RECEIVED BY TIC AUG 28 1972

UCRL - 74040
PREPRINT

Conf-720929--1

This is a preprint of a paper intended for publication in a journal or proceedings. Since changes may be made before publication, this preprint is made available with the understanding that it will not be cited or reproduced without the permission of the author.

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

INTERACTION OF INTENSE PHOTON BEAMS WITH PLASMAS (II)

Ray E. Kidder

July 18, 1972

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

To be presented at the Japan-U.S. Seminar on Laser
Interaction With Matter, Kyoto, Japan, Sept. 25-29, 1972.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

leg

INTERACTION OF INTENSE PHOTON BEAMS WITH PLASMAS (II)^{*}

Ray E. Kidder

ABSTRACT

The elementary properties of light waves in a plasma are briefly reviewed, with particular attention to the Maxwell stresses. Two examples of optical stress effects are treated: Steady-state self-focusing of light beams (transverse stress), and the density jump resulting from light pressure (longitudinal stress).

The use of focused laser light to achieve super-high plasma density by rapid isentropic compression of a pellet of solid deuterium-tritium is discussed, an exact analytic expression for the required pressure-time profile being given for the case of spherical isentropic homogeneous compression.

The application of laser-heated DT pellets to the production of electric power is considered, emphasizing the importance of extremely high pellet compression. The requirements that must be met by the laser and laser-heated pellets are described in the context of a practical fusion power plant.

^{*}Work performed under the auspices of the U.S. Atomic Energy Commission

I. Introduction

In the summer of 1969, the author gave a series of lectures at the International School of Physics "Enrico Fermi" in Varenna, Italy, on the Interaction of Intense Photon and Electron Beams with Plasmas [1]. These lectures were motivated by the rapid advances being made in high-power short-pulse laser technology [2], by means of which sufficient focused light intensity could be achieved to permit the heating of plasmas to temperatures of many kilovolts, and to bring about a variety of nonlinear intensity-dependent effects. They attempted to provide an introductory account of the linear and nonlinear physical processes thought to be most significant in the light-plasma interaction, and of some of the properties and hydrodynamic behavior of laser-heated plasmas. (A very brief discussion was also given of plasma heating by means of an intense burst of relativistic electrons).

During the three years that have elapsed since these lectures were given, the rapid rate of advance of laser technology has continued. One of the most notable of these advances has been the development of the pulsed CO₂ TEA (Transverse Electric Discharge at Atmospheric Pressure) laser [3]. These lasers are now capable of producing 1200 joules in a 20 usec pulse [4], 90 joules in a 50 nsec pulse [5], or 4 joules in a pulse less than 2 nsec in duration [6]. A high-energy short-pulse amplifier system is now under construction with an output capability of 1000 joules in the nanosecond region [7]. The expectation that these TEA lasers may be able to produce nanosecond pulses with reasonable efficiency (~ 10%) at a moderate pulse repetition rate (10-100 pps), and at modest capital cost (< \$5000 per installed kilowatt of (average) optical output power), has suggested the possibility that laser-heated pellets of thermonuclear fuel may someday provide a practical means of generating commercial electric power [8].

The plan of this paper is to update the Varenna lectures of three years ago by considering the application of lasers to controlled fusion (an application that seemed considerably more remote at that time), and by discussing

*Work performed under the auspices of the U.S. Atomic Energy Commission.

CONTENTS

1.	Introduction	1
2.	Properties of Light Waves in a Plasma	2
2.1	Energy Density and Intensity	3
2.2	Electromagnetic Stresses and the Plasma Equation of Motion	3
2.3	Radiation Pressure	4
2.4	Steady-State Self-Focusing of Light Waves in a Plasma	6
3.	Hydrodynamics of Laser-Produced Plasmas: Some Simple Analytical Results	8
3.1	Achievement of Extremely High Plasma Density by Rapid Isentropic Compression	8
3.2	Density Jump Due to Radiation Pressure	12
4.	Fusion Power From Laser-Heated Pellets	14
4.1	Fusion Power Plant Energy Schematic	14
4.2	The Lawson Condition	16
4.3	The Lawson Condition Applied to a Freely Expanding Plasma	17
4.4	Summary	20
	References	21
	Figure Captions	22
	Figures 1 - 7	

the simple hydrodynamic theory of how to achieve the high fuel compressions that are necessary to the pellet-fusion approach. In addition, the effects of light pressure on laser-produced plasmas will be considered in an elementary fashion, and illustrated by two examples. This latter topic is included because, although the Varenna lectures dealt at some length with the energetics of the light-plasma interaction, they did not deal with the interesting effects of the light waves' momentum.

We shall begin with a brief review of the properties of electromagnetic waves propagating in a fully-ionized plasma medium.

