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COMPARISON AND ANALYSIS OF THEORETICAL DOPPLER-COEFFICIENT RESULTS FOR FAST REACTORS

by

R. N. Hwang and K. Ott

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

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R. N. Hwang and K. Ott*

Reactor Physics Division

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*On leave from Kernforschungszentrum Karlsruhe, Germany, within the framework of the USAEC-Euratom Scientist Exchange Program

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COMPARISON AND ANALYSIS OF THEORETICAL DOPPLER-COEFFICIENT RESULTS FOR FAST REACTORS

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R. N. Hwang and K. Ott

ABSTRACT

Comparison and analysis of the theoretical Doppler coefficient for fast reactors as calculated independently at ANL and KFK were made. The factors that affect the magnitude as well as the temperature behavior of the Doppler coefficient were examined in great detail. The results of this study are indicative of the possible causes of significant discrepancies in the previous comparison of this kind.¹

I. INTRODUCTION

In the past few years there has been a great deal of theoretical work on the Doppler effect for large fast reactors. Refined methods of calculating the Doppler coefficient of a fast system have become available. At the same time, our knowledge of resonance data has also been significantly improved. One is, therefore, led to believe that good agreement among the Doppler coefficients as calculated by different laboratories for identical reactor systems can be achieved. An extensive comparison of this type proposed by D. Okrent¹ was presented at the Argonne Fast Reactor Conference in 1965. Ten different laboratories contributed independently the calculated results in which the Doppler coefficients for various systems were included. Comparisons of the results showed surprisingly and unexpectedly large discrepancies. These discrepancies were shown in terms of the difference of a particular result and the median value defined as the average over the results of all laboratories. The discrepancies of all results ranged from about -85% to +60% of the median value. Such disagreement is certainly unsatisfactory and requires further consideration.

These comparisons were based on the Doppler coefficients at a given temperature. Undoubtedly, still larger discrepancies may be expected at other temperatures of practical importance. For fast reactor studies, the range of temperature of practical interest varies from room temperature to about 3600°K. The Doppler coefficient may vary by a factor as much as 30 or more within this temperature range, especially when the sharp rise in temperature is accompanied by the loss of sodium.

Hence, the incorrect prediction of the temperature behavior of the Doppler coefficient in the normal operating temperature range may lead to much more serious errors in the event of power excursion. For this reason, Froelich and Ott² have carried out a more detailed comparison in which not only the magnitudes, but also the temperature behaviors of the Doppler coefficients as obtained by two independent calculations were considered. The latter comparison, however, led only to preliminary conclusions.

In recent experiments,^{3,4} the temperature behavior of the Doppler reactivity effect has been under serious consideration. The significant improvement in the accuracy of these experiments makes possible the detailed studies of the true temperature behavior of the Doppler coefficient instead of by $1/T$ fitting, which was widely used in the past. The use of $1/T$ fitting is believed to be inconclusive, since the temperature behavior of the Doppler coefficient is characterized by the spectrum of a particular system under consideration. Furthermore, a different temperature distribution $(1/T)^{1.1}$ for instance, may give just as good a fit to the measured values as $1/T$ within the limit of experimental errors and the limited temperature range considered.

The purpose of this paper is to extend the comparison of at least two independent results into greater detail. In this comparison, the temperature, energy, and enrichment dependence of the Doppler coefficient have been carefully examined. It is believed that the results of this comparison will give some indication of possible causes responsible for the large discrepancies mentioned previously. In particular, it will be shown that two independent calculations with the best available data and methods as carried out at ANL and KFK, respectively, indeed give excellent agreement.

II. GENERAL DESCRIPTION OF THE COMPARISON

For convenience, some bare systems typical of fast reactor composition as proposed by Okrent¹ were again used in the present work. To simplify the problem further, the simplest cases with no Pu^{240} and no fission product present were chosen for the composition. Table I gives the concentrations in atom per cc for the constituents of the systems under consideration. The ratios γ of fertile to fissile materials were taken to be 9.0, 7.0, 5.0 and 2.5, respectively.

