SCALING IN $p\pi^- \rightarrow pX^-$ AND AN ESTIMATE OF THE
TRIPLE-POMERANCHUKON COUPLING*

Frank E. Paige and Ling-Lie Wang
Brookhaven National Laboratory, Upton, New York 11973

ABSTRACT

We show that the CERN-IHEP data on $p\pi^- \rightarrow pX^-$ at 25 and 40 GeV/c exhibit scaling in s'/s . We give an estimate of the triple-Pomeranchukon coupling and discuss the possibility of its measurement at NAL.

A CERN-IHEP collaboration has recently measured the diffractive inclusive cross section for $p\pi^- \rightarrow pX^-$ at incident momenta of 25 and 40 GeV/c as a function of missing mass $(s')^{1/2}$. In this note we point out that the cross section $d\sigma/dtd(s'/s)$ exhibits approximate scaling in s'/s , and that a small non-scaling term is also present. See Fig. 1. To estimate the theoretically interesting triple-Pomeranchukon (PPP) vertex, we have made a three-term Regge pole fit to the data. This fit indicates that the contribution of the PPP term at present energies is comparable to that of the non-scaling term. From this we conclude that a detailed fit will be necessary to isolate the PPP term even at NAL energies.

Consider the diffractive inclusive reaction $a+b \rightarrow c+X$ with $s = (p_a+p_b)^2$, $s' = (p_a+p_b-p_c)^2$, and $t = (p_a-p_c)^2$, in the end-of-spectrum region, where s' and s/s' are both large while t is small. As has been discussed by many authors,² the Regge pole model predicts that (c.f. Fig. 2)

$$\begin{aligned} \frac{d\sigma}{dtd\frac{s'}{s}} \sim & \frac{1}{16\pi} \sum_{ij} \beta_{a\bar{c}i}(t) \beta_{a\bar{c}j}^*(t) g_{ijP}(t) \beta_{b\bar{b}P}(0) \left(\frac{s}{s'}\right)^{\alpha_i(t)+\alpha_j(t)-1} \\ & + \frac{1}{16\pi} \sum_{ij} \beta_{a\bar{c}i}(t) \beta_{a\bar{c}j}^*(t) g_{ijM}(t) \beta_{b\bar{b}M}(0) \\ & \left(\frac{s}{s'}\right)^{\alpha_i(t)+\alpha_j(t)-\alpha_M(0)} \left(\frac{s}{s_0}\right)^{\alpha_M(0)-1} \end{aligned} \quad (1)$$

Here P and M refer to the Pomeranchukon and meson trajectories respectively, and it has been assumed that $\alpha_P(0) = 1$. The Regge residues $\beta_{a\bar{c}i}(t)$ are related to the $aa \rightarrow cc$ cross section by

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$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left| \sum_i \beta_{aci}^2(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)} \right|^2. \quad (2)$$

The various terms in Eq. (1) will be denoted in consecution by (ijP) and (ijM). For fixed s'/s and sufficiently large s only the (ijP) terms are important, and so $d\sigma/dtd(s'/s)$ is a function only of s'/s and t even if several terms are included in the sum.

In Fig. 1 we plot $d\sigma/dtd(s'/s)$ at 25 and 40 GeV/c using the CERN-IHEP data. (The cross section has been averaged over the interval $0.17 \text{ GeV}^2 < -t < 0.35 \text{ GeV}^2$.) Evidently the data at the two energies agree quite well, but the 40 GeV/c data are somewhat lower. This difference provides a direct measure of the non-scaling term.