2. Properties of Light Waves in a Plasma

The dispersion relation relating the frequency ω and the wave number k of a light wave in a plasma (neglecting absorption) is

$$(ck)^2 = \epsilon\omega^2 = \omega^2 - \omega_p^2, \quad (2.1)$$

where ϵ is the dielectric constant of the plasma medium, c is the velocity of light, and ω_p is the electron plasma frequency ($\omega_p^2 = 4\pi e^2 N_e / m$). The dielectric constant ϵ can be written in terms of the critical electron number density for light propagation N_{ec} according to

$$\epsilon = 1 - (\omega_p/\omega)^2 = 1 - (N_e/N_{ec}), \quad (2.2)$$

where N_e is the electron number density of the plasma. The phase velocity v_p and group velocity v_g of light waves in a plasma are given by

$$v_p = \omega/k = c/\sqrt{\epsilon} > c, \quad (2.3)$$

$$v_g = d\omega/dk = c^2/v_p = c\sqrt{\epsilon} < c, \quad (2.4)$$

The magnetic induction B is (approximately) proportional to the electric intensity E according to

$$B = \sqrt{\epsilon} E. \quad (2.5)$$

2.1 Energy Density and Intensity

The intensity I and energy density U associated with the light wave are given in terms of the mean-squared values of the fields according to

$$\begin{aligned} I &= (c/4\pi) \langle EB \rangle \\ &= \sqrt{\epsilon} c \langle E^2 \rangle / 4\pi = v_g U, \end{aligned} \quad (2.6)$$

$$\begin{aligned} U &= \{ [d(\omega\epsilon)/d\omega] \langle E^2 \rangle + \langle B^2 \rangle \} / 8\pi \\ &= U_r + U_e = \langle E^2 \rangle / 4\pi, \end{aligned} \quad (2.7)$$

where U_r is the energy density associated with the electromagnetic field itself,

$$U_r = \{ \langle E^2 \rangle + \langle B^2 \rangle \} / 8\pi = (1+\epsilon) \langle E^2 \rangle / 8\pi, \quad (2.8)$$

and U_e is the kinetic energy density of the plasma electrons due to their forced oscillation in the electromagnetic field,

$$U_e = N_e \langle KE \rangle_{osc} = (1-\epsilon) \langle E^2 \rangle / 8\pi. \quad (2.9)$$

2.2 Electromagnetic Stresses and the Plasma Equation of Motion

The Maxwell stress tensor for a plasma medium is given by

$$\underline{\underline{S}} = \langle \epsilon \underline{E} \underline{E} + \underline{B} \underline{B} - \frac{1}{2} (E^2 + B^2) \underline{I} \rangle / 4\pi, \quad (2.10)$$

and its divergence is:

$$\underline{\nabla} \cdot \underline{\underline{S}} = (1/2) (\epsilon - 1) \underline{\nabla} U + \partial \underline{g} / \partial t, \quad (2.11)$$

where $\underline{g}$ is the electromagnetic momentum density

$$\underline{g} = c \langle \underline{E} \times \underline{B} \rangle / 4\pi c. \quad (2.12)$$

The law of conservation of total (matter plus field) momentum then reads,

$$\rho \dot{\underline{u}} + \partial \underline{g} / \partial t = \underline{\nabla} \cdot \underline{\underline{S}} + \underline{\nabla} \cdot \underline{P}. \quad (2.13)$$

Assuming that the material stress tensor $\underline{P}$ is isotropic, and using Eq. (2.11), we obtain finally the plasma equation of motion,

$$\rho \dot{\underline{u}} = - (1/2) (1 - \epsilon) \underline{\nabla} U - \underline{\nabla} p, \quad (2.14)$$

where p is the plasma pressure, ρ is the plasma density, and $\dot{\underline{u}}$ is the plasma acceleration [9].

2.3 Radiation Pressure

In the special case in which plane light waves are propagating parallel to a plasma density gradient in, say, the z -direction, we may write

$$\rho \dot{u} = - \partial / \partial z \{ \langle E^2 \rangle + \langle B^2 \rangle \} / 8\pi - \partial p / \partial z, \quad (2.15)$$

$$= - \partial (p + U_r) / \partial z. \quad (2.16)$$

Eqs. (2.14) and (2.15) are generally valid. Eq. (2.16) follows from Eq. (2.14) or (2.15) in the special case we are considering, in which the propagation is assumed to be adiabatic so that dI/dz is zero. In this case the presence of the light wave is accounted for by simply adding a pressure p_r , the "radiation pressure", equal to the electromagnetic energy density U_r , to the matter pressure p in the equation of motion.

As the light wave propagates into plasma of increasing density, ϵ diminishes, tending toward zero as the light wave approaches the surface of reflection where the electron density assumes the critical value N_{ec} for which $\omega_p = \omega$. The radiation pressure p_r given by Eq. (2.8) can be written,

$$p_r = (1/2) (\epsilon^{1/2} + \epsilon^{-1/2}) I/c, \quad (2.17)$$

in which form it would appear that the radiation pressure becomes infinite at the surface of reflection. This result, however, requires for its validity that the adiabatic condition be satisfied; that the fields change slowly in traveling one wavelength.

