The Pair-Field Susceptibility of Superconductors*

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Recent experimental^{1,2} and theoretical work^{3,4,5} has shown that the susceptibility associated with the superconducting transition can be determined in a simple d.c. tunneling experiment. Both Scalapino⁴ and Takayama⁵ have presented detailed calculations of an excess current due to pair tunneling in superconductor-insulator normal metal junctions. This current is a measure of the susceptibility in the normal metal. In the following we present the extent of agreement between experiment and theory and indicate areas where further work should be pursued.

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In Scalapino's calculation the part of the dc current arising from fluctuations of $\Delta(x)$ is given by

$$(1) \quad I_1(V, H) = \frac{4e|\bar{c}|^2 A}{d\hbar} \operatorname{Im} \chi(\omega, q)$$

$\bar{c}$ is an average coupling strength, d is the thickness of the normal metal which is taken to be a film of thickness less than the temperature dependent coherence length. The quantity A is the area of the junction perpendicular to the direction of current flow. $\operatorname{Im} \chi(\omega, q)$ is the imaginary part of the Fourier transform of the pair-field susceptibility which is a complicated expression involving the digamma function.

The quantities ω and q are determined by the dc voltage and the magnetic field through relations

$$(2) \quad q = \frac{2e}{\hbar} H \left(d/2 + \lambda' \right), \quad 2eV = \hbar\omega$$

λ' is the London penetration depth of the superconducting half of the junction. Values of the parameters V and H are always small enough to permit χ to be written as:

$$(3) \quad \chi^{-1}(\omega, q) = N(0) \epsilon \left[1 - \frac{i\omega}{\Gamma_0} + \xi^2(T) q^2 \right]$$

where $\Gamma_0 = \frac{8}{\pi} \frac{k_B T_c}{\hbar} \epsilon$, $\xi(T) = \xi(0) \epsilon^{-1/2}$ and $\epsilon = \frac{T - T_c}{T_c}$

are the pair relaxation frequency, coherence length, and the reduced temperature respectively, and $N(0)$ is the single-spin density of states.

The resulting dc current is

$$(4) \quad I_c(V, H) = \frac{4e |\bar{C}|^2 A}{d \kappa N(0) e} \frac{\omega / \Gamma_0}{[1 + \xi^2(T) q^2]^2 + (\omega / \Gamma_0)^2}$$

with $\bar{C}$ given by

$$(5) \quad \bar{C} = (R_N A)^{-1} \ln \left| \frac{4\Delta'}{\Delta} \right|$$

R_N is the reduced junction resistance such that in the normal state the current is $e^2 V (\kappa R_N)^{-1}$, Δ' and Δ are the energy gaps of the superconducting side of the junction and the normal side when in the superconducting state.

Takayama's results are very similar to Scalapino's, differing only in the constant $\bar{C}$ and in the details of the dependence of the current on the magnetic field. In equation (4) $q^2 \xi^2(T)$ is replaced by $\frac{q'^2 \xi^2(T)}{3}$ with

$$(6) \quad q' = \frac{2e}{\kappa} (d/2) H$$

It must be pointed out that neither calculation takes into account the thermal noise associated with the quasiparticle tunneling channel. Treatment of this feature might be expected to be essential for the

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description of the current-voltage characteristic in regions of negative dynamical resistance near T_c . A second complication which is not included has to do with the possibility of a proximity effect³ between the superconductor and the normal metal across the barrier. It has been suggested that the Josephson coupling energy might induce a steady equilibrium value for the real part of the order parameter in the normal metal above what ordinarily would be considered to be its transition temperature.

We have measured I-V characteristics of aluminum-aluminum oxide lead tunneling junctions biased from a constant current source. The details of sample preparation, characterization, and measurement have been described in detail elsewhere.² A major departure from previous practice was the fitting of the quasiparticle current-voltage characteristic⁶ to the leading term of:

$$(7) \quad I_g = \frac{2C_{NN}}{e} \Delta \sum_{n=1}^{\infty} (-1)^{n+1} K_1 \left(\frac{n \Delta}{k_B T} \right) \sinh \left(\frac{n e V}{k_B T} \right)$$

The fit was carried out over a range of voltages for which the pair current due to fluctuations was small. Equation 7 was then used to compute the quasiparticle background current at voltages in the region of interest.

With a minicomputer these calculations could be carried out "on-line" with excess current-voltage characteristics available immediately upon subtraction of the computed quasiparticle "background" current from the measured current. As in previously reported work, junction edges were all masked. Temperatures were determined using a calibrated germanium thermometer purchased from the CryoCal Corporation. As before, extensive

tests were carried out to determine the quality of the junctions.

In Figure 1 we show excess-current-voltage characteristics at several temperatures.

