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SOME PRELIMINARY CONSIDERATIONS TO THE NEUTRON ELECTRIC DIPOLE
MOMENT EXPERIMENT AT ORNL: INCLUDING A SUMMARY OF SYMMETRY
PRINCIPLES FOR ELEMENTARY PARTICLES

W. B. Dress

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Physics Division

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W. B. Dress *

ABSTRACT

The first part (section *1 through *3) review the discrete symmetries of particle physics and briefly discuss some of the more important experiments establishing the violations of these symmetries. Section *4 briefly suggests the need for a new measurement of the electric-dipole moment of the neutron. Section *5 is concerned with the neutron spectrum obtained from a bent neutron conducting tube as a function of various experimental parameters. In section *6, the transition probability for a split coil magnetic resonance spectrometer is derived; this probability is then averaged over the neutron spectrum obtained in *5. In sections *7 and *8, the design problems of the required electric and magnetic fields are considered in detail.

* Presently an Oak Ridge Graduate Fellow from Harvard University under appointment from Oak Ridge Associated Universities (research originated while a National Science Foundation Cooperative Fellow from Harvard University).

*1. SYMMETRIES

An obvious symmetry of classical electromagnetic theory is its invariance under the change in the sign of the charge -- this is so taken for granted in courses in electromagnetism, that it is rarely mentioned. At times, however, a consequence of this symmetry is emphasized: namely that Maxwell's equations hold for both signs of the charge. (An empiricist may here point out that the symmetry is a consequence of the charge independence of Maxwell's equations.)

In order to study the consequences of charge conjugation in quantum mechanics, we postulate a unitary operator, C , which changes the sign of the electric charge:

$$C \psi(Q, \vec{r}, \vec{s}) = \psi(-Q, \vec{r}, \vec{s})$$

where Q is the charge, $\vec{r}$ the space coordinates, and $\vec{s}$ the spin. For the electromagnetic interaction to be invariant under the operation C (this is required by correspondence to the classical Maxwell equations) we must have

$$j^\mu A_\mu \xrightarrow{C} j^\mu A_\mu$$

Thus $A_\mu \xrightarrow{C} -A_\mu$ since $j^\mu \xrightarrow{C} -j^\mu$.

An expansion of the potential in terms of creation and annihilation operators leads to the result

$$CaC^{-1} = -a \quad \text{and} \quad Ca^\dagger C^{-1} = -a^\dagger$$

as well as the usual commutation relations.

So for an n-photon state,

$$C|n\rangle = C \prod_1 a^\dagger(k_1, \lambda_1) |0\rangle = C a^\dagger(1) C^{-1} a^\dagger(2) \dots a^\dagger(n) C^{-1} C |0\rangle = (-)^n |n\rangle$$

if the vacuum state is assumed even under the operation C. If the invariance of the Hamiltonian under C is assumed, then this result is equivalent¹ to Furry's theorem.² This may be illustrated by the gamma decay of the neutral pion:

$$\pi^0 \rightarrow n \gamma.$$

Now the process $\pi^0 \rightarrow 2\gamma$ is known to occur,³ thus the decay $\pi^0 \rightarrow 3\gamma$ is strictly forbidden if the interaction is invariant under charge conjugation, and not just suppressed by a factor of $\alpha = 1/137$. This has recently been verified by Cline and Dowd (Phys. Rev. Letters 14, 530 (1965)). They looked for the decay at rest of

$$K^+ \rightarrow \pi^+ + \pi^0$$

in the Lawrence Radiation Laboratory heavy-liquid bubble chamber. They searched film containing about two hundred thousand $K^+ 2\pi$ decays, and obtained only 74 events where three electron-positron pairs pointed towards the decay vertex. These events were analyzed carefully, and it was concluded that they were either background or did not satisfy the $K^+ 2\pi$ decay kinematics. Their results expressed as an upper limit on the 3γ decay mode give

$$\frac{\pi^0 \rightarrow 3\gamma}{\pi^0 \rightarrow 2\gamma} < 3.8 \times 10^{-4}$$

¹L. Michel, Nuovo cimento 10, 319 (1953).

²W. F. Furry, Phys. Rev. 51, 125 (1937).

³Panofsky et al., Phys. Rev. 80, 94 (L) (1950).

which is about 20 times smaller than it would be if C were not conserved in the decay.

The symmetry operation most closely related to a mirror (or the "layman's mirror" as Wigner calls it in his recent Scientific American article) is that of spatial inversion of the coordinates through a point; i.e., a polar vector such as $\vec{r}$ is transformed into $-\vec{r}$ under the parity operation. The mirror operation itself is a combination of the parity operation and a rotation of π degrees about the normal to the plane of the mirror. The first and most obvious result of parity conservation is Laporte's Rule which states that dipole transitions can take place only between states such that $\Delta l = 1$. This follows from the fact that the spherical harmonics are eigenstates of parity with eigenvalues $(-)^l$. Another consequence of parity conservation is that nuclei cannot possess odd electric moments and even magnetic moments.⁴

A concept foreign to classical physics, and not usually introduced in atomic physics, is intrinsic parity. As an example, consider the reaction

A meaningful question is: what is the parity of the π^- with respect to the deuteron parity? Since the capture takes place from an atomic state of deuterium,⁵ conservation of angular momentum, and Fermi statistics for the neutrons imply that the final state is a 3P_1 . Thus if parity is conserved in the interaction, the parity of the π^- relative to the deuteron is negative.⁶ Now the parity of the π^0 meson was determined

⁴N. F. Ramsey, Molecular Beams, Oxford University Press, 1956, p. 58.

⁵Doede et al., Phys. Rev. 129, 2808 (1963).

⁶Panofsky, Aamodt, and Hadley, Phys. Rev. 81, 565 (1951).

to be odd with respect to the vacuum in the decay $\pi^0 \rightarrow 2\gamma$,⁷ thus, if the pion is really a charge triplet, the parity of the π^- with respect to the vacuum is odd, and hence the nucleon parity is even.

A less obvious result of the conservation of parity comes from Dirac theory. The Dirac equation is

$$(\gamma_\mu \partial_\mu + m) \psi = 0.$$

Under parity, $\vec{x} \rightarrow -\vec{x}$, and $\psi \rightarrow S\psi$, where S is a unitary operator. Thus

$$(-\gamma_k \partial'_k + \gamma_4 \partial_4 + m) S^\dagger \psi' = 0$$

so multiplying on the left by S , we find

$$S\gamma_k S^\dagger = -\gamma_k$$

$$S\gamma_4 S^\dagger = \gamma_4$$

Thus the parity operator in Dirac theory may be written as

$$P = \eta \gamma_4$$

where $|\eta| = 1$. In particular, consider a particle at rest

$$\psi(E > 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{P} \eta \gamma_4 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \eta \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$\psi(E < 0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{P} \eta \gamma_4 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\eta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

⁷Plano et al., Phys. Rev. Letters 3, 525 (1959).

Thus the parity of a particle is opposite to the parity of its anti-particle for spin $1/2$.

A prediction based on this result is that the parity of the 2γ wavefunction in electron-positron decay must be odd, and therefore the wavefunction must be of the form $\vec{k} \cdot (\vec{e}_1 \times \vec{e}_2)$ where $\vec{e}$ is the photon polarization vector and $\vec{k}$ the relative momentum of the two photons. Now the general wavefunction is of the form

$$\psi = \alpha \vec{e}_1 \cdot \vec{e}_2 + \beta \vec{k} \cdot (\vec{e}_1 \times \vec{e}_2)$$

so if parity is conserved and the above result is assumed, $\vec{e}_1$ will be perpendicular to $\vec{e}_2$. This prediction was confirmed in 1950 by Wu and Shakhnov (Phys. Rev. 77, 136 (1950)).

Time reversal as a mirror seems to be alien to everyday concepts as judged by its frequent appearance in motion pictures when a startling effect is desired. In classical mechanics, however, the motion of a system is symmetric under the operation $t \rightarrow -t$ if the Lagrangian is quadratic in the velocities. In general, the physical properties of a system are assumed to be conserved under a transformation. This requires for an operator T that

$$|\langle u' | v' \rangle|^2 = |\langle Tu | Tv \rangle|^2 = |\langle u | T^\dagger T | v \rangle|^2 = |\langle u | v \rangle|^2$$

so that

$$T^\dagger T = 1.$$

However it is permissible to have either

$$\langle Tu | Tv \rangle = \langle u | v \rangle$$

or

$$\langle Tu | Tv \rangle = \langle v | u \rangle = \langle u | v \rangle^*.$$

In the second case T is called antiunitary, a term due to Wigner,⁸ who showed that the time-reversal operator in quantum mechanics must be of this nature.