TABLE I. Atomic Concentrations (atom/cc)

Cases	γ	Pu^{239}	U^{238}	Na	SS	O
1	9.0	0.00072	0.00648	0.0123	0.011	0.0144
2	7.0	0.00090	0.0063	0.0123	0.011	0.0144
3	5.0	0.0012	0.0060	0.0123	0.011	0.0144
4	2.5	0.00206	0.00514	0.0123	0.011	0.0144

Table II shows the results of previous comparison for these systems as taken from Okrent's paper. The average absolute values of the deviation from the median are as much as 37-40% at $T = 1100^\circ\text{K}$. Still larger deviations might be expected if the comparisons had also been made for other temperatures, as was pointed out by Froelich and Ott.²

TABLE II. Comparison of Different Calculations of Doppler Coefficients for Identical Reactors (from ANL-7120)

$\frac{U^{238}}{Pu^{239}} =$	$\left(T \frac{dk}{dT}\right)_{1100^\circ\text{K}}$			Deviation from Average in %		
	5	7	9	5	7	9
AI	0.0067			+29		
ANL	0.0049	0.0092	0.0140	-6	0	-3.5
APDA	0.0055			+6		
BNL	0.0009	0.0018	0.0030	-83	-80	-79
B&W	0.0068	0.0125	0.0184	+31	+36	+27
France	0.0034	0.0083	0.0116	-35	-10	-20
GE	0.0046	0.0093	0.0150	-12	+1	+3.5
Karlsruhe	0.0052	0.0098	0.0149	0	+6	+2.8
UK	0.0077	0.0147	0.0223	+48	+60	+54
UNC		0.0060	0.0100		-35	-31
Median Value	0.0052	0.0092	0.0145	37	40	38

In the present work, a much more detailed comparison of the Doppler coefficients of these four bare systems, calculated independently at Argonne National Laboratory and Kernforschungszentrum Karlsruhe, will be given. It should be noted that some errors in the ANL cross-section Code MC² used in the ANL calculations for previous comparison have subsequently been discovered. This code is still undergoing correction and was not used in the calculations discussed in this paper. This explains inconsistencies between the ANL results in Table II and those calculated for this paper. In addition, two other results calculated in the following ways are presented along with ANL and KFK results:

- (1) Calculations were directly based on ABN⁷ group constants and using original ABN self-shielding factors.
- (2) Spectra were based on ABN group constants, but the temperature derivatives of the reaction cross section $\left(\frac{\partial \Sigma_f}{\partial T} \text{ and } \frac{\partial \Sigma_a}{\partial T}\right)$ calculated with the Karlsruhe Doppler Program were used.

The ABN set is used with original $1/E$ weighting in all groups. The former will be referred to as ABN results, whereas the latter will be referred to as ABN-KFK results in the later discussions.

It should be mentioned that no attempt is made in this paper to compare directly the group constants used in the individual calculations. The detailed comparison of group constants which were generated by different approximations in the treatment of the weighting spectrum and resonance self-shielding is an extremely difficult task. Furthermore, differences in the group boundaries and microscopic nuclear parameters make the direct comparison of group constants even more difficult. It is, therefore, more meaningful to compare only those quantities that may either directly or indirectly affect the magnitudes of the Doppler coefficient and its temperature behavior.

III. DETAILED COMPARISON AND ANALYSIS OF ANL AND KFK RESULTS

A. Comparison of Bucklings

Critical calculations were made by iterating the bucklings until these systems became critical. All cases done at ANL were based on the ELMOE program⁵ so that resonances of light materials were treated accurately.

112-7567

Fig. 1. B^2 versus Fertile-to-fissile Ratio

Figure 1 shows values of B^2 calculated at ANL and KFK for four cases with different fertile-to-fissile ratios. The buckling values of ANL and KFK are in excellent agreement in all cases under consideration. Consequently, the corresponding critical sizes are also in excellent agreement. However, noticeable discrepancies were observed when the ABN set was used in the calculation. The main reason for such discrepancies is believed due to the difference in fission cross sections in the energy region between ~ 10 keV and 100 keV. Fission cross sections

for both ANL and KFK sets were based on the more recent measurements by White,⁶ which are substantially lower in the keV region. The fission cross sections of ABN were based on the older measurement.⁷

B. Flux and Adjoint Flux in the Doppler Region

One of the most essential problems in the calculation of the Doppler coefficient is the estimation of the flux and adjoint flux in the energy region where the Doppler effect is significant. The variations of the low-energy flux and the adjoint flux will affect not only the magnitude of the Doppler coefficient, but also its variation as a function of temperature.

