Recent Experimental Studies of the $K\pi$ Interaction

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I will try to survey the recent experimental work relating to $K\pi$ scattering. After a brief description of the techniques, I will concentrate on compiling, comparing and interpreting results and understanding discrepancies. For other discussions of a review type, I refer you to the earlier review talks by Peter Schlein at Philadelphia in 1968⁽¹⁾ and Argonne in 1969,^(2,3) his Erice lectures in 1970,⁽⁴⁾ the talk by Gordon Kane at Philadelphia in 1970,⁽⁵⁾ and the talk by Victor Henri at Bologna in April, 1971.⁽⁶⁾

$K\pi$ SCATTERING RELATIONS

First, let me summarize the relations used for phase shift analyses. If we knew the angular distribution $d\sigma/d\Omega$ for real on-shell $K\pi$ scattering, we could decompose it in a spherical harmonic (Y_l^0) expansion according to

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left[A_0 + A_1 Y_1^0(\theta) + A_2 Y_2^0(\theta) + \dots \right]$$

where q is the momentum of either the K or π in the $K\pi$ center of

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mass, and the A_ℓ are the moments of the angular distribution. If only S and P amplitudes are present, these amplitudes are related to the moments by

$$A_0 = |S|^2 + 3 |P|^2$$

$$A_1 = 12 \sqrt{\frac{\pi}{3}} \operatorname{Re} SP^*$$

$$A_2 = 12 \sqrt{\frac{\pi}{5}} |P|^2$$

or equivalently, they are related to the cross section and the normalized moments $\langle Y_\ell^0 \rangle = A_\ell / 4\pi A_0$ by

$$\begin{aligned} \sigma &= \frac{4\pi}{2} (|S|^2 + 3 |P|^2) \\ \langle Y_1^0 \rangle &= \frac{1}{\sqrt{3}\pi} \frac{3 \operatorname{Re} SP^*}{|S|^2 + 3 |P|^2} \\ \langle Y_2^0 \rangle &= \frac{1}{\sqrt{5}\pi} \frac{3 |P|^2}{|S|^2 + 3 |P|^2} \end{aligned}$$

The isotopic spin decomposition of the amplitudes into $T = 1/2$ and $T = 3/2$ states is as follows.

$$\begin{aligned} K^+ \pi^- &\rightarrow K^+ \pi^- & S^{+-} &= \frac{2}{3} S_{1/2} + \frac{1}{3} S_{3/2} \\ K^+ \pi^- &\rightarrow K^0 \pi^0 \\ K^+ \pi^0 &\rightarrow K^0 \pi^+ \} & S^{00} &= -\frac{\sqrt{2}}{3} S_{1/2} + \frac{\sqrt{2}}{3} S_{3/2} \\ K^+ \pi^0 &\rightarrow K^+ \pi^0 & S^{+0} &= \frac{1}{3} S_{1/2} + \frac{2}{3} S_{3/2} \\ K^+ \pi^+ &\rightarrow K^+ \pi^+ & S^{++} &= S_{3/2} \end{aligned}$$

If you switch the (+) and (-) charge superscripts in these reactions you have the decomposition for an incoming K^- . The P, D, and higher angular momentum waves have the same isotopic spin decomposition as S. So far, all $K\pi$ phase shift analyses have been restricted to $K\pi$ masses from threshold to about 1.2 GeV, a region in which elastic S and P waves appear to provide an adequate description of the data.

If there is no inelasticity then each amplitude is related to a single real phase shift δ_{ℓ}^{2I} .

$$S_{1/2} = e^{i\delta_0^1} \sin \delta_0^1; P_{1/2} = e^{i\delta_1^1} \sin \delta_1^1$$

$$S_{3/2} = e^{i\delta_0^3} \sin \delta_0^3; P_{3/2} = e^{i\delta_1^3} \sin \delta_1^3$$

These four phase shifts can be determined from the angular distribution information ($\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$, or A_1 and A_2) for any two $K\pi$ reactions which have different isotopic spin decompositions. The cross section information (σ or A_0) can then be used as an additional constraint to resolve the ambiguities, which I'll speak of later.

This is all simple and non-controversial. Life is made more complicated by the fact that we don't have any real pion or kaon targets and must resort to scattering kaons on the virtual pions that surround a target nucleon.

Fortunately such scatterings are major contributors to certain reactions. We use these reactions and our knowledge of the dynamics of pion exchange to extract $K\pi$ scattering information.

For example, the reaction $K^+ p \rightarrow K^+ \pi^- \pi^+ p$ has a large contribution from the process described by the pion-exchange diagram:

Near the pion-exchange pole, the experimentally determined differential cross section $d^4\sigma(K^+ \pi^- \pi^+ p)/dtdMdm d\Omega$ for this process is related to the unknown differential cross section for $K\pi$ scattering $d\sigma(K^+ \pi^-)/d\Omega$ by (7)

$$\frac{d^4\sigma(K^+ \pi^- \pi^+ p)}{dt dM dm d\Omega} \xrightarrow{(t \rightarrow \mu^2)} \frac{M^2 Q \sigma(\pi^+ p) m_q^2}{4\pi^3 \hbar^2 c^2 m_p^2 p_{lab}^2 (t - \mu^2)^2} \frac{d\sigma_{K^+ \pi^- (m)}}{d\Omega}$$

where m and q (M and Q) are the effective mass and internal momentum at the $K^+ \pi^- (\pi^+ p)$ vertex, t is the four momentum transfer squared carried by the exchanged pion, μ is the pion mass, m_p is the proton mass, p_{lab} is the K^+ beam momentum, Ω is the K scattering solid angle in the $K\pi$ rest system, and $\sigma(\pi^+ p)$ is the experimentally determined $\pi^+ p$ elastic cross section. If everything in the previous relation except $d\sigma_{K^+ \pi^- (m)}/d\Omega$ is shifted to the left hand side, we obtain the extrapolation function

$$F(t, M, m, \Omega) = \frac{4\pi^3 \hbar^2 c^2 m_p^2 p_{lab}^2 (t - \mu^2)^2}{M^2 Q \sigma(\pi^+ p) m_q^2} \frac{d^4\sigma(K^+ \pi^- \pi^+ p)}{dt dM dm d\Omega}$$

in which everything is measurable and which has the property that for fixed M , m , and Ω ,

$$F(t, M, m, \Omega) \rightarrow \frac{d\sigma_{K^+\pi^-(m)}}{d\Omega} \quad \text{as } t \rightarrow \mu^2.$$

We fit an analytic function in t , usually a polynomial, to the experimentally determined F in some (M, m, Ω) bin, and take its value at $t = \mu^2$ for $d\sigma_{K^+\pi^-(m)}/d\Omega$. Instead of extrapolating in bins of Ω , most analyses make a moment expansion of $d^4\sigma/dtdMdm d\Omega$ and extrapolate the unnormalized moments (or the cross section and normalized moments). For example, the Brussels-CERN-UCLA Collaboration⁽⁸⁾ uses the normalized moment expansion

$$F(t, M, m, \Omega) = \frac{4\pi \hbar^2 c^2 m_p^2 p_{\text{lab}}^2 (t - \mu^2)^2}{M^2 Q \sigma(\pi^+ p) m^2 q} \frac{d\sigma(K^+\pi^-\pi^+ p)}{dt dM dm} \sum_{\ell m} \langle Y_{\ell}^m(m, t) \rangle Y_{\ell}^m(\Omega)$$

which approaches:

$$\sigma_{K^+\pi^-(m)}$$

$$\langle Y_{\ell}^m(m) \rangle_{K^+\pi^-}$$

as $t \rightarrow \mu^2$.