Since the spin vector $\vec{S}$ transforms to $-\vec{S}$ under time reversal, and the linear momentum also anticommutes with T , a representation for the operation of time reversal is complex conjugation followed by a rotation of π degrees about the y axis in spin space in the representation in which S_z is the observable:⁹

$$T = e^{-i\pi S_y} K.$$

Thus we must have

$$T^2 = \left(e^{-i\pi S_y} K \right)^2 = e^{-2\pi i S_y} K^2 = e^{-2\pi i S_y}$$

since $S_y^* = -S_y$. Thus $T^2 = 1$ for integral spin and -1 for half integral spin. A consequence of this result is Kramers degeneracy which says that if the Hamiltonian is invariant under T , then an electron with spin $1/2$ has at least a two-fold degeneracy since $T^2 = -1$.

In the β -decay of the neutron, one can construct an observable which is odd under time reversal. This is

$$\langle \vec{\sigma}_n \rangle \cdot (\vec{P}_e \times \vec{P}_\nu)$$

which is equal to¹⁰

⁸E. P. Wigner, Goettinger Nachrichten 31, 546 (1932).

⁹A. Messiah, Quantum Mechanics, Wiley and Sons, New York, 1962, p. 670.

¹⁰S. J. Sakurai, Invariance Principles and Elementary Particles, Princeton University Press, 1964, p. 107.

$$-\vec{P}_{\text{recoil}} \cdot (\langle \vec{\sigma}_n \rangle \times \vec{P}_e)$$

and is non-zero if there are more recoil protons on one side or the other of the plane with normal $\langle \vec{\sigma}_n \rangle \times \vec{P}_e$. Experimentally, it was found that the coefficient of the above term is 0.04 ± 0.05 which indicates that the decay is symmetrical under time reversal.¹¹

At this time, the most important result of time-reversal invariance (in the light of possible non-invariance under parity) is a selection rule forbidding an electric dipole moment interaction of an elementary particle. This result is obtained by noting that the interaction of an electric dipole with an external electric field is proportional to $\vec{S} \cdot \vec{E}$, but $\vec{S}$ is odd and $\vec{E}$ is even under time reversal; so the proportionality constant must be zero.

True to the prognostication of a footnote,¹² the combination of the mirrors of parity and charge conjugation (called the CP mirror by Wigner)

¹¹Burgy et al., Phys. Rev. 120, 1829 (1960).

¹²Wick, Wightman, and Wigner, Phys. Rev. 88, 101 (1952)

The footnote mentioned is on page 104, and reads:

"It is, of course, true that if, in addition to the inversion I as it is usually understood, (i.e., such that the sign of the charges is preserved, and E is polar) one believes in the operation of charge conjugation C as an exact symmetry property of nature it becomes arbitrary whether one regards I or CI as the inversion law. Adopting CI, however, none of the states of atoms or nuclei, that one considers normally, would be a state of definite parity (states of definite parity would involve superpositions of states containing protons and anti-protons, etc.). The definition thus is not a convenient one to adopt. That C is an exact symmetry property is moreover still far from proved. The disturbing possibility remains that C and I are both only approximate and CI is the only exact symmetry law. This would force us to regard the electric field as an axial vector. This possibility, however, seems rather remote at the moment."

has been a most fruitful one. The most far-reaching consequence of CP invariance is the "behavior of neutral particles under"¹³ the combined inversion. As an example, consider the K meson. Strange mesons with mass around 500 MeV were known to exist with one positive charge, one negative charge, and in the neutral state. However, in order that the production of strange particles be charge independent, the isotopic spin of the K meson must be one half. Thus the K is an iso-doublet, K^+ and K^0 ; with antiparticles K^- and $\bar{K}^0$. Since the strangeness of K^0 and $\bar{K}^0$ are different, it is reasonable to assume that they are not the same particle. We then define

$$|K_1^0\rangle = (|K^0\rangle + CP|K^0\rangle)/\sqrt{2}$$

$$|K_2^0\rangle = (|K^0\rangle - CP|K^0\rangle)/\sqrt{2}$$

where the $|K^0\rangle$ and $|\bar{K}^0\rangle$ are eigenstates of strangeness. By choosing an appropriate phase factor for $|K^0\rangle$, $|K_1^0\rangle$ and $|K_2^0\rangle$ are eigenstates of CP. Thus, since the production process conserves strangeness, the creation of a K^0 (or a $\bar{K}^0$) meson corresponds to the creation of a K_1^0 or a K_2^0 meson with equal probability. The illuminating characteristic of defining a K_1^0 and a K_2^0 in this manner appears when the decay of the neutral meson is examined. Since strangeness is not conserved in strange particle decay, and if CP invariance is assumed, the physical particles which decay are the K_1^0 and the K_2^0 and not the K^0 and $\bar{K}^0$. Now K_1^0 and K_2^0 are eigenstates of CP with eigenvalues plus and minus one, so their decay

¹³M. Gell-Mann and A. Pais, Behavior of Neutral Particles Under Charge Conjugation, Phys. Rev. 97, 1387 (1955). This paper was written before the violation of charge conjugation was discovered, so the symmetry C was assumed instead of CP which is used above.

modes as well as lifetimes and masses will be different. This prediction was confirmed by establishing the existence of a long-lived neutral K meson whose decay products were different than those of the short-lived meson.¹⁴

Another prediction based on CP invariance is that elementary particles cannot possess electric dipole moments. The argument is similar to the one for T invariance in that the spin is odd with respect to CP (since the spin is even under P and is "odd with respect to charge" while the electric field is even under both C and P.¹⁵ Thus the interaction would change sign upon CP inversion.

The final mirror considered is the combination CPT. Invariance under a CPT reflection can be demonstrated for all field theories which assume:¹⁶

1. Invariance under proper Lorentz transformations,
2. Interaction densities are local and constructed out of field operators and derivatives of the field operators,
3. "Normal" spin-statistics relation,
4. Interaction densities are properly symmetrized or antisymmetrized,
5. Interaction densities are Hermitian.

The most obvious consequences of CPT invariance are the equality of masses

¹⁴Landé et al., Phys. Rev. 103, 1901 (1956).

Bardou et al., Annals of Physics 5, 156 (1958).

The second reference is a comprehensive report on the cloud chamber experiments on the neutral long-lived K meson at Columbia University, and includes a review of predictions based on the above particle-mixture theory as well as experimental confirmations.

¹⁵L. Landau, Nuclear Physics 3, 127 (1957).

¹⁶J. J. Sakurai, op. cit., p. 142.

and lifetimes of particles and antiparticles.¹⁷ For example, consider a particle state at rest, then if the Hamiltonian is invariant under CPT

$$\begin{aligned}
 m &= \langle \psi | m | \psi \rangle = \langle \psi | H | \psi \rangle = \langle T\psi | THT^\dagger | T\psi \rangle = \langle \psi | THT^\dagger | \psi \rangle \\
 &= \langle CP\psi | CPT H T^\dagger P^\dagger C^\dagger | CP\psi \rangle = \langle \bar{\psi} | H | \bar{\psi} \rangle = \langle \bar{\psi} | \bar{m} | \bar{\psi} \rangle \\
 &= \bar{m}
 \end{aligned}$$

Which states that the mass of ψ is equal the mass of $\bar{\psi}$. These predictions have been verified by experiment in several cases:

1. $\frac{\text{Lifetime}(\pi^+ \rightarrow \mu^+ + \nu)}{\text{Lifetime}(\pi^- \rightarrow \mu^- + \bar{\nu})} = 1.004 \pm .007$ ¹⁸
2. $\frac{\text{Lifetime}(\mu^+)}{\text{Lifetime}(\mu^-)} = 1.000 \pm .001$ ¹⁹
3. $\frac{\text{Mass}(\pi^+)}{\text{Mass}(\pi^-)} = 1.0021 \pm .0027$ ²⁰

¹⁷See, for example, Lee, Yang, and Oehme, Phys. Rev. 106, 340 (1957) and Lüders and Zumino, Phys. Rev. 106, 385 (1957).

