```

      STRYA2=(H2OMAS(I)*KWVTRY+GASMAS(I)*EGTRY+
1      (1.-PROSTY)*RHOSOL*ESTRY/PROSTY-ENERGY(I))
2      /(DLTRY*EVAP)
C      ..NOW, GUESS THAT THE TEMPERATURE HAS CHANGED BY
C      SOME SMALL AMOUNT. FOR THIS TEMPERATURE,
C      CALCULATE THE CORRESPONDING VALUES OF THE WATER
C      SATURATION FROM WATER CONTINUITY AND FROM THE
C      FIRST LAW..
      TPLUS=TEMPK(I)+DEL TAT
      IF (TPLUS.LT.TCRIT) GO TO 212
      DELTAT=-DEL TEM
      TPLUS=TEMPK(I)+DEL TAT
212     CONTINUE
      CALL WATERD(TPLUS,DLTRY,K)
      CALL VAPORP(TPLUS,PTRY,K)
      DWVTRY=UPERD(TPLUS,PTRY)
      STRYB1=(H2OMAS(I)-DWVTRY)/(DLTRY-DWVTRY)
      CALL WATERE(TPLUS,ELTRY,K)
      EGTRY=AIRNRG(TPLUS)
      EVVTRY=UPERE(TPLUS)
      ESTRY=SOLIDE(TPLUS)
      EVAP=EVVTRY-ELTRY
      STRYB2=(H2OMAS(I)*EVVTRY+GASMAS(I)*EGTHY
1      +(1.-PROSTY)*RHOSOL*ESTRY/PROSTY-ENERGY(I))
2      /(DLTRY*EVAP)
C      ..NOW, USE A LINEAR INTERPOLATION TO ESTIMATE THE
C      TEMPERATURE AT WHICH THE WATER SATURATION VALUES
C      CALCULATED FROM THE TWO RELATIONS WOULD AGREE..
      TEMPK(I)=TEMPK(I)+DEL TAT*(STRYA1-STRYA2)/
1      (STRYB2-STRYB1+STRYA1-STRYA2)
213     CONTINUE
C      ..SET OTHER PROPERTY VALUES..
      CALL VAPORP(TEMPK(I),PPSIWV(I),K)
      RHOVAP(I)=UPERD(TEMPK(I),PPSIWV(I))
      CALL WATERD(TEMPK(I),RHOLIQ(I),K)
      SW(I)=(H2OMAS(I)-RHOVAP(I))/(RHOLIQ(I)-RHOVAP(I))
      IF (SW(I).LT.1.) GO TO 215
      SW(I)=1. $ GASMAS(I)=0. $ RHOGAS(I)=0. $ GO TO 221
215     IF (SW(I).LE.0.) SW(I)=0.
      RHOGAS(I)=GASMAS(I)/(1.-SW(I))
221     NRGGAS(I)=AIRNRG(TEMPK(I))
      NRGVAP(I)=UPERE(TEMPK(I))
      CALL WATERE(TEMPK(I),NRGLIQ(I),K)
      PPSIG(I)=RHOGAS(I)*41.6317*TEMPK(I)
      PPSI(I)=PPSIWV(I)+PPSIG(I)
      P(I)=PPSI(I)/PSTAR
250     CONTINUE
251     CONTINUE
      IF (ISAVE.GT.N) ISAVE=N
      TAU=TAU+DEL TAU
300     CONTINUE
      WRITE (3,700) TAU
      WRITE (3,705) (PPSI(I),I=1,ISAVE)
      WRITE (3,710) (PPSIWV(I),I=1,ISAVE)
      WRITE (3,715) (TEMPK(I),I=1,ISAVE)
      WRITE (3,720) (SW(I),I=1,ISAVE)
400     CONTINUE
700     FORMAT (/,10HTIME TAU =,F6.3,/)
705     FORMAT (26H PRESSURE DISTRIBUTION: /,6(X,F9.3),
1     /,5(X,F9.3))
710     FORMAT (26H PARTIAL PRESSURE OF VAPOR: /,

