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SOME ELECTROMAGNETIC EFFECTS OF HIGH DENSITY
GAMMA FLUX INTERACTING WITH MATTER

by

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Clayborne Dudley Taylor

May 1965

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SOME ELECTROMAGNETIC EFFECTS OF HIGH DENSITY
GAMMA FLUX INTERACTING WITH MATTER

by

Clayborne Dudley Taylor
New Mexico State University
University Park, New Mexico

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ABSTRACT

A metal slab is considered to be bombarded normally by a flux of gamma rays. As a result of this bombardment electrons are scattered from the slab with a distribution of velocities. This distribution is determined by the energies of the incident gamma rays and the slab thickness. An approximation to the velocity distribution is obtained for the case of gamma rays with the energy of 1.02 Mev--corresponding to the average energy of the gamma rays emitted from a nuclear explosion.

A nonlinear integral equation describing the motion of the electrons egressing from a slab is derived. It is found to depend on (1) the flux density of electrons egressing from the slab, and (2) the distribution of the forward components of the electrons' velocities upon egressing from the slab, i. e., the velocity components along the normal to the slab. It is proved that the nonlinear integral equation can be solved by a method of successive approximations. Two special cases are treated: (1) the flux density of the egressing electrons is considered to be approximately the electron flux density that is scattered from the slab by the gamma flux from a nuclear explosion, and the electrons are considered to egress from the slab with the same velocity--the average velocity of the electrons scattered from the slab by the gamma flux from a nuclear explosion, and (2) all the electrons are considered to egress from the slab at once with a distribution of forward components of the velocities which crudely approximates the distribution for electrons scattered from the slab by the gamma flux from a nuclear explosion. After obtaining these equations of motion, the resulting electric fields are then found.

It is shown that the formulation for the treatment of a slab being bombarded by gamma rays can be applied in treating the RF radiation from a high altitude nuclear explosion. The results of this treatment are compared with those of the theory by Karzas and Latter, and a discussion of factors determining the RF radiation is presented.

A monopole antenna is considered to be bombarded by a flux of gamma rays propagating normally to the axis of the antenna. As a result of this bombardment, electrons are scattered from the antenna, thus charging the antenna positively. This induces a current in the load impedance of the antenna--appearing as if the antenna were receiving a transient RF signal. An expression for the time dependent voltage across the load impedance of the antenna is obtained by assuming that every electron scattered from the antenna never returns and that the antenna capacitance is the d. c. capacitance.

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CHAPTER I

INTRODUCTION

The advent of nuclear technology has stimulated interest in the interaction of a high density gamma flux with matter. A knowledge of this interaction can, for example, be applied in determining methods for detecting nuclear explosions [1, 2, 3, 4, 5]. The work presented here is a study of this interaction and a study of some electromagnetic phenomena that occur when a high density gamma flux is incident upon a metal object.

The source of a high density gamma flux is a nuclear chain reaction such as that producing a nuclear explosion. In a nuclear explosion gamma rays are produced both directly by the fission process and indirectly by inelastic scattering of neutrons which are emitted in the fission process. It is estimated that about 0.03% of the total yield of a nuclear explosion is produced as gamma rays, which have an average energy of about 1 Mev [4]. According to this estimate there are about 7.5×10^{21} gamma rays produced per kiloton of yield. Thus a nuclear explosion is capable of producing a very high density gamma flux.

A high density gamma flux incident upon an object will scatter electrons from it; this results in the object being positively charged. Some of the electrons scattered from the object may return because of the coulomb attraction between the electron and the positively charged object. These accelerating electrons radiate provided that their distribution is asymmetric.

First it is of interest to obtain either the velocity distribution or the energy distribution of electrons that are scattered from an object which is irradiated by a high density gamma flux. The irradiated object is considered to be an infinite metal slab which is irradiated uniformly over the surface by unidirectionally propagating gamma rays. An approximation to the velocity distribution of electrons that are scattered from the slab is obtained for monoenergetic incident gamma rays of energies less than 10 Mev. Also a crude approximation is obtained for the distribution of the forward components of the electron velocities.*

In obtaining the motion of the electrons after their emergence from the slab, the various mechanisms of energy losses are discussed. However, to avoid the complexities of dielectric breakdown, the medium about the slab must be considered to be a vacuum. A determination of this motion requires the treatment of a complicated self-consistent field problem. However, a nonlinear integral equation which describes this motion is derived based on the knowledge of the flux density of the egressing electrons and the distribution of the forward components of their velocities. It is rigorously proved that the integral equation can be solved by a method of successive approximations. Two special cases are treated: (1) the flux density of the egressing electrons is considered to be approximately that which is scattered out of the slab by the flux of gamma rays from a