¹⁸Bardon et al., Phys. Rev. Letters 16, 775 (1966).

¹⁹Meyer et al., Phys. Rev. 132, 2693 (1963).

²⁰Crowe, Nuovo Cimento 5, 541 (1957).

*2. VIOLATED SYMMETRIES

The first indication that parity conservation might be violated in weak interactions was the so-called τ - θ puzzle: The equal masses and lifetimes indicate that τ and θ are the same particle. But τ^+ decays into three pions while θ^+ decays into two pions. In 1956, Lee and Yang (Phys. Rev. 104, 254 (1956)) suggested that the problem be resolved by assuming that parity is not necessarily conserved in the (weak) decay. They then proceeded to examine the consequences of this assumption in β -decay, hyperon decay, meson decay, and muon decay. As if in response to this suggestion, a flood of experiments confirming the non-conservation of parity appeared almost immediately. In the very next volume of the Physical Review, at least three such experiments were reported (the first two in adjacent letters).

Taking them in order, then, we find:

1. β -decay in Co^{60} : Wu et al. (Phys. Rev. 105, 1413 (1957))

It was found that $\langle \vec{\sigma} \cdot \vec{P}_e \rangle < 0$ or in the angular distribution,

$1 + \alpha P \cos \theta$ -- where the z-axis is defined by the cobalt spin and P is the polarization -- α was negative and definitely non-zero (they were not able to obtain an accurate value for α however).

2. Meson decay and muon decay: Garwin, Lederman, and Weinrich (Phys. Rev. 105, 1415 (1956)) and Friedman and Telegdi (Phys. Rev. 105, 1681 (1957))

If parity is not conserved in both the decays

$$\pi \rightarrow \mu + \nu \quad \text{and} \quad \mu \rightarrow e + \nu + \bar{\nu}$$

then the μ will be polarized if the π decays at rest. This follows from the fact that parity conservation implies that $\langle \vec{\sigma}_\mu \cdot \vec{P}_\mu \rangle = 0$.

Thus the decay of the π at rest will provide a source of polarized muons. Then the quantity $\langle \vec{\sigma}_\mu \cdot \vec{p}_e \rangle$ can be measured as above. The value of αP in the angular distribution was found to be $-1/3$ with accuracy of 10%.

3. Hyperon decay: Crawford et al. (Phys. Rev. 108, 1102 (1957)) and Eisler et al. (Phys. Rev. 108, 1353 (1957))

In the decay of the Λ^0 produced in the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$, consider the observable $\langle \vec{p}_\pi' \cdot (\vec{p}_\pi \times \vec{p}_\Lambda) \rangle$ where the prime refers to the decay pion in $\Lambda^0 \rightarrow N + \pi$. This quantity is odd under spatial inversion. The experiment is then to see if there are as many decay pions emitted above as below the plane defined by the cross product. Experimentally, more pions were emitted upward (in the direction of $\vec{p}_\pi \times \vec{p}$) which shows parity is not conserved in the decay.

4. β -decay of the free neutron: Burgy et al. (Phys. Rev. 120, 1829 (1960))

The Argonne group measured the parameters α , β , and γ in the free neutron decay where the decay probability is proportional to

$$1 + a \vec{p}_e \cdot \vec{p}_\nu + \alpha \langle \vec{\sigma}_n \rangle \cdot \vec{p}_e + \beta \langle \vec{\sigma}_n \rangle \cdot \vec{p}_\nu + \gamma \langle \vec{\sigma}_n \rangle \cdot (\vec{p}_e \times \vec{p}_\nu).$$

In the case of parity conservation, α and β should be zero, while γ is zero under time-reversal. They found $\alpha = -0.114 \pm 0.019$ and $\beta = 0.88 \pm 0.15$ which clearly shows parity non-invariance.

If T invariance is assumed, the CPT theorem then asserts that charge-conjugation invariance is also violated in the above processes. As an example not depending on the CPT theorem, consider the helicity of electrons

in the decay $\mu \rightarrow e + \nu + \bar{\nu}$. Assume the interaction is invariant under C, then the helicity of the positron in μ^+ decay must be the same as the helicity of the electron in the μ^- decay since the helicity does not change under charge conjugation ($\vec{\sigma} \rightarrow \vec{\sigma}$ and $\vec{p} \rightarrow \vec{p}$).¹ Experimentally, it is found that the helicity of the positron is about one and the helicity of the electron about minus one,² which shows the violation of charge conjugation.

Until just about two years ago, the invariance of all interactions under CP was accepted even though experiments were undertaken to set an upper limit on the CP non-conserving amplitude.³ However an experiment at Brookhaven⁴ proved a little too sensitive to be in the best interests of CP invariance: 45 ± 9 decays of the K_2^0 meson (the state odd under CP) were determined to be of the 2π mode which is forbidden by CP symmetry. The experiment analyzed charged particles emerging in coincidence from a beam of neutral K mesons (at a distance of some $300 K_1^0$ decay lengths from the point of production -- corresponding to about 19 meters at a momentum of 1.1 GeV/c) with respect to their momentum assuming them to be pions. If the two pions came from a two-body decay, the vector sum of their momenta should be in the direction of the K meson beam. This turns out to be a sensitive test since, in the mass range covering the K meson mass, the group found a strong peak in the graph of the number of events plotted against $\cos \theta$ in the range $.99995 < \cos \theta < 1.00000$. After disposing of

¹J. J. Sakurai, op. cit., p. 129.

²Culligan et al., Nature 180, 751 (1957).

³Abanshian et al., Phys. Rev. Letters 13, 243 (1964). The authors conclude with the remark that their experiment "places an upper limit on CP violation of 5×10^{-6} , with at least a suggestion that CP may actually be violated by this amount."

⁴Christenson, Cronin, Fitch, and Turlay, Phys. Rev. Letters 13, 138 (1964).

several possibilities which could conceivably lead to their results but not be a 2π decay of the K_2^0 meson, they give the branching ratio as

$$\frac{K_2^0 \rightarrow \pi^+ + \pi^-}{K_2^0 \rightarrow \text{all charged modes}} = (2.0 \pm 0.4) \times 10^{-3}$$

and conclude with "the K_2^0 meson is not a pure eigenstate of CP."

Several hypotheses have been put forth to explain the observed 2π mode within a framework of exact CP symmetry. One such hypothesis is that the K_1^0 mesons do not decay according to a simple exponential law; however M. V. Terent'ev (Soviet Physics Uspeki 8, 445 (1965)) shows how such an explanation is inadequate to account for the observed branching ratio. Another hypothesis put forth by Bernstein, Cabibbo, and Lee (Physics Letters 12, 146 (1964)) shows how such an asymmetry effect might arise from interaction with a new, long-ranged, and extremely weak vector field in which K^0 and $\bar{K}^0$ have opposite potential energies -- by coupling to the hypercharge, for example.⁵ This theory predicts that the parameter measuring the amount of CP violating amplitude vary as $\sqrt{1 - v^2/c^2}$.⁶ Thus for a K meson momentum of 10 Gev/c one expects the ratio to be about

$$2 \times 10^{-3} \times \frac{E(10 \text{ Gev/c})}{E(1.1 \text{ Gev/c})} \approx .17.$$

An experiment was performed with K momentum of 10 Gev/c by De Bouard et al. (Physics Letters 15, 58 (1965)) who report a branching ratio of $(3.5 \pm 1.4) \times 10^{-3}$, which shows that such a long-range weak force is not

⁵Bell and Perring, Phys. Rev. Letters, 13, 348 (1964).

⁶See the above mentioned article by Terent'ev which is a comprehensive discussion of the 2π decay considering both possibilities of CP violation as an explanation and examining theories which conserve CP.

present. Still another confirmation of the 2π decay which also showed that the weak long-ranged force is not present is by Galbraith et al. (Phys. Rev. Letters 14, 383 (1965)). They obtained a value of $R = (2.08 \pm 0.35) \times 10^{-3}$ at a momentum of 5 Gev/c.

*3. FURTHER DEVELOPMENTS

The surprising thing about the violations of the symmetries is that order, instead of chaos, has resulted -- thus far; and particle physics seems to be much richer because of the broken mirrors.