```

```

1 6(X,F9.3),/,5(X,F9.3)
715 FORMAT (25H TEMPERATURE DISTRIBUTION,/,
1 6(X,F9.3),/,5(X,F9.3))
720 FORMAT (17H WATER SATURATION,/,11(X,F5.3),/,
1 10(X,F5.3))
999 CONTINUE
CALL EXIT(1)
END
FUNCTION SUPERP (TK,RHOCGS)
C
C CALCULATES PRESSURE OF SUPERHEATED STEAM, GIVEN
C THE TEMPERATURE AND DENSITY, USING CLAUSIUS
C EQUATION OF STATE P(V-B)=RT. B IS CHOSEN SO THAT
C PROPERTIES AGREE WITH TABULATED PROPERTIES
C OF DRY SATURATED STEAM EXCEPT NEAR THE
C CRITICAL POINT. ALSO SEE SUPERD AND SUPERH. TK IS
C TEMPERATURE IN DEGREES KELVIN, RHOCGS IS DENSITY IN
C G./CC., SUPERP IS PRESSURE IN LBF./SQ.IN.
C
B=-8.8 $ R=66.9353
SUPERP=R*TK/(1./RHOCGS-B)
RETURN
END
SUBROUTINE VAPORP (TK,VP,K)
C
C CALCULATES VAPOR PRESSURE, GIVEN THE TEMPERATURE,
C USING THE RELATION ON PAGE 141 OF STEAM TABLES BY
C KEENAN, KEYES, HILL, AND MOORE. RETURN (K=1)
C AND NO VAPOR PRESSURE CALCULATION IF TEMPERATURE
C EXCEEDS CRITICAL TEMPERATURE. TK IS TEMPERATURE
C IN DEGREES KELVIN. VP IS VAPOR PRESSURE IN
C LBF/SQ.IN.
C
DIMENSION F(8)
TC=TK-273.15
TCRIT=374.136
IF (TCRIT.GT.TC) GO TO 100
K=1
RETURN
100 CONTINUE
K=0
PCRIT=220.88
TAU=1000./TK
F(1)=-741.9242 $ F(2)=-29.72100
F(3)=-11.55286 $ F(4)=-0.8685635
F(5)=0.1094098 $ F(6)=0.439993
F(7)=0.2520658 $ F(8)=0.05218684
DUMMY=0.0
DO 200 I=1,8
J=I-1
200 DUMMY=DUMMY+F(I)*(0.65-0.01*TC)**J
DUMMY=DUMMY*TAU*.00001*(TCRIT-TC)
P=PCRIT*EXPF(DUMMY)
VP=14.5038*P
RETURN
END

```

FUNCTION SUPERD (TK,PPSI)

C  
C  
C  
C  
C  
C  
C  
C  
C  
C  
C  
CALCULATES THE DENSITY OF SUPERHEATED STEAM, GIVEN  
THE TEMPERATURE AND PRESSURE, USING CLAUSIUS  
EQUATION OF STATE  $P(V-B)=RT$ . B IS CHOSEN SO THAT  
PROPERTIES AGREE WITH TABULATED PROPERTIES  
OF DRY SATURATED STEAM EXCEPT NEAR THE  
CRITICAL POINT. ALSO SEE SUPERH AND SUPERP. TK IS  
TEMPERATURE IN DEGREES KELVIN. PPSI IS PRESSURE IN  
LBF./SQ.IN., SUPERD IS DENSITY IN G./CC.

B=-8.8 \$ R=64.9353  
SUPERD=1./(B+R\*TK/PPSI)  
RETURN  
END  
SUBROUTINE WATERD (TK,DFCGS,K)

C  
C  
C  
C  
C  
C  
C  
C  
C  
CALCULATES THE DENSITY OF SATURATED WATER,  
GIVEN THE TEMPERATURE, USING EQUATION 18 ON PAGE  
21 OF STEAM TABLES BY KEENAN AND KEYES. RETURN  
(K=1) AND NO CALCULATION IF TEMPERATURE EXCEEDS  
CRITICAL TEMPERATURE. TK IS TEMPERATURE IN DEGREES  
KELVIN. DFCGS IS THE DENSITY IN G./CC.

TC=TK-273.15  
TCRIT=374.136  
IF (TCRIT.GT.TC) GO TO 100  
K=1  
RETURN  
100 CONTINUE  
K=0  
VCRIT=3.1975  
A=-0.3151548  
B=-0.001203374  
C=7.48908/10.\*\*13  
D=0.1342489  
E=-0.003946263  
X=TCRIT-TC  
ROOT3=X\*\*(1./3.)  
VFCGS=(VCRIT+A\*ROOT3+B\*X+C\*X\*X\*X)/(1.+D\*ROOT3+E\*X)  
DFCGS=1./VFCGS  
RETURN  
END  
FUNCTION AIRNRG (TK)