That is, we require

$$|d\epsilon/dz|/\epsilon \ll k, \quad (2.18)$$

or

$$\epsilon \gg \epsilon_{MIN} = |d\epsilon/dz|/k = 1/kH, \quad (2.19)$$

where H is the scale height of the density gradient near the reflecting surface,

$$H = |dN_e/dz|/N_{ec}. \quad (2.20)$$

Using the fact that $k = \sqrt{\epsilon} k_0$, we obtain the result,

$$\epsilon_{MIN} = (k_0 H)^{-2/3}, \quad (2.21)$$

where $k_0 (= \omega/c = 2\pi/\lambda_0)$ is the vacuum wavenumber of the light wave. An estimate of the peak radiation pressure exerted against the reflecting surface can then be obtained by setting ϵ equal to ϵ_{MIN} in Eq. (2.17). That is

$$p_r \sim (k_0 H)^{1/3} (I/2c), \quad (N_e \sim N_{ec}), \quad (2.22)$$

provided that $(k_0 H)^{1/3}$ is much larger than unity. If, for example, H were 160 times the vacuum wavelength of the light, the factor $(k_0 H)^{1/3}/2$ would equal five, a modest enhancement of the vacuum value of the radiation pressure (I/c).

2.4 Steady-State Self-Focusing of Light Waves in a Plasma

The optical stresses exerted on a plasma by a light beam act in such a manner as to expel the plasma from the region of high light intensity, as can be seen from the equation of motion (2.14) of the plasma. The resulting decrease in plasma density causes an increase in the refractive index $n (= \sqrt{\epsilon})$ of the plasma. That is, the plasma becomes a convergent lens where the beam is localized, and focuses the beam.

The magnitude of this effect can be readily obtained as follows. The plasma pressure p is given by

$$p = N_e k T_e, \quad (2.23)$$

where T_e is the electron temperature, and where for simplicity we ignore the ion pressure ($T_e \gg T_i/Z$). Assuming further that the plasma motion caused by the light beam takes place isothermally, Eq. (2.14) can be written in the steady state ($\dot{u}=0$) as

$$\nabla \cdot [(I/nI_{NL}) + k n N_e] = 0, \quad (2.24)$$

where I_{NL} is the light intensity at which the energy of the oscillatory motion of the electrons is equal to the energy of their random thermal motion,

$$I_{NL} = 2N_{ec}ckT_e, \quad (2.25)$$

an intensity at which nonlinear optical effects would be expected to be well-advanced. [1].

The solution of Eq. (2.24) is

$$N_e = N_{e0} \exp (- I/n I_{NL}), \quad (2.26)$$

where N_{e0} is the electron density of the unperturbed plasma, indicating that if I is large compared with nI_{NL} , the plasma is virtually completely expelled from the region of the beam.

If, on the other hand, we consider a light beam containing "hot-spots" or small-scale increases in intensity δI , and consider the weakly nonlinear case in which the intensity I is small compared with I_{NL} (or more precisely, $I/nI_{NL} \ll 2\epsilon/(1-\epsilon)$), then corresponding variations in the electron density δN_e

$$\delta N_e/N_e = - \delta I/nI_{NL}, \quad (2.27)$$

and refractive index δn

$$\delta n = [(1-\epsilon)/2\epsilon] \delta I/I_{NL}, \quad (2.28)$$

will be produced, and will result in steady-state self-focusing of hot-spots of radius R_0 in a distance z_{SF} ,

$$z_{SF} \sim R_0 \sqrt{n/\delta n} = R_0 / \sqrt{[(1-\epsilon)/2\epsilon] (\delta I/nI_{NL})}, \quad (2.29)$$

provided the power content $\delta P (= \pi R_0^2 \delta I)$ of the hot-spot exceeds a threshold value P_c

$$\delta P \gg P_c \sim [\sqrt{\epsilon} / (1-\epsilon)] (mc^3/e^2) kT_e \quad (2.30)$$

so that diffraction spreading can be overcome.

3. Hydrodynamics of Laser-Produced Plasmas: Some Simple Analytical Results

3.1 Achievement of Extremely High Plasma Density by Rapid Isentropic Compression

It is well-known that extremely high pressures can be achieved by focusing an intense laser pulse on a solid surface in vacuum [1]. The high pressure is derived from the violent blowoff of the strongly-heated surface matter into the vacuum, in the same manner that a rocket derives thrust from its exhaust.