As an example, if we allow parity to be violated in weak interactions, the two-component theory of the neutrino becomes acceptable and leads to verifiable predictions which had not been made previously as well as introducing a useful order into the decays of leptons. For zero mass, the Dirac equation reduces to two decoupled equations -- one for positive-energy neutrinos (say) and one for negative-energy antineutrinos:

$$(\vec{\sigma} \cdot \vec{\nabla} + \partial_0) \nu = 0$$

$$(-\vec{\sigma} \cdot \vec{\nabla} + \partial_0) \bar{\nu} = 0$$

Now, since spins and momenta are opposite for particle and antiparticle,

$$\text{helicity } (\bar{\nu} \text{ with } E > 0) = \text{helicity } (\nu \text{ with } E < 0);$$

however, from the above equations we may write

$$\text{helicity } (\bar{\nu} \text{ with } E < 0) = - \text{helicity } (\nu \text{ with } E > 0)$$

with the result

$$\text{helicity } (\bar{\nu}) = - \text{helicity } (\nu)$$

So the neutrino and antineutrino are different particles if parity is

not conserved. This result can also explain the observed helicity of electrons in polarized muon decay (helicity of $e^\pm$ is about $\pm v/c$ experimentally.¹⁾ The observed magnitude of the helicity can be predicted by a Hamiltonian which is constructed to violate parity:²⁾

$$H_{\text{int}} = (\bar{p} \gamma_\mu n)(\bar{e} \gamma_\mu (C_V + C'_V \gamma_5) \nu) + \text{H.C.}$$

where C'_V is the parity-violating amplitude. The helicity of the electron based on this Hamiltonian turns out to be

$$- \frac{2 \operatorname{Re}(C_V C_V'^*)}{|C_V|^2 + |C_V'|^2} \frac{P_e}{E_e}$$

which agrees with experiment for $C_V = C'_V$.

Perhaps the most productive result of the violation of C and P in weak interactions is the hypothesis made by Landau (cited above) that CP is the exact symmetry law. This assumption has been quite useful and thought to be quite accurate (until 1964), however, new understanding from the supposed violation of CP in the 2π decay of the K_2^0 meson is for the future to determine.

¹Culligan et al., op. cit.

²J. J. Sakurai, op. cit., p. 76.

*4. A NEUTRON ELECTRIC DIPOLE MOMENT EXPERIMENT?

That part of elementary particle physics concerned with symmetry principles seems to have come to a halt with the publication of the paper by Christenson et al. in 1964. It is true that a flurry of new proposals, theories, ideas, and suggestions has flooded the journals since then, but one gets the impression that those concerned with the problem are most avidly awaiting new experimental results which confirm or deny time-reversal non-invariance or CP violation in a specific manner to a specific degree. One such experiment is in progress at Oak Ridge National Laboratory and is concerned with providing a new measurement of the electric dipole moment of the neutron. The motivation for this experiment is the above-mentioned paper of Christenson et al. and a number of theoretical calculations of the moment based on various models and assumptions. A summary of these calculations is given in a paper by Boulware (Nuovo Cimento, 40, 1041, 1965). The estimates range from 10^{-21} cm x e to 10^{-28} cm x e, where e is the charge on the electron. Those values above about 3×10^{-23} cm x e should be accessible to the experiment now under way.

In another paper, Feinberg (Phys. Rev., 140, B1402, 1965) considers C and T violations in electromagnetic interactions of hadrons and shows that nucleons should possess electric dipole moments of the order of 10^{-19} cm x e, which is comparable to the existing upper limit of 5×10^{-20} cm x e set by Smith, Purcell, and Ramsey in 1950 (Phys. Rev. 108, 120, 1957).

*5. Neutron Spectrum

The source of neutrons is the HB-2 beam hole of the Low Intensity Test Reactor (LITR). A neutron-conducting pipe conveys the neutrons through the reactor shield (about four meters thick). The tube terminates in a cavity filled with D₂O as a moderator. Scattering in the surrounding aluminum liner will probably raise the neutron temperature to about 100°C,¹ since the dimensions (about three inches) of the moderator are not larger than several neutron mean free paths.

The four meter neutron-conducting pipe is bent with a radius of curvature of 30 meters; thus the high-energy neutrons and gamma rays will pass through the wall of the pipe and be absorbed in the reactor shield. Just outside the shield, there is another bent section of neutron-conducting pipe - it is some 20 inches long and has a radius of curvature of one meter. This section acts as a low-pass filter to the neutrons, and determines the main characteristics of the neutron spectrum. Since the low-pass filter section has a radius of curvature smaller than that of the preceding section through the reactor shield, the neutrons exiting from it will not have been affected by the four meter section (except for reflectivity losses). Thus we consider a flux of neutrons of 3.0×10^{13} per cm² per second with a Maxwell-Boltzmann energy distribution at the entrance to the low-pass filter section.² Since this section is a bent pipe and not just a window, there is a certain admittance factor which depends upon the geometry of the section and the material of the reflecting surfaces.

¹P. D. Miller, A Low Pass Filter for Cold Neutrons to be Used in a Search for a Neutron Electric Dipole Moment, ORNL-TM-1149, p. 11, Oak Ridge National Laboratory, June, 1965.

²Ibid., p. 4.

A calculation similar to that done by Maier-Leibnitz and Springer (Reactor Science and Technology 17, 217 (1963)) gives³

$$\text{Admittance} = \frac{\rho h \gamma_c}{\pi} f$$

where

$$f = f_{<} = \sin \gamma_c - \gamma_c \cos \gamma_c - (1-d/\rho)^2 - \cos^2 \gamma_c \\ + \cos \gamma_c \cos^{-1} \left(\frac{\cos \gamma_c}{1-d/\rho} \right) \text{ for } E < E_c,$$

and

$$f = f_{>} = \sin \gamma_c - \gamma_c \cos \gamma_c \text{ for } E > E_c.$$

E is the neutron energy

ρ is the radius of curvature

h is the height of the neutron-conducting tube

d is the width of the neutron conducting-tube
(the curved surfaces are a distance d apart)

E_c is the energy such that $\cos \gamma_c = 1-d/\rho$

γ_c is the critical glancing angle defined by

$$\cos \gamma_c = \sqrt{1 - \frac{\lambda_{Na}^2}{\pi}}$$

= index of refraction.⁴

The spectrum of neutrons emerging from the low-pass filter is then

³Ibid., p. 7.

⁴M. L. Goldberger and F. Seitz, Phys. Rev. 71, 294 (1947).

$$\frac{dZ}{dE} = \frac{\rho h \gamma_c}{\pi} f \frac{I_0}{kT} \frac{E}{kT} e^{-E/kT}$$

where we have taken

$$T = 393^\circ\text{K}$$

$$\rho = 100 \text{ cm}$$

$$h = 10 \text{ cm}$$

$$d = 1 \text{ cm}$$

$$I_0 = 3.0 \times 10^{13}$$

The walls of the neutron-conducting pipe are nickel coated (coherent scattering length of nickel is: $a = + 1.03 \times 10^{-12} \text{ cm}$) -- thus nickel must be used in the relation for E_c . However, the portion of the apparatus in the magnetic field region has neutron reflecting walls of aluminum (whose chief virtue is that it is non-magnetic), thus aluminum (coherent scattering length: $a = + 3.45 \times 10^{-13} \text{ cm}$) determines the point where the critical angle is no longer $\pi/2$. The admittance factor for the apparatus as a whole is then a composite of an admittance factor for an aluminum pipe and one for a nickel pipe. This situation is shown on the following graph. A straight-line approximation for f was used for computation purposes. The corresponding neutron spectra are shown on the next graph.

We now consider the effect of the reflectivity being less than one. In the following, it is assumed that the reflectivity is energy independent. This is a reasonable approximation, since non-zero absorption of the walls is due to two effects: neutron capture by the wall material which goes as $1/v$ or λ , and effects due to surface imperfections which go as $1/\lambda$. A reflectivity of .98 for a polished nickel surface is assumed,⁵ and a lower

⁵P. D. Miller, op. cit, p. 8.

Fig. 1

value of .8 for the aluminum surface. These numbers are very uncertain, but not unreasonable (which is about all one can say for their use).

