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A 200-WATT CONDUCTION-COOLED REACTOR POWER SUPPLY FOR SPACE APPLICATION

by

Donald R. MacFarlane

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A 200-WATT CONDUCTION-COOLED REACTOR POWER SUPPLY
FOR SPACE APPLICATION

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Donald R. MacFarlane

Reactor Engineering Division

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NOMENCLATURE

LIMITATIONS OF HEAT REMOVAL BY CONDUCTION (See Section IV)

t	temperature, °C
a	outer radius of sphere, cm
r	radius, cm
q	total heat generated, w
k	thermal conductivity, w/(cm)(°C)
$\overline{Q}$	average power density, w/cm ³
$\overline{P}$	average specific power, w/gm
ρ	density, gm/cm ³

THERMOELECTRIC CONVERTER DESIGN (See Section V)

R	resistance, ohms
E_0	generator output voltage, v
ΔT	temperature difference between hot and cold junction, °C
α	$S_p - S_n$ = Seebeck coefficient of couple, v/°C
S	Seebeck coefficient of leg of couple, v/°C
X	packing fraction
N	number of thermoelements in generator
I	generator current, amp
P_0	power output of generator, w
ℓ	length of thermoelement arms, cm
A_t	total area of thermoelectric material normal to heat flow, cm ²
ρ	electrical resistivity, ohm-cm
k	thermal conductivity, w/(cm)(°C)
q	heat input to generator, w
T	absolute temperature, °K
r_t	thermal resistance, °C/w
η	thermal efficiency
W	weight of thermoelectric material
d	density of thermoelectric material, gm/cm ³

SUBSCRIPTS

L	load
g	generator
n	negative leg
p	positive leg
h	hot junction

RADIATOR DESIGN (See Section VI)

T	absolute temperature, °K
r	radius, cm
σ	Boltzman constant = 5.67×10^{-12} w/(cm ²)(°K) ⁴
ϵ	emissivity
k	thermal conductivity, w/(cm)(°C)
x	thickness of uniform fin, cm
q	total heat radiated, w
y	tapered fin half thickness, cm
ϕ	angular displacement in plane of fin
θ	angular displacement in plane perpendicular to plane of fin
t	thickness of spherical shell, cm
q/A	heat flux at inside surface of heat sink

A 200-WATT CONDUCTION-COOLED REACTOR POWER SUPPLY FOR SPACE APPLICATION

by

D. R. MacFarlane

ABSTRACT

The limited supply of relatively long-half-life isotopes having a reasonably high power density and the low conversion efficiencies obtainable with thermoelectric devices has so far limited the power output of isotope-fueled sources of electric power to several tens of watts. In addition, the high cost of the available isotopes results in a very large expense for isotope-fueled generators producing several hundred watts. It appears that a small, minimum-weight, conduction-cooled reactor is an attractive alternate to the isotope-fueled power supplies in the 200-w size range.

The proposed reactor is a small, high-density fast core of U^{233} surrounded by a beryllium reflector. This approach, generally speaking, gives a reactor that is more compact and of lighter weight than can be obtained with a moderated system having a softer neutron spectrum. In the reactor design, the path of heat flow is from the core to the inner reflector and then to the thermoelements in close contact with the inner reflector. The reject heat flowing from the thermoelement cold junctions enters the outer portion of the reflector, which acts as the heat sink and conducts the reject heat to the large, circular, tapered fin radiator which is attached to the reflector.

Survey physics calculations for various reactor systems fueled with U^{235} , U^{233} , and Pu^{239} are reported. Some limits imposed on the system design by the thermoelectric generator are discussed, and the problem of radiator design for the space environment is treated in some detail. No attempt is made to present a detailed final design of the power supply; rather, the report is restricted to a general delineation of the limits imposed by various parameters and a resulting final conclusion as to the performance limits of small conduction-cooled reactors in this size range.

I. INTRODUCTION

The selection of the type of power supply for a given space mission depends on the power level required and the mission duration. Generally speaking, the chemically fueled power supplies are the preferred energy sources for mission durations up to about one week. Within this domain on the time scale there is competition between the various chemically fueled systems, with the chemical dynamic converters and heat engines winning out at the higher power levels above about 1-10 kw, and batteries and fuel cells being more suitable at the lower power levels. For mission durations greater than one week, the chemical systems yield to solar power and energy from nuclear sources. Here there is competition between the nuclear fission sources and solar power supplies at different power levels. At lower power levels (0-100 w), solar static (photovoltaic cells) and radioisotope-powered energy sources are the best choice. Nuclear fission sources and solar dynamic (solar collectors and heat engines) systems are the most attractive in the intermediate power range; above power levels of about 50 kw, nuclear reactor power supplies are the only practical systems.

The question of concern in this report is the competition between nuclear reactor power supplies and radioisotope-fuel supplies in the low and intermediate range of power for mission durations of the order of one year. The actual determination of this transition point is somewhat different than the others in the time vs power domain because it is influenced by the actual quantities of radioisotopes which are available, in addition to the performance characteristics of the systems in question. First of all, the number of isotopes which are suitable for power application are relatively few and, secondly, the quantities available relative to power generation needs are small. As a study of Fig.1 will show, the specific powers of the isotopes which are suitable for power generation are, in some cases, greater than those for reactor power supplies above the heat-transfer-limited power density. This means that these isotopes would be competitive with reactor power supplies into the multimewatt power range if they were available in sufficient quantity. The maximum size of power supply which can be constructed as based on isotope heat is, therefore, determined by the considerations of isotope availability and considerations of the hazards of handling megacuries of radioactivity in a highly concentrated form. An additional factor, which may or may not be determining, is economics. The isotopes are quite expensive at present, and indeed this factor may force the selection of an alternate and cheaper system, even at the expense of performance. (The term performance as used throughout the report in this context is referring to the specific power of a given system or w/gm.)

The production quantities of some of the various isotopes suitable for power generation which are more readily available are given in Table 1. From these data it is apparent that the maximum size of power supply which would be constructed if a total year's production of Ce^{144} were available would be one or two kilowatts electric. It is clear that only Sr^{90} , Cs^{137} , and

Ce^{144} can be considered for power supplies in the several-hundred-watt-electric size range and that only a few such supplies can be built per year with present supply of these isotopes. The economic comparison of the isotope heat sources with a small reactor is interesting. Also listed in the Table 1 is the total cost of each of the radioisotopes for a power supply producing 200 w of electric power. As a figure for comparison, the fuel for a small reactor which might require a loading of 5-10 kg of fissionable material at a cost of \$15 per gram would cost from \$75,000 to \$150,000. Thus it appears that there is some economic incentive for using reactor-powered heat sources over isotope sources in such small sizes of power supplies.

Fig. 1

Specific Weights of
Reactor Heat Sources
and Isotope Heaters
(No Shielding)

Table 1

PRODUCTION RATES OF RADIOISOTOPES FOR POWER SOURCES

Isotope	Fuel Form	Half-life	Specific Power (w/gm)	Annual Oak Ridge Capacity (Ref 2) 1961				Cost \$/curie (Oak Ridge 1962)	Cost of Isotope for 200-watt Electric Supply (\$)
				curies/year	curies/watt	watts/year	watts/electric*		
Cs-137	CsCl	33 yr	0.323	500 000 [†]	320	1,562	78	0.50	640,000
Sr-90	SrTiO ₃	28 yr	0.112	250 000 [†]	153	1,634	82	0.75	459,000
Ce-144	CeO ₂	285 d	0.303	4 x 10 ⁶	128	31,250	1563	0.75	384,000
Pm-147	Pm ₂ O ₃	2.5 yr	0.167	500 000	2700	185	9	-	-

[†]New isotope production facilities at Hanford will produce an additional 1×10^6 c of Cs^{137} and 2×10^6 c of Sr^{90} by 1963

*Assuming a conversion efficiency of 5%

A further consideration in favor of a reactor is that the radiation hazard is considerably less since the amount of activity involved for isotope-powered heat sources in the 5000-w thermal range are in megacuries, whereas a reactor operating at 5000 w for one year contains only kilocuries of activity a few minutes after shutdown. Although the radioactive hazards from the destruction and dispersion of a reactor power supply are therefore less by a factor of 1000 than those of a radioisotope supply, it must be admitted that the shielding problem during operation may be more difficult with the reactor system.