In the application of lasers to the fusion of pellets of thermonuclear fuel, it is necessary that the pellet be compressed to very high density [10], a requirement that is discussed more fully in the next section dealing with fusion power from laser-heated pellets. We are therefore led to enquire how the high pressure derived from laser-induced blowoff can be efficiently employed to compress fusionable pellets to high density. The problem can be divided into two parts: What pressure-time profile should be applied to achieve high compression, and what laser output power-time profile should be employed to achieve the desired pressure profile?

The desired pressure profile can be derived from a consideration of a spherical isentropic homogeneous compression, the isentropic character of the compression guaranteeing that the work expended to achieve a given compression is minimized. The analysis is similar to that given in reference [1] describing the homogeneous expansion of an isothermal sphere into a vacuum, and is as follows.

A homogeneous compression is one in which the element of volume d^3x in the Lagrangian description, in which $\underline{x}(t)$ denotes the instantaneous position vector of a mass point, decreases everywhere at the same rate. That is,

$$d^3x = h^3(t)d^3x_0, \quad (3.1)$$

where $\underline{x}_0 = \underline{x}(t=0)$, and $h(t)$ is a dimensionless scale factor. Writing Eq. (3.1) in spherical co-ordinates we obtain immediately,

$$r(r_0, t) = r_0 h(t), \quad (3.2)$$

$$u(r_0, t) = \dot{r} = r_0 \dot{h}(t). \quad (3.3)$$

From the law of mass conservation we may write

$$dm = \rho r^2 dr = dm_0 = \rho_0 r_0^2 dr_0, \quad (3.4)$$

which together with Eq. (3.2) implies that

$$\rho(r_0, t) = \rho_0(r_0)/h^3(t). \quad (3.5)$$

The equation of motion is,

$$\ddot{u} = -r^2(dp/dm) = -(r^2/\rho_0 r_0^2)(dp/dr_0). \quad (3.6)$$

We wish to write dp/dr_0 in terms of dp_0/dr_0 . For that purpose we invoke the relation between pressure p and density ρ along an isentrope of an ideal gas,

$$\begin{aligned} p(r_0, t) &= a(r_0) \rho^\gamma(r_0, t) \\ &= p_0(r_0)/h^{3\gamma}(t). \end{aligned} \quad (3.7)$$

Making use of these relations, the equation of motion can be written,

$$\begin{aligned} h^{3\gamma-2} \ddot{h} &= -(1/\rho_0 r_0) dp_0/dr_0 \\ &= -1/t_c^2 = \text{constant}, \end{aligned} \quad (3.8)$$

where the r.h.s. of Eq. (3.8) is a function of r_0 alone and the l.h.s. is a function of t alone. Since r_0 and t are independent Lagrangian variables, each side of the equation must be constant.

Assuming that $\gamma = 5/3$, which describes either a fully degenerate Fermi electron gas or a fully ionized plasma, and that the fluid is initially at rest ($\dot{h}(0)=0$), the solution of the temporal part of Eq. (3.8) is

$$h^2 = 1 - \tau^2, \quad (\tau = t/t_c). \quad (3.9)$$

This result shows that t_c is the "total collapse" time, i.e., the time at which the plasma radius shrinks to zero. Assuming for simplicity that the initial plasma state is one of uniform entropy ($da/dr_0 = 0$), the solution of the spatial part of Eq. (3.8) becomes

$$y_0(r_0) = p_0(r_0)/p_0(0) = (1 + \beta x_0^2)^{5/2}, \quad (3.10)$$

where

$$\beta = (R_0/c_0 t_c)^2/3, \quad (3.11)$$

$$x_0 = r_0/R_0, \quad (0 \leq x_0 \leq 1), \quad (3.12)$$

$$c_a^2 = \gamma p_0(0)/\rho_0(0). \quad (3.13)$$

The desired pressure profile follows immediately from Eqs. (3.7) and (3.9),

$$p(R, t) = p(R, 0)/(1 - \tau^2)^{5/2} \quad (3.14)$$

where $R(= h(t)R_0)$ denotes the plasma radius. A graph of the pressure profile and of the initial pressure distribution is shown in Fig. 1.

The initial state of the plasma required by our solution is not a uniform state, as evidenced by Eq. (3.10). However, computer calculations with the WAZER code [11] have verified, as one might expect, that if the compression is carried out slowly enough that β is less than unity, the pressure profile we have derived can be applied to a plasma in an initially uniform state without producing any significant increase in entropy.

Having determined the desired pressure profile, we turn to the question of the laser output power profile required to produce it.