The neutron energy range may be divided into three regions where different approximations to the absorption coefficient may be made. The first, or low-energy range, is such that the critical angle is $\pi/2$ or greater -- here the approximation is that of a straight pipe where the number of bounces, m , a neutron undergoes if it enters at an angle θ to the axis is

$$m = \frac{L}{w} \tan \theta,$$

where L is the length and w is the spacing between sides of the pipe.

Thus the attenuation factor for the entire apparatus is⁶

$$A_I = A_{dI}(\frac{\pi}{2}) \cdot A_h(\frac{\pi}{2}) = \int_0^{\pi/2} \cos \phi e^{g_d \tan \phi} \cdot \frac{\int_0^{\pi/2} e^{g_h \tan \theta} d\theta}{\pi/2}$$

where $g_d = \frac{L_n}{d} \ln r_n + \frac{L_a}{d} \ln r_a + \frac{L_b}{d} \ln r_n$, and similar expression for g_h .

The integrals average the attenuation, r^m , over all possible entrance angles.

L_b is the length of the low-pass filter section

L_n is the length of the sections which are nickel coated

L_a is the length of the aluminum section

r_n is the reflectivity of nickel

r_a is the reflectivity of aluminum

The factor $A_h(\gamma_c)$ occurs in the attenuation in all three regions because the surfaces corresponding to the dimension h are planar.

In the high energy region, the only neutrons able to traverse the

⁶ Ibid., p. 10.

Fig. 2

low-pass filter are those making garland reflections.⁷ The number of bounces for a high energy neutron ($E > E_c$) entering at an angle θ to the axis, is given by

$$m = L_b / 2\rho\theta$$

Thus the attenuation factor for the entire apparatus is

$$A_{III} = A_{dIII}(\gamma_c) \cdot A_h(\gamma_c) = \frac{A_h(\gamma_c)}{\sin \gamma_c} \int_0^{\gamma_c} \cos \theta \exp \left\{ \tan \theta \left[\frac{L_n}{d} \ln r_n + \frac{L_a}{d} \ln r_a \right] + L_b \frac{\ln r_n}{2\rho\theta} \right\} d$$

In the intermediate-energy region, a portion of the neutrons make only garland reflections, and the rest can collide with both walls. The neutrons which collide with both walls undergo

$$m = L_b / \rho(\theta - \phi)$$

bounces, where $\cos \phi = \frac{\rho}{\rho-d} \cos \theta$.

The attenuation factor for the entire apparatus is then

$$A_{II} = A_{dIII}(\gamma^*) \cdot A_h(\gamma_c) + \frac{A_h(\gamma_c)}{\sin \gamma_c} \int_{\gamma^*}^{\gamma_c} \cos \theta \exp \left\{ \tan \theta \left[\frac{L_n}{d} \ln r_n + \frac{L_a}{d} \ln r_a \right] + L_b \frac{\ln r_n}{\rho(\theta - \phi)} \right\} d$$

where $\cos \gamma^* = 1-d/\rho$ (corresponding to E_c).

The angular distribution of the neutrons is given by the integrands of the above expressions, and looks like

⁷The term is used to indicate those paths involving collisions with the outside wall only. See H. Maier-Leibnitz and T. Springer, Reactor Science and Technology (Journal of Nuclear Energy Parts A/B) 17, 220 (1963).

Fig. 3

ORNL-DWG 66-2971

Fig. 4

ORNL-DWG 66-2972

Fig. 5

If we now insert a section of pipe at an angle α to the axis of the conducting pipe

those neutrons striking the wall of the new section at an angle greater than γ_c will not be reflected. Thus the angular distribution will be asymmetric:

where the shaded portion is those neutrons not reflected.

In the actual apparatus, this section at an angle α is the polarizing magnet. Varying α as a parameter gives us a sensitive control over the average velocity of neutrons exiting the apparatus. A computer program which performed the calculations indicated above was used to find the best value for α . The results are shown on the preceding graphs.

Since the width of the resonance line is proportional to the time spent in the apparatus, or inversely proportional to the average velocity $\langle v \rangle$ of the neutrons; and the accuracy of a measurement depends upon the square root of the number of neutrons counted, Z_0 ; we may define a figure of merit as

$$\sqrt{Z_0} / \langle v \rangle$$

Fig. 6

The best choice for the angle α will be that which maximizes this figure of merit. As seen on the graph, this occurs for $\alpha \approx 4^\circ$. Pleasantly enough, at 4° , the intensity is about 4700 neutrons per second reaching the detector (for a transition probability of one), and the average velocity is about 80 meters per second.

The neutron spectrum with the polarizing magnet at an angle of 4° is on the following graph.

Fig. 7

*6. TRANSITION PROBABILITY

The energy of a neutron in an external magnetic field, $\vec{H}$, is $-\vec{\mu} \cdot \vec{H}$, where $\vec{\mu}$ is the magnetic dipole moment of the neutron. If the magnetic field is a rotating field perpendicular to a stationary field in the form $\vec{H} = H_0 \hat{z} + H_1 (\hat{x} \cos \omega t - \hat{y} \sin \omega t)$, the Hamiltonian may be written as

$$\mathcal{K} = -\gamma \hbar I_z H_0 - \gamma \hbar I_x H_1 \cos \omega t + \gamma \hbar I_y H_1 \sin \omega t.$$

Introducing the unit vectors

$$e_{\pm} = (\hat{x} \pm i\hat{y}) / \sqrt{2}, \quad \hat{e}_0 = \hat{z};$$

we may write

$$\vec{I} = I_z \hat{e}_0 + I_+ \hat{e}_+ + I_- \hat{e}_-$$

where

$$I_{\pm} = (I_x \pm iI_y) / \sqrt{2}$$

so that

$$\langle I m' | I_{\pm} | I m \rangle = \sqrt{\frac{1}{2}(I \mp m)(I \pm m + 1)} \delta_{m, m' \pm 1}.$$

The Hamiltonian is then

$$\mathcal{K} = -\gamma \hbar I_z H_0 - \gamma \hbar H_1 I_- e^{i\omega t} / \sqrt{2} - \gamma \hbar H_1 I_+ e^{-i\omega t} / \sqrt{2}$$

or in matrix form

$$\mathcal{K} = \mathcal{K}_0 + V = \begin{pmatrix} q & 0 \\ 0 & p \end{pmatrix} + b \hbar \begin{pmatrix} 0 & e^{i\omega t} \\ e^{-i\omega t} & 0 \end{pmatrix},$$

where

$$q = -\gamma \hbar H_0 / 2$$

$$p = +\gamma \hbar H_0 / 2$$

$$b = -\gamma H_1 / 2.$$

$\mathcal{K}_0$ is then the interaction of a neutron with a constant field, H_0 , along the z-axis; while V represents the interaction with a clockwise-rotating magnetic field of magnitude H_1 in the x,y-plane.

If $|\psi\rangle$ represents the state of the system at any time, t , and $|\psi_0\rangle$ is the state at time $t = t_0$, then

$$|\psi\rangle = U(t, t_0) |\psi_0\rangle$$

where the evolution operator, $U(t, t_0)$, satisfies the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} U(t, t_0) = \mathcal{K}(t) U(t, t_0)$$

with the initial condition $U(t_0, t_0) = 1$. For convenience in the calculation, we let $U = U_0 U_I$, where

$$i\hbar \frac{\partial}{\partial t} U_0(t, t_0) = \mathcal{K}_0 U_0(t, t_0) \text{ with } U_0(t_0, t_0) = 1.$$