In some of the analyses to be discussed, the extrapolation function is modified by form factors in an effort to weaken its dependence on t . Frequently q and Q are taken to be the off-shell momenta, that is the incoming rather than the outgoing momenta, at the $K\pi$ and πp vertices, respectively.

Similar extrapolation procedures can be used for three-body final states, for example, $K^-\bar{p} \rightarrow K^-\pi^+n$.

The elementary-pion-exchange relation for this diagram is

$$\frac{d^3\sigma}{dt\,dm\,d\Omega}(K^-\pi^+n) \xrightarrow{(t-\mu^2)} \frac{m_q^2 g_{\pi NN}^2 (-t)}{4\pi m_p^2 (t-\mu^2)^2} \frac{d\sigma_{K^-\pi^+}(m)}{d\Omega}$$

Here, however, one should be more careful in his choice of extrapolation quantities because, as Kane⁽⁵⁾ points out, the absorption model, or any model which allows natural parity exchange, predicts that the extrapolation function for $d\sigma_{K^-\pi^+}/d\Omega$ has a pole at $t = 0$ and that the normalized moments have poles somewhere between the physical region and $t = \mu^2$. It is safer to utilize an extrapolation function for $-t\,d\sigma_{K^-\pi^+}/d\Omega$ or its unnormalized moments since they do not have these poles.

REACTIONS

Now let's look at the analyses that have been done. To give you an idea of the extent to which the most useful pion-exchange reactions have been exploited, I shall list the reactions suggested for study by Schlein.⁽³⁾ These are reactions using either a K^+ or K^- beam scattering on protons or neutrons. Charge exchange to the nuclear system is required to avoid ω exchange. No more than one unseen neutral is allowed. In addition to final states of the type $K\pi N$ we also consider $K\pi\Delta^{++}$ and $K\pi\Delta^-$. Reactions involving a Δ^0 or Δ^+ are not listed because the $\pi^+N\Delta$ vertex is weaker by a factor of 2 in these charge states and results in a less clean Δ signal than

for Δ^{++} or Δ^- reactions. I shall of course also list any other reactions which have been used in analyses of the $K\pi$ interaction. I shall then list all of the analyses of which I am aware.

T = 3/2 ANALYSES

List and Results

First let's look at the pure $T = 3/2$ analyses, that is, analyses of $K^+\pi^+$ or $K^-\pi^-$ systems. The relevant reactions, analyses, and references are given in Table I.

Table I. $T = 3/2$ Reactions and Analyses

Reaction	Group - Momentum	Ref.
$K^-p \rightarrow K^-\pi^- \Delta^{++}$	Col. - Neb. - Iowa 4.6, 5.0	9
	UCLA-LRI 2.05, 2.63	10
	Amsterdam-Nijmegen 4.2	11, 12
$K^-n \rightarrow K^-\pi^- p$	SABRE 3.0	13, 14
	ANL-NW-Kan. 5.5	15
	JHU 12.6	16
$K^+_p \rightarrow K^+\pi^+ n$ $\rightarrow K^+\pi^+ \Delta^0$	ANL 2.5	17
	Brussels-CERN 5.0	18
$K^+_n \rightarrow K^+\pi^+ \Delta^-$		

No analyses have been done using $K^+n \rightarrow K^+\pi^+\Delta^-$, perhaps because it is a one-constraint deuterium reaction.

The extrapolated cross section for $T = 3/2$ $K\pi$ elastic scattering resulting from six of these eight analyses is shown as a function of $K\pi$ mass in Fig. 1. The references for these analyses can be found in Table I. They all utilized Chew-Low extrapolation. In some cases Durr-Pilkahn or exponential form factors were also tried, but the differences resulting from their inclusion were within the extrapolation errors.

The $T = 3/2$ $K\pi$ cross section appears to be roughly constant and around 2 or 3 mb for $K\pi$ masses less than 1.2 GeV. Above 1.2 GeV the Amsterdam-Nijmegen result⁽¹¹⁾ is higher than the Argonne-Northwestern-Kansas⁽¹⁵⁾ result and the Johns Hopkins University result.⁽¹⁶⁾ The latter two analyses were done using $K^-n \rightarrow K^-\pi^-p$ with the differential cross section constrained to pass through zero at $t = 0$. This may give an underestimate of the errors and could bias the results. The Amsterdam-Nijmegen results were obtained from $K^-p \rightarrow K^-\pi^-\Delta^{++}$ where background from K^{*0} and Δ^0 coupled with the fact that $t_{\min}$ is larger than 0.2 GeV^2 for the two highest $K\pi$ mass points, may account for the somewhat larger values of $\sigma(K^-\pi^-)$ obtained.

Discrepancies

The earlier Colorado-Nebraska-Iowa analysis⁽⁹⁾ of $K^-p \rightarrow K^-\pi^-\Delta^{++}$ gave a considerably larger cross section (11 ± 5 mb at 0.8 GeV) than

those shown in Fig. 1. In that analysis, the pole approximation, i.e., the form given earlier with no t dependence other than that of the propagator, was assumed to be exact in the physical region. Their result is to be expected because analyses of the reactions

$pp \rightarrow p \pi^- \Delta^{++}$ (19), $\pi^+ p \rightarrow \pi^+ \pi^- \Delta^{++}$ (20), and $K^+ p \rightarrow K^+ \pi^- \Delta^{++}$ (21) showed that t dependent form factors were required to obtain good fits to $d\sigma/dt$. The pole approximation would have led to much too small $d\sigma/dt$ values. Conversely, the Colorado-Nebraska-Iowa analysis utilizing the pole approximation would be expected to require too large a $K^+ \pi^-$ cross section in order to bring $d\sigma/dt$ up to its observed magnitude.