With this background, it is worthwhile to study carefully the lower limit of power level for power supplies based on a reactor heat source. The next section of the report gives the design summary of a conduction-cooled reactor for producing 200 w of electric power. The proposed reactor system is competitive performance-wise with a Sr^{90} -fueled power supply and also appears to be cheaper.

Section III of the report gives the results of physics calculations for various reactor systems fueled with U^{235} , U^{233} , and Pu^{239} . The calculations are of a survey nature. Reactors with homogeneously distributed moderator and also fast-reflected systems were studied.

The balance of the report is concerned with the conceptual design and optimization of a conduction-cooled reactor system for producing 200 w of electric power. The power supply is designed for operation in space, and the problem of radiator design for this environment is treated in some detail. Consideration is given to the optimization of the radiator in terms of power dissipated per unit weight, both from the standpoint of the radiator geometry and the operating temperature. No attempt is made herein to present a detailed final design of the power supply; rather, the report is restricted to a general delineation of the limits imposed by various parameters and a resulting final conclusion as to the performance limits of small conduction cooled reactors in this size range.

II. REACTOR SYSTEM DESIGN SUMMARY

A sketch of the proposed reactor is shown in Fig. 2. This conceptual design was developed with the idea of providing a very simple power supply in space wherein all the heat removal is by conduction and radiation heat transfer, and, hence, there is no need for pumps, piping, coolant supplies, and associated hardware. The simplicity thus achieved is a means of obtaining improved system reliability for long-term unattended operation.

Fig. 2. Conduction-cooled Reactor System

Basically, the reactor is a small, high-density fast core surrounded by a beryllium reflector. This approach, generally speaking, gives a reactor that is more compact and of lighter weight than can be obtained with a moderated system having a softer neutron spectrum. In the reactor design shown, the path of heat flow is from the core into the inner reflector and then to the thermoelements in close contact with the inner reflector. The reject heat flowing from the thermoelement cold junctions enters the outer portion of the reflector, which acts as the heat sink and conducts the reject

heat to the large, circular, tapered-fin radiator which is attached to the reflector. The beryllium reflector thus serves a dual purpose in acting as a neutron reflector and as a heat sink for the radiator.

The calculations and designs presented herein are based on spherical geometry. This was chosen for convenience in analysis, but there is no loss of generality since the conclusions apply to a cylindrical reactor as well. The various parameters of the proposed reactor system are summarized in Table 2. The total weight of the 200-w power supply is 51 kg.

Table 2

REACTOR SYSTEM DESIGN SUMMARY

<u>Reactor</u>	
Fuel	U^{233}
Cold clean critical loading (kg)	4.3
Reflector material	beryllium
Reflector thickness (cm)	~10
Radius of core (cm)	4
Outer radius of reflector (cm)	15
Thermal power (w)	5000
Core center temperature ($^{\circ}C$)	764
Reactor weight (kg)	28
Control system	Separation of halves of reactor
<u>Thermoelectric Generator</u>	
Thermoelectric material	PbTe
Power output (w)	200
Output voltage (v)	12
Hot junction temp ($^{\circ}C$)	593
Cold junction temp ($^{\circ}C$)	343
Thermal efficiency (%)	4.0
Number of couples	207
Length of couple arms (cm)	0.68
Dia of n-couple (cm)	1.18
Dia of p-couple (cm)	1.57
Inner radius of generator (cm)	10
Outer radius of generator (cm)	11
Weight of thermoelectric material (kg)	4.0
Total weight of generator (kg)	10.0
<u>Radiator</u>	
Material	beryllium
Temperature ($^{\circ}C$)	343 (616 $^{\circ}K$)
Total power radiated (w)	4800
Fin thickness at base (cm)	5.0
Fin thickness at tip (cm)	0.10
Fin diameter (cm)	70
Fin weight (kg)	13
Total weight of power supply (kg)	51
Specific power of power supply (w/kg)	3.9

A. Reactor

The reactor is a solid sphere of U^{233} surrounded by a beryllium reflector with a nominal thickness of 10 cm. The reactor thermal power is 5000 w, and this gives a power of 200 w electric at a thermoelectric generator efficiency of 4%. The core center temperature for conduction cooling at this power level is 764°C . The reactor has no control rods and reactivity control is accomplished by moving the two halves of the reactor together or apart as required for startup and shutdown.

B. Thermoelectric Generator

The design of the generator is based on thermoelectric material of lead telluride, operating at hot junction temperature of 593°C and a cold junction temperature of 343°C . As shown in Fig. 2, the thermoelectric generator is located at an intermediate point in the beryllium reflector, where the heat flux coming from the reactor is low enough for the thermoelectric material to withstand without cracking. The portion of the beryllium reflector on the cold-junction side of the generator acts as a heat sink to conduct heat to the radiator fin. The total weight of the thermoelectric generator is 10 kg.

C. Radiator

As shown in Fig. 2, the radiator consists of a single, tapered, cylindrical fin attached to the outer surface of the reflector. Optimization studies described in a later section of the report show that this configuration gives the most efficient radiator for this particular power supply. The radiator temperature is 343°C (616°K) and the radiator fin weight is 13 kg. In this design 40% of the reject heat is dissipated from the surface of the sphere and 60% is radiated by the fin.

D. Control System

A simple and obvious method of controlling the reactor is to build it in two halves and move the halves apart by suitable control mechanisms. This method has the advantage of eliminating control rods and the problems associated with them, namely, those of flux perturbation and of cooling the rods. A method consisting of splitting the reactor through the center of the fin is probably the more advantageous from a mechanical design standpoint. The radiator fin itself can form the mounting plate for the necessary control motor and structure which will be required to effect the relative movement of the two reactor halves. Furthermore, once the reactor is brought to power, there should be relatively small amount of control needed by a mechanical control system, since the negative temperature coefficient of the system will effectively hold the power level. It is

possible, however, to devise a relatively simple temperature-sensing system which measures the temperature difference across a certain segment of the reactor core or reflector and feeds this as an input to the control system. Since this temperature difference is proportional to the total power of the system, a signal which is proportional to reactor power is provided. It is assumed that it is possible to build a simple control system which will contribute only a few kilograms to the total weight.

III. PHYSICS SURVEY

A. Homogeneously Distributed Moderator

The problem of finding a critical reactor system of minimum total weight was investigated by use of the DSN transport theory code to calculate U^{235} - and Pu^{239} -fueled systems at low moderator-to-fuel ratios. The following moderators were studied: lithium-7 hydride, zirconium hydride, light water, and beryllium. In all cases, the fuel was assumed to be homogeneously distributed in the moderator. The calculations were based on spherical geometry and criticality at room temperature. A sixteen-group cross-section set* was used throughout the calculation. The objective of these physics calculations was the selection of a critical reacting system of minimum total mass having suitable physical properties for the removal of at least 5000 w of thermal power by conduction cooling.