Then the interaction part of the evolution operator satisfies

$$i\hbar \frac{\partial}{\partial t} U_I(t, t_0) = V_I U_I(t, t_0) \text{ with } U_I(t_0, t_0) = 1,$$

where

$$V_I = U_0^\dagger V U_0 \quad \text{and} \quad U_0^\dagger U_0 = 1.$$

Since $\mathcal{K}_0$ is time-independent, we may write U_0 formally as

$$U_0(t, t_0) = e^{-i(t-t_0)\mathcal{K}_0/\hbar}$$

Now with

$$V_I = b\hbar \begin{pmatrix} 0 & e^{i\omega_0 t_0 + i(\omega - \omega_0)t} \\ e^{-i\omega_0 t_0 - i(\omega - \omega_0)t} & 0 \end{pmatrix}$$

where $\omega_0 = (p-q)/\hbar = \gamma H_0$, the matrix elements are

$$(U_I)_{11} = (U_I)_{22}^* = e^{i\alpha(t-t_0)} [\cos\lambda(t-t_0) - \frac{i\alpha}{\lambda} \sin\lambda(t-t_0)]$$

$$(U_I)_{21} = -(U_I)_{12}^* = -\frac{ib}{\lambda} e^{i\alpha(t-t_0) - i\omega t + i\omega_0(t-t_0)} \sin\lambda(t-t_0)$$

where

$$\alpha = (\omega - \omega_0)/2 \text{ and } \lambda^2 = b^2 + \alpha^2.$$

The matrix elements for $U(t, t_0)$ are then

$$U_{11} = e^{i\omega(t-t_0)/2} [\cos\lambda(t-t_0) - \frac{i\alpha}{\lambda} \sin\lambda(t-t_0)] = U_{22}^*$$

$$U_{21} = -\frac{ib}{\lambda} e^{-i\omega(t+t_0)/2} \sin\lambda(t-t_0) = -U_{12}^*,$$

and the transition probability is simply

$$P_{p,q} = |\langle p|U(t, t_0)|q\rangle|^2$$

where we have taken $|\psi_0\rangle = |q\rangle$, and $|p\rangle$ as the final state. If the neutron passes through a rotating-field region in time $\tau = t - t_0$, the probability of it being in state $|p\rangle$ when it emerges -- if it was in state $|q\rangle$ upon entering -- is

$$P_{p,q} = |\langle p|U_I(\tau)|q\rangle|^2 = |(U_I)_{21}|^2 = \frac{b^2}{\lambda^2} \sin^2 \lambda \tau.$$

In the Ramsey method of separated oscillatory fields, the neutron spends

a time τ in each of the rotating-field regions, and a time T in the homogeneous-field region between the rotating-field regions. The evolution operator in this case is the product of three operators, each expressing the evolution of the system in one of the three regions:

$$U = U_3(\tau)U_2(T)U_1(\tau)$$

where

$$U_1(\tau) = U_{01}(\tau)U_{11}(\tau) = e^{-i\tau\mathcal{H}_0/\hbar} U_{11}(\tau)$$

$$U_2(T) = U_{02}(T) = e^{-iT\mathcal{H}_0/\hbar}$$

$$U_3(\tau) = U_{03}(\tau)U_{13}(\tau) = e^{-i\tau\mathcal{H}_0/\hbar} U_{13}(\tau).$$

The matrix elements of U_{11} and U_{13} are identical to the matrix elements for U_I above -- however the correct initial times t_{01} and t_{03} , must be used. In this case

$$t_{01} = t_0, \text{ and } t_{03} = t_0 + \tau + T.$$

If the field, H_0 , is not constant spatially between the oscillatory-field regions, the neutron will experience a time-varying magnetic field as it traverses the space between the rf coils. Assuming that we can analyze the field in a manner such that over the region ΔL_j , the value of the field is H_{0j} , the time-evolution operator connecting the wavefunction in region ΔL_{j-1} with that in region ΔL_{j+1} will be

$$e^{-i\Delta t_j \mathcal{H}_{0j}/\hbar}$$

where $\Delta t_j = \Delta L_j / v$ and $\langle m | \mathcal{K}_{Oj} | m' \rangle = \pm \gamma \hbar H_{Oj} / 2 \delta_{mm'}$, and v is the neutron velocity. The total evolution operator corresponding to U_2 is then

$$\prod_j U_{O2}(\Delta t_j) = \prod_j e^{-i \Delta t_j \mathcal{K}_{Oj} / \hbar} = e^{-\frac{i}{\hbar} \sum_j \Delta t_j \mathcal{K}_{Oj}}.$$

Since $T = \sum_j \Delta t_j$, and $L = vT$, we have

$$\frac{1}{T} \sum_j \Delta t_j \mathcal{K}_{Oj} = \frac{v}{L} \sum_j \frac{\Delta L_j}{v} \mathcal{K}_{Oj} = \bar{\mathcal{K}}_O$$

which is the spatial average of $\mathcal{K}_O$. In this case, $\bar{\mathcal{K}}_O = -\gamma \hbar \bar{H}_O / 2$, where $\bar{H}_O$ is the spatial average (between the oscillatory-field regions) of the stationary field, H_O . Thus in matrix form,

$$U_2 \rightarrow \begin{pmatrix} e^{i \bar{\omega}_O T/2} & 0 \\ 0 & e^{-i \bar{\omega}_O T/2} \end{pmatrix}$$

where $\bar{\omega}_O = \gamma \bar{H}_O$.

The final complication considered here is that of introducing a phase shift between the two rf coils: suppose the second rotating field leads the first by a phase angle δ . Then the matrix U_2 will have the off-diagonal element

$$(U_2)_{21} = -\frac{i b}{\lambda} e^{-i \omega (3\tau + T)/2 - i \delta} \sin \lambda t.$$

The transition probability is then

$$\begin{aligned}
\delta^P_{p,q} &= |\langle p | U_3(\tau+T+\tau, \tau+T) U_2(T) U_1(\tau, 0) | q \rangle|^2 \\
&= |(U_3)_{21} (U_2)_{11} (U_1)_{11} + (U_3)_{22} (U_2)_{22} (U_1)_{21}|^2 \\
&= 4 \frac{b^2}{\lambda^2} \sin^2 \lambda \tau \left\{ \cos \lambda \tau \cos \frac{1}{2} [(\omega - \bar{\omega}_0)T + \delta] - \frac{\alpha}{\lambda} \sin \lambda \tau \sin \frac{1}{2} [(\omega - \bar{\omega}_0)T + \delta] \right\}.
\end{aligned}$$

Note that we have assumed that the expressions α and λ are the same for both U_1 and U_3 -- this is equivalent to assuming that the neutron experiences the same field H_0 in both rotating-field regions (as well as assuming the frequency of both rotating fields to be the same).

Near resonance, where $|\alpha| \ll b$, we may write

$$\delta^P_{p,q} \approx \sin^2 2b\tau \cos \frac{1}{2} [(\omega - \bar{\omega}_0)T + \delta].$$

With the aid of the trigonometric identities used by Ramsey (Molecular Beams, Appendix D, Oxford University Press, 1956), a form for the transition probability near resonance which is linear in the trigonometric functions of the time is

$$\begin{aligned}
\delta^P_{p,q} &= \frac{1}{4} - \frac{1}{2} \frac{\cos 4b\tau}{2} + \cos \delta \left\{ \frac{1}{2} \frac{\cos(\bar{\omega}_0 - \omega)T}{2} - \frac{1}{4} \frac{\cos(4b\tau + (\bar{\omega}_0 - \omega)T)}{2} \right. \\
&\quad \left. - \frac{1}{4} \frac{\cos(4b\tau - (\bar{\omega}_0 - \omega)T)}{2} \right\} + \sin \delta \left\{ \frac{1}{2} \frac{\sin(\bar{\omega}_0 - \omega)T}{2} \right. \\
&\quad \left. + \frac{1}{4} \frac{\sin(4b\tau - (\bar{\omega}_0 - \omega)T)}{2} - \frac{1}{4} \frac{\sin(4b\tau + (\bar{\omega}_0 - \omega)T)}{2} \right\}.
\end{aligned}$$

Since the above expressions for the transition probability are valid only at a particular velocity, $v = \ell/\tau = L/T$, where ℓ and L are the lengths of the rotating-field regions and the constant-field region between the rf coils; we must average the transition probability over the neutron spectrum. Thus we compute

$$\langle \delta_{p,q}^P(\omega) \rangle = \int_0^{\infty} \frac{dZ}{dV} \delta_{p,q}^P(\omega, V) dV;$$

or in terms of the energy

$$\langle \delta_{p,q}^P(\omega) \rangle = \int_0^{\infty} \frac{dZ}{dE} \delta_{p,q}^P(\omega, E) dE,$$

where

$$\frac{dZ}{dE} = A(\gamma_c) \frac{\rho h \gamma_c}{\pi kT} I_0 \frac{E}{kT} e^{-E/kT}$$

as in *5.