Figures 2 and 3 show the variations of flux per unit lethargy for two extreme cases, normalized to $\int_0^{\infty} \phi(E) dE$, with $y = 9.0$ and 2.5 , respectively. KFK and ABN results are readily comparable since they were based on the same group boundaries. As one can see, KFK spectra are generally higher than ABN spectra in the high-energy regions, whereas the former become lower than the latter in the groups of lower energy. The reason for this deviation is essentially the different weighting in the calculation of the removal cross sections ($1/E$ in ABN and a typical spectrum of a 1000-MWe, sodium-cooled reactor for KFK).

112-7565

Fig. 2. Spectra at 900°K for $y = 9.0$

112-7571

Fig. 3. Spectra at 900°K for $y = 2.5$

From Figs. 2 and 3, it is clear that a direct comparison of ANL spectra with those of KFK and ABN is rather difficult, because of the difference in the group structures. A more instructive comparison can be obtained by rearranging the KFK and ABN spectra into ANL group structure. Figures 4 and 5 show the corresponding flux per unit lethargy based on group constants of three different origins after being rearranged into ANL group structure. The total areas under each spectra were preserved.

The rearranged histogram has been calculated by integration of the original histogram over the ANL group structure. It is seen that the agreement between ANL and KFK values is exceedingly good with the exception of the energy region at and near the sodium resonance. In the ANL calculations, the resonance self-shielding effect of light materials was treated accurately by use of the ELMOE code.⁵ Consequently, the ANL flux is lower than both the KFK and the ABN values in the group in which the sodium resonance occurs, whereas the ANL flux becomes higher in the two next groups below the sodium resonance due to the effect of resonance scattering.

112-7566

Fig. 4. Rearranged Spectra at 900°K for $y = 9.0$

112-7572

Fig. 5. Rearranged Spectra at 900°K for $y = 2.5$

112-7577

Fig. 6. Adjoint Flux for Various Groups

Figure 6 shows the variations of the low-energy adjoint flux for the $y = 2.5$ case only. Excellent agreement between the ANL and the KFK values was observed in the energy region beyond 1 keV; the ABN values are consistently higher than both ANL and KFK values. The close agreement between the ANL and the KFK adjoint flux in the keV region is not surprising since the fission cross sections for both sets were based on the same measured values given in Ref. 6. In the energy region below about 1 keV the ANL values are consistently lower than both the KFK and the ABN values. The discrepancies are believed to be due entirely

to the difference in Pu^{239} fission cross sections. For this comparison, old Pu^{239} parameters,⁸ which were known to underestimate the fission cross sections⁹ significantly in the low-energy end, were used in the calculation of ANL set. Better agreement in the low-energy adjoint flux would be possible had the newer Pu^{239} parameters¹⁰ been used in the calculations.

C. Total dk/dT

With the exception of the direct calculation based on Monte Carlo techniques, there are three methods by which one can calculate the Doppler reactivity effect of a given system. They are:²

- (1) successive k calculation using reaction cross sections at two different temperatures;
- (2) perturbation method using the change in the group reaction cross section ($\delta\Sigma_f$ and $\delta\Sigma_a$);
- (3) perturbation method using the temperature derivative of the group reaction cross sections $\left(\frac{\partial\Sigma_f}{\partial T} \text{ and } \frac{\partial\Sigma_a}{\partial T}\right)$.

The first two methods give the change in reactivity, δk , as the temperature changes from an initial temperature T_1 to a final temperature T_2 , whereas the third method directly gives the Doppler coefficient $\partial k/\partial T$ at a given temperature. Studies of the Doppler effect at ANL up to the present time have been based exclusively on the first two methods.

The third method has been generally used at KFK. In order to compare directly the calculations done at ANL and KFK, one needs to convert δk to dk/dT , or vice versa. This conversion requires an assumption regarding the temperature law. As in Ref. 2, the following relation was used:

$$\frac{dk}{dT} = \frac{C}{T^x}, \quad (1)$$

where both free parameters C and x are assumed to be temperature independent. Integrating Eq. (1), one has

$$\begin{aligned} (\delta k)_{T_1 \rightarrow T_2} &= \frac{C}{x-1} \left[\frac{1}{T_1^{x-1}} - \frac{1}{T_2^{x-1}} \right] \quad \text{for } x \neq 1 \\ &= C \ln \frac{T_2}{T_1} \quad \text{for } x = 1. \end{aligned} \quad (2)$$