The Brussels-CERN analysis⁽¹⁸⁾ also obtained a $T = 3/2$ cross section much larger (around 100 mb at 0.8 GeV) than those shown in Fig. 1. They analyzed $K^+ p \rightarrow K^+ \pi^+ \Delta^0$, a reaction not expected to be amenable to extrapolation to the pion-exchange pole because of the domination of the final state by K^{*0} and Δ^{++} production. In principle it should be possible to extrapolate away these "background" contributions if a sufficiently high order extrapolation polynomial is used. However, the signal to background ratio is so small that this reaction is probably useless for obtaining $K^+ \pi^+$ cross section information. The Brussels-CERN group has in fact redone the extrapolation with a quadratic rather than linear extrapolation function and found that the results fell to around 10 mb with errors sufficiently large to be compatible with the results shown in Fig. 1. (22)

Two of these experiments, the Amsterdam-Nijmegen Collaboration⁽¹²⁾ using $K^-p \rightarrow K^- \pi^- \Delta^{++}$ and the SABRE Collaboration,⁽¹⁴⁾ using $K^-n \rightarrow K^- \pi^- p$ also report $T = 3/2$ S- and P-wave phase shifts as a function of $K\pi$ mass. Neither had sufficient statistics to extrapolate the angular distribution so they simply used the normalized $K\pi$ moments in the physical region. Both reactions used have background problems, which certainly affect the $K\pi$ angular distribution. The SABRE collaboration made corrections for $K^- \Delta^0$. The moments are in agreement for the two analyses. The phase shifts obtained from these moments and the extrapolated cross sections using the on-shell $K\pi$ scattering relations for S and P waves are shown in Fig. 2.

δ_0^3 is small and δ_1^3 is negligible up to 1.0 or 1.1 GeV. Note that the sign of the phase shifts cannot be determined from these experiments. The signs of δ_0^3 and δ_1^3 could be changed simultaneously without changing the $K^- \pi^-$ cross section or moments. The negative sign shown in Fig. 2 has been chosen to agree with the results of analyses involving both $T = 1/2$ and $T = 3/2$ amplitudes, where the sign is determined from interference with the K^* (892).

These phase shift results should be regarded with caution since the χ^2 for the phase shift fits was high, typically from 5 to 15 for one degree of freedom. Bakker⁽¹⁴⁾ suggests that this could result from having neglected off-mass-shell effects or from having incorrectly treated the background. These problems could both be eliminated by

extrapolation to the pion exchange pole. However, for a meaningful extrapolation, the statistics in these experiments would probably have to be increased by a factor of three or more. Counter experiments could do well here.

INELASTIC $K\pi$ SCATTERING ANALYSES

List of Analyses

Now let's look at the analyses of inelastic $K\pi$ scattering.

Table II gives the relevant reactions.

Table II. Inelastic Reactions and Analyses

	Reaction	Group-Momentum-Channel	Ref.
$T = 3/2$ only	$K^- p \rightarrow \Delta^{++} (\)^{--}$ $K^- n \rightarrow p (\)^{--}$ $K^+ p \rightarrow n (\)^{++}$ $K^+ n \rightarrow \Delta^- (\)^{++}$		
$T = 1/2$ and $T = 3/2$	$K^+ p \rightarrow \Delta^{++} (\)^0$ $K^+ n \rightarrow p (\)^0$ $K^- p \rightarrow n (\)^0$ $K^- n \rightarrow \Delta^- (\)^0$	UCLA 7.3 ($K^0 \pi^+ \pi^-$, $K^0 \pi^0 \pi^+$) CERN 5, 8 ($K^0 \pi^+ \pi^-$)	1 24

The parentheses stand for $K\pi\pi$, $K\pi\pi\pi$ or any other inelastic channel open to $K\pi$ scattering. The reactions with a final state Δ^{++} or proton can be used even with inelastic channels which have a missing neutral, (e.g., $K^+ \pi^- \pi^0$, $K^0 \pi^0 \pi^+ \pi^-$) and can be used in missing mass experiments to measure the total $K\pi$ cross section. As you can see from Table II, not much has been done. The field is wide open here, both for analyses of existing data, and for counter experiments to measure the total $K\pi$ cross section.

UCLA Result

The UCLA result, given by Schlein⁽¹⁾, is shown in Fig. 3. Here reactions of the type $K^+ p \rightarrow \Delta^{++} ()^0$ are used. The mass spectra for $K^0 \pi^+ \pi^-$ and $K^0 \pi^+ \pi^- \pi^0$ systems produced with low momentum transfer to the recoil Δ^{++} are compared with a curve which approximately represents the sum of the mass spectra for the $K^+ \pi^-$ and $K^0 \pi^0$ systems produced in the same way. At low momentum transfers, this is about the same as comparing $K^+ \pi^- \rightarrow K^0 \pi^+ \pi^-$ and $K^+ \pi^- \rightarrow K^0 \pi^+ \pi^- \pi^0$ with $K^+ \pi^- \rightarrow (K^+ \pi^- + K^0 \pi^0)$. Colton et al.⁽²³⁾ have tested such comparisons utilizing pp reactions. Though there are other $K\pi$ inelastic channels not shown in Fig. 3, the indication is that the inelasticity is small below 1.2 GeV.

Mettel Result

In a more recent study done by Denyse Mettel⁽²⁴⁾ at the International K^+ Collaboration CERN Summer Workshop in 1970, the

cross section for $K^+ \pi^- \rightarrow K^0 \pi^+ \pi^-$ was obtained by Chew-Low extrapolation from the CERN 5 GeV/c and 8 GeV/c $K^+ p \rightarrow K^0 \pi^+ \pi^- \Delta^{++}$ data. The results are shown in Fig. 4. This cross section appears compatible with zero and less than 1 mb below 1.2 GeV/c, compared with a minimum of 15 mb for $K^+ \pi^- \rightarrow K^+ \pi^- + K^0 \pi^0$ in this mass region. The curve is the contribution to this cross section expected from the D-wave $K^*(1420)$ assuming a branching ratio $K^*(1420) \rightarrow K\pi/\text{ALL} = 0.5$ and assuming for the purpose of isotopic spin decomposition that the $K^*(1420)$ decays to $K\pi\pi$ only via $K^*\pi$. The $K^*(1420)$ appears to account for a large part of the $K^0 \pi^+ \pi^-$ cross section in the mass region around 1.4 GeV.

T = 1/2 and 3/2 ANALYSES

List of Analyses

Now let's look at the elastic scattering reactions and analyses involving both T = 1/2 and T = 3/2 amplitudes. These are given in Table III.

Table III. $T = 1/2$ and $3/2$ Reactions and Analyses

Reaction	Group-Momentum	Ref.
$K^+ p \rightarrow K^+ \pi^- \Delta^{++}$ $\rightarrow K^0 \pi^0 \Delta^{++}$	UCLA 7.3	21
	Brussels-CERN 5	18
	Brussels-CERN-UCLA (WDST)	8
	JHU 5.5	25
	JHU (WDST)	26
	LRL 12.6 (in progress)	27
	UCLA 7.3	21
	Brussels-CERN 5	18
	Brussels-CERN-UCLA (WDST)	8
	JHU (WDST)	26
	LRL 12.6 (in progress)	27
$K^+ n \rightarrow K^+ \pi^- p$ $\rightarrow K^0 \pi^0 p$	UCLA-CIT-NAL 2.0 (in progress)	28
	LRL 12	29
	UCLA-CIT-NAL 2.0 (in progress)	28
$K^- p \rightarrow K^- \pi^+ n$ $\rightarrow \bar{K}^0 \pi^- p$	ANL-EFI 5.5	30
	UCLA-CIT-NAL 2.0 (in progress)	28
	ANL-EFI 5.5	30
$K^- n \rightarrow K^- \pi^+ \Delta$		

Clearly a great deal of work has been done here. I have no results to report for the analyses listed as "in progress," but perhaps Schlein will fill us in on the latest results of the UCLA-Cal. Tech.-NAL experiment. See References 27 and 28 for descriptions of these experiments.

