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AND INDUCED $0^{\pm}$ MESONS

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COUPLING CONSTANT RELATIONS FOR 1^+
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When a current conservation is broken in a spontaneous (or almost spontaneous) fashion, induced bosons with spin zero and small mass are expected to arise in such a way as to keep the matrix element of the current free of divergence. On two previous occasions^{1,2} one of us (Y.N.) has derived a relation between the coupling constant of the induced 0^+ meson and that of the 1^+ meson which is assumed to dominate the form factor of the current (vector or axial vector). This relation was given by

$$|f|^2 = |G|^2 / m^2$$

and

$$|\lambda|^2 = m^2$$

if the effective interaction Lagrangian is defined as

$$\mathcal{L}_{int} = (f \partial_\mu \phi + G V_\mu) j_\mu - \lambda \partial_\mu \phi V_\mu + h.c.$$

Here j_μ is the current; ϕ and V_μ represent scalar and vector

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fields (of either parity, depending on j_μ); m is the mass of the vector meson.

It has come to our attention that the above derivation of Eq. (1) is marred by an error of sign so that actually the f and λ terms become non-Hermitian. The purpose of this note is to show that a model Lagrangian can be constructed which leads to a modified but meaningful relation

$$|f|^2 = |G|^2 / m^2 (Z_1 - 1) \quad (1)$$

where Z_1 is the current renormalization constant.

We start from the Lagrangian density*

* This may be interpreted as a model in which the self-energy effect due to virtual baryon pair on the field ϕ is replaced effectively by that of the field V_μ . If we use instead a Lagrangian in which the first term is $(\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial_\mu V_\nu - \partial_\nu V_\mu)/4$, there will be only one scalar field (ϕ) as a dynamical variable. In this case we do not obtain a relation like Eq. (13) between f_0 and g_0 .

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \partial_\mu V_\nu \partial_\mu V_\nu - \frac{m_0^2}{2} V_\mu V_\mu - \frac{1}{2} \partial_\mu \phi \partial_\mu \phi - \frac{1}{2} \mu_0^2 \phi^2 \\ & - \lambda V_\mu \partial_\mu \phi + f_\mu [\lambda g_0 V_\mu + f_0 \partial_\mu \phi] \end{aligned} \quad (2)$$

The current j_μ is regarded as an external source, and the fields V and ϕ are assumed to be real for simplicity. Equation (2) gives rise to the field equations

$$\begin{aligned} (\square - m_0^2) V_\mu - \lambda \partial_\mu \phi &= -\lambda g_0 j_\mu \\ (\square - \mu_0^2) \phi + \lambda \partial_\mu V_\mu &= f_0 \partial_\mu j_\mu \end{aligned} \quad (3)$$

We are interested in the limit $m_0 \gg \mu_0 \rightarrow 0$. In this limit the Lagrangian is invariant under the displacement $\phi \rightarrow \phi + \text{const}$, so that there exists a conserved current

$$\begin{aligned} J_\mu &\equiv \delta L / f_0 \delta(\partial_\mu \phi) = j_\mu - (\lambda/f_0) V_\mu - (1/f_0) \partial_\mu \phi \\ \partial_\mu J_\mu &= 0 \end{aligned} \quad (4)$$

In the absence of external currents this system has three types of excitations: 1) Transverse vector field V_μ^t ($\partial_\mu V_\mu^t = 0$) of mass $m = m_0$; 2) Two scalar modes ϕ_i ($i = 1, 2$) of mass κ_i accompanied by induced $V_{\mu i} = -\lambda \partial_\mu \phi_i / (m_0^2 - \kappa_i^2)$. The transverse mode is not coupled to ϕ , so its mass and wave function are not renormalized. On the other hand, the original ϕ field is mixed with $\partial_\mu A_\mu$ to form new scalar excitations. Their masses satisfy

$$(\mathcal{X}^2 - m_0^2)(\mathcal{X}^2 - \mu_0^2) + \lambda^2 \mathcal{X}^2 = 0,$$

or

$$\left. \begin{matrix} \mathcal{X}_1^2 \\ \mathcal{X}_2^2 \end{matrix} \right\} = \frac{1}{2} \left\{ m_0^2 + \mu_0^2 - \lambda^2 \pm [(m_0^2 + \mu_0^2 - \lambda^2)^2 - 4m_0^2 \mu_0^2]^{\frac{1}{2}} \right\} \quad (5)$$