The results are given in Table 3 and Fig. 3. As shown, only the Li^7H - and light-water-moderated systems show a reduction in weight over a bare unmoderated sphere of U^{235} . Addition of beryllium or zirconium hydride moderators causes an increase in the total weight of the reactor over the bare sphere of the U^{235} (fast) system. Likewise with the plutonium-fueled system studied, namely, plutonium moderated with

Table 3

RESULTS OF DSN CRITICAL-RADIUS* CALCULATIONS FOR U^{235} AND Pu^{239} IN VARIOUS MODERATORS

Problem Number	Fuel	Enrichment (%)**	Moderator	Mol Ratio R (mols mod / mol fuel)	Critical Radius (cm)	Fuel Loading (kg)	Total Weight Fuel + Mod (kg)
92	U^{235}	90	Li^7H	1	11.8	68.5	70.8
93				5	16.5	64.7	75.6
35				10	17.8	44.9	60.1
42				50	18.1	10.2	27.5
36		100		100	18.3	5.4	23.4
43		90		200	20.5	3.8	29.3
88			ZrH_{10}	1	12.8	73.0	101.6
95				2	14.9	75.3	134.5
89				5	18.2	66.0	195.5
94				15	20.0	32.2	221.7
39				50	20.2	10.3	212.7
40				100	21.2	6.0	240.8
41				200	23.7	4.2	335.1
90			H_2O	1	12.2	58.3	62.8
91				5	15.3	34.1	47.1
46				10	15.3	18.2	32.2
47				50	15.5	4.0	19.4
48				100	16.8	2.6	22.3
86			Be	1	10.5	64.8	67.3
87				5	14.2	76.4	91.0
52				10	16.8	75.8	104.9
44		90		50	24.0	52.9	154.3
38		100		100	25.3	32.1	155.1
45		90		200	28.4	22.8	197.0
56	Pu^{239}	96		5	10.0	27.6	32.8
57				10	12.9	35.3	48.7
58				50	21.4	38.2	110.5
59				100	25.8	34.2	163.4
60				200	31.0	30.1	257.4
49	U^{235}	90	none	-	8.9	55.7	55.7
50	Pu^{239}	96	none	-	5.0	10.1	10.1

*Spherical geometry

**Balance is U^{238} for U^{235} systems and Pu^{240} for the Pu^{239} systems

*Set 201 worked out by C. Cohn from the Hansen and Roach data.(3)

beryllium, the addition of beryllium moderator increases the total weight of the reactor over that of a bare sphere of plutonium (fast) system. Since these systems are under moderated, their neutron spectra are epithermal with most of the neutron flux above group eight (see Table 5 for group structure definition).

Fig. 3. Total Critical System Weight for U^{235} - and Pu^{239} -fueled Homogeneous Reactors Using Various Moderators

As shown in Fig. 3 the Li^7H - and light-water-moderated systems give relatively low weights for reactor systems of about 24 kg and 19 kg, respectively. Unfortunately, both these systems have drawbacks which make them less attractive than their physics parameters would indicate. Lithium hydride has a very low thermal conductivity, which prohibits the removal of the desired quantity of power by conduction cooling. Obviously, any light-water-moderated system requires some sort of pressure vessel for the attainment of high operating temperatures. This drawback plus the fact that relatively limited amounts of power can be removed from a light-water-moderated, homogeneous uranium system by conduction cooling immediately prompts one to think of alternate systems. As shown by the results of the calculations, the other moderators, namely, beryllium and zirconium hydride, lead to heavier systems than would be attainable with the use of just bare unmoderated spheres of fissionable material. Therefore, to obtain a system of minimum total mass it is undesirable to use these moderators.

B. Reflected Fast Systems

Since the reflector savings in small fast reactor systems are approximately equal to the reflector thickness for moderate reflector thicknesses, it appears that a weight saving can be obtained by use of a low-density reflector such as beryllium; consequently, this possibility was investigated next. Calculations were done for fast reactor systems fueled with Pu^{239} , U^{233} , and U^{235} metals, and the oxides and carbides of these fuels for bare spheres, and spheres reflected with 2, 5, and 10 cm of beryllium. Beryllium is superior to all the other materials considered as reflectors because of its high melting point, light weight, and good thermal conductivity. The results of these calculations are given in Table 4 and are plotted in Fig. 4. The curve for plutonium is based on the density of alpha-phase plutonium, 19.7 gm/cm^3 . Because of difficulty of stabilizing this phase at the elevated temperatures required for power generation, the curve for plutonium metal systems has a rather academic interest, at least in the present

Table 4

RESULTS OF DSN CRITICAL-RADIUS* CALCULATIONS FOR U^{235} , U^{233} , AND Pu^{239} FAST REACTORS WITH BERYLLIUM REFLECTORS

Problem Number	Fuel	Enrichment (%)**	Beryllium Reflector Thickness (cm)	Overall Critical Radius (cm)	Fuel Loading (kg)	Total Weight of System (kg)	Core Power Distribution $\left(\frac{P_{\text{center}}}{P_{\text{edge}}}\right)$
85	U^{235}	100	0	8.4	46.0	46.0	-
82	↓	↓	2	9.0	27.8	30.9	2.3
83	↓	↓	5	11.0	17.3	26.0	1.6
84	↓	↓	10	15.3	12.0	38.8	1.06
69	U^{235}C	90	0	12.5	96.3	101.3	-
78	U^{233}	100	0	5.8	15.2	15.2	-
79	↓	↓	2	6.8	8.9	10.5	2.2
80	↓	↓	5	9.2	6.0	11.5	1.6
81	↓	↓	10	13.8	4.3	24.1	1.2
74	U^{233}C	↓	0	8.4	29.1	30.6	-
75	↓	↓	2	9.0	16.9	20.7	2.2
76	↓	↓	5	11.0	10.3	19.3	1.5
77	↓	↓	10	15.1	6.6	32.6	1.04
50	Pu^{239}	96	0	5.0	10.1	10.1	-
53	↓	↓	2	6.2	6.2	7.4	2.1
54	↓	↓	5	8.8	4.5	9.3	1.8
55	↓	↓	10	13.5	3.4	22.0	1.4
68	Pu^{239}C	↓	0	7.7	22.5	23.7	-
71	↓	↓	2	8.5	13.3	16.6	2.2
72	↓	↓	5	10.6	8.7	17.5	1.7
73	↓	↓	10	15.0	6.0	31.4	1.2
66	$\text{Pu}^{239}\text{O}_2$	↓	0	9.5	31.4	35.6	-
64	$\text{Pu}^{239}\text{Fe}_3$	↓	0	11.2	42.3	71.9	-
65	$\text{Pu}^{239}\text{Ni}_3$	↓	0	10.7	38.8	67.4	-

*Spherical geometry

**Balance is U^{238} for U^{235} systems and Pu^{240} for the Pu^{239} systems

connection. However, it does serve to show the minimum weight for any fast system reflected with beryllium. As shown in the figure, it is possible to obtain a critical reactor system with a total mass of only 7.2 kg.

Fig. 4. Total Critical System Weight for Solid Spherical Fast Reactors with Varying Beryllium Reflector Thickness

C. Neutron Spectra and Flux Distribution in Reflected Fast Systems

The neutron spectrum data for several of the fast-reflected cores are given in Table 5. The spectra are quite hard at the center of the core, with practically all the flux being in the first eight energy groups. As shown, there is some spectrum softening at the edge of the core next to the reflector, and this softening is greater with the thicker reflector, as expected. With the thicker (10-cm) reflector, the spectrum shift gives

a considerably flatter core power distribution than would be expected from considerations of geometric flux flattening alone. For example, the center-to-edge flux ratio for a U^{235} metal core with a 10-cm beryllium reflector is 1.81, whereas the corresponding power ratio is 1.06. As shown by the data of Table 4, this effect is apparent for all the systems with a 10-cm reflector.