World Data Summary Tape (WDST)

The reaction $K^+p \rightarrow K\pi\Delta^{++}$ has been studied extensively, and this is in part due to the existence of the World Data Summary Tape of the International K^+ Collaboration containing almost 80,000 $K^+p \rightarrow K^+\pi^-\pi^+p$ and 20,000 $K^+p \rightarrow K^0\pi^0\pi^+p$ events with beam momenta ranging from 2.5 to 12.6 GeV/c. Table IV, taken from the Brussels-CERN-UCLA Collaboration analysis,⁽⁸⁾ shows the contributions of the various laboratories to the WDST, along with the cross sections reported by these laboratories.

Brussels-CERN-UCLA Analysis

The Brussels-CERN-UCLA analysis⁽⁸⁾ selected Δ^{++} events from these two reactions and extrapolated to obtain the on-shell cross sections and normalized moments of the angular distributions for $K^+\pi^- \rightarrow K^+\pi^-$ and $K^+\pi^- \rightarrow K^0\pi^0$ as a function of $K\pi$ mass. To check the method, they selected K^{*0} events from the $K^+p \rightarrow K^+\pi^-\pi^+p$ reaction and extrapolated to obtain the cross section and normalized moments of the angular distribution for $\pi^+p \rightarrow \pi^+p$. The crosses in Fig. 5 show the results of

Table IV. Data from International K^+ Collaboration

Laboratory	Ref.	Momentum GeV/c	$K^+ p \rightarrow K^+ \pi^- \pi^+ p$		$K^+ p \rightarrow K^0 \pi^0 \pi^+ p$	
			Number of events	Cross section (mb)	Number of events	Cross section (mb)
Illinois	31	2.53	2 723	2.18 ± 0.17		
Illinois	31	2.76	4 586	2.27 ± 0.18		
Brussels-CERN Collaboration	32	2.95	1 108	2.13 ± 0.11	788	2.20 ± 0.19
Illinois	31	3.2	4 823	2.17 ± 0.17		
Brussels-CERN Collaboration	33	3.43	1 848	2.15 ± 0.11	556	2.19 ± 0.20
Chicago	34	4.27	1 838	2.00 ± 0.20		
Berkeley, LRL	35	4.6	2 668	1.80 ± 0.20	759	1.65 ± 0.20
Brussels-CERN Collaboration	18	4.97	6 241	1.95 ± 0.10	3 077	1.68 ± 0.10
Johns Hopkins	25, 26	5.44	9 518	1.71 ± 0.20	900	1.44 ± 0.15
UCLA	21	7.3	11 619	1.75 ± 0.15	2 279	1.37 ± 0.15
Brussels-CERN Collaboration	36	8.25	4 780	1.36 ± 0.08	2 721	1.33 ± 0.09
Berkeley, LRL	35	8.92	7 559	1.16 ± 0.20	2 272	0.66 ± 0.20
Birmingham-Glasgow- Oxford Collaboration	37	10.	14 265	1.19 ± 0.13	4 620	0.86 ± 0.16
Rochester	38	12.6	3 619	0.97 ± 0.09	834	0.57 ± 0.07
Total			77 267		18 806	

the $\pi^+ p$ cross section extrapolation. The curve represents real $\pi^+ p$ elastic scattering data.⁽³⁹⁾ The two extrapolations shown differ only in the form factors used. The upper one uses Benecke-Dürr (BD) form factors,⁽⁴⁰⁾ while the lower one uses q^{off} in the place of q in the one-pion-exchange formula given earlier.

Note that the errors shown on the extrapolated points are statistical and do not include the uncertainties in the cross sections of Table IV. In addition, cross section extrapolations are sensitive to the assumed form of the extrapolating function, as is obvious from the 10% difference at the Δ^{++} peak between the two extrapolations shown.

Figure 6 shows the extrapolated $\pi^+ p$ moments from the same reaction. Again the curves represent real $\pi^+ p$ elastic scattering data.⁽⁴¹⁾ Figure 7 shows the same moments except that here they are not extrapolated but are simply averaged over low momentum transfers $|t_{KK}^*| < 0.3 \text{ GeV}^2$. The extrapolation makes some improvements, especially in $\langle Y_1^0 \rangle$ at low masses. The agreement of the extrapolated moments with the real $\pi^+ p$ data is taken as an indication that the same type of extrapolation applied to the $K\pi$ moments in $K^+ p \rightarrow K\pi\Delta^{++}$ will give something that resembles the on-shell $K\pi$ scattering moments.

The extrapolated $K\pi$ moments are shown in Figs. 8 and 9. The small values of $\langle Y_3^0 \rangle$ and $\langle Y_4^0 \rangle$ below 1.2 GeV show that D-wave is negligible in that region. The rapid variation in $\langle Y_1^0 \rangle$ around 0.9

GeV can be attributed to interference between a significant S wave and the $K^*(890)$ P wave. The drop-off of $\langle Y_2^0 \rangle$ to either side of the $K^*(890)$ also indicates the presence of S wave. The difference between the $K^+ \pi^-$ and $K^0 \pi^0$ moments indicates the presence of $T = 3/2$ phase shifts.

The S- and P-wave phase shifts in the region from threshold to 1.2 GeV obtained from $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ of Figs. 8 and 9 are shown in Fig. 10. No inelasticity was allowed. δ_1^3 was assumed to be zero. The good χ^2 values and the resemblance of δ_1^1 to the expected P-wave Breit-Wigner are taken as evidence for this assumption. Strong correlations between δ_0^1 and δ_1^1 in the region below 0.8 GeV make their independent determination difficult unless the $K\pi$ extrapolated cross section is used along with the moments.

Solution A for δ_0^3 gives a $T = 3/2$ cross section in agreement with the direct determinations from $K^+ \pi^+$ and $K^- \pi^-$ experiments. Solution B gives much too large a cross section. Note that the sign of all three phase shifts could be changed simultaneously. The choice that $90^\circ < \delta_1^1 < 180^\circ$ for masses greater than 0.9 GeV in accordance with the expected Breit-Wigner behavior results in a negative δ_0^3 in that mass region.

Two solutions labeled "down" (1) and "up" (2) are found for δ_0^1 . These solutions are also shown displaced by 180° (Solutions 1' and 2'). In the regions above 0.95 GeV and below 0.8 GeV $K\pi$ mass, Solution (1, 1') is associated with Solution A and Solution (2, 2') is

associated with Solution B. Therefore only Solution (1, 1') is allowed in these regions. In the $K^*(890)$ region an ambiguity exists which allows any combination of δ_0^1 and δ_0^3 solutions. Two possibilities therefore exist for δ_0^1 . The first is Solution 1 all the way, and the second is Solution 1 to 0.8 GeV, then Solution 2 to 0.95 GeV, then Solution 1' the rest of the way, i.e., a narrow S-wave resonance in the $K^*(890)$ region.