C  
C  
C  
C  
C  
C  
C  
C  
C  
C  
CALCULATES THE INTERNAL ENERGY OF AIR, GIVEN THE  
TEMPERATURE, USING A LINEAR RELATION BETWEEN THE  
TEMPERATURE AND THE SPECIFIC HEAT AT CONSTANT  
VOLUME. RELATION OBTAINED BY FIT TO DATA IN GAS  
TABLES BY KEENAN AND KAYE. TK IS TEMPERATURE IN  
DEGREES KELVIN. CSUBV IS SPECIFIC HEAT AT CONSTANT  
VOLUME IN CAL./G.K. (OR BTU./LBM.R.). AIRNRG IS  
INTERNAL ENERGY IN CAL./G.

AIRNRG=.164\*TK+.000019\*TK\*TK  
RETURN  
END

FUNCTION SUPERE (TK)

C  
C CALCULATES THE INTERNAL ENERGY OF WATER VAPOR AT  
C LOW PRESSURE, GIVEN THE TEMPERATURE, USING A  
C QUADRATIC FIT TO DATA IN STEAM TABLES BY KEENAN,  
C KEYES, HILL, AND MOORE. TK IS TEMPERATURE IN  
C DEGREES KELVIN. U IS INTERNAL ENERGY IN BTU./LBM..  
C SUPERE IS INTERNAL ENERGY IN CAL./G..  
C

TR=1.8\*TK  
U=868.664+.2907\*TR+.00004\*TR\*TR  
SUPERE=U/1.8

RETURN

END

SUBROUTINE WATERE (TK,NRG,K)

C  
C CALCULATES THE INTERNAL ENERGY OF LIQUID WATER,  
C GIVEN THE TEMPERATURE, USING A QUADRATIC FIT TO  
C DATA IN STEAM TABLES BY KEENAN, KEYES, HILL, AND  
C MOORE. TK IS TEMPERATURE IN DEGREES KELVIN. U IS  
C INTERNAL ENERGY IN BTU./LBM.. NRG IS INTERNAL  
C ENERGY IN CAL./G. RETURN (K=1) AND NO CALCULATION  
C IF TEMPERATURE EXCEEDS CRITICAL TEMPERATURE.  
C

REAL NRG

TCRIT=647.286

IF (TCRIT.GT.TK) GO TO 100

K=1

RETURN

100 CONTINUE

K=0

TR=1.8\*TK

U=-448.1+.8432\*TR+.00014\*TR\*TR

NRG=U/1.8

RETURN

END

FUNCTION SOLIDE (TK)

C  
C CALCULATES THE INTERNAL ENERGY OF THE STEMMING  
C MATERIAL, GIVEN THE TEMPERATURE, USING A CONSTANT  
C SPECIFIC HEAT. TK IS TEMPERATURE IN DEGREES KELVIN.  
C SOLIDE IS INTERNAL ENERGY IN CAL./G.  
C

SOLIDE=.25\*TK

RETURN

END





```

DIMENSION MU(2),M(2),X(2)
REAL M,MU,MU1,MU2
MU(1)=MU1 $ MU(2)=MU2
X(1)=X1 $ X(2)=1.-X1
M(1)=28.969 $ M(2)=18.016
VISMIX=0.
DO 200 I=1,2
DENOM=0.
DO 100 J=1,2
DENOM=DENOM+X(J)*(1.+(MU(I)/MU(J))**.5)*
1 (M(J)/M(I))**.25)**2/(8.*(1.+M(I)/M(J))**.5
100 CONTINUE
200 VISMIX=VISMIX+X(I)*MU(I)/DENOM
RETURN
END
FUNCTION VISLIQ (TK)
C
C CALCULATES VISCOSITY OF LIQUID WATER, GIVEN THE
C TEMPERATURE, AT PRESSURES NEAR VAPOR PRESSURE USING
C CURVE FIT TO DATA ON PAGE 113 OF STEAM TABLES BY
C KEENAN, KEYES, HILL, AND MOORE. TK IS TEMPERATURE
C IN DEGREES KELVIN. VISLIQ IS VISCOSITY IN
C MICROPOISE.
C
TC=TK-273.15
VISLIQ=272000./TC
RETURN
END
SUBROUTINE PERM (SW,PERMW,PERMG)
C
C WHEN WATER SATURATION EXCEEDS IRREDUCIBLE WATER
C SATURATION, CALCULATES RELATIVE PERMEABILITIES OF
C WATER, PERMW, AND GAS, PERMG, USING RELATIONS GIVEN
C IN WYLLIE AND GARDNER, WORLD OIL 146, (1958). THESE
C RELATIONS ARE BASED ON DRAINAGE CYCLE TESTS.
C OTHERWISE, RELATIVE PERMEABILITIES OF GAS AND WATER
C ARE 1. AND 0., RESPECTIVELY. SW IS WATER SATURATION.
C SWI IS IRREDUCIBLE WATER SATURATION.
C
SWI=0.1
IF (SW.GT.SWI) GO TO 100
PERMW=0.0 $ PERMG=1.0
RETURN
100 SWSTAR=(SW-SWI)/(1.-SWI)
PERMW=SWSTAR**3
PERMG=(1.-SWSTAR)**3
RETURN
END