From a knowledge of $(\delta k)_{T_1 \rightarrow T_2}$ and $(\delta k)_{T_2 \rightarrow T_3}$, the determination of dk/dT can be made quite readily. In particular, x is determined by the following transcendental equation:

$$\frac{\delta k_{T_1 \rightarrow T_2}}{\delta k_{T_2 \rightarrow T_3}} = \frac{T_2^{1-x} - T_1^{1-x}}{T_3^{1-x} - T_2^{1-x}} \quad (x \neq 1). \quad (3)$$

A computer program has been written at ANL in order to determine the parameter x in Eq. (3).

Tables III through V sum the numerical results obtained at ANL and KFK, respectively, using group constants of three different origins. Both the accidental and the self-overlap effect of resonance of fissile and fertile isotopes were taken into account in all three calculations. Table III shows δk 's calculated at ANL by use of method 2. Since the group flux and the adjoint flux are sensitive to the temperature in the low-energy groups, the first-order perturbation theory may introduce errors. Instead of being obtained by the method of Henry,¹¹ δk 's given here are simply the averages of the values in which the perturbed and unperturbed group fluxes and adjoint fluxes were used, respectively. Comparison of this result with successive k calculations showed no significant difference in all cases considered. Tables III and IV show values of dk/dT at 300 and 900°K, respectively, for four different systems. Excellent agreement between ANL and KFK values was observed for all cases considered. ABN-KFK values are generally higher than both ANL and KFK values.

TABLE III. δk 's Calculated at ANL

Cases	y	δk (750-300°K)	δk (1100-750°K)
1	9.0	-0.01494	-0.006052
2	7.0	-0.009578	-0.003905
3	5.0	-0.005073	-0.002022
4	2.5	-0.001092	-0.000406

TABLE IV. $(dk/dT)_{900^\circ K} \times 10^6$

Cases	y	ANL	KFK	ABN-KFK
1	9.0	-17.13	-16.59	-18.89
2	7.0	-10.99	-10.89	-12.63
3	5.0	-5.82	-5.83	-6.92
4	2.5	-1.18	-1.26	-1.60

TABLE V. $(dk/dT)_{300^\circ K} \times 10^6$

Cases	y	ANL	KFK ^a	ABN-KFK
1	9.0	-54.92	-53.30	-61.78
2	7.0	-35.21	-35.70	-42.58
3	5.0	-19.25	-19.80	-24.37
4	2.5	-4.37	-4.55	-6.24

^aNo KFK values are available. Values listed here are calculated from KFK values at 900°K and the x -dashed curve of Fig. 3 in Ref. 2.

D. x Values

The quantity x is a measure of the temperature behavior of the Doppler coefficient. It has been shown in Ref. 2 that the variations of x values calculated at various laboratories are generally in poor agreement. The preliminary results calculated recently by Gitter and Segev¹³ show even greater discrepancies than those discussed

previously. A small error in either the Doppler-broadened cross sections or the Doppler reactivity change may significantly affect the x values. In other words, good agreement in δk or dk/dT at a given temperature may not insure that x values will agree. The poor agreement in x values implies that the calculated Doppler coefficient for extremely high temperature will be quite different.²

Figure 7 shows the variations of x values as the function of the fertile-to-fissile ratio y . Agreement in x values between ANL and KFK

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Fig. 7. x versus Fertile-to-fissile Ratio

is generally good, but certainly not as good as those in dk/dT . For comparison, x values calculated from ABN spectra and ABN reaction cross sections were also included in Fig. 7. Significant discrepancies occur not only in the magnitude but also in the general behavior of x values when ABN results are considered. If one assumes that two independent calculations give identical dk/dT at room temperature, a discrepancy of 0.5 in the x value implies a deviation in dk/dT at 3000°K by a factor of $(10)^{0.5}$ or 3.16. It is interesting to note that x is also extremely sensitive to small effects such as p-wave contributions and the overlap effect of U^{238} resonances. These effects will not only significantly change the magnitude of x , but also change the slope of x with respect to y , as indicated in Fig. 7. It is believed that the difference in the treatment of these two effects may be partially responsible for the large discrepancies in x values given in Ref. 2. Furthermore, the difference in Pu^{239} resonance parameters, which determines the relative contribution of the positive component of the Doppler coefficient, may also cause discrepancies in x values. To clarify the general behavior of the x value, a detailed discussion will be given in the following.