In Figure 2 we show the variation of the reciprocal of the square root of the dynamical conductance as a function of temperature. It should be noted that the curve is a linear function of temperature over almost 500 mK. These results can be taken as evidence that the dominant tunneling process involves specular transmission of electrons across the barrier rather than diffuse transmission. If the transmission were diffuse, then the reciprocal of the dynamical conductance rather than the square root of the reciprocal would vary linearly with temperature.^{5,7} Also shown in Figure 2 is the variation of the dc Josephson current with temperature below T_c . It should be noted that the data above T_c and that taken below T_c extrapolate to the same temperature if data taken in the immediate vicinity of T_c (1.800 to 1.815 K) are ignored. In this region there is a zero-voltage current (the detectable voltage signal is less than 0.5 nV). We do not know whether these results which are typical of junctions containing a disordered aluminum electrode are a manifestation of "critical behavior" or of the peculiar form of the proximity effect mentioned earlier. The results may also be the consequence of geometric inhomogeneities in the aluminum films. At temperatures just above those at which there is an apparent zero-voltage current there is an additional difficulty in obtaining the full excess-current-voltage characteristic as a negative dynamical resistance results in switching when the junction is current biased. In this range (from 15 mK to 40 mK above T_c) a model dependent

determination of the peak current can be made which is consistent with data taken at higher temperatures where there is no switching. The measured value of $\frac{dI_p^{-1}}{dT}$ is $1.57 \times 10^8 \text{ A}^{-1} \text{ K}^{-1}$. Values computed from references 4 and 5 are $2.00 \times 10^8 \text{ A}^{-1} \text{ K}^{-1}$ and $2.94 \times 10^9 \text{ A}^{-1} \text{ K}^{-1}$ respectively.⁸ These results suggested that the magnitude of the excess current which can be easily computed from $\frac{dI_p^{-1}}{dT}$ is only in qualitative agreement with theory. However, the calculation of $\frac{dI_p^{-1}}{dT}$ from theory is of limited applicability as it depends on values of parameters which are imprecisely known for dirty aluminum films. On the other hand, once the magnitude of the peak current is determined at a particular temperature, it is possible to show that the excess current-voltage characteristics at that temperature and at all others except in the immediate vicinity of T_c are consistent with the theory. The variation of the peak voltage with temperature which is not shown here is in excellent agreement with theory except in the immediate vicinity of T_c . This constitutes a determination of the pair relaxation frequency, a quantity which depends only on the transition temperature and fundamental constants.

The magnetic field dependences of excess current-voltage characteristic are shown in Figure 3 along with the field dependence of the inverse of the peak current. The excess current-voltage characteristics are quantitatively identical with the curves in Figure 1. The linear variation of I_p^{-1} with H^2 is consistent with equation (4). Although the experimental value of $\frac{dI_p^{-1}}{dH^2}$ appears to be in agreement with calculations based on reference 5, the agreement must be considered to be qualitative as the calculation depends on the magnitude of the coherence length which involves ℓ , the mean free path, a quantity known only qualitatively. The coherence length was calculated from $\xi(T) = 0.85(\xi_0 \ell)^{1/2} \epsilon^{01/2}$ with $\xi_0 = 16,000 \text{ \AA}$ and, ℓ computed from $\rho \ell = 1.6 \times 10^{-11} \Omega\text{-cm}$.⁹ The abrupt increase in

I_p^{-1} as a magnetic field is initially applied is not explained by theory and suggests that at finite q and ω that fluctuations are somewhat weaker than predicted.

In summary, the data, over the range of parameters investigated, except in the immediate vicinity of T_c appear to be quantitatively consistent with the theory. The difficulties appearing in the immediate vicinity of T_c require further experimental and theoretical study.

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FIGURE CAPTIONS

Fig. 1. Excess current-voltage characteristics of junction Al2-5. Curves a, b, and c correspond to $T = 1.84726$, 1.93276 and 2.09892 K, respectively. The solid line is the theoretical excess current obtained by fitting the theory to the experimental values of the peak current I_p and the peak voltage V_p . The deviation of experiment and theory at high voltages can be removed by a model dependent analysis which corrects the quasiparticle current at high voltages for a small pair current which would be present.

Fig. 2. The center figure shows $\left(\frac{dI}{dV}\right)^{-1/2}_{V=0}$ vs. T for junction Al2-5 over the entire temperature range covered. The left insert shows in detail the departure close to T_c of the curve from a linear temperature dependence. The right insert shows the behavior of the maximum zero voltage current near the transition temperature of $1.801 \pm .001$ K.

Fig. 3. The left figure shows excess current-voltage characteristics in a magnetic field at a constant temperature $T = 1.86015$ K. The curves labelled by o, x, $\square$, + correspond to 0, 40.8 81.3, 122 Oe respectively. The right insert is I_p^{-1} vs. H^2 at $T = 1.85202$ K (upper curve), and $T = 1.84726$ K (lower curve.)

Slopes $\frac{dI_p^{-1}}{dH^2}$ determined from the data are $.845 \times 10^3 \text{ A}^{-1} \text{ Oe}^{-2}$ (upper)

and $.797 \times 10^3 \text{ A}^{-1} \text{ Oe}^{-2}$ (lower) respectively. Using reference 4 we compute slopes of $8.47 \times 10^3 \text{ A}^{-1} \text{ Oe}^{-2}$ and $8.64 \times 10^3 \text{ A}^{-1} \text{ Oe}^{-2}$. From reference 5 the slopes are 0.87×10^3 and 0.88×10^3 .

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