```

SUBROUTINE SUPERT (TK,RHOV,RHOG,RHOS,PROSTY,ENERGY,K)

C  
C CALCULATES THE TEMPERATURE. USING THE STORAGE TERMS  
C IN THE PROGRAM WITH WATER SATURATION ASSUMED ZERO.  
C TEMPERATURE OBTAINED BY SOLUTION OF QUADRATIC  
C EQUATION SINCE INTERNAL ENERGY SUBROUTINES GIVE  
C ENERGIES AS QUADRATICS IN TEMPERATURE. RETURN (K=1)  
C WITHOUT CALCULATION IF CALCULATED TEMPERATURE WOULD  
C BE COMPLEX OR LESS THAN TRIPLE POINT TEMPERATURE.  
C THIS SUBROUTINE MUST BE CHANGED IF AIRNRG, SUPERE,  
C OR SOLIDE ARE CHANGED.  
C

A=.000072\*RHOV+.000019\*RHOG

B=.2907\*RHOV+.164\*RHOG+(1.-PROSTY)\*.25\*RHOS/PROSTY

C=868.664\*RHOV/1.8-ENERGY

DISC=B\*B-4.\*A\*C

IF (DISC.GT.0.) GO TO 100

K=1

RETURN

100 CONTINUE

TK=(-B+SQRTF(DISC))/(2.\*A)

IF (TK.GT.273.16) GO TO 200

K=1

RETURN

200 CONTINUE

K=0

RETURN

END

## Appendix C – Properties Used in Calculations

With the exception of ideal gas equation-of-state calculations for air, all air, steam and liquid water properties are calculated in subprograms shown in Appendix B.

Sources for the property relations are noted in the body of this report and as comments in the program listing. This appendix shows how these calculated values compare with tabulated values of the properties. The comparisons are shown on the following figures, where the subprograms used in the calculations are noted as well as sources of tabulated values. No measured relative permeabilities are shown because these results have significant scatter.