We impose the reality condition $m_0^2 + \mu_0^2 - \lambda^2 \geq 2m_0\mu_0$, or $(m_0 - \mu_0)^2 \geq \lambda^2$. Then

$$0 < \mathcal{X}_1^2 < \mathcal{X}_2^2. \quad (6)$$

In the limit $m_0 \gg \mu_0$ we have

$$\begin{aligned} \mathcal{X}_2^2 &\approx m_0^2 - \lambda^2 \\ \mu^2 = \mathcal{X}_1^2 &= \mu_0^2 m_0^2 / \mathcal{X}_2^2 \approx \mu_0^2 m_0^2 / (m_0^2 - \lambda^2) \end{aligned} \quad (7)$$

We shall next compute the renormalization of the wave function and of the coupling constant for these two scalar modes. The renormalized scalar field ϕ^R is defined by

$$\phi_i^R = Z_{3i}^{-1/2} \phi_i \quad (8)$$

in such a way that it behaves like a free quantized field of mass $\mathcal{X}_i$. Thus Z_3 is found by requiring that when ϕ_i together with the induced $V_{\mu i}$ is substituted in the Hamiltonian

$$H = \frac{1}{2} \int d^3x \left[(\partial\phi/\partial t)^2 + (\partial V_\mu/\partial t)(\partial V_\mu/\partial t) + \nabla\phi \cdot \nabla\phi + \nabla V_\mu \cdot \nabla V_\mu + \right. \\ \left. + 2\lambda \nabla\phi \cdot \underline{V} + \mu_0^2 \phi^2 + m_0^2 V_\mu V_\mu \right]$$

one quantum of excitation has the energy $E_1 = (\underline{k}^2 + \chi_1^2)^{1/2}$. In this fashion we obtain**

** Z_3 is larger than one because two scalar modes of opposite metric (ϕ and $\partial_\mu V_\mu$) are mixed.

$$Z_{3i} = 1 - \lambda^2 m_0^2 / (m_0^2 - \chi_i^2)^2 \quad (9)$$

The scalar mode of mass χ_2 actually has $Z_3 < 0$ and is a ghost state, i.e. has negative energy. However, this mode may be uncoupled from the external current and hence all physical processes. Since the interaction of j_μ with a scalar mode is $L_{int} = j_\mu [f_0 \partial_\mu \phi_i + \lambda g_0 V_{\mu i}]$, this condition is given by

$$f_0 - \lambda g_0 / (m_0^2 - \chi_2^2) = 0 \quad (10)$$

The other scalar mode has $Z_3^{-1} = 1 - \frac{\lambda m^2}{(\frac{m^2 - \mu^2}{2})^2}$ and renormalized coupling constant defined by $f j_\mu \cdot \partial_\mu \phi_1^R = j_\mu [f_0 \partial_\mu \phi_1 + g_0 \lambda V_{\mu 1}]$ is given by

$$f = z_3^{1/2} [1 - \lambda^2 g_0 / f_0 (m^2 - \mu^2)] f_0 \quad (11)$$

In addition, for small momentum transfer ($\partial_\mu \rightarrow 0$) we have $V_\mu = \frac{\lambda g_0 j_\mu}{m_0^2}$ from Eq. (3), and the current $j_\mu \rightarrow z_1^{-1} j_\mu$, where

$$z_1^{-1} = \left(1 - \lambda_0^2 g_0 / f_0 m_0^2\right) \quad (12)$$

In the limit $\mu_0 \rightarrow 0$ Eqs. (10-12) become

$$f_0 = g_0; \quad z_3^{-1} = z_1^{-1} = (1 - \lambda^2 / m_0^2); \quad f = z_1^{-1/2} f_0 \quad (13)$$

The coupling constant of the transverse vector field, $G = \lambda g_0$ and the coupling constant of the scalar field are therefore related by

$$f^2 \approx f_0^2 z_1^{-1} = G^2 / m^2 (z_1 - 1) \quad (14)$$

while the renormalized scalar mass μ is found from Eq. (6) to be

$$\mu^2 = \mathcal{M}_1^2 \approx \mu_0^2 z_1 \quad (15)$$

Equation (14) replaces the old relation $f^2 = G^2 / m_0^2$.

One undesirable feature about J_μ , Eq. (7), is that the

form factor reduces to 1 for large momentum transfer, instead of approaching zero, as seems to be the actual case. This defect may be remedied by defining the physical current to be

$$\begin{aligned}
 Q_\mu &\equiv J_\mu + \partial_\nu F_{\nu\mu} / g_0 \lambda \\
 &= Z_1^{-1} \frac{(-m_0^2)}{\square - m_0^2} \left[\delta_{\mu\nu} - \partial_\mu \partial_\nu / m_0^2 \right] f_\nu \\
 &\quad - \frac{1}{\square - \mu^2} \frac{(m_0^2 - \lambda^2 - \mu^2)}{(m_0^2 - \mu^2)} \partial_\mu \partial_\nu f_\nu
 \end{aligned}$$

$$(F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu) \quad (16)$$

which has the "correct" form factor $F(q^2) = Z_1^{-1} m^2 / (m^2 + q^2)$.