If we assume that the pressure $p(R,t)$ exerted against the plasma is proportional to the two-thirds power of the light intensity [1], and that the light is totally absorbed at the Lagrangian surface (of radius R), then

$$\begin{aligned} \Psi/\Psi_0 &= (R/R_0)^2 I/I_0 \\ &\sim 1/h^{11/2} = 1/(1-\tau^2)^{11/4}, \end{aligned} \quad (3.15)$$

where Ψ is the total laser output power that is focused on the plasma. In reality, however, the situation is more complicated than this due to the effects of electron heat conduction and plasma ablation. Computer calculations which take these effects into account have been performed [12], with the result that the exponent on the r.h.s. of Eq. (3.15) should be approximately equal to 2 (instead of 2.75), i.e.,

$$\Psi/\Psi_0 \approx 1/(1-\tau^2)^2. \quad (3.16)$$

A graph of the required power profile Ψ/Ψ_0 is included in Fig. 1. Computer results (obtained with the WAZER code) of the plasma particle trajectories and density distribution resulting from the application of the laser power profile of Eq. (3.16) are shown in Figs. 2 and 3, respectively. Particle trajectories obtained by applying the ideal pressure profile given by Eq. (3.14) are shown in Fig. 4 for comparison. Fig. 2 shows that the prescribed laser power profile of Eq. (3.16) has resulted in a sufficient departure from the ideal isentropic pressure profile of Eq. (3.14) to

generate a mild shock. The result of this increase in entropy is that the plasma temperature at the time of peak compression is approximately three times as high as it would have been had the compression been ideal.

3.2 Density Jump Due to Radiation Pressure

Optical stress self-focusing of a light beam, considered earlier in Section (2.4), is an example of an effect which depends on forces exerted on the plasma in a direction perpendicular to the direction of propagation of the beam. Let us now consider an effect which depends on forces exerted in a direction parallel to the direction of light wave propagation.

We consider a plane isothermal expansion of a plasma half-space (of initial density ρ_0) into a vacuum (to the right), and ask how the flow is modified by the absorption of light (incident from the right) at a plane of specified density ρ_c ($< \rho_0$), the critical density for light propagation. We assume for simplicity that, other than being absorbed at the plane ρ_c , the light wave is unaffected by the plasma. The effect of the light wave, then, is merely to give rise to a constant radiation pressure $p_r (= I/c)$ to the right of the plane ρ_c . (We are disregarding the enhancement factor contained in Eq. (2.22)). We shall further assume that the (electron) thermal conductivity is high enough to justify our assumption of isothermal flow, but that the (ion) fluid viscosity is negligible so that the deposition of the light wave's momentum can be taken as being sharply localized. (These assumptions are somewhat self-contradictory, but not seriously so if the electron temperature is very large compared with the ion temperature at the surface where the light is absorbed, a condition which will generally prevail if the electron temperature is high).

The resultant flow can be determined exactly. It consists of four regions as shown in Fig. 5: An undisturbed region (A), a simple isothermal rarefaction (B), a region of uniform flow (C), and a second simple isothermal rarefaction (D). As can be seen, the effect of the light wave's momentum

is to pile up the plasma upstream and to produce a density jump (rarefaction shock) discontinuity at the plane of absorption separating regions (C) and (U).

The magnitude of the density jump $\Delta\rho$ is given by

$$(\Delta\rho)/\rho_c = [a + \sqrt{a(4+a)}] / 2, \quad (3.17)$$

where

$$a = p_r/p_c, \quad (3.18)$$

$p_c (= \rho_c c_i^2)$ is the plasma pressure at the plane of absorption, and c_i is the isothermal speed of sound in the plasma. The density jump satisfies the inequalities

$$a < (\Delta\rho)/\rho_c < a + 1, \quad (3.19)$$

and for small a is given by

$$(\Delta\rho)/\rho_c \approx \sqrt{a} \quad (a \ll 1). \quad (3.20)$$

This latter result shows that the density jump tends to be rather significant even at relatively low light intensity since it is proportional to only the square-root of the radiation pressure p_r .

Further analytical results are as follows, where the meaning of the subscripts is defined in accordance with Fig. 5.

$$\rho_3/\rho_c \equiv 1, \quad U_0/c_i = -1, \quad (3.21)$$

$$U_1/c_i = \ln(\rho_0/\rho_1) - 1, \quad (3.22)$$

$$U_2/c_i = \ln(\rho_0/\rho_1) - \rho_c/\rho_2, \quad (3.23)$$

$$u_1/c_i = u_2/c_i = \ln(\rho_0/\rho_1). \quad (3.24)$$

U_i ($i = 0, 1, 2$) denotes the velocity of the point (i) in the rest frame of the undisturbed fluid (to the left). $u_1 (= u_2)$ denotes the velocity of the fluid in region (C).

The existence of such a radiation pressure-induced density jump could possibly influence significantly the nature and rate of growth of plasma instabilities excited in the region of the density jump.