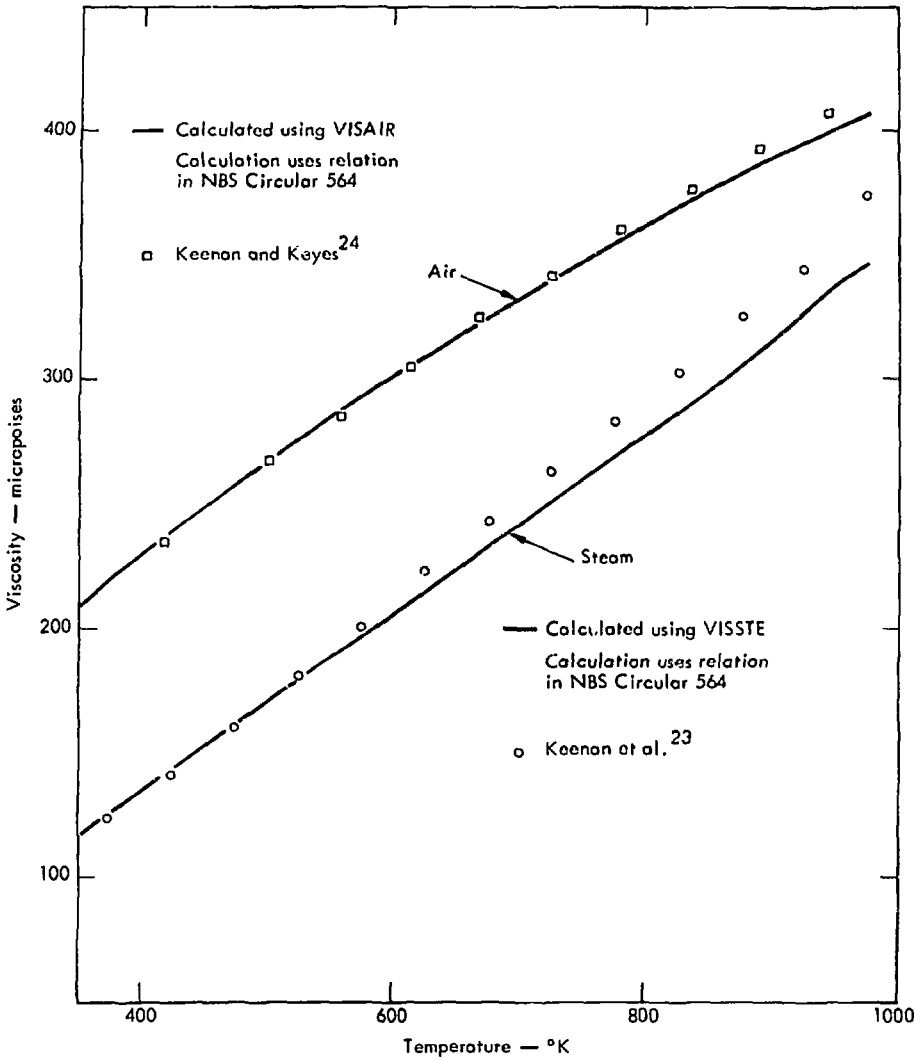


Fig. C-1. Air and steam viscosity.

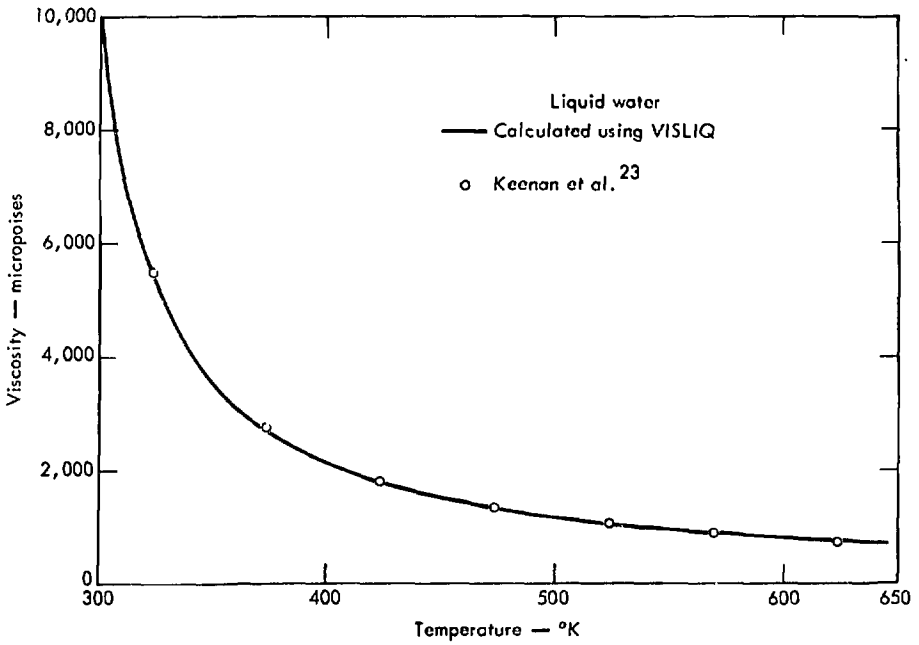


Fig. C-2. Liquid water viscosity.