Table 5
NEUTRON SPECTRA FOR FAST BERYLLIUM-REFLECTED CORES

Problem Number		75		77		82		84	
Fuel		U ²³³ C		U ²³³ C		U ²³⁵		U ²³⁵	
Reflector Thickness (cm)		2		10		2		10	
Group	Energy Range	ϕ _j		ϕ _j		ϕ _j		ϕ _j	
		Center	Edge	Center	Edge	Center	Edge	Center	Edge
1	3-∞ Mev	0.142	0.109	0.133	0.074	0.127	0.096	0.122	0.063
2	1.4-3	0.274	0.243	0.261	0.186	0.242	0.211	0.236	0.162
3	0.9-1.4	0.152	0.147	0.147	0.116	0.151	0.143	0.147	0.113
4	0.4-0.9	0.227	0.234	0.223	0.194	0.254	0.258	0.247	0.207
5	0.1-0.4	0.168	0.191	0.178	0.197	0.196	0.218	0.203	0.216
6	17-100 kev	0.033	0.059	0.048	0.101	0.030	0.060	0.040	0.103
7	3-17	0.002	0.013	0.008	0.050	0.0005	0.011	0.004	0.050
8	0.55-3	0	0.003	0.001	0.028	0	0.002	0.0004	0.028
9	100-550 ev	↓	0.0006	0	0.017	↓	0.0007	0	0.017
10	30-100	↓	0.00012	↓	0.009	↓	0.00009	↓	0.009
11	10-30	↓	0	↓	0.006	↓	0	↓	0.007
12	3-10	↓	↓	↓	0.005	↓	↓	↓	0.005
13	1-3	↓	↓	↓	0.003	↓	↓	↓	0.004
14	0.4-1	↓	↓	↓	0.002	↓	↓	↓	0.003
15	0.1-0.4	↓	↓	↓	0.002	↓	↓	↓	0.003
16	Thermal (0.025)	↓	↓	↓	0.009	↓	↓	↓	0.009

D. Conclusions

These results show that it is possible to obtain relatively light-weight reactors when considering fast systems with a beryllium reflector. All the systems with the exception of those fueled with U^{235} were shown to have a minimum between 2 and 4 cm of reflector. The U^{233} - and Pu^{239} -fueled systems are interesting ones from the standpoint of providing reactors of minimum weight. In considering which of the two materials would be the best choice for a small power plant, a comparison of the physical properties of the fuels is also necessary (see Section IV). Plutonium, unfortunately, has a very low melting point, and therefore it is not suitable for this application, since the quantity of power which could be removed by conduction without melting the core is relatively low. On the other hand, U^{233} is a satisfactory fuel and would meet the requirement on heat removal. Hence, metallic U^{233} metal is a clear choice over plutonium metal.

The U^{233} carbide likewise has a good thermal conductivity and a much higher melting point than uranium metal; hence, it is also an interesting system for consideration. The situation with regard to plutonium carbide is somewhat unclear at this time. Plutonium carbide has a lower melting point than uranium carbide, although it is still reasonably high. The real uncertainty is the thermal conductivity of plutonium carbide. The final determination of the usefulness of plutonium carbide must await good measurements of its thermal conductivity.

The oxides of both plutonium and U^{233} have too low a thermal conductivity to be useful in conduction-cooled reactors of this type.

IV. LIMITATIONS OF HEAT REMOVAL BY CONDUCTION

A. Unreflected Heat Source

As already stated, in order to have a simple power plant in space, the reactor should not require auxiliary coolant loops and therefore should be cooled by conduction of heat to the thermoelectric conversion elements on its surface. Reject heat from the thermoelements is then dissipated by radiation. It is assumed that the reactor is a sphere having thermoelements in good contact with its surface. The reactor surface temperature is 593°C at the thermoelectric generator hot junction.

The solution of the heat-conduction equation for a sphere with the sine r/r_0 heat-generation rate (unreflected sphere) is given by Carslaw and Jaeger⁽⁴⁾ as

$$t = \frac{A_0 a}{\pi k} \frac{a}{\pi r} \sin \frac{\pi r}{a} + t_0 \quad , \quad (1)$$

where

$$A_0 = q/4a^2 \quad ; \quad t_0 = \text{temperature at outer boundary of sphere} \quad .$$

By Eq. (1), the total temperature difference across the sphere is obtained as

$$\Delta t = \frac{q}{4\pi k a} \quad ,$$

so that

$$q = 4\pi a k \Delta t \quad (2)$$

since

$$\lim_{r \rightarrow 0} \left(\frac{a}{\pi r} \sin \frac{\pi r}{a} \right) = 1.0 \quad .$$

The average power density in a sphere with the sine function heat generation is given by

$$\bar{Q} = q/V = 3k\Delta t/a^2 \quad . \quad (3)$$

The average specific power $\bar{P}$ is obtained by dividing by the density,

$$\bar{P} = 3k\Delta t/\rho a^2 \quad . \quad (4)$$

Thus, Eqs. (2), (3), and (4) characterize a spherical heat source in terms of its total temperature difference, thermal conductivity, and radius. For a sphere with uniform heat generation, the temperature difference is reduced by a factor of two for a given total heat q ; hence, the factor in Eq. (2) would be 8 instead of 4 for the case of uniform generation. Equations (2) and (3) are plotted in Fig. 5, and Eq. (4) is plotted in Fig. 6.

Fig. 5. Power Density for Spherical Heat Sources - Heat Removal by Conduction (Sphere Surface Temp = 593°C)

Fig. 6. Specific Power for Spherical Heat Sources - Heat Removal by Conduction (Sphere Surface Temp = 593°C)

The solid lines in Fig. 5 give average power densities $\bar{Q}$ for various values of the parameter $k\Delta t$. The dotted lines give the total heat removed from the sphere. Figure 6 gives the average specific power $\bar{P}$ for various values of the parameter $k\Delta t/\rho$.

The minimum radii for various reactor systems are determined by criticality considerations and are given in Tables 3 and 4. The maximum values of $k\Delta t$ for the various systems are based on a sphere surface temperature of 593°C and the center temperature at the melting point of the material. The values of these parameters are listed in Table 6. These values are also plotted in Figs. 5 and 6 to show the relative positions of the various systems. The horizontal bars in the figures show the ranges of permissible total power and power density up to center melting for minimum critical spheres of the various materials. The dotted extensions of the bars show the increase which is obtained if power generation has a uniform rather than sine function distribution. The results of Fig. 5 show that it is possible to obtain a total thermal power in the range of several tens of kilowatts by conduction cooling from a number of systems, if a core center temperature near the melting point is permitted. The power densities are fairly low, however, the highest being about 20 w/cm^3 .

Table 6

THERMAL CONDUCTIVITIES AND MELTING POINTS OF FUEL AND REFLECTOR MATERIALS

Material	Uranium	Uranium Carbide	Uranium Oxide	Plutonium*	Plutonium Carbide	Plutonium Oxide	Be	ZrH	LiH	SrTiO ₃
Melting Point, $^\circ\text{C}$	1132	2400	2800	680	1654	2400	1284	1093 ~	688	1670
Thermal Conductivity, $\text{w/(cm)}(^\circ\text{C})$, at 300°C	0.306 400 0.331 500 0.364 600 0.398 700 0.435 800 0.477 900 0.536	0.251 ↓	0.050 0.040 0.037 0.033 0.030 0.027 0.025	0.138 0.159 0.188 0.209	0.063† ↓	assumed same as UO ₂	1.49 1.40 1.32 1.24 1.15 1.07 0.99	0.225 ↓	0.0510 0.0453 0.0414	0.045 ↓
$k\Delta t$ (max) - 593 to M.P., w/cm	243	454	55	18	67	45	691	113	3.8	49
ρ , gm/cm^3	18.8	12.3	10.0	19.7	12.3	10.0	1.85	6.0	0.71	5.0
$\frac{k\Delta t}{\rho}$, $(\text{w/cm}^2)/\text{gm}$	13	37	5	1	5	4	374	19	5	10

*Pu, 1% Al alloy; personal communication from R. Dunworth

**Since ZrH does not have a sharp melting point, the temperature is taken at the point where the vapor pressure is 5 atm

† Estimated from electrical conductivity measurements, personal communication from O. Kruger

Figure 6 gives similar plots for the specific power of the various reactors. As shown, the highest specific power obtainable with conduction cooling is considerably below a high-power-density, liquid-metal-cooled core.