To facilitate computations, we define¹

$$I(x) = \frac{1}{2} \int_0^{\infty} \frac{dZ}{dU} \cos \sqrt{\frac{x}{U}} dU$$

$$K(x) = \frac{1}{2} \int_0^{\infty} \frac{dZ}{dU} \sin \sqrt{\frac{x}{U}} dU$$

where $U = E/kT$. The transition probability averaged over the neutron spectrum is then²

¹c.f. Ramsey, op.cit., Section V.3.2 and Appendix D.

²Ramsey, ibid., p 132.

Fig. 8

$$\begin{aligned}
\langle \delta^P_{p,q} \rangle = & \frac{1}{4} - \frac{1}{2} I(4b\ell/\alpha) + \cos\delta \left[\frac{1}{2} I(\lambda L/\alpha) - \frac{1}{4} I(4b\ell/\alpha + \lambda L/\alpha) \right. \\
& - \frac{1}{4} I(4b\ell/\alpha - \lambda L/\alpha) \left. \right] + \sin\delta \left[\frac{1}{2} K(\lambda L/\alpha) \right. \\
& - \frac{1}{4} K(4b\ell/\alpha + \lambda L/\alpha) + \frac{1}{4} K(4b\ell/\alpha - \lambda L/\alpha) \left. \right]
\end{aligned}$$

where now $\alpha = \sqrt{2kT/m}$ and $\lambda = \bar{\omega}_0 - \omega$.

A computer program was written to calculate a table of values for $I(x)$ and $K(x)$ for the spectrum in *5. As a check, the Maxwell-Boltzmann spectrum was substituted for the theoretical spectrum of *5, and the numbers obtained were compared to Table D.I in Ramsey's Molecular Beams.³ $K(x)$ for a Maxwell-Boltzmann distribution and for the distribution in *5 are shown on the preceding graph where the abscissa has been adjusted so that the first zero of both functions occurs at the same point. The graphs are very similar, but the Maxwell-Boltzmann $K(x)$ oscillates more rapidly, has larger extrema, and does not decay as rapidly as the $K(x)$ for the spectrum of *5. Qualitatively, one would then expect the transition probability to have less oscillations than in a corresponding transition probability for a Maxwell-Boltzmann distribution (as shown on the following graph). It is of interest to note that the spectrum of *5 may be fitted closely to a Maxwell-Boltzmann distribution of the same intensity. The best fit is obtained with a temperature of $T = .242^\circ\text{K}$. If we define a temperature for the spectrum in *5 by

$$\text{most probable velocity} = \sqrt{2kT/m},$$

we obtain $T = .218^\circ\text{K}$ for $\langle v \rangle = 79.71 \text{ m/sec}$. This comparison is shown on the next graph.

³Ramsey, *ibid.*, p 425.

Fig. 9

Since we now have a theoretical value for the slope of the transition probability at resonance, and an expected neutron intensity, we may estimate the size of an electric dipole moment which we can just observe. If the two rf coils have a phase shift of 90° between them, and we are at resonance, the number of neutrons counted in one hour will be

$$N = 3600 \text{ sec} \times 4700 \frac{\text{neutrons}}{\text{sec}} \times (\sim .5) = 8.5 \times 10^6.$$

So counting for one hour with the electric field one way, and one hour with the field reversed, we have a total number of counts of

$$N_+ - N_- = N_0(P_+ - P_-)$$

or

$$\frac{\Delta N}{N_0} = \Delta P$$

which is the change in transition probability upon reversing the electric field. The error in ΔP is

$$\epsilon(\Delta P) = \frac{\epsilon(\Delta N)}{N_0} = \frac{\sqrt{\epsilon^2(N_+) + \epsilon^2(N_-)}}{N_0} \approx \frac{\sqrt{2N}}{N_0} \approx 1/\sqrt{N_0}$$

since $N_0 \approx 2N$. Now, we can see an effect if

$$\Delta P > \epsilon(\Delta P),$$

but at resonance

$$\frac{dP}{d\nu} = \frac{.1436}{2} \text{ per cycle per second}$$

Fig. 10

so

$$\Delta P \approx .1436 \Delta v/2$$

and

$$\Delta v > \frac{2}{.1436} \sqrt{N_0} \approx 3.4 \times 10^{-3} \text{ cps.}$$

We may define a "gyro-electric" ratio, γ_e , by $\vec{\mu}_e = e\vec{D} = h \vec{I} \gamma_e$ where $\vec{\mu}_e$ is the electric dipole moment of the neutron. Then the resonant frequency due to the electric field is given by

$$\omega = \gamma_e E$$

or

$$\Delta\omega = \gamma_e \Delta E = \frac{eD}{Ih} \Delta E = \frac{2e}{h} \Delta E$$

Thus

$$D = \frac{\Delta v}{\Delta E} \frac{h}{2e}$$

and we will see an effect if (assuming an electric field of 200 kv/cm)

$$D > \frac{3.4 \times 10^{-3}}{4 \times 10^5} \frac{6.625 \times 10^{-34}}{2 \times 1.6 \times 10^{-19}} \approx 1.8 \times 10^{-23} \text{ cm.}$$

In actual practice, the errors will be somewhat worse than the pure statistical errors, so a limit of about 5×10^{-23} cm is more realistic. This is a factor of 1000 lower than the limit set by Smith, Purcell, and Ramsey in 1949.⁴

⁴J. H. Smith, E. M. Purcell, and N. F. Ramsey, Experimental Limit to the Electric Dipole Moment of the Neutron, Phys. Rev. 108, 120 (1957).

*7. FIELD INHOMOGENEITY

The specifications of the design of the magnet which supplies the field, H_0 , are determined by the amount of broadening tolerable in the neutron magnetic resonance line. Line broadening may result from two independent types of magnetic field inhomogeneities. The first depends on the fact that neutrons following different paths will experience different average fields in both the oscillatory-field region and the space in between. The second effect is due to the neutron experiencing varying fields along its flight path.

To get an idea of the size of the first effect, assume that the pole pieces are perfect, homogeneous, magnetic conductors -- then we can define a magnetic potential which obeys Laplace's equation in the region between the poles:

If the gap, $2d$, is much smaller than the width of the plates (and their length is much larger than their width), the approximation by a semi-infinite planar boundary-value problem is appropriate.

The mapping, $w(z)$, is the Schwarz-Christoffel transformation defined by

$$\frac{dz}{dw} = A \frac{\sqrt{w-1}}{w}$$

where we have placed the right-angle vertex at $w = 1$, the left-extended vertex at $w = 0$, and the remaining vertex at the point ∞ . If the origin in the z -plane is placed at the image of $w = 1$, we obtain the mapping explicitly as

$$z = \frac{2di}{\pi} \left\{ \sqrt{w-1} + \frac{1}{2} \ln \frac{1 + i \sqrt{w-1}}{1 - i \sqrt{w-1}} \right\}$$

A harmonic function, analytic in the upper-half plane, satisfying the boundary conditions is

$$W = - \frac{V_0}{\pi} \ln w + i V_0$$

The field is then found from

$$i \frac{dW}{dz} = i \frac{\partial U}{\partial x} - \frac{\partial V}{\partial y} - i \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} = -1 H_y + H_x$$

and

$$\frac{dW}{dZ} = \frac{dW}{dw} \frac{dw}{dZ} = - \frac{V_0}{\pi w} \frac{w}{A \sqrt{w-1}},$$

so

$$H_x - i H_y = - \frac{V_0}{d \sqrt{w-1}}.$$

The variation of H with position in the gap for a value of $y = -4.25/9$ gap lengths (corresponding to the dotted line drawn in the following sketch) is shown on the next graph.

If we define a field inhomogeneity as

$$I = \sqrt{H_x^2 + (H_0 - H_y)^2} / H_0$$

we find that it is of the order of a few parts in 10^4 in the shaded region of the following sketch. An estimate of the tolerable field inhomogeneity may be made from the Uncertainty Principle, which determines the natural width of the resonance line:

$$\Delta\nu \Delta t \sim 1$$

where

$$\Delta\nu = \gamma \Delta H \text{ and } \Delta t \approx L / \langle v \rangle$$

Then

$$\Delta H \sim \frac{\langle v \rangle}{\gamma L} \approx \frac{80 \frac{\text{m}}{\text{sec}}}{(1.84 \times 10^4 / \text{sec gauss}) (2\text{m})} \approx 2.2 \times 10^{-3} \text{ gauss}$$

which is an order of magnitude larger than the inhomogeneity within the shaded region.