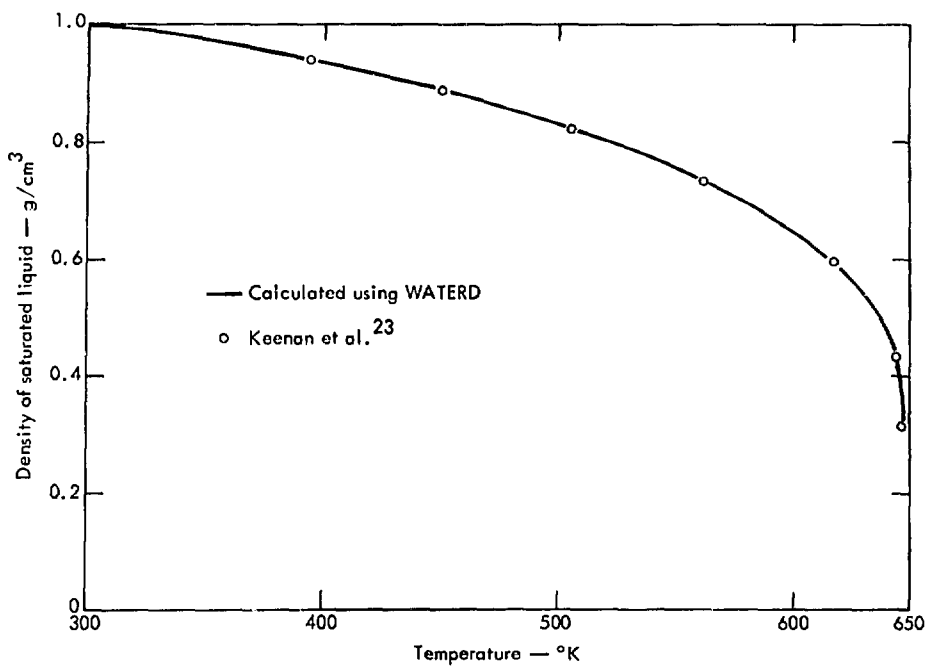


Fig. C-3. Liquid water density.

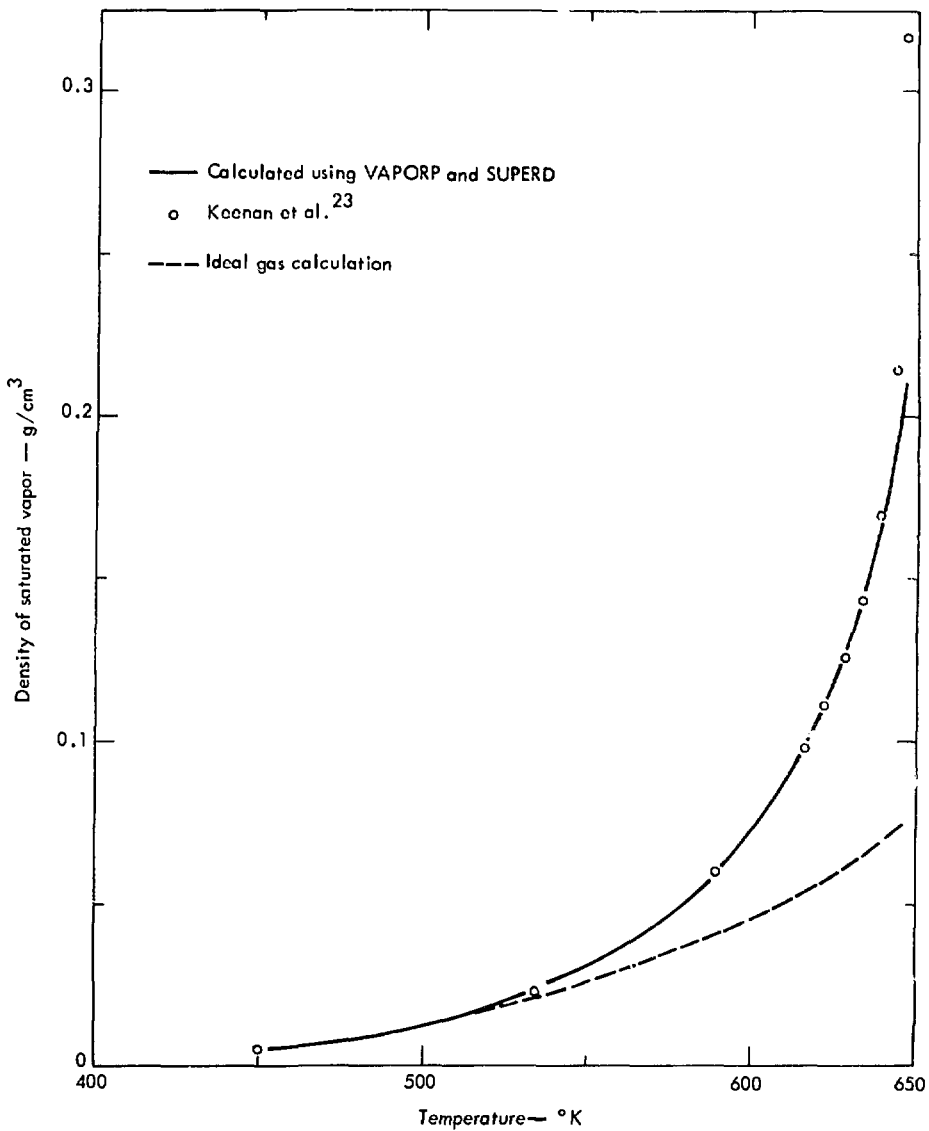


Fig. C-4. Saturated water vapor density.