The $K^*(890)$ region was then studied in detail (in 20 MeV overlapping bins) using only $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ for the $K^+\pi^-$ events since the $K^0\pi^0$ events have a much bigger mass resolution. The δ_0^3 was assumed negative and such that the $T = 3/2$ $K\pi$ cross section equalled 1.8 mb. The δ_1^1 was assumed to be a P-wave K^* Breit-Wigner with mass 0.891 GeV and width 0.050 GeV. Again, δ_1^3 was assumed to be zero and no inelasticity was allowed. The δ_0^1 resulting from these assumptions is shown in Fig. 11. The two possible solutions are clearly seen. Both give good agreement with the $K^+\pi^-$ moments shown in Fig. 12(a-b).

Extrapolations were also made to obtain the $K\pi$ cross section and this information has not yet been used in the phase shift determinations. Figure 12(c) shows the extrapolated $K^+\pi^-$ cross section obtained using Benecke-Dürr form factors. Other form factors were tried and resulted in variations of as much as 20% from the 90 mb peak value shown here.

The cross sections computed from both the "down" and "up" solutions are also shown on Fig. 12(c). Both appear to peak at a lower $K\pi$ mass than the extrapolated cross section. The "up" solution gives a somewhat broader peak at a lower mass. For the "down" solution, the peak shift clearly results from the unfortunate choice of $M_{K^*} = 0.891$ GeV. For the "up" solution the peak is further shifted and broadened by the S-wave resonance on the low mass side of the K^* .

To see if this shift could be corrected without destroying the agreement with the moments, a simple parameterization of the phase shifts, capable of representing either solution was used. The "down" solution δ_0^1 is represented by an effective range approximation

$$\cot \delta_0^1 = \frac{1}{qa} + \frac{r}{2} q.$$

The "up" solution is represented by adding a Breit-Wigner phase shift to this slowly varying "background" phase shift. δ_0^3 corresponds to a constant cross section and $\delta_1^3 = 0$. δ_1^1 is a P-wave Breit-Wigner. The mass and width of the S- and P-wave resonances were varied in an effort to match the cross section and moments for both elastic and charge exchange scattering.

When resolution effects are included, the data are adequately reproduced by many combinations of S- and P-wave parameters with the separation between the resonant masses $m_P - m_S$ ranging from 0 to 20 MeV and the S-wave width from 0 to 30 MeV. Figure 13 shows an example. The curve in part (a) is an example of the δ_0^1 "up"

solution parameterization with the indicated background effective range parameters and a Breit-Wigner of mass 888 MeV and width 10 MeV. The δ_0^1 results of Fig. 11 are also shown in part (a). The parameterized S-wave resonance of this example has been shifted to a higher mass and is narrower than that suggested by the Fig. 11 values. Parts (b)-(g) show the extrapolated cross section and moments for elastic ($K^+\pi^- \rightarrow K^+\pi^-$) and charge exchange ($K^+\pi^- \rightarrow K^0\pi^0$) scattering. The curves shown in (b)-(g) are computed from the δ_0^1 of (a) and P-wave Breit-Wigner of mass 898 MeV and width 46 MeV. δ_0^3 corresponds to 3 mb and $\delta_1^3 = 0$. The dashed curves include the effects of a Gaussian resolution function. The 7 MeV resolution for the $K^+\pi^-$ mass is from the 7.3 GeV/c UCLA experiment⁽⁴²⁾ and the 25 MeV resolution is chosen to give agreement with the $K^0\pi^0$ mass peak width for the entire WDST sample.

The dashed curves are in fairly good agreement with the extrapolated values. The discrepancy of σ^{CE} could result from the inadequate representation of the resolution function for the combined $K^0\pi^0$ data of Table IV by a simple 25 MeV Gaussian. Since σ^{EL} has been brought into good agreement with the extrapolated values while retaining the agreement in the moments, we conclude that an "up" solution exists.

Johns Hopkins University Analysis

Johns Hopkins University also did phase shift analyses^(25, 26) utilizing their own data on $K^+p \rightarrow K^+\pi^- \Delta^{++}$ at 5.5 GeV/c and that of the International K^+ Collaboration WDST when it contained about half the statistics shown in Table IV. In this analysis they determine δ_1^1 directly from the unnormalized moment of $\cos^2\theta$ in the $K^+\pi^-$ angular distribution, extrapolated to the pion pole. This moment is proportional to $\sin^2\delta_1^1$ assuming that $\delta_1^3 = 0$ and that only S and P waves are present. Figure 14 shows their result for δ_1^1 from the JHU data alone and from the WDST. Good agreement is observed with a Breit-Wigner of mass 895 MeV and width 50 MeV. They attribute the small discrepancy to a δ_1^3 which is negative and less than 10° throughout the mass range studied.

They then assume $\delta_1^3 = 0$ and determine δ_0^1 and δ_0^3 from the appropriate combinations of the $\cos\theta$ and $\cos^2\theta$ moments (extrapolated) of the $K^+\pi^-$ and $K^0\pi^0$ angular distributions, and from the values of δ_1^1 shown in Fig. 14. The resulting δ_0^1 and δ_0^3 values are shown in Fig. 15. Two solutions are found for each phase shift. The third solution, indicated by a prime is just one of the others incremented by 180° . The two solutions for δ_0^1 are related by

$$\delta_0^{1\text{ II}} = \delta_1^1 - \delta_0^{1\text{ I}} - \frac{\pi}{2} .$$

Similarly the two solutions for δ_0^3 are related by

$$\delta_0^3{}^A = \delta_1^1 - \delta_0^3{}^B - \frac{\pi}{2}.$$

These phase shift transformations leave $\text{Re } SP^*$ and $|P|^2$ unchanged but change $|S|^2$.

The cross sections computed from the four possible combinations of these solutions are compared with the extrapolated $K^+\pi^-$ cross section in Fig. 16. It is possible to jump from one solution to another at around 900 MeV because the phase shifts cross or are close at that mass. Below 900 MeV solutions IA and IB look good, and above 900 MeV IA and IIB look good. However, the small value of the $T = 3/2$ cross section observed in $K^+\pi^+$ and $K^-\pi^-$ experiments is incompatible with δ_0^3 solution A below 900 MeV and B above 900 MeV. The remaining solution, IB below 900 MeV and IA above, is shown in Fig. 17. The cross section for this solution is seen to be in good agreement with the extrapolated cross section in Fig. 18.

The Johns Hopkins group thus finds only one solution. It is compatible with the "down" solution of the Brussels-CERN-UCLA group.