B. Reflected Heat Source

For the reflected fast systems it is also necessary to consider the temperature drop through the beryllium reflector to the thermoelement hot junction. As shown in Section V, the smallest permissible inner radius for the thermoelectric generator is 10 cm; hence, the thickness of the reflector between the reactor core and the generator for each of the reactors listed in Table 4 is 10 cm minus the radius of the particular reactor.

Values of the temperature difference from the edge of the core to the thermoelement hot junction are plotted in Fig. 7. The equation giving this temperature difference for spherical geometry is

$$\Delta t = \frac{q}{4\pi k} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (5)$$

Through use of the results of Figs. 5 and 7 and the data of Table 6, it is possible to calculate the core center temperature for the beryllium-reflected systems at various power levels. The results of these calculations are given in Table 7. From these results it is evident that removal

Fig. 7. Temperature Difference Across Beryllium Reflector as a Function of Core Radius for Various Reactor Power Levels

of 5000 w of thermal power by conduction from the minimum weight U^{233} metal, U^{233} carbide, or U^{235} metal-fueled fast reactors is feasible. Plutonium oxide and carbide systems, based on assumptions as to their respective thermal conductivities, appear to be very close to the limit at an output of 5000 w. Furthermore, the very high centerline temperature required for the plutonium oxide presents a greater problem with respect to core stability and containment.

Table 7

CORE CENTER TEMPERATURE FOR VARIOUS CONDUCTION-COOLED
CORES WITH A BERYLLIUM REFLECTOR*

Core Material	Melting Point (°C)	Critical Radius (cm)	Core Center Temperature (°C)		
			5,000 w	10,000 w	20,000 w
U^{233} metal	1132	3.8	764	935	>M.P.
$U^{233}C$	2400	5.1	783	973	1353
U^{235} metal	1132	5.3	707	821	1049
PuO_2	2400	9.5**	2263	>M.P.	-
PuC	1654	5.0	1260	>M.P.	-

*Based on $t = 593^\circ C$ at $r = 10$ cm in the beryllium reflector

**No beryllium reflector

V. DESIGN OF THERMOELECTRIC CONVERTER

The most convenient and practical power-conversion device to use in the sub-kilowatt power range for the generation of electric power is the thermoelectric converter; the advantage is the lack of moving parts and, hence, the simplicity and potentially greater reliability associated with the elimination of moving parts. There are a large number of thermoelectric materials under study today for power-conversion application. There is, however, only one material, lead telluride, which is truly operational from the standpoint of having operated satisfactorily in power supplies and having fairly well-known operating limits and properties. It is true that there are a number of materials having higher figures of merit and higher upper limits of operating temperature, and hence potentially higher conversion efficiency than lead telluride, so that there is the possibility of improved performance when these materials are tested and proven. For the present study, however, in order to keep the designs of the system closer to the areas of proven technology, the design of the thermoelectric converter is based on lead telluride.

A. Equations for Maximum Power Output

The equations are developed using the following assumptions:

1. no electric contact resistance at couple-junction interfaces;
2. no thermal resistances at hot- and cold-junction interfaces.

It can be shown⁽⁵⁾ that the maximum power output is obtained from a thermoelectric generator when the internal resistance of the generator is matched to the load resistance, that is, when

$$R = R_L = R_g \quad .$$

This leads to the following equation for the voltage of a power-maximized generator:

$$E_0 = \frac{1}{2} \Delta T \alpha N \quad . \quad (6)$$

The current is given by

$$I = N \alpha \Delta T / 2R \quad . \quad (7)$$

This leads to the following expression for the power output:

$$P_0 = I^2 R = (N \alpha \Delta T)^2 / 4R \quad . \quad (8)$$

The internal resistance of the generator is given by

$$R_g = \frac{N^2 \ell}{A_t} (1+y) \left(\frac{\rho_n}{y} + \rho_p \right) \quad , \quad (9)$$

where

$$y = \sqrt{\rho_n / \rho_p}$$

for maximum power output and the thermal resistance of the generator is given by

$$r_t = \frac{\ell}{A_t} \left(\frac{1+y}{y k_n + k_p} \right) \quad . \quad (10)$$

The heat input to the generator is given by the following expression:

$$q_{in} = \underbrace{N \alpha_h T_h I}_{\text{Peltier Heat}} + \underbrace{(\Delta T / r_t)}_{\text{Conduction}} - \underbrace{\frac{1}{2} I^2 R}_{\text{Joule Heat}} \quad . \quad (11)$$

Substitution of Eqs. (7), (9), and (10) into Eq. (11), and the assumption that α_h is equal to α_{avg} lead to the following expression for the heat input to the generator:

$$q_{in} = \frac{A_t \Delta T}{\ell(1+y)} \left[\frac{4\alpha^2 T_h + 8(y k_n + k_p) \left(\frac{\rho_n}{y} + \rho_p \right) - \Delta T \alpha^2}{8 \left(\frac{\rho_n}{y} + \rho_p \right)} \right] \quad . \quad (12)$$

Substitution of Eq. (9) into Eq. (8) gives the following expression for the power output of the generator:

$$P_0 = \frac{A_t \alpha^2 \Delta T^2}{4 \ell (1+y) \left(\frac{\rho_n}{y} + \rho_p \right)} \quad . \quad (13)$$

Division of Eq. (13) by Eq. (12) gives the following expression for the efficiency of the generator:

$$\eta = \frac{2 \Delta T}{4 T_h + \frac{8}{\alpha^2} (y k_n + k_p) \left(\frac{\rho_n}{y} + \rho_p \right) - \Delta T} \quad . \quad (14)$$

Rearrangement of Eq. (13) gives the following expression for the length of the couples arms of the generator:

$$\ell = \frac{A_t \alpha^2 \Delta T^2}{4P_0 (1+y) \left(\frac{\rho_n}{y} + \rho_p \right)} \quad (15)$$

The total volume of thermoelectric material is then given by

$$\ell A_t = \frac{(A_t \alpha \Delta T)^2}{4P_0 (1+y) \left(\frac{\rho_n}{y} + \rho_p \right)} \quad (16)$$

and the weight of the thermoelectric material is then given by multiplying Eq. (16) by the density of the thermoelectric material:

$$W = \frac{d(A_t \alpha \Delta T)^2}{4P_0 (1+y) \left(\frac{\rho_n}{y} + \rho_p \right)} \quad (17)$$

These equations characterize the thermoelectric power supply in terms of the properties of thermoelectric material, the temperature difference across the thermoelements, the hot-junction temperature, and the input heat flux to the thermoelements.

B. Results of Design Calculations

The above equations were used to calculate the performance limits of a power supply surrounding a spherical heat source and producing 200 w at 12 v. The negative couple used was lead telluride doped with 0.10% PbI_2 and the positive couple lead telluride doped with 1.0% sodium. Figures 8, 9, and 10 give properties of these materials as a function of temperature. For the calculations, average properties were determined for each cold-junction temperature and a constant hot-junction temperature of 593°C.

Fig. 8. Plot of Seebeck Coefficient vs Temperature for PbTe Couple

Fig. 9. Plot of Thermal Conductivity vs Temperature for PbTe Couple

Fig. 10. Plot of Resistivity vs Temperature for PbTe Couple

Figures 11 and 12 give results of the calculations with Eqs. (14), (15), and (17). Figure 11 is a series of curves computed for a packing fraction of the thermoelectric material around the spherical heat source of 0.75. This is close to the maximum packing fraction which can be expected from a physical standpoint in an actual generator. Figure 12 gives a similar series of curves for a packing fraction of 0.50. In both curves the ordinate is the total weight of the generator in kilograms and the abscissa is the temperature difference across the thermoelements and/or the efficiency. As can be seen from the curves of Figs. 11 and 12, change of the packing fraction from 0.75 to 0.50 merely shifts the set of curves corresponding to the various heat-source radii downward. The significance of this is that some flexibility is available in matching the size of the heat source to the desired generator configuration by varying this param-

eter. In a practical generator, a packing fraction of 0.50 is probably the more reasonable number to use. Obviously, too small a packing fraction is undesirable, since this increases the possibility of direct heat leakage from the hot to the cold junction via the insulation surrounding the thermoelements. For values of the packing fraction used in these calculations, this heat leakage is small enough to be neglected.