4. Fusion Power From Laser-Heated Pellets

In order that a laser-heated pellet of thermonuclear fuel represent a practical source of fusion power, it must satisfy two criteria: It must be brought to a high temperature, and its specific surface area ($= 3/\rho k$) must be reduced to a low value. A high temperature is required to provide an adequate rate of thermonuclear reaction. A low specific surface area is required to reduce energy lost from the surface of the pellet (principally hydrodynamic expansion losses) to an acceptable value as compared with energy gained throughout the mass of the pellet by thermonuclear fusion. This latter requirement may be immediately translated into a requirement for high pellet compression, since the specific surface area S of a pellet of any given mass M decreases with increasing pellet density ρ ($S \propto 1/\rho^{2/3}$).

We shall first determine what is practical by a brief consideration of the energy schematic of a fusion power plant, and then see what this implies in terms of pellet and laser characteristics.

4.1 Fusion Power Plant Energy Schematic

The energy schematic of a fusion power plant employing laser heating of the thermonuclear fuel is shown in Fig. 6. The numbers given in Fig. 6 for laser power, electric power, and waste heat are based on assuming 40% conversion of heat into electricity (for both reactor and laser heat), and either 5% or 50% (numbers in parentheses apply to the latter efficiency) conversion of

electricity into laser light. (The corresponding efficiency of conversion of heat into light is therefore 3.2% or 25%, respectively), The 40% efficiency represents modern steam-electric plant performance.

A conversion efficiency of 30% (electricity into light) has been achieved with the electric discharge CO₂ laser in the free-running oscillator mode of operation (long-pulse mode), and efficiencies approaching 50% have recently been reported with an electric discharge CO laser at liquid nitrogen temperature. It is anticipated that 5% efficiency or better can be achieved in the energy-storage mode of operation (short-pulse mode) of these lasers.

The conclusion that emerges from an inspection of the power flow diagram of Fig. 6 is that the thermonuclear power release must be large compared with the laser output power required to heat the fuel, i.e.,

$$P_N/P_L = \alpha \gg 1. \quad (4.1)$$

As the laser efficiency ranges between 5% and 50%, the smallest acceptable value of the multiplication factor α ranges between 100 and 10. With these values, the laser represents more than 40% of the power plant load, and the overall plant efficiency (important in coolant cost and thermal pollution considerations) is only 28%. We will therefore assume that an $\alpha \epsilon_L$ product of at least 5, where ϵ_L is the laser efficiency, is a requirement of a laser-heated fusion plant.

The laser that is required must be a reliable unit providing very high average power output (an appreciable fraction of the electrical output of the fusion power plant). It is also desirable that it be efficient, so that the requirement for a large thermonuclear multiplication factor α can be somewhat relaxed.

4.2 The Lawson Condition

The time τ that a DT plasma must be maintained at temperature T and (ion) number density n to yield $(\alpha/\beta\epsilon)$ times its heat content is easily found to be,

$$n\tau = (\alpha/\beta\epsilon)F(T), \quad (4.2)$$

where

$$F(T) = 12 \text{ kT} / \langle \sigma_{DT} v \rangle Q. \quad (4.3)$$

The quantity $\langle \sigma_{DT} v \rangle$ is the Maxwell-averaged DT fusion reaction cross section; and $Q (= 22.2 \text{ MeV})$ is the energy release per DT reaction, including the exothermic neutron capture reaction in a tritium-breeding lithium blanket.

The factor F is a function of temperature alone, as shown in Fig. 7, having the value $5 \times 10^{13} \text{ sec/cm}^3$ at a temperature of 10 keV.

If we now identify ϵ as that fraction of the laser light that is converted into heat content of the plasma characterized by (n, T) , and allow for the possibility that the heat content may be multiplied β -fold by thermonuclear self-heating of the plasma, we may identify the quantity α in Eq. (4.2) with the ratio P_N/P_L of the thermonuclear output power to laser input power as defined by Eq. (4.1).

A conventional (relatively low density, non laser-heated) fusion reactor is characterized by $\beta \sim 1$, $\epsilon \sim 1$. To compensate for a 40% steam-

electric plant efficiency, α must exceed 2.5 in order to provide a net power gain. If the fuel temperature is 10 keV, we find from Eq. (4.2) that $n\tau \sim 10^{14}$ sec/cm³. This is the often-quoted "Lawson Condition" for the production of fusion power.

Assuming that the same $\beta\epsilon$ product (~ 1) applies to the laser-heated fusion reactor, but using values of α ranging between 10 and 100 as arrived at in the preceding section, we require that

$$5 \times 10^{14} < n\tau < 5 \times 10^{15} \text{ sec/cm}^3 \quad (4.6)$$

for a practical laser fusion reactor.

4.3 The Lawson Condition Applied to a Freely Expanding Plasma

We will now specialize our discussion to the case of laser-heated solid DT pellets, an attractive approach to controlled fusion because no magnetic fields are required for plasma confinement. The use of lasers to heat lower density ($n \sim 10^{17}$ cm⁻³) magnetically confined plasmas has been considered elsewhere [13].