Argonne-Enrico Fermi Institute Analysis

The Argonne-Enrico Fermi Institute analysis⁽³⁰⁾ has used $K^-p \rightarrow K^-\pi^+n$ and $K^-p \rightarrow \bar{K}^0\pi^-p$ at 5.5 GeV/c to obtain off-mass-shell $K\pi$ phase shifts. I think Yuta will talk about this later, so I'll just

show the result for comparison in Fig. 19. Part (a) is the δ_0^3 magnitude taken from Cho, et al.,⁽¹⁵⁾ and used in this analysis. δ_1^1 and δ_2^1 were taken to be K^* (890) and K^* (1400) Breit-Wigners. δ_1^3 and δ_2^3 data were assumed to be zero. The results of the analysis are that δ_0^3 is negative and that two solutions for δ_0^1 shown in Fig. 19 (b) can be found. (c) and (d) give the ρ and ω contributions to the $K^+\pi^-$ and $\bar{K}^0\pi^-$ reactions respectively.

In addition to the δ_0^1 solutions shown in Fig. 19 (b) Yuta states that the "up" (black dot) solution could be joined to the "down" (white dot) solution plus 180° at around 1 GeV. The "down" solution passes through 90° at around 1.2 GeV. However they express reservations about the solutions above 1 GeV because of increasing elasticity and the $I = 3/2$ p-wave amplitude.

Compilation of δ_0^1 Results

Figure 20 shows the δ_0^1 results from the three analyses just discussed. (This figure was prepared after the talk, and is included here to complement Mercer's remark in the question period. In addition to solution I (down) points of Johns Hopkins, I have included those points of their solution II (up) which are compatible with their cross section extrapolation. If one accepts the large jump at 850 MeV to which Mercer refers, then Johns Hopkins has an "up" solution.) The results for both the "down" and "up" solutions appear

quite compatible for the three experiments. One should be very cautious in interpreting the errors shown on δ_0^1 . The Brussels-CERN-UCLA analysis used overlapping bins so each event appears twice. In addition, in determining δ_0^1 , all analyses made restrictive assumptions on the other phase shifts. The JHU analysis errors should be most representative since they made the least restrictive assumptions, assuming only that $\delta_1^3 = 0$ and that the scattering is purely elastic. Even there, the error estimates do not include possible systematic effects arising from the extrapolation. The Brussels-CERN-UCLA errors should not be more than a factor $\sqrt{2}$ smaller than the Johns Hopkins errors. The curves indicate my own subjective confidence level.

This δ_0^1 result is quite similar to the $T = 0$ S-wave $\pi\pi$ phase shift result in the ρ region. Each has a "down" solution, which is slowly varying in the vicinity of the P-wave resonance, and each has an "up" solution, which resonates just below the P-wave resonance. The existence of $J^P = 0^+$ resonant (up) solutions so close to the $J^P = 1^-$ K^* and ρ is undoubtedly more than coincidence.

My suspicion is that the continuous, slowly varying "down" solutions are real, and that the "up" solutions are ambiguous solutions generated from the "down" solutions and the resonant P-wave K^* and ρ . The phase shift transformation mentioned when discussing the Johns Hopkins analysis could generate such ambiguous solutions. It seems much less likely that nature would provide us with rapidly varying

resonant S-wave phase shifts (i.e., real "up" solutions) which, when combined with the rapidly varying resonant P-wave phase shifts, would yield ambiguous solutions as continuous and slowly varying as the "down" solutions. However, this is just a feeling, and no solid evidence exists to support it, at least in the $K\pi$ case. (I'll leave $\pi\pi$ to Gutay.) Therefore, at the moment, the existence of a narrow $T = 1/2$ S-wave $K\pi$ resonance near the $K^*(890)$ is still an open question. The existence of the scalar $K\pi$ resonance 1.1-1.2 GeV mass and large (~ 0.4 GeV) width which we suggested at UCLA in 1968⁽²¹⁾ also still appears to be an open question. We will have to push the phase shift analyses into the inelastic region to find out if δ_0^1 continues rising past 90° .

ABOVE 1.1 GeV $K\pi$ MASS

A phase shift analysis in the inelastic region could also shed additional light on the reports of $K\pi$ mass peaks in the region from 1080 to 1260 MeV⁽⁴³⁾ and on the reports of large $J^P = 0^+$ and 1^- contributions in the vicinity of the $J^P = 2^+ K^*(1420)$. Johns Hopkins University⁽⁴⁴⁾ suggests the existence of a $J^P = 1^-$ resonance, or at least a significant amount of 1^- under the $K^*(1420)$ in the reaction $K^+p \rightarrow K^+\pi^- \Delta^{++}$ at 5.5 GeV/c. Fu et al.⁽³⁵⁾ suggest significant S- and P-wave contributions in this region using the same reaction at 9 GeV/c.

More recently, Firestone et al. (29) presented evidence for an S (or perhaps P)-wave resonance just below the $K^*(1420)$ based on their analysis of $K^+n - K^+\pi^-p$ at 12 GeV/c. Figure 21 shows the Dalitz plot for this reaction. They concentrate on the $K^*(1420)$ region, the horizontal band at about 2 GeV^2 . The effect of the vertical N^* band on the K scattering angle in the $K\pi$ rest frame can be understood by remembering that the longitudinal decay cosine for the $K\pi$ system scales linearly from +1 to -1 as one moves from left to right along the K^+ band. Since the longitudinal decay cosine is almost equal to the K scattering angle for peripheral $K\pi$ systems, the N^* events contribute only to very forward K scattering angles. They claim that their results are not altered when these events are removed. Figures 22(a) and (b) show the K scattering angle for the two regions of $K^+\pi^-$ mass (a) 1.3-1.4 GeV and (b) 1.4-1.5 GeV. They attribute the striking difference between these two distributions to a large S-wave contribution in the region from 1.3-1.4 GeV. Figures 22(c) and (d) show that polar and equatorial cuts made to enhance D- and S-wave, respectively, give $K^+\pi^-$ mass peaks at 1440 and 1370 Mev, respectively. They give these as approximate masses for the $K^*(1420)$ and the new S-wave resonance. I view these results cautiously because all of the above effects could result from the interference of a significant but constant S wave with the $K^*(1420)$ D-wave. However, they give two intriguing results that could not. Their $K^0\pi^+\pi^-$

mass peak [Fig. 22(e)] from $K^+ n \rightarrow K^0 \pi^+ \pi^- p$ is at 1440 ± 5 MeV consistent with their D-wave peak in $K^+ \pi^-$. Note that parity conservation prevents S-wave $K\pi$ from coupling to $K\pi\pi$. Also, their estimate of the number of S-wave events, obtained from the Legendre coefficients of the angular distribution, peaks in the 1.3-1.4 GeV bin [Fig. 22(f)]. These results are only suggestive of $K\pi$ scattering effects because high momentum transfer events were included in the analysis.

Firestone plans to try an extrapolation and $K\pi$ phase shift analysis with these data. The three body reaction at 12 GeV/c permits them to get quite close to $t = 0$ in the $K^*(1400)$ region and permits them to make a fairly clean separation of N^* events. I'll look forward to seeing their results.