It should be pointed out that the data given by Figs. 11 and 12 really correspond to a minimum weight figure for the generator, since it includes only the weight of thermoelectric material and the weight of the insulation surrounding the thermoelement plus about a 10% additional weight allowance for structural material. In some of the experimental generators built so far, the total weight of the generator has run as high as four or five times the weight of the thermoelectric material contained in it; however, in those designs no particular attempt was made to minimize weight. It is believed, therefore, that the weights of thermoelectric materials given in Figs. 11 and 12 multiplied by a factor of two or three would give reasonable values for the weight of a complete generator.

The two really important parameters in limiting the performance of a generator are the hot-junction temperature and the permissible

Fig. 11. Weight of Lead Telluride Thermoelectric Material for a 200-w Generator; Packing Fraction $X = 0.75$

Fig. 12. Weight of Lead Telluride Thermoelectric Material for a 200-w Generator; Packing Fraction $X = 0.50$

temperature gradient along the length of the thermoelement. The hot-junction temperature, of course, influences the overall thermal efficiency of the generator; therefore, it is desirable to have this as high as possible. For the calculations given in Figs. 11 and 12 the hot junction was taken to be constant at 593°C , about the upper operating limit for lead telluride. The second parameter, namely, the permissible temperature gradient in the thermoelement, influences the weight of the generator for a given power output. With lead telluride, the limit is a gradient of about $300^{\circ}\text{C}/\text{cm}$. Values greater than this produce excessive stresses in the material and thereby decrease its operating lifetime greatly. This gradient corresponds to an input heat flux of about 5 or 6 w/cm^2 for lead telluride. It appears that this sort of limitation will be found in most thermoelectric materials, since the strengths of most of the semiconductor materials have been likened to that of plaster, and it does not appear that they will be able to withstand the large temperature gradients which are necessary to obtain a really high power density.

C. Integration of Generator with Reactor

It is also necessary to consider the limitations imposed by the thermoelectric converter on the selection of a particular reactor system. This limitation results from the fact that the maximum heat flux which can be accepted by the thermoelectric converter from a heat source imposes a minimum radius for the heat source. This maximum heat flux input results from the maximum temperature-gradient limitation of the thermoelements. As a look at Figs. 11 and 12 shows, reduction of the radius of the heat source at a given thermal efficiency requires a corresponding reduction in the lengths of the thermoelements, and these two effects combine to give a lower total weight of thermoelectric material for a given power output. For an outer radius of the heat source of 10 cm, with a packing fraction of 0.5, and a ΔT across the thermoelectric generator of 250°C , Fig. 12 gives the required length of the thermoelements as 0.7 cm. This length gives a gradient of about $360^{\circ}\text{C}/\text{cm}$. The conclusion is that the inside radius of the 200-w thermoelectric generator for a spherical configuration cannot be less than about 10 cm.

None of the beryllium-reflected reactor cores listed as giving attractive minimal-weight systems have a radius for the fissionable material portion of the core as large as 10 cm (see Table 4). All of these systems, therefore, can be combined with the thermoelectric generator by placing the generator in the beryllium reflector. The minimum inner radius of 10 cm selected for the generator plus the length of the thermoelements of 0.7 cm gives an outer radius of 10.7 cm for the generator. If a heat-sink thickness of 3 to 4 cm of beryllium is required, this brings the outer radius of the system to about 14 or 15 cm.

Further discussion will center on the U^{233} -fueled system which has a weight of 24.1 kg and a critical loading of 4.3 kg U^{233} . Since the critical radius of the small fast core is only 3.8 cm, there is ample space for the addition of more fuel which will be required for the excess reactivity to compensate for temperature change and due to the probable addition of some alloying element with the uranium fuel to form a more stable fuel alloy. This is generally true of the other systems also, since their critical radii are less than the 10-cm minimum radius for the thermoelectric generator. From the data in Fig. 12 it can be seen that the total thermoelectric material required for the generator weighs about 4 kg. Assuming that $2\frac{1}{2}$ times the weight of thermoelectric material is required as additional structural material in the generator, this gives a total weight for the generator of 10 kg. The characteristics of the thermoelectric generator are tabulated in the design summary (Table 2).

An additional problem which must be considered in connection with the thermoelectric elements is the radiation damage which they may incur in operation. This problem has been studied by J. C. Danko et al.,⁽⁷⁾ and, although it is not possible to draw final conclusions from their work, the results do indicate that the thermoelectric generator will not be adversely affected by the radiation provided the coldest part of the thermoelectric material is above a minimum annealing temperature. In the U^{233} -fueled reference design reactor (see Table 2), the peak flux at the core center is 6.8×10^{12} n/(cm²)(sec). For a year of operation this is an integrated flux of $\sim 2 \times 10^{20}$ nvt. Danko et al. irradiated lead telluride to integrated fast flux doses of 1.5×10^{20} nvt and found that most of the fast neutron damage was annealed out on reheating at 300°C. While these results do not permit a final judgment to be made, they are encouraging.

VI. RADIATOR DESIGN

A. General Criteria for Space Radiator Design

In the foregoing discussions of reactor systems and design of thermoelectric converters for a small power supply, all of the calculations were based on spherical geometry. Accordingly, the calculations discussed in this section are concerned with the problem of dissipating the reject heat from a spherical heat source. In the conduction-cooled reactor producing 5000 w thermal, the reject heat flux which must be removed from the generator cold junction is larger by about a factor of three than that which can be dissipated by a reasonable radiator temperature. This means that the radiating surface must be increased or extended by means of fins. Since it is desirable to maintain as simple a configuration as possible, a logical design to achieve this is to provide a spherical heat sink at the generator cold junction and to allow the heat to be conducted from this sink into fins placed on its surface, and thereby dissipated to space.

Because of the current interest in space power systems, a fair amount of work has been done on the design of radiators for these systems. Of particular interest in the present connection is a paper by Sparrow et al.,⁽⁸⁾ in which they show that the maximum performance for space radiators is generally obtained with flat plate or planar geometry. This result is due to the view factor considerations and mutual irradiation between fins which reduces the power dissipated per unit weight when the geometry of the radiator is not planar. The results of the analysis of Sparrow et al. indicate that the radiator with highest performance is a single radial fin extending out from the equator of the sphere. Fortunately, this design is also easier to analyze than a multifinned radiator with mutual interactions between fins.

Figure 2 shows the model and geometry upon which the following analysis is based. Two cases are considered, namely, a radial fin of constant cross section from base to tip, and a radial fin with uniformly varying cross section from the base to the tip. It was realized upon analyzing the case of uniform cross section that considerable advantage would be gained in tapering the fin, since the outer portions of the fin furthest from the heat source are obviously less effective radiating surfaces. Increasing the temperature gradient slightly at this point by tapering the fin does not greatly affect the heat transfer performance, but does give substantial weight saving. The net effect is an increase in power radiated per unit weight of radiator. It is likely that some further optimization can be effected by employing a nonlinear tapering function; however, this avenue was not investigated in the present work. In any event, it does not appear that further incremental gains will be large compared with those obtained in going from a fin of uniform cross section to the uniformly tapered fin.

B. Development of Equations

The following assumptions were used in deriving the equations used to calculate the performance of the radiators:

1. The usual assumption in fin calculations was made, namely, that the temperature gradient across the fin thickness is small and, relative to the gradient along its length, can be neglected.
2. The temperature dependence of the properties of the fin material is neglected insofar as average properties for the temperatures involved are used.
3. Mutual radiation between the fin and the spherical heat source can be neglected, since, in general, the outer radius of the fin is three to five times the radius of the heat source. Support for the validity of this assumption is given by the fact that the area of a cone formed by drawing the tangent from the sphere to the tip of the fin is greater than 90% of the area of the fin and sphere combined, for most cases considered.
4. The fin radiates to a temperature of 0°K.
5. The boundary condition at the base of the fin is $T = T_b$ at $r = r_1$.
6. The usual boundary condition at the outer tip used in fin problems, namely, that no heat is transferred through the tip of the fin, or $dT/dr = 0$ at $r = r_2$.