The magnitude of this broadening may be estimated by averaging the

Fig. 11

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Fig. 12

transition probability over the values of H_y in the region of interest. The effect of a non-zero value for H_x would contribute to a further broadening, since the neutrons would tend to precess about the resultant H ; however, this resultant is at an angle of smaller than 10^{-4} radians to H_0 , and has an average value of zero due to the symmetry of the magnet.

For an H_0 of ten gauss, corresponding to a resonant frequency of about 29.2 kilocycles per second, an average of over 24 points in the region of interest gives a broadening of $1/15$ in the dispersion-type line shape using a phase shift of 90° between the rf coils. This may not seem to be serious until one looks at the maximum slope of the resonance line. The results of this calculation are on the following graph.

Since the slope of the line is decreased by about 33%, we are inclined to pursue the problem further in hopes that this might be significantly reduced. The first possibility that comes to mind is to add a second vertex to the above Schwarz-Christoffel transformation:

The transformation then becomes

$$\frac{dZ}{dw} = A(w-1)^{\beta} (w-B)^{-\alpha} (w-0)^{-1}.$$

Choosing the origin at the point indicated results in

$$Z(w) = A \int_0^w \frac{w'-1}{w'} \left(\frac{w'-1}{w'-B} \right)^{\alpha} dw'$$

In the w -plane, we have the situation

with the potential

$$W = -\frac{V_0}{\pi} \ln w + i V_0.$$

The field is given by

$$H_x - i H_y = i \frac{dW}{dZ} = i \frac{dW}{dw} \frac{dw}{dZ} = i \frac{-V_0}{\pi w} \frac{w}{A \sqrt{w-1}} \left(\frac{w-B}{w-1} \right)^{\alpha}.$$

To find the values of A and B , we use a method due to M. E. Rose (Phys. Rev. 53, 715 (1938)). First, expand the field in powers of w :

$$H_x - i H_y = -\frac{iV_0}{\pi iA} \cdot B^{\alpha} \left\{ 1 - \frac{2-2\alpha \cdot B-B}{2B} w + \dots \right\}$$

Fig. 13

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Fig. 14

Now for $w \rightarrow 0^+$, $z \rightarrow +\infty$ where the field must be a uniform $H_y = V_o/d$; this gives

$$-i V_o/d = -i H_y = -i \frac{V_o}{\pi i A} B^\alpha,$$

so

$$A = \frac{dB^\alpha}{\pi i}.$$

To choose B, note that a perfectly homogeneous field corresponds to $H_x = 0$ and $H_y = V_o/d$; thus if we choose the coefficient of the first power of w in the above expansion to be zero, we have chosen the best value for B, since most of the inhomogeneity is due to this first order term as $|w| < 1$. Thus the inhomogeneity will be of second order if

$$2 - 2\alpha B - B = 0,$$

or

$$B = 2\alpha/(2\alpha + 1).$$

An investigation with a computer showed that the inhomogeneity is quite insensitive to the particular choice for α as long as the above relationship holds. (In the z -plane, B corresponds to the hypotenuse of the right triangle with angle $\pi\alpha$.) Accordingly we chose an angle which was easy to machine -- $\pi\alpha = 60^\circ$, or $\alpha = 1/3$. The variation of H_x and H_y with this type of field shim are shown on the preceeding graph. When a similar average over the region of interest is performed, the results (within a few parts in 10^8) are the same as those for a homogeneous field.

Another source of inhomogeneity of the first type is due to the pole pieces being non-parallel. Suppose the surface of the pole piece

is flat over the width to .001 inch. Then, since the spacers separating the pole faces are six inches apart, the pole faces will be out of parallel by .002 inch out of six inches in the worst case. The boundary-value problem is then:

A Schwarz-Christoffel transformation of the boundary gives

$$H_x - i H_y = -i H(R) \frac{R}{r} e^{-i\phi}$$

where $H(R)$ is the value of the field for $z = R e^{i\phi}$, and $\phi_R = 0$. Thus to zero order (since ϕ is a small angle)

$$H_y(r) = H(R) \frac{R}{r} \quad \text{and} \quad H_x = 0.$$

Performing an average of the transition probability over the shaded region for $\pi\beta = 1/6000$, we obtain a broadening of less than 1/15 and the slope at resonance is reduced by about 2%.

The second effect -- due to the field varying along the neutron's path -- can be shown to be small, so we need not bother calculating it. In the first place, this effect averages to zero over most of the path, since the rf fields are applied only to small regions (about ten centimeters) at the beginning and end of the two meter homogeneous-field region.

Secondly, if the pole material is reasonably homogeneous, the gap large, and the pole faces smooth; the variation of the field along a neutron path for the length of an oscillating field region will be small. The larger the gap, the farther the region of interest is from the pole pieces and the local variations of the surface and material have less effect. Since the inhomogeneity due to the finiteness of the pole pieces is a function of the distance into the gap, and for a distance of 0.6 into the magnet the inhomogeneity is of the order of one part in 10^4 , we could choose the gap to be 9 cm (distance of edge of shaded region from $x = 0$) divided by 0.6 gap units, or 15 cm without appreciable broadening of the line. However, a gap of 9 cm was chosen so that the inhomogeneity due to the finiteness of the pole pieces was much less than the inhomogeneity due to the pole pieces being tilted at an angle of $1/6000$ radians.

If the variations on the surface are of size δ , they will influence the region of interest by something on the order of

$$\Delta H = H_0 \delta / 9 \text{ cm.}$$

The smoothness of the surfaces (finish) is specified to be 4 microinches, thus

$$\frac{\Delta H}{H} \sim 2 \times 10^{-6}$$

which is negligible compared to 10^{-4} .

*8. PROBLEMS OF THE ELECTRIC FIELD

In the ten months since the initiation of the experiment, the only existing thing which may be called experimental is our pilot model of the electrostatic system. It is composed of the vacuum pump and the high-voltage power supplies which will be used on the final apparatus, as well as a special chamber containing two aluminum plates five inches wide and one foot long in the configuration of the common parallel-plate capacitor with a gap of one centimeter.

There are four areas of problems: high-voltage cable, vacuum feed-throughs, stand-off insulators supporting the aluminum plates, and the plates themselves. All problems (in the pilot model) seem to have been solved satisfactorily with the exception of the vacuum feed-throughs. The initial problem was the breakdown of the high-voltage cable (which was solved by going to a larger diameter) and a shield-termination effect. A spark between the plates would initiate breakdown of the air around the shield termination near the vacuum feedthrough. When this happened, a spark would at times leap across the fluted ceramic of the feedthrough and generally cause an over-current shut-down of the power supplies. The solution to this problem was to encase the last 20 inches of the cable leading to the feedthrough in a transformer-oil bath. When this was done, we could put higher voltages on the plates, which caused the stand-off insulators (which lift the plates some three cm off ground) to break down internally. When removed, they showed a number of burnt spots in each end, and one was cracked. The solution to this problem was to use fused-quartz insulators in place of the ceramic ones. Thus, we were once more able to go up in voltage. This, of course, brought a further problem to view: the aluminum plates soon gave out -- that

is, breakdown occurred continuously between the plates. The solution to this problem was to anodize the plates and introduce helium at a pressure of about 10^{-3} torr into the chamber as suggested by the rather comprehensive paper on electrostatic breakdown by F. Rohrbach (Some Studies of High Voltage Vacuum Breakdown Across Large Gaps, CERN 64-50 N.P.A. Division, Meyrin, Nov. 9, 1964). We were then able to attain 200 kilovolts across the one cm gap for periods on the order of an hour without serious breakdown. Naturally, another problem then developed -- the vacuum feedthroughs were breaking down internally -- i.e., from the center conductor to the inside metal wall. This (at 100 kv each) created a copious supply of X-rays as well as raising the current to a factor of ten to twenty above the quiescent stage of about 40 microamps. This meant that the voltage on the plates dropped to something near zero as there are large resistors (about 500 megohms) between the power supplies and the feedthroughs. There have been several solutions to this problem suggested, but we haven't had time to try them as yet.

All in all, we feel confident that we will eventually be able to maintain a field of 200 kv/cm for the final apparatus.

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