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FIGURE CAPTIONS

- Figure 1. $T = 3/2$ $K\pi$ elastic cross section. See Table I for references and the reactions studied.
- Figure 2. $T = 3/2$ $K\pi$ phase shifts from references 12 and 14. Figure from Brussels-CERN-UCLA collaboration. ⁽⁸⁾
- Figure 3. $K^+\pi^-$ inelastic scattering. The histograms and curves are mass spectra for the parenthesized $K\pi$ systems in the reactions indicated. Low momentum transfer ($t < 0.5 \text{ GeV}^2$) to the recoil Δ^{++} is required. From Schlein. ⁽¹⁾
- Figure 4. Cross section for $K^+\pi^- \rightarrow K^0\pi^+\pi^-$ obtained by Chew-Low extrapolation of the CERN 5 GeV/c and 8 GeV/c data on the reaction $K^+p \rightarrow K^0\pi^+\pi^- \Delta^{++}$. This is a preliminary result from D. Mettel. ⁽²⁴⁾
- Figure 5. $\sigma(\pi^+p)$ versus mass obtained by linear extrapolation of $K^+p \rightarrow K^{*0}\pi^+p$ to the pion pole using Benecke-Dürr ⁽⁴⁰⁾ and Chew-Low (q^{off}) form factors. The curves represent real π^+p elastic scattering data. ⁽³⁹⁾ Brussels-CERN-UCLA Collaboration, WDST. ⁽⁸⁾
- Figure 6. Extrapolated π^+p moments from $K^+p \rightarrow K^{*0}\pi^+p$. Quadratic extrapolations with $|\chi_{KK^*}| < 0.4$ and 1.0 GeV^2 are shown. The curves are real $\pi^+p \rightarrow \pi^+p$ scattering moments calculated from the phase shifts

in Reference 41: Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 7. Off-shell $\pi^+ p$ moments averaged over low momentum transfers $|t_{KK^*}| < 0.3 \text{ GeV}^2$. The curves are the same as those in Figure 6. Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 8. $K^+ \pi^-$ extrapolated moments vs mass. Quadratic extrapolation with $|t| < 1 \text{ GeV}^2$. Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 9. $K^0 \pi^0$ extrapolated moments vs mass. Quadratic extrapolation with $|t| < 1 \text{ GeV}^2$. Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 10. δ_0^1 , δ_0^3 and δ_1^1 obtained from the $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ moments of Figs. 8 and 9, assuming $\delta_1^3 = 0$. Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 11. Solutions for δ_0^1 using only $K^+ \pi^-$ moments, with the assumptions described in the text. Brussels-CERN-UCLA Collaboration, WDST. (8)

Figure 12. Comparison of δ_0^1 solutions of Fig. 11 with $K^+ \pi^-$ extrapolated moments (a-b) and extrapolated cross section (c). Brussels-CERN-UCLA Collaboration, WDST. (8)

- Figure 13. (a) The curve is an example of the δ_0^1 "np" solution parameterization described in the text. The Fig. 11 values are also shown. (b-g) curves computed from this parameterization are compared with the elastic (from Fig. 12) and charge exchange cross section and moments. Brussels-CERN-UCLA Collaboration, WDST. (8)
- Figure 14. δ_1^1 determined from the unnormalized moment of $\cos^2 \theta$ in the $K^+ \pi^-$ angular distribution in $K_p^+ \rightarrow K^+ \pi^- \Delta^{++}$ extrapolated to the pion pole for (a) the JHU 5.5 GeV/c data and (b) the WDST data. Johns Hopkins, WDST. (26)
- Figure 15. δ_0^1 and δ_0^3 determined from the $\cos \theta$ and $\cos^2 \theta$ moments of the $K^+ \pi^-$ and $K^0 \pi^0$ angular distribution extrapolated to the pion pole, and δ_1^1 of Fig. 14. Johns Hopkins, WDST. (26)
- Figure 16. Comparison of extrapolated cross section with those computed from the four possible combinations of δ_0^1 and δ_0^3 solutions of Fig. 15. Johns Hopkins, WDST. (26)
- Figure 17. Solution from δ_0^1 and δ_0^3 remaining after requiring agreement with extrapolated $K^+ \pi^-$ and $T = 3/2$ cross sections. Johns Hopkins, WDST. (26)

- Figure 18. Comparison of extrapolated cross section with that computed from the phases shifts of Fig. 17.
Johns Hopkins, WDST. ⁽²⁶⁾
- Figure 19. (a) $|\delta_0^3|$ taken from Cho et al. ⁽¹⁵⁾ (b) δ_0^1 solutions, (c, d) ρ and ω exchange parameters for $K^-p \rightarrow K^+\pi^+n$ and $K^-p \rightarrow \bar{K}^0\pi^-p$ respectively. Argonne-Enrico Fermi Institute. ⁽³⁰⁾
- Figure 20. Comparison of the δ_0^1 results from the Johns Hopkins ⁽²⁶⁾ Brussels-CERN-UCLA, ⁽⁸⁾ and Argonne-Enrico Fermi Institute ⁽³⁰⁾ $K\pi$ phase-shift analyses. Those points of the Johns Hopkins analysis which are on the fast rising part of the up solution are the points of their Solution II which cannot be ruled out by cross section arguments. The curves are my own estimate of the limits within which we now know δ_0^1 .
- Figure 21. Dalitz plot for $K^+n \rightarrow K^+\pi^-p$ at 12 GeV/c. Firestone, et al. ⁽²⁹⁾
- Figure 22. (a) K scattering angle in the $K\pi$ rest frame ($\cos \theta = \hat{K}_{in} \cdot \hat{K}_{out}$) for $K\pi$ masses 1.3-1.4 GeV. (b) Same for 1.4-1.5 GeV. (c) and (d) $K\pi$ mass spectrum for polar and equatorial selections on $\cos \theta$. (e) $K^0\pi^+\pi^-$ mass from $K^+n \rightarrow K^0\pi^+\pi^-p$ at 12 GeV/c. (f) S-wave $K^+\pi^-$ events versus mass. a, b, c, d, and f are from $K^+n \rightarrow K^+\pi^-p$ at 12 GeV/c. Firestone, et al. ⁽²⁹⁾

Figure 23.