With the use of these assumptions, the following differential equation can be written for the temperature distribution in a fin of uniform cross section:

$$\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} - AT^4 = 0 \quad , \quad (18)$$

where

$$A = 2\sigma\epsilon/kx \quad .$$

Equation (18) is the usual type of nonlinear differential equation encountered in the design calculations of space radiators. Of ultimate interest in the solution is the total power dissipated by a given fin, and this quantity is given by the following equation:

$$q = 4\pi\sigma\epsilon \int_{r_1}^{r_2} rT^4(r) dr \quad . \quad (19)$$

Hence, having solved for the temperature distribution by means of Eq. (18), it is possible to calculate the total power dissipated by a fin by means of Eq. (19).

The differential equation describing the temperature distribution in the uniformly tapered fin is obtained by a method similar to that given by Janicke and Just⁽⁹⁾ for a straight (non-circular) tapered fin. The equation for a circular tapered fin is

$$y(r) \left(\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right) + \frac{dT}{dr} \frac{dy(r)}{dr} - AT^4 = 0 \quad (20)$$

where

$$y(r) = y_1 - \frac{y_1 - y_2}{r_2 - r_1} (r - r_1) \quad .$$

In a similar manner, the total power dissipated by a fin can then be calculated by use of Eq. (19).

C. Results of Fin Calculations

Equations (18), (19), and (20) were programmed for solution on the Pace analog computer.* Beryllium metal is an obvious choice for the radiator material since it has a high melting point, a good thermal conductivity, and is very light in weight. All of the results reported here are based on beryllium as the radiator material and a radiator surface emissivity of 0.85. Calculations were run for each type of fin at two different base temperatures: 600°K and 700°K.

1. Uniform Fin

The results of calculations for the fin of uniform cross section are given in Table 8. A series of curves representing these calculations are presented in Figs. 13 and 14. These figures are plots of specific power (w/gm) dissipated vs the total radiator power. As can be seen from the curves, a radiator with a given fin thickness shows a rapidly decreasing performance as the total power increases for the thinner fins and less rapidly decreasing performance with increasing power for the thicker fins. Indeed, for the very thin fins, an upper power limit is approached rapidly, and increasing the diameter of the fin has practically no effect on the total power dissipated. This result is due to the fact that the temperature gradient has reduced the temperature at the tip of the fin to such a low value that negligible quantities of heat are being radiated by the additional surface. The curves for the two temperatures are similar in shape, and the only difference is that the curves of Fig. 14, for a temperature of 700°K, are shifted upward.

*Work done by Louis C. Just

Table 8

SUMMARY OF CALCULATIONS FOR FINS OF UNIFORM CROSS SECTION

	Inner Radius (cm)	Outer Radius (cm)							
	12	22	32	42	52	62	72	82	92
Temperature (°K)	600	543	456	391	344	307	279	256	238
Fin Thickness (cm)		0.78	0.74	0.74	0.74	0.74	0.70	0.70	0.70
Fin Weight (gm)		1.458	3.789	6.975	11.020	15.930	20.360	26.690	33.750
Total Power									
Radiated (w)		1.002	1.546	1.787	1.900	1.966	2.012	2.033	2.052
Specific Radiator Power (w/gm)		0.69	0.41	0.26	0.17	0.12	0.099	0.076	0.061
Temperature (°K)	600	567	501	442	395	357	327	304	282
Fin Thickness (cm)		1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4
Fin Weight (gm)		2.919	8.025	13.930	22.020	31.820	42.930	53.470	67.670
Total Power									
Radiated (w)		1.126	2.019	2.515	2.790	2.956	3.060	3.159	3.195
Specific Radiator Power (w/gm)		0.39	0.25	0.18	0.13	0.093	0.071	0.059	0.047
Temperature (°K)	600	593	570	537	504	472	444	422	396
Fin Thickness (cm)		7.8	7.8	7.4	7.4	7.4	7.4	7.3	7.3
Fin Weight (gm)		14.500	39.800	69.700	110.300	159.300	216.000	279.100	352.900
Total Power									
Radiated (w)		1.277	2.947	4.503	5.774	6.706	7.456	8.170	8.412
Specific Radiator Power (w/gm)		0.088	0.074	0.065	0.052	0.042	0.035	0.029	0.024
Temperature (°K)	700	608	493	415	363	321	293	266	247
Fin Thickness (cm)		0.80	0.75	0.75	0.73	0.73	0.71	0.71	0.71
Fin Weight (gm)		1.490	3.850	7.090	10.900	15.700	20.710	27.100	34.300
Total Power									
Radiated (w)		1.652	2.324	2.572	2.699	2.772	2.808	2.832	2.844
Specific Radiator Power (w/gm)		1.1	0.60	0.36	0.25	0.18	0.14	0.11	0.083
Temperature (°K)	700	644	552	476	420	378	344	317	294
Fin Thickness (cm)		1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4
Fin Weight (gm)		2.990	7.930	14.200	22.500	31.400	42.600	55.800	68.600
Total Power									
Radiated (w)		1.943	3.165	3.619	4.025	4.200	4.291	4.363	4.382
Specific Radiator Power (w/gm)		0.65	0.40	0.26	0.18	0.13	0.10	0.078	0.064
Temperature (°K)	700	686	648	601	558	516	481	450	422
Fin Thickness (cm)		8.0	8.0	8.0	7.8	7.8	7.5	7.5	7.5
Fin Weight (gm)		14.900	41.100	75.600	115.600	167.000	219.700	287.600	363.700
Total Power									
Radiated (w)		2.336	5.066	7.317	8.945	10.220	11.030	11.550	11.950
Specific Radiator Power (w/gm)		0.16	0.12	0.097	0.077	0.061	0.050	0.040	0.033

Fig. 13. Specific Power Dissipated vs Total Power for Uniform Fin - $T = 600^{\circ}\text{K}$.

Fig. 14. Specific Power Dissipated vs Total Power for Uniform Fin - $T = 700^\circ\text{K}$.

The plots for various fin thickness at a given temperature form a family of curves. The locus of the points of maximum performance for the various fin thicknesses is readily determined by drawing a line tangent to all the individual curves. This locus of maximum performance is shown dotted in the figures. Thus, for any given total power dissipated, there is an optimum in thickness and length which will give the largest power dissipated per unit weight.

Results for the solution of this problem, namely, the circular fin of uniform cross section, are also reported in the literature by Chambers and Somers,⁽¹⁰⁾ who obtained solutions with the digital computer. The results reported here check well with their results.

2. Tapered Fin

The results of calculations for the tapered fins are summarized in Table 9. The method used was the same as that for the fin of uniform cross section, that is, the temperature distribution for a fin of a given thickness was calculated at varying outer radii and then this result used to calculate the total power dissipated by the fin. In order to obtain a stable solution with the analog computer, it was necessary to maintain a finite fin thickness at the outer radius. This thickness was taken as 0.10 cm in all the calculations. The thickness parameter for the tapered fins, therefore, is the thickness at the fin base. The results of the calculations are given in Figs. 15 and 16 for the two fin-base temperatures of 600°K and 700°K . The shapes of the curves follow the same general pattern as those for the fins of constant cross section, except that the curves for the smaller fin thicknesses do not approach a vertical asymptote so sharply as the outer radius is increased. The curves representing the locus of points of maximum performance are also shown on

the curves of Figs. 15 and 16. A comparison of Figs. 13, 14, 15, and 16 shows that the improvement in power dissipation per unit weight of the tapered fin over the fin of constant cross section is a factor of about 1.6 in the range of 5000 w. This factor is relatively constant throughout the power range from 2000 to 8000 w and is the same for the 600°K and 700°K radiators. These results show, as would be expected, that a significant reduction in radiator weight can be obtained by the use of the tapered fin radiator.