Let us now apply our results to the actual situation in which ϕ is the known pseudoscalar octet and V_μ the conjectured axial vector octet. Before breaking the $SU(3)$ symmetry, the couplings f and g in general consist of two parts, the symmetric (d-type) f_s, g_s and the antisymmetric (f-type) f_a, g_a . We can then deduce from the above argument that $f_s : f_a = g_s : g_a$, $Z_{1s} = Z_{1a}$.

The symmetry breaking may be introduced in the Lagrangian through f_0, g_0, m_0, μ_0 and λ .*** In this case, Z_1 will be

*** A priori, λ may have d-type and f-type symmetry breaking terms.

different for π , $K(\bar{K})$ and η , and the coupling constant relation (15) between f and G will hold for each vertex πNN , $\pi \Xi \Xi$, $K \Lambda N$, etc., separately, except that in all the formulas λ^2 , f^2 , g^2 must be replaced by $|\lambda|^2$, $|f|^2$, $|g|^2$.

Take, in particular, the pion case. The pion-nucleon coupling is known to have the value

$$f_{\pi NN}^2 / 4\pi = 0.08 / \mu_\pi^2 \quad (17)$$

In our model there is no a priori way to fix Z_1 . On the other hand, $Z_1(\pi)$ may be determined from the β -decay in conjunction with the V-A and SU(6) theories.^{3,4} Z_1 is then given by

$$\begin{aligned} Z_1^{-1} &= (-G_A/G_V) : 5/3 = (1.18 \pm 0.02) \times 3/5 \\ &= 0.71 \pm 0.01 \end{aligned}$$

$$\lambda^2 / m_\phi^2 = 0.29 \pm 0.01 \quad (18)$$

Thus

$$G_{\tilde{\rho} NN}^2 / m_{\tilde{\rho}}^2 \approx (0.29/0.71) f_{\pi NN}^2 = 0.41 f_{\pi NN}^2 \quad (19)$$

where $\tilde{\rho}$ denotes the $T = 1$, $Y = 0$ axial vector meson. We also find from Eqs. (14), (15), (17) and (18)

$$\mu_{0\pi}^2 = 0.71 \mu_{\pi}^2$$

$$f_0^2/4\pi = (f^2/4\pi)/0.71 = 0.11/\mu_{\pi}^2 \quad (20)$$

If μ_0 is common to all mesons, then from Eq. (15)

$$Z,(\pi)/Z,(K) = \mu_{\pi}^2/\mu_K^2$$

$$Z,(\eta)/Z,(\pi) = \mu_{\eta}^2/\mu_{\pi}^2$$

(21)

Further, if the bare coupling f_0 is SU(3)-symmetric, the renormalized couplings squared of K and η to any baryon pair must be reduced by the same ratios (21) relative to the pion coupling, whereas the couplings of the same meson to different baryons will maintain SU(3) symmetry.[†]

[†] For example, $|f_{\Lambda NK}/f_{\pi NN}|^2 = |f_0 \Lambda NK/f_0 \pi NN|^2 \mu_{\pi}^2/\mu_K^2 \sim 1/14$ if the SU(6) prediction $f_0 \Lambda NK/f_0 \pi NN = 3\sqrt{3}/5$ is taken.

It is conceivable that the deviation from simple universality of the leptonic decay of baryons may be due to the renormalization effects. However, Eq. (21) leads to too much suppression ($= \mu_{\pi}^2/\mu_K^2$) of the $\Delta S \neq 0$ axial vector current

relative to the $\Delta S = 0$ current, as compared to the value $\sim \mu_\pi/\mu_K$ which is experimentally required.^{5,6} (As for the leptonic decay of pseudoscalar mesons, the above assumption leads to the right ratio μ_π/μ_K for the K and π decay amplitudes.⁷)

Finally, we remark that our formulation naturally applies also to the vector (1^-) and the induced scalar (0^+) mesons.¹ If the current j_μ (not regarded as external) is strictly conserved by itself, we expect $Z_1 = 1$, $\lambda = 0$, so that $f^2 \sim G^2/m^2(Z_1-1) \rightarrow \infty$ if G^2 remains finite. However, the coupling of the induced scalar meson is effectively zero since $\partial_\mu j_\mu = 0$. On the other hand, in a case like the $\Sigma\Lambda$ transition current where $\partial_\mu j_\mu \propto (M_\Sigma - M_\Lambda) \neq 0$, G^2 must be zero. This is consistent with the prevalent view that the vector current is purely of the so-called f-type.

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