To obtain the PPP coupling we must make a fit to the data. Since the cross section at the larger values of s'/s is quite flat, a term in Eq. (1) which scales and which has $\alpha_i(t) + \alpha_j(t) \approx 1$ must dominate there: this is the MMP term since $\alpha_M(0) \approx 0.5$. Since scaling is not exactly satisfied, a small non-scaling term, presumably PPM, must be present. We therefore take³⁻⁵

$$\begin{aligned} \frac{d\sigma}{dtd\frac{s'}{s}} = & G_{PPP}(t) \left(\frac{s}{s'}\right)^{2\alpha_P(t)-1} + G_{MMP}(t) \left(\frac{s}{s'}\right)^{2\alpha_M(t)-1} \\ & + G_{PPM}(t) \left(\frac{s}{s'}\right)^{2\alpha_P(t)-\alpha_M(0)} \left(\frac{s}{s_0}\right)^{\alpha_M(0)-1}, \end{aligned} \quad (3)$$

where $\alpha_P(t) = 1+0.5t$, $\alpha_M(t) = 0.5+1.0t$, with t in GeV^2 , and

$$G_{RRR'}(t) = \frac{1}{16\pi} \beta_{pPR}^2(t) \beta_{\pi\pi R'}(0) g_{RRR'}(t). \quad (4)$$

Other terms could certainly be present; we exclude them only for simplicity. Then the non-scaling part of the cross section is given solely by the last term in Eq. (3). Taking as an average value $t = -0.26 \text{ GeV}^2$, we find that the best fit is given by

$$\left. \begin{aligned} G_{PPP}(t) &= 2.4 \text{ GeV}^{-2} \\ G_{MMP}(t) &= 75 \text{ GeV}^{-2} \\ G_{PPM}(t) &= 42. \text{ GeV}^{-2} \end{aligned} \right\} \quad \text{at } t = -0.26 \text{ GeV}^2. \quad (5)$$

The contribution of each term is shown in Fig. 2. Since the shapes of the PPP and PPM terms are so similar, the relative magnitudes are not well determined.⁶ It seems safe to conclude, however, that a non-zero PPM term is present and that the PPP term does not dominate in any region at presently available energies.

As the energy is increased, the PPP and MMP contributions remain fixed on the scaling plot while the PPM term decreases like $(s)^{\alpha_M(0)-1} = (s)^{-0.5}$. If our estimate of the various terms is correct, then a significant PPM contribution will remain even at $p_{lab} \sim 200$ GeV/c, while the PPP contribution becomes larger than the MMP one only for $s'/s \lesssim 0.05$. (c.f. Fig. 1.) To extract the PPP term it will therefore be necessary to do experiments at various energies with consistent absolute normalizations and to make a detailed fit.

For internal consistency of the Regge pole model it is required that $g_{PPP}(t)$ vanishes linearly at $t = 0$ if $\alpha_P(0) = 1.6, 7$. Therefore a turnover of the PPP term is anticipated near $t = 0$. To estimate the position of this turnover, we take

$$g_{PPP}(t) = g_0(-t)e^{c_2 t}, \quad (6)$$

where g_0 and c_2 are constants. As found from the elastic cross section,⁸

$$\beta_{\overline{PPP}}(t) = \beta_{\overline{PPP}}(0)e^{c_1 t}, \quad \beta_{\overline{PPP}}(0) = 10, \quad c_1 = 1.5 \text{ GeV}^{-2} \quad (7)$$

Assuming rather arbitrarily that $c_2 = c_1 = 1.5 \text{ GeV}^{-2}$, we find that the PPP contribution has a peak at

$$t = \frac{-1}{2c_1 + c_2 + 2\alpha'_P \ln \frac{s}{s'}} \approx -0.125 \text{ GeV}^2 \text{ for } \frac{s}{s'} = 20. \quad (8)$$

This peak is certainly kinematically accessible since $t_{\min} \approx -m_p^2 (\frac{s'}{s})^2$. The observation of it, while no doubt difficult, would be a useful test of current theoretical ideas.^{6,7}

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FIGURE CAPTIONS

- Fig. 1 Scaling plot for CERN-IHEP data. (Only a few representative points from Ref. 1 are included.) The curves are our fit. It deviates from the data for larger s'/s , but the Regge-pole model is expected to apply only for fairly small values of s'/s .
- Fig. 2 Regge pole diagrams for the inclusive cross section at the end of the spectrum.

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FIG. 1

FIG. 2

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