Table 9

SUMMARY OF CALCULATIONS FOR TAPERED FINS

	Inner Radius (cm)	Outer Radius (cm)							
	12	22	32	42	52	62	72	82	92
Temperature (°K)	600	569	505	445	398	358	328	300	280
Fin Thickness at base (cm)		3.0							
Fin Weight (gm)		2.788	6.808	12.080	18.570	26.300	35.290	45.470	57.320
Total Power Radiated (w)		1.155	2.215	2.935	3.329	3.674	3.916	4.000	4.121
Specific Radiator Power (w/gm)		0.41	0.33	0.24	0.18	0.14	0.11	0.088	0.072
Temperature (°K)	600		525	472					
Fin Thickness at base (cm)			4.5	4.5					
Fin Weight (gm)			10.060	17.820					
Total Power Radiated (w)			2.463	3.395					
Specific Radiator Power (w/gm)			0.25	0.19					
Temperature (°K)	600	582	538	489	444	409	378	349	326
Fin Thickness at base (cm)		6.0							
Fin Weight (gm)		5.460	13.310	23.580	36.320	51.280	68.640	88.810	110.900
Total Power Radiated (w)		1.229	2.633	3.722	4.515	5.120	5.628	5.907	6.113
Specific Radiator Power (w/gm)		0.23	0.20	0.16	0.12	0.10	0.082	0.067	0.055
Temperature (°K)	600		555	514					
Fin Thickness at base (cm)			9.0	9.0					
Fin Weight (gm)			19.820	35.250					
Total Power Radiated (w)			2.820	4.230					
Specific Radiator Power (w/gm)			0.14	0.12					
Temperature (°K)	700	647	555	480	423	378	343	313	290
Fin Thickness at base (cm)		3.0							
Fin Weight (gm)		2.788	6.808	12.080	18.570	26.300	35.290	45.470	57.320
Total Power Radiated (w)		2.033	3.553	4.436	4.884	5.332	5.562	5.695	5.834
Specific Radiator Power (w/gm)		0.73	0.52	0.37	0.26	0.20	0.16	0.13	0.10
Temperature (°K)	700		584	512					
Fin Thickness at base (cm)			4.5	4.5					
Fin Weight (gm)			10.060	17.820					
Total Power Radiated (w)			4.043	5.247					
Specific Radiator Power (w/gm)			0.40	0.29					
Temperature (°K)	700	669	601	535	489	437	398	368	343
Fin Thickness at base (cm)		6.0							
Fin Weight (gm)		5.460	13.310	23.580	36.320	51.280	68.640	88.810	110.900
Total Power Radiated (w)		2.227	4.382	5.876	6.857	7.674	8.134	8.485	8.781
Specific Radiator Power (w/gm)		0.41	0.33	0.25	0.19	0.15	0.12	0.096	0.079
Temperature (°K)	700		626	568					
Fin Thickness at base (cm)			9.0	9.0					
Fin Weight (gm)			19.820	35.250					
Total Power Radiated (w)			4.817	6.815					
Specific Radiator Power (w/gm)			0.24	0.19

Fig. 15. Specific Power Dissipated vs Total Power for Tapered Fin - $T = 600^\circ\text{K}$.

Fig. 16. Specific Power Dissipated vs Total Power for Tapered Fin - $T = 700^\circ\text{K}$.

D. Heat-sink Calculations

A look at Fig. 2 shows that the heat-flow path is not quite as simple as the foregoing calculations have treated it and that it is actually necessary for heat to flow through the spherical heat sink to the base of the fin. Since

the heat radiated directly by the sphere is only of the order of 25-50%, the balance being dissipated by the fin, there will be temperature gradients in the heat sink. It is desirable that these temperature gradients be minimal so that the cold-junction temperature of those thermoelements furthest from the fin will be not greatly increased, with a corresponding reduction in their electrical output. An analysis of this problem was made. The equation for solution of this heat flow problem was derived on the basis of the following assumptions:

1. small temperature gradients in the ϕ and r directions;
2. constant heat flux at the inside surface of the heat sink;
3. as in the analysis of the fin, neglect of the mutual irradiation between fin and sphere;
4. radiation by the sphere to 0°K ;
5. a boundary condition $T = T_b$ at $\theta = \pi/2$;
6. the boundary condition at the pole of the sphere, $dT/d\theta = 0$.

With these considerations, the equation for the temperature distribution in a thin spherical shell corresponding to the heat sink is as follows:

$$\frac{d^2T}{d\theta^2} + \cot\theta \frac{dT}{d\theta} - AT^4 + B = 0 \quad , \quad (21)$$

where

$$A = \sigma\epsilon r^2/kt$$

$$B = (q/A)r^2/kt$$

$$r = \text{mean shell radius} \quad .$$

Equation (21) was also programmed for solution on the Pace analog computer,* and solutions were obtained for the cases given in Table 10. From these results it appears that a heat-sink thickness of 4-5 cm is adequate to assure a reasonably isothermal heat sink. It is recognized, however, that some loss in performance will result even though it does not seem to be a really difficult problem. This is because, as discussed earlier, all of the reactors which appear to be attractive as heat sources use the beryllium reflectors of at least 5-cm thickness. Hence, this beryllium used for the cold-junction heat sink serves the additional purpose as a reflector for the reactor.

*Work done by Louis C. Just

Table 10

RESULTS OF TEMPERATURE DISTRIBUTION CALCULATIONS
IN A SPHERICAL SHELL HEAT SINK

Shell Mean Radius (cm)	Heat Flux Input to Inside of Shell (w/cm ²)	Shell Thickness (cm)	Temperature at Base of Fin (°K)	Maximum Shell Temperature (°K)
11 ↓	3.8 ↓	2.4	600	678
		4.6	600	642
		2.5	700	762
		5.0	700	732

E. Radiator Optimization

Since the weight of the reactor and converter are not dependent on the radiator temperature, the optimization of system weight can be done independently of these parts of the system. Figure 17 gives the results of optimization of the radiator weight from calculations based on the data of Figs. 15 and 16. As shown by the curve, the minimum weight of the radiator fin for the 200-w power supply is about 9 kg at a temperature of 630°K.

Fig. 17. Radiator Fin Weight as a Function of Radiator Temperature.

VII. CONCLUSIONS

The design of a small, conduction-cooled fast reactor for producing 200 w of electric power appears to be feasible with essentially no extensions of existing technology. Furthermore, such a power supply shows an increase in specific power over the 500-w SNAP-10 system by almost a factor of 3, i.e., 3.9 w/kg against 1.4 w/kg.

It is recognized that there are a number of design areas which need further investigation. These include compatibility of materials, irradiation damage, development of control systems, and development of a thermoelectric generator with a long lifetime and high reliability. The latter requirement is not particular to this system but is a need for space power supplies in general. Probably the key factor in determining final feasibility is the radiation damage to the thermoelements, and it is recommended that this be investigated initially in any development program.

As discussed in previous sections of the report, there is an upper power level for the conduction-cooled systems which is imposed by the limitations of the heat flux that the thermoelements can stand without cracking. As the reactor size is increased, its surface-to-volume ratio decreases and, hence, the surface heat flux increases. Thus, upper limits to the reactor size and power level are quickly reached. From this point of view, then, there is incentive to have a thermoelectric material capable of withstanding larger temperature gradients. In the proposed design, for example, the power output can be doubled with no changes, except by the addition of a larger radiator, if the temperature gradient in the thermoelements can be doubled. The obvious alternate approach to obtaining a higher power level is to use several of the small systems in parallel.

In summary, it does not appear that any of the design and development problems are particularly difficult. It is believed that the proposed system can be developed with a relatively small amount of effort.

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