It is well known that the temperature derivative of the reaction cross section of particular isotope assumes its asymptotic distributions of $T^{-1/2}$ and $T^{-3/2}$ in the low- and high-energy regions, respectively. Therefore, one expects a variation of x between these two asymptotic limits in the intermediate-energy region. However, there are two principal effects which may make x value exceed these two limits:*

- (1) strong compensation of the negative and positive components of the system, e.g., U^{238} and Pu^{239} ;

*Other effects, such as the multilevel effect and the crystalline binding effect, that may also significantly affect the temperature behavior are not discussed here.

(2) overlap effect in the energy region where resonances are neither strongly overlapping nor completely isolated.

These two factors are discussed separately as follows.

Assume that the total Doppler coefficient is given by

$$\frac{dk}{dT} = \frac{C}{Tx} = \frac{C_1}{Tw} - \frac{C_2}{Tz}, \quad (4)$$

where C_1/T^w and C_2/T^z are the negative and positive components, respectively. For large fast reactors, the latter quantity is always smaller than the former and the value of z is nearly always equal to its asymptotic value of $3/2$ beyond about 1 keV. The value of x is not only dependent on the relative contribution of these two components, but also dependent on the relative magnitudes of w and z . For the case $w < z$, the larger is the positive contribution the smaller x becomes. On the other hand, the reverse is true when $w > z$. For this reason the discrepancies in the Pu^{239} Doppler contribution alone could account for the discrepancies in dk/dT of two different origins at various temperatures even if the flux and adjoint flux were identical.

Another factor that may significantly affect the value of x is the overlap effect of U^{238} resonances in the intermediate-energy region between about 10 to 50 keV. Even though the inclusion of the overlap effect of U^{238} in the intermediate region has very little effect on the total dk/dT at low temperatures, the effect on the x values is noticeable, as shown in Fig. 7. The reason is that the contribution of the overlap terms (with the

112-7568

Fig. 8. First-order Overlap Effect

first-order term equal to $e^{(s)}$) is a sharply increasing function of temperature in the intermediate-energy region, while the contribution of nonoverlap terms (with the first-order term equal to $E^{(s)}$) is relatively insensitive to temperature. Figure 8 shows the variation of $e^{(s)}$ versus the ratio of Doppler width and average level spacing taken from Ref. 12. The result of combining overlap and nonoverlap terms gives rise to a sharp increase of x value in the intermediate-energy region. It is in this intermediate-energy region that x may exceed its asymptotic value of $3/2$. The value of x rapidly approaches its asymptotic value of $3/2$ in the

112-7570

Fig. 9. Variation of x versus Energy for $y = 9.0$

because larger discrepancies in the "localized" values of $\partial k(u)/\partial T$ might compensate one another over the large energy span under consideration. In order to investigate the question one step further, a detailed study of the calculated Doppler coefficients has been carried out on the group-wise basis.

Two extreme cases, with $y = 2.5$ and $y = 9.0$, respectively, were examined. Since the multigroup energy boundaries used at ANL and KFK are entirely different, the following comparison is made on $\partial k(u)/\partial T$ per group lethargy width. Figures 10 and 11 show the variations of $(\partial k(u)/\partial T)_{900^\circ K}$ per unit lethargy for $y = 9.0$ and 2.5 , respectively. The total dk/dT is equal to the area under the histogram. The agreement between ANL and KFK results is exceedingly good for all energy groups. Small discrepancies in each energy group

112-7510

Fig. 10. Group-wise Doppler Coefficient at $900^\circ K$ for $y = 9.0$

high-energy region, where the Doppler width becomes comparable to or greater than the average spacing. Figure 9 shows the variation of x values for $y = 9.0$ as calculated at ANL. It is seen that x reaches its maximum value near 25 keV for this particular case.

$$E. \quad \frac{\partial k(u, T)}{\partial T}$$

The excellent agreement between ANL and KFK results in total dk/dT could be somewhat deceiving

112-7511

Fig. 11. Group-wise Doppler Coefficient at $900^\circ K$ for $y = 2.5$

may be attributed to the differences in the localized flux and adjoint flux, and also in the relative magnitude of Pu^{239} contribution. Relatively larger discrepancies were found when ABN-KFK values were considered. The deviations of ABN-KFK from both ANL and KFK values are considerably higher for the case of $\gamma = 2.5$.