$\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ as a function of t for a series of $K^+ \pi^-$ mass intervals in the reaction $K^+ p \rightarrow K^+ \pi^- \Delta^{++}$. Curves are described in the text. Brussels-CERN-UCLA Collaboration, WDST.⁽⁸⁾

DISCUSSION

- Derrick Aren't δ_1^3 and δ_0^3 coupled? Suppose that you assume $\delta_1^3 = 0$, then δ_0^3 would increase to account for the cross section.
- Trippe The coupling is not so simple. δ_1^3 is associated with non-isotropic $K\pi$ angular distributions. In the analyses involving both $T = 1/2$ and $T = 3/2$ phase shifts, it is coupled to δ_1^1 through the A_2 moment. Of course it is also coupled to δ_0^3 and δ_0^1 via σ and the A_1 moment. It's hard to make a general statement about which coupling is stronger.
- Poirier What is the curve on Fig. 3? ($K^0 \pi^+ \pi^- \Delta^{++}$, $K^0 \pi^+ \pi^- \pi^0 \Delta^{++}$ Mass Plots)
- Trippe The curve is taken from the UCLA scalar $K\pi$ resonance paper⁽²¹⁾ and it corresponds approximately to the sum of the $K\pi$ mass spectra for $K^+ p \rightarrow K^+ \pi^- \Delta^{++}$ and $K^+ p \rightarrow K^0 \pi^0 \Delta^{++}$.
- Mercer I don't think the Johns Hopkins and CERN results are all that different because it looks to me like the two sets of solutions lie very close to each other. The whole question is whether you can make that sudden jump from one to the other. When we did the analysis,

the world data tape was much smaller than now, and so the data could not be divided so finely.

We had no evidence of such a rapid increase and our main argument was just that it was unlikely that you could jump from one solution to the other by 40° like that in a matter of 40 MeV.

Trippe I see - so you think that there is a chance that you could connect the two? .

Mercer If you could make the argument about resolution function stronger. We still have this problem that none of the other solutions agree well with the extrapolated cross section.

Kane How about $K\pi$ s-wave resonances? Have you said anything about the unitarity limit in discussing a $K\pi$ s-wave resonance at the $K^*(890)$? What if you really had an s- and p-wave resonance both approaching the unitarity limit?

Trippe Then you would certainly get a higher cross section than the p-wave unitarity limit, but in fact the extrapolations for the $K\pi$ cross section tend to come out about 10-20 mb higher which is what you would expect if you had an s and p-wave resonance.. However that is not in itself evidence for an s-wave

resonance because the cross section extrapolations have that much uncertainty.

Kane If it had been lower, it would have been evidence against it?

Trippe Yes, if it had been much lower.

Diebold What should be done in the future? Should more experimental data be taken or are these world data tapes sufficient?

Trippe There are problems in using this tape because it is difficult to do cross section extrapolations when all the kinematic factors are different, all the resolution functions are different etc. so I am looking forward to large statistics experiments. I think large statistics experiments at one momentum, where the kinematic boundaries don't change so you don't incur those problems, like the 12 GeV/c LRL group A experiment should be a better place to do this kind of analysis. Also, I would like to see certain counter experiments like missing mass, say if you could detect the Δ or N and measure its momentum well, then you could do the total $K\pi$ cross section accurately.

Diebold If you subdivide the WDST data into energy regions, looking for consistency in the moments, what do you

find?

Trippe We find that the moments as a function of p_{lab} , that is the $K\pi$ angular distributions, are consistent between all laboratories within errors. That allows us to combine the data with confidence that we are not doing anything outrageous. The extrapolations for cross sections, especially for the $K^0\pi^0$ channel, are a different matter. When I take a single laboratory, there are just not enough statistics to do it. That's why a high statistics experiment would be useful.

Poirier Which data is not included? Is Yuta's data included for instance?

Trippe No, this is only K^+ with 4 prongs and 2 prong with Vees, no K^- and $K\pi N$ data is in this data pool.

Lynch I was interested in whether all the people that do these phase shift analyses incorporate the same assumptions. In particular, do they all do their extrapolations out to some rather large values of t and do they all use the Jackson frame for angular distributions?

Trippe They all use the Jackson frame. We⁽⁸⁾ went out to 1 (GeV/c)^2 . We also went out to 0.4 (GeV/c)^2 . Generally, they are lower than 0.4 (GeV/c)^2 .

Musgrave You only talked about the $m = 0$ moments. Is there anything occurring in the illegal moments?

Trippe The illegal moments (i.e. the $m \neq 0$ which are illegal for π exchange without absorption) extrapolate to zero at the pole (if you do the same kind of extrapolation as we did with the $m = 0$ moments) but it's not a very strong constraint because these moments are required to be zero at $t = 0$ rather than at $t = m_\pi^2$, just because at that point ϕ the azimuthal angle becomes undefined so its not a very strong constraint.

Gelfand Have you done your extrapolation taking into account the effects of the form factor in different partial waves?

Trippe No, we haven't put any form factors into the phase shift analysis. What we have done is to take the off-shell moments which have some t dependence. We have looked at them. We have seen that none of the t -dependences anywhere in the physical region are very rapidly varying. We have simply fit the off-shell moments to a quadratic in $(t - \mu^2)$ without any assumption about form factors. If you put in a sufficiently high polynomial to fit the moments, you are taking into account all kinds of form factors.

Gelfand Well, for example, the coefficients of the polynomial are the same for the $\langle Y_2^0 \rangle$ moment and the $\langle Y_1^0 \rangle$ moment. The extrapolations should be different because in the $\langle Y_2^0 \rangle$ moment the P-wave has got a form factor in it in the Dürre-Pilkahn model and the S-wave does not.

Trippe Yes, that's right. We independently extrapolate the $\langle Y_2^0 \rangle$ and the $\langle Y_1^0 \rangle$ moment, so they have different t-dependences.

Gelfand How different are the t dependences? $\langle Y_2^0 \rangle$ should have a more rapid t dependence than $\langle Y_1^0 \rangle$.

Trippe The $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ moments are not radically different in their t-dependences, but there are certainly differences. (Figure 23 taken from the Brussels-CERN-UCLA analysis⁽⁸⁾ was not shown at the talk but has been added to provide a more complete answer to Gelfand's question. $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$ are shown as a function of t for a series of $K^+ \pi^-$ mass intervals. The curves are one standard deviation above and below the quadratic extrapolation function. The values at $t = -0.02 \text{ GeV}^2$ are the extrapolated values of Fig. 8.)

Gutay

One cannot make a general statement. The threshold region can be something completely different than the K^* and ρ region. In the ρ region you see essentially the same momentum transfer dependence.

Fig. 1

XBL 715 6333

--- S wave phase shift corresponding to a $I = \frac{3}{2}$ cross section of 1.6 mb

◊ From $K^- p \rightarrow K^- \pi^- \Delta^{++}$ at 4.2 GeV (Amsterdam-Nijmegen)

◊ From $K^- n \rightarrow K^- \pi^- p$ at 3.0 GeV (S.A.D.R.E.)

$K\pi$ scattering- $I = \frac{3}{2}$ phase-shifts

Fig. 2

Fig. 3

Fig. 4

XBL 715 6337

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

$K^+\pi^-$ Extrapolated moments in 20 MeV bins

$|t| < 1.0 \text{ GeV}^2$

Fig. 10

Fig. 10

Fig. 12

XBL 715 6336

Fig. 13. a-b-c

XBL 715 6334

Fig. 13. d-e-f-g

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Fig. 18

Fig. 19

Fig. 20

XBL 718-4171

Fig. 21

XBL7012-7393

Fig. 22.
a-b-c-d-f

$K^+\pi^-$ MOMENTS EXTRAPOLATION

Fig. 23