355
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MASTER

DCX-2 PERFORMANCE CURVES

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T. K. Fowler

OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

for the

U.S. ATOMIC ENERGY COMMISSION

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DCX-2 PERFORMANCE CURVES

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ABSTRACT

This report gives performance curves of steady state ion density versus injected currents for a survey of various parameters. The parameters are E_M , the energy of injected H_2^+ ions; L , the ion path length; and factors h and Γ , where h/Γ is the fraction of the undissociated portion of the molecular beam not absorbed and Γ is taken as 2. Performance curves are also computed assuming the energy transfer rates to be enhanced by arbitrary factors of 10 and 100.

Analyses of the variation of steady state ion density with injected current for gas break-up trapping devices have previously been given by Simon¹ and Mackin.² A more recent paper by Fowler and Rankin³ (herein referred to as F-R) has made use of ion energy distributions obtained as solutions of the Fokker-Planck equations to provide proper averages of the products of velocity with ion loss and ion destruction cross sections ($\overline{\sigma_\ell v}$ and $\overline{\sigma_d v}$) assumed constant in the former papers. These averages (which are functions of α , the fraction of "burnout" of the neutrals)

¹A. Simon, J. of Nuclear Energy: Part C, 1(4), 215 (July 1960).

²R. J. Mackin, Jr., Nuclear Fusion 1(2), 131 (March 1961).

³T. K. Fowler and M. Rankin, Energy Distribution of Ions and Electrons in DCX After Burnout: Oracle Code Eddie, ORNL-3161 (August 29, 1961).

were then used with equations governing neutrals to give a DCX-2 performance curve for one particular set of parameters (Fig. 6 of F-R). This report extends the F-R report first by surveying $\overline{\sigma_\ell v}$ and $\overline{\sigma_d v}$ for various values of E_M ; the energy of injected H_2^+ ions; the ion path length, L ; and factors h and Γ , defined as follows.

Since molecular ions are a source of neutrals, it has been proposed that the undissociated portion of the beam be absorbed. h/Γ is the fraction not absorbed, which, together with dissociated ions, eventually result in the return to the system of neutral atoms, Γ in number per molecular ion.

We have also computed performance curves assuming energy transfer rates (E.T.R.) to be enhanced by an arbitrary factor β , chosen to be 10 and 100. This has been done to indicate the possible effect of collective phenomena not accounted for with our transfer rates taking into account binary collisions only. Having in mind mechanisms such as the Harris instability in which ion cyclotron motion couples to electron plasma oscillations, we have increased by the factor β only the ion-electron transfer rate, and the electron scattering rate appearing in the electron loss term.

The set of simultaneous equations whose solution (as a function of the parameter α) gives the performance curves are as follows:

$$(1) \quad hI + \theta_0 N_0 + (\Gamma - h) n_+^2 \overline{\sigma_\ell v} V = f(\hat{n}_0 - \alpha n_+) \frac{v_0}{\ell} V + \theta_0 \hat{n}_0$$

$$(2) \quad (\hat{n}_0 - \alpha n_+) \frac{v_0}{\ell} = \alpha n_+^2 \overline{\sigma_d v}$$

$$(3) \quad \frac{I}{V} \left\{ \frac{(\hat{n}_0 + n_+) \sigma_B L}{1 + (\hat{n}_0 + n_+) \sigma_B L} \right\} = n_+^2 \overline{\sigma_\ell v}$$

$$(4) \quad \bar{n}_0 = \alpha n_+$$

where $\bar{n}_0$ = average internal neutral gas (atom) density; $\hat{n}_0$ = average external neutral gas (atom) density; and N_0 = neutral density in absence of beam or plasma. These equations differ from those given in F-R only by the generalization of equation (1) by the introduction of h . We take $\Gamma = 2$.