Figures 12 and 13 shows the variations of $\partial k(u)/\partial T$ at 300°K . For this temperature, KFK values are not available. Again much larger discrepancies between ANL and ABN-KFK were found for the case $\gamma = 2.5$ than for $\gamma = 9.0$. The difference in x is responsible for such discrepancies. Figures 10 through 13 seem to suggest that the Pu^{239} component of $\partial k(u)/\partial T$ in ANL values may have been higher than KFK values below 1 keV. Better agreement would be possible had the same set of plutonium parameters been used in the calculations.

112-7576

Fig. 12. Group-wise Doppler Coefficient at 300°K for $\gamma = 9.0$

112-7575

Fig. 13. Group-wise Doppler Coefficient at 300°K for $\gamma = 2.5$

It is interesting to note from Figs. 10 through 13 that the relative contribution of $\partial k(u)/\partial T$ is significantly shifted toward the energy region below the sodium resonance, when the temperature is raised. From Fig. 9, it is not difficult to see that the sharp increase of x in the high-energy region is responsible for the shift. The shift in the relative importance of $\partial k(u)/\partial T$ at high temperature is extremely important in fast reactor safety studies. For the case of complete loss of sodium in the event of power excursion, the spectrum is also hardened and shifted toward the high-energy region. The combination of the spectral shift and the shift in $\partial k(u)/\partial T$ caused by temperature rise will significantly reduce

the absolute magnitude of the Doppler effect. Such an effect is particularly important for the large systems in which the sodium-void coefficients are expected to be positive due to the relatively small leakage components. Figure 14 shows the variations of spectrum for the no-sodium case as compared to the full-sodium case for $\gamma = 9.0$. Figure 15 shows the variations of $\partial k(u,T)/\partial T$ at $T = 300$ and 900°K for $\gamma = 9.0$ under the assumption of complete loss of sodium as calculated at ANL. The total Doppler coefficients were found to be 29.36×10^{-6} and 7.86×10^{-6} for $T = 300$ and 900°K , respectively. The corresponding α value became 1.2 as compared to 1.05 for the same system with full sodium. It is quite clear that the incorrect prediction of the temperature behavior of dk/dT may lead to significant error in the magnitude of the Doppler coefficient at high temperature, especially when the loss of sodium is considered.

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Fig. 14. Spectral Variation for Cases with and without Sodium

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Fig. 15. Group-wise Doppler Coefficient at Two Different Temperatures with Complete Loss of Sodium

F. Conclusions

From the results discussed in the preceding section, one is led to the following conclusions:

1. Excellent agreement between ANL and KFK results was observed in:
 - (a) criticality calculations;
 - (b) spectra;

(c) total $dk(T)/dT$;

(d) temperature dependence of dk/dT (in terms of x values)

for all systems under consideration. The maximum discrepancies in the critical bucklings and the total dk/dT for all systems under consideration are 8.4 and 6.5%, respectively. The agreement in the spectra calculations are equally good except for the energy region at and near the sodium resonance, due to the difference in the treatment of resonance shielding effect for light scatterers.

2. Reasonably good agreement between ANL and KFK results were found also in group-wise $dk(u,T)/dT$ for all systems. Small discrepancies in x and $dk(u,T)/dT$ were believed to be attributed to the difference in Pu^{239} contributions and the "localized" difference in flux and adjoint flux. Better agreement might be possible if the same Pu^{239} resonance parameters had been used.

For two completely independent calculations of this type, the agreement between all ANL and KFK results, in general, can be considered to be exceedingly good. The authors are inclined to believe that such good agreement is not at all coincidental, but rather is a consequence from the use of the well-accepted data and well-developed computational techniques. In our opinion, the discrepancies in the previous comparisons may be caused by the use of older data or less accurate computational methods in preparing group constants, as well as in the treatment of the Doppler broadening of the reaction cross sections. It is important to realize, however, that the excellent agreement between ANL and KFK results does not imply the correctness of these values when compared to the true Doppler coefficients. The correctness of the calculated values depends, to a great extent, upon the advancement of our present knowledge in nuclear data and further improvement of our existing computational techniques.

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