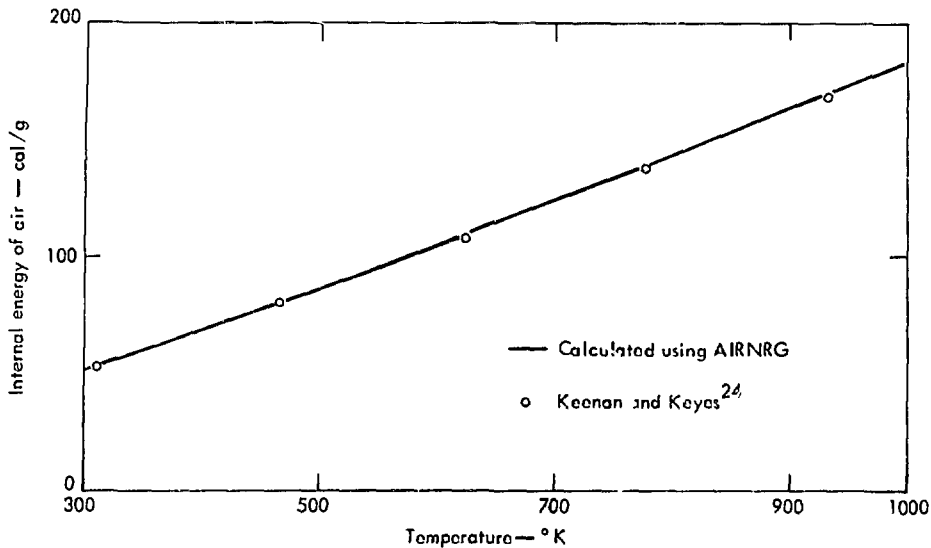


Fig. C-5. Air internal energy.

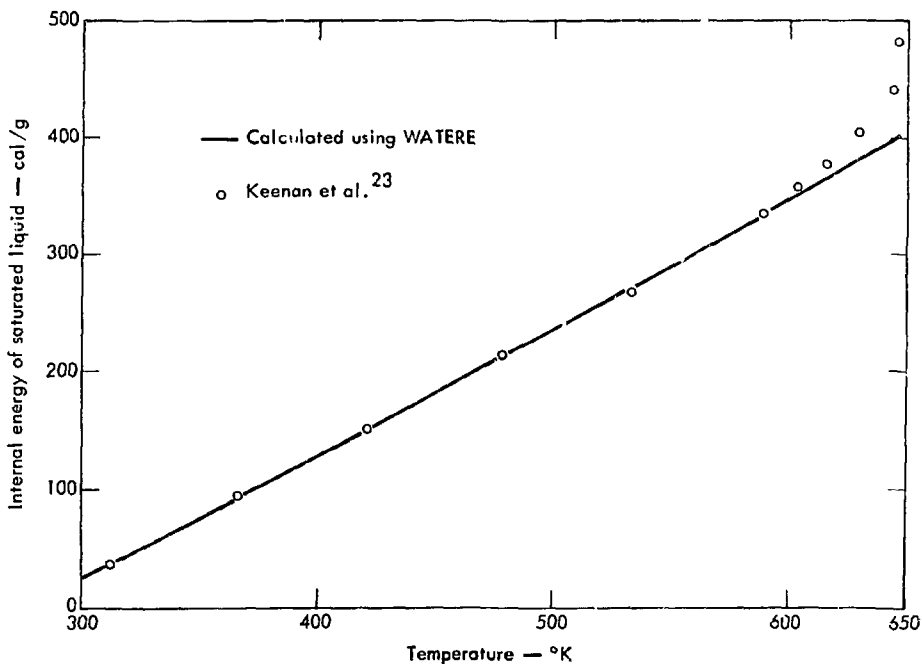


Fig. C-6. Saturated liquid water internal energy.