A cubic equation, $An_+^3 + Bn_+^2 + Cn_+ + D = 0$, results from combining the four above equations. Here

$$A \equiv \alpha \frac{\bar{\sigma}_d v}{v_0} \frac{\ell}{\Gamma} \left(\frac{\alpha \bar{\sigma}_d v f}{\Gamma} + \frac{\alpha \theta_0 \bar{\sigma}_d v}{IV} \frac{\ell}{v_0} - \bar{\sigma}_d v \right)$$

$$B \equiv (\alpha+1) \left(\alpha \frac{\bar{\sigma}_d v f}{\Gamma} + \frac{\alpha \theta_0 \bar{\sigma}_d v}{IV} \frac{\ell}{v_0} - \bar{\sigma}_d v \right) + \alpha^2 \frac{\bar{\sigma}_d v}{v_0} \frac{\ell}{IV} \frac{\theta_0}{\Gamma}$$

$$C \equiv \alpha \frac{\bar{\sigma}_d v}{v_0} \frac{\ell}{\Gamma} \left(- \frac{\theta_0 N_0}{IV} \right) + (\alpha+1) \left(\frac{\alpha \theta_0}{IV} \right) - \frac{h}{\Gamma} \frac{\bar{\sigma}_d v}{\sigma_B L}$$

$$D \equiv (\alpha+1) \left(- \frac{\theta_0 N_0}{IV} \right)$$

Physical solutions of (1)-(4) are those with n_+ , $\hat{n}_0$, $\bar{n}_0$ and I positive. If α and n_+ are positive, then by (2)-(3) the other quantities are also positive. Thus, α having been defined positive, $n_+ > 0$ is sufficient for a meaningful solution. Now if coefficient A alone is positive, also $B > 0$, and since $D < 0$, there is one and only one positive root, n_+ . We have found that if $A < 0$, there is no positive solution.

The condition $A > 0$ places a lower limit on the burnout fraction α , approximately

$$(5) \quad \alpha_{\min} \sim \frac{\Gamma}{f} \frac{\bar{\sigma}_d v}{\sigma_B L}$$

This limitation on burnout can be understood as follows. In the limit of large current, the dissociation is complete, and the molecular beam

is the principal source of neutrals, all of which are pumped by the plasma. The neutrals are supplied at rate $(\Gamma/f)I$ and trapped ions are supplied at rate I , each proportional to I . Thus, $\alpha = \bar{n}_0/n_+$ is given by Γ/f times the reciprocal of the ratio of loss rates, which is just equation (5). Once this limit is reached, α remains the same for all greater I .

Points on the performance curves as a function of α were calculated by an IBM 7090 code. Two numerical difficulties may be mentioned. For small values of α approaching the limiting condition, degeneration of the cubic equation to a quadratic equation takes place with a loss in accuracy of the solution to the cubic. In this case, solutions are easily found from the quadratic equation. For small α and h , equation (1) is subject to errors in taking the difference of large, nearly equal, quantities and I is more reliably calculated from equation (3).

Figure 1 gives $\overline{\sigma_\ell v}$ and $\overline{\sigma_d v}$ as functions of α for $E_M = 200, 400, 600$, and 800 kev. Figure 2 gives $\overline{\sigma_\ell v}$ and $\overline{\sigma_d v}$ as functions of α for $E_M = 600$ kev and for the enhanced energy transfer rates of 10 and 100.

Performance curves are included as follows:

Figure 3: Variation of molecular ion energy for $L = 50$ m
and $h = 2$.

Figures 4-7: Variation of path length for $h = 2$ and $E_M = 200, 400, 600$, and 800 kev.

Figures 8-11: Variation of path length for $h = .001$ and $E_M = 200, 400, 600$, and 800 kev.

Figure 12: Variation of energy transfer rate for $h = 2$ and $E_M = 600$ kev.

The parameters $l/v_o = .1389 \times 10^{-3}$ sec, $f = .8$, $V = 10^5 \text{ cm}^3$, $\theta_o = 10^6 \frac{\text{cm}^3}{\text{sec}}$, and $N_o = 7 \times 10^9 (\frac{\#}{\text{cm}^3})$ are the same in all graphs.

For convenience in understanding the influence of various parameters, we give the following table of upper critical currents.

Table I. Critical Current (U.C.C.)

Variation With Injection Energy, E_M		Variation With Path Length, L	
For L = 50 m		For h = 2 and $E_M = 600$ kev	
E_M (kev)	U.C.C. (ma)	L (meters)	U.C.C. (ma)
800	19	100	20
600	73	50	73
400	920	20	420
200	29,000	10	1450
Variation With L, Undissociated Ions Absorbed		Variation With Arbitrary Increase in Energy Transfer Rate (E.T.R.) by Factor β	
For h = .001 and $E_M = 600$ kev		For h = 2, $E_M = 300$ kev, and L = 50 m	
L (meters)	U.C.C. (ma)	β	U.C.C. (ma)
100	3.3	1	73
50	6.8	10	330
20	17	100	1500
10	33		

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