The confinement time τ of a laser-heated (pellet) plasma freely expanding in vacuum is proportional to the time required for a rarefaction wave, traveling with (isothermal) sound speed c_i , to traverse the plasma radius R . Taking the proportionality factor to be 1/4, the relation between R and τ becomes

$$R = 4c_i\tau. \quad (4.7)$$

Since the ion density n is equal to $(2.4 \times 10^{23})\rho$, ρ being the plasma density, we obtain that

$$n\tau = (6 \times 10^{22}/c_i) \rho R. \quad (4.8)$$

Assuming a temperature of 10 keV, then $c_i = 9 \times 10^7$ cm/sec, $F = 5 \times 10^{13}$ sec/cm³, and we obtain from Eqs. (4.2) and (4.8)

$$n\tau = (5 \times 10^{13}) \alpha/\beta\epsilon, \quad (4.9)$$

or

$$\rho R = (0.075) \alpha/\beta\epsilon. \quad (4.10)$$

Using the values $\alpha = 10$, $\beta\epsilon = 1$, we obtain the equivalent results;

$$n\tau = 5 \times 10^{14} \text{ sec/cm}^3, \quad \rho R = 0.75 \text{ gm/cm}^2. \quad (4.11)$$

We can now write approximate relations for the fuel mass M , the confinement time τ , and the required laser pulse energy W in terms of the dimensionless parameters $\alpha, \beta\epsilon$, and η , where η is defined as the ratio of the plasma density to that of solid DT (0.2 gm/cm³).

$$M \approx (4\alpha/\beta\epsilon)^3/\eta^2 \quad \text{mgm}, \quad (4.12)$$

$$\tau \approx (\alpha/\beta\epsilon)/\eta \quad \text{nsec}, \quad (4.13)$$

$$W \approx (4\alpha)^3/(\beta\epsilon)^4 \eta^2 \quad \text{MJ}. \quad (4.14)$$

It should be noted that the required pulse energy W is a sensitive function of the parameters $\alpha, \beta\epsilon$, and η which will vary widely depending on the particular manner in which the pellet is heated, and one should ultimately rely on detailed computer calculations such as those reported in reference [12]. (This may account for the widely varying estimates of required laser pulse energy that have been obtained by different investigators).

If a reasonable approximation to isentropic compression of a solid DT pellet can be achieved by the method described earlier in Section (3.1), it becomes possible to consider pellet compressions η as large as 10^4 . The work w per electron of the pellet (regarded as fully-degenerate electron gas) that must be expended to achieve (isentropically) a compression η is given by

$$w = 3\eta^{2/3} \text{ eV/electron,} \quad (4.15)$$

which amounts to only 1.4 keV per electron at a compression of 10^4 , an entirely acceptable cost in energy invested.

With these words of caution, let us illustrate these results with three examples in Table I below: "Breakeven" ($\alpha = 1$, $\beta\epsilon = 0.5$, $\eta = 10$), "Power Plant A" ($\alpha = 10$, $\beta\epsilon = 1/2$, $\eta = 10^3$), and "Power Plant B" ($\alpha = 50$, $\beta\epsilon = 1$, $\eta = 10^4$).

TABLE I

	R (Microns)	τ (psec)	W (MJ)
Breakeven	700	200	10
Power Plant A	70	20	1
Power Plant B	20	5	0.1

Power plant A would require a laser producing megajoule pulses with 50% efficiency according to our criterion that $\alpha\epsilon_L \geq 5$, whereas Power plant B would require a laser producing 100 kilojoule pulses with 5% efficiency, a much more modest requirement. The necessity for extremely high fuel-compression is well-illustrated in this comparison.

4.4 Summary

The approach to controlled fusion via laser-heated pellets is attractive because no magnetic confinement of the plasma is required. The walls of the fusion reactor can consist of liquid lithium, situated an appropriately large distance from the reacting thermonuclear pellet. No coils, superconducting or otherwise, need be employed. The problems of heating and confinement are effectively solved. However, as is usually the case, one has to pay a price.

The price is an efficient repetitively-pulsed laser producing pulses in the nanosecond-megajoule range of good beam quality, whose average power represents a substantial fraction of the electric generating capacity of the power plant. Recent calculations concerning the coupling of the laser pulse with the pellet plasma suggest that the wavelength of these pulses may need to be relatively short, that is, considerably less than the 10 microns of the CO_2 lasers [14].

Although such a laser would represent a very large advance beyond the present state of the art, there does not appear to be any obstacle that would, in principle, rule it out. However as Rose has pointed out [15], today's short-pulse lasers are fragile and expensive devices, and probably the most difficult task will be that of making a laser (that produces peak powers of 10^{15} watts) both cheap enough and reliable enough for practical power plant use.