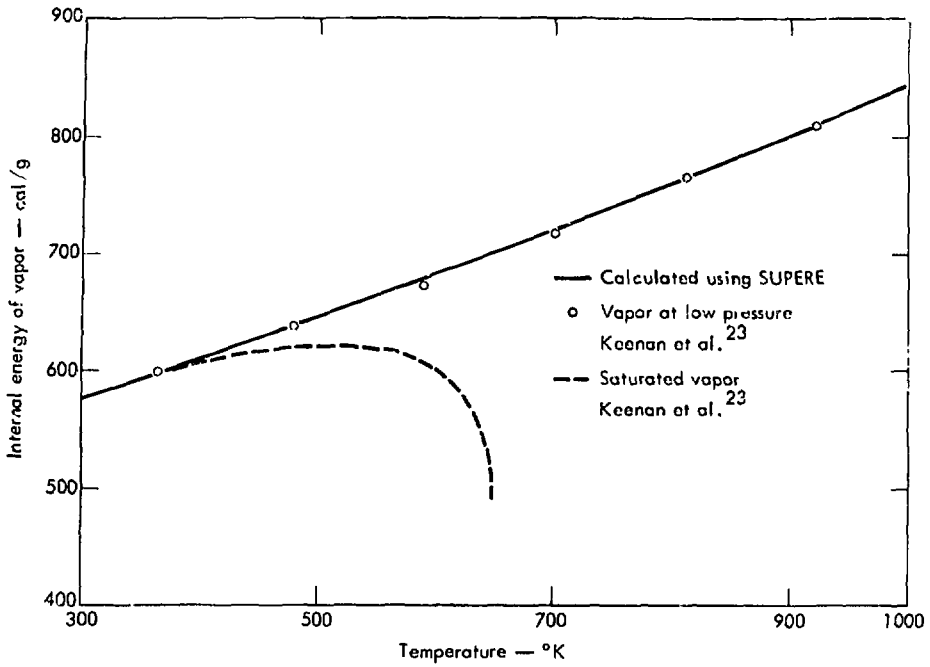


Fig. C-7. Internal energy of water vapor at low pressure.

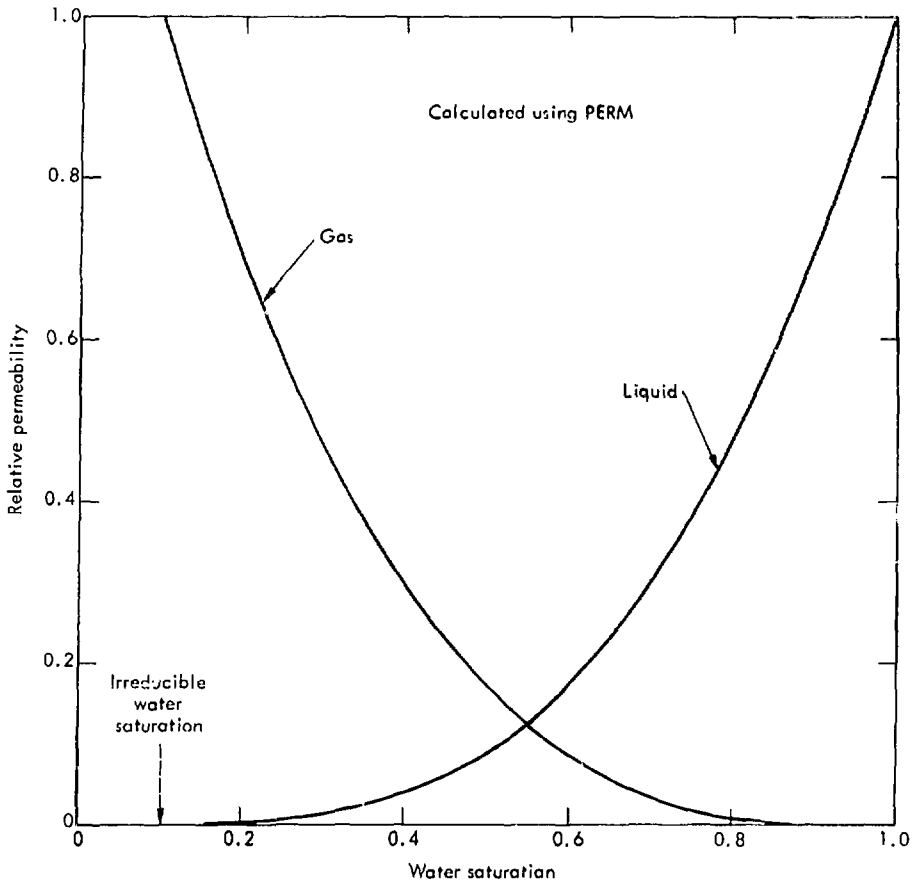


Fig. C-8. Relative permeability of gas and liquid.

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