ACKNOWLEDGEMENTS

The author is greatly indebted to S. Podner, J. Eddleman, J. Kuckolls, J. Scofield, and J. Shearer for many invaluable discussions and clarifications, and to A. Cecil for carrying out the computer calculations in his customary expert manner.

REFERENCES

- [1] P. Caldirola and H. Knoepfel (Eds.), Physics of High Energy Density, Academic Press, New York (1971).
- [2] R. E. Kidder, Nuclear Fusion 8, 3 (1968).
- [3] A. J. Beaulieu, Appl. Phys. Lett., 16, 504 (1970).
- [4] J. D. Daugherty, Seventh International Quantum Electronics Conference, Montreal, Canada, May 8-11 (1972), paper Q1.
- [5] M. C. Richardson et al, Seventh International Quantum Electronics Conference, Montreal, Canada, May 8-11 (1972) paper Q10.
- [6] F. Rheault et al, Seventh International Quantum Electronics Conference, Montreal, Canada, May 8-11 (1972), paper Q2.
- [7] C. A. Fenstermacher, Symposium on High Power Molecular Lasers, Quebec, Canada, May 15-17 (1972).
- [8] M. J. Lubin and A. P. Fraas, Scientific American 224, 21 (1971).
- [9] See, for example, Landau and Lifshitz, Electrodynamics of Continuous Media, Pergamon Press, New York (1960).
- [10] R. E. Kidder, Proceedings of the Esfahan Symposium on Fundamental and Applied Laser Physics, August 29 - September 5 (1971). (To be published).
- [11] R. E. Kidder and W. S. Barnes, WAZER: A One-dimensional, Two-temperature Hydrodynamic Code, UCRL-50583, January 31, (1969).
- [12] J. Nuckolls et al, Seventh International Quantum Electronics Conference, Montreal, Canada, May 8-11 (1972). (To be published).
- [13] J. M. Dawson et al, The Use of Long-Wavelength, High-Powered Lasers For Controlled Thermonuclear Fusion, Paper CN-28/D-13, IAEA Conference on Controlled Fusion, Madison, Wisconsin, (1971). To be published.
- [14] R. E. Kidder and J. W. Zink, Nuclear Fusion 12, 325 (1972).
- [15] D. J. Rose, Science 172, 797 (1971).

FIGURE CAPTIONS

- Fig. 1. Normalized pressure profile $y(t)$ [$= p(R,t)/p(R,0)$] and initial pressure distribution $y_0(r_0)$ [$= p_0(r_0)/p_0(0)$], vs. dimensionless time τ ($=t/t_c$) and Lagrangian radius $x_0(=r_0/R_0)$, of an ideal homogeneous isentropic compression. (The parameter β equals 0.435). Also shown is the normalized laser output power profile $\phi(= \Psi/\Psi_0)$ approximating that required to achieve isentropic compression to high density.
- Fig. 2. Computer results (WAZER) showing particle trajectories $r(r_0,t)$ (cm) vs. time t (nsec) resulting from laser output power profile: $\Psi(\text{watts}) = (16\pi \times 10^{10})/[1-(t/t_c)^2]^2$, $t_c = 9.756$ nsec, $R_0 = 0.04$ cm, $\beta = 0.435$, $\rho_0 = 0.2$ gm/cm³ (DT), $T_0 = 10$ eV. (Note presence of unwanted shocks).
- Fig. 3. Computer results (WAZER) showing plasma density $\rho(\text{gm/cm}^3)$ vs. normalized Lagrange radius $x_0(= r_0/R_0)$ when $t = 9.539$ nsec, $\Psi = 2.6 \times 10^{14}$ watts.
- Fig. 4. Computer results (WAZER) showing particle trajectories $r(r_0,t)$ (cm) vs. time t (nsec) resulting from the ideal pressure profile of Eq. (3.14) and using the initial pressure distribution of Eq. (3.10). $t_c = 9.756$ nsec, $\beta = 0.435$. (Note complete absence of shocks).
- Fig. 5. Log of plasma density [$\log(\rho/\rho_c)$] vs. dimensionless distance $z/c_1 t$ exhibiting radiation pressure-induced density jump (solid lines, $\rho_0/\rho_c = 2$, $a = p_r/p_c = 0.314$). Dashed line represents unperturbed density profile ($a=0$).

FIGURE CAPTIONS (cont'd)

Fig. 6. Power flow diagram for laser reactor. Numbers denote power in arbitrary units. Numbers not in parenthesis correspond to a laser efficiency of 5%, those in parenthesis correspond to an efficiency of 50%. P_L and P_N denote laser output power and thermonuclear power respectively.

Fig. 7. Graph of the Lawson Factor $F(\text{sec}/\text{cm}^3)$ versus Ion Temperature $T(\text{keV})$ for equimolar DT fuel.

Fig. 1.

Fig. 2.

Fig. 3

Fig. 4.

Fig. 5.

Fig. 7.