* The forward component of an electron's velocity is the component along the normal to the slab.

nuclear explosion, and all the electrons are considered to emerge from the slab with the same velocity (the average velocity of the electrons scattered from the slab by gamma rays from a nuclear explosion), and (2) all the electrons are considered to egress from the slab at once with a velocity distribution which crudely approximates that of the electrons which are scattered from the slab by the gamma rays from a nuclear explosion. After obtaining the electron motions in each case the resulting electric fields are obtained.

The formulation of the foregoing problem can also be applied to the treatment of the RF radiation from a nuclear explosion by choosing an appropriate model for the nuclear device. A convenient model is a hemispherical shell with a point source of gamma rays at its center. By using this model Karzas and Latter [4] have made a rough approximation to the coherent radiation field. The formulation developed in this paper leads to accurate results for this coherent radiation field; these results are then compared to those obtained by Karzas and Latter. Also a discussion of the factors determining this radiation is presented.

A problem that is related to the previous problem is treated last. It is the problem of finding the signal produced in a receiving antenna by gamma ray bombardment. A monopole antenna is considered to be bombarded by a flux of gamma rays propagating normally to the axis of the antenna. The direct effect of the bombardment is the charging of the antenna. Then the antenna's receiving circuit is excited by the resulting flow of charge through it. A time voltage is produced across the antenna load impedance as if the antenna were

receiving a transient RF signal. By neglecting the presence of the electrons scattered from the antenna and assuming the antenna's capacitance is the d.c. capacitance, an expression is obtained for the time dependent voltage across the antenna load impedance.

CHAPTER II

AN INFINITE SLAB IRRADIATED UNIFORMLY OVER ONE SURFACE BY A GAMMA FLUX

To obtain quantitative results in the theoretical study of an object that is irradiated by a high density gamma flux, a suitable situation must be selected. An infinite slab irradiated uniformly over one surface is the most convenient situation to formulate. And fortunately this formulation can be applied to obtain approximate results for the irradiation of some finite objects. Three particular applications of this formulation are given in the succeeding chapters: obtaining the electric field produced by electrons which are scattered from a slab by gamma rays, obtaining the radiated field of these accelerated electrons, and obtaining the signal produced in a monopole antenna by a removal of electrons from it.

Throughout this chapter and the succeeding chapters, proofs and lengthy derivations will be relegated to the appendices.

2.1 Interaction Processes of Gamma Rays with Matter

There are three principal processes through which gamma rays interact with matter--photoelectric effect, Compton scattering, and electron-positron pair production. The dominant process depends upon the energy of the gamma rays and the nature of the absorbing material. The energy ranges of the dominant processes are shown in Figure 2.1 by plotting the energies of gamma rays where two of the interaction cross sections are equal for the particular atomic number, Z , of the interacting material. As stated previously, the average

FIGURE 2.1. Relative importance of the three major types of gamma ray interaction (from Evans [19]).

energy of the gamma rays that are emitted from a nuclear explosion is about 1 Mev. Thus they interact with matter principally through the Compton effect for all but the most dense substances. Throughout this chapter it is assumed that the gamma rays that are emitted from a nuclear explosion interact with matter solely through the Compton scattering process.

One of the earliest successes of the Dirac electron theory was the treatment of the Compton scattering process by Klein and Nishina [6]. This classic formulation is in remarkable agreement with experiment. Thus their results are used here in the treatment of gamma rays interacting with matter. Since their work is so well known, the development of their results is not presented in this paper.

According to the Compton scattering process, gamma rays in penetrating matter are scattered within the medium by inelastic collisions with atomic electrons. The Klein-Nishina theory is worked out for the scattering of gamma rays with free electrons; however, their results are valid for scattering with atomic electrons provided that the scattering process gives these electrons much more energy than their binding energy. Thus the scattered electrons' velocity distribution as derived from the Klein-Nishina theory is accurate for the Compton scattering of atomic electrons only when the scattered electrons' speeds are much greater than $\frac{c}{137} Z$, the speed of the innermost electrons of a Bohr atom, where Z is the atomic number of the scattering atoms and c is the speed of light.

The velocity distribution of the free electrons that are

scattered by gamma ray photons is proportional to the differential cross section obtained from the Klein-Nishina theory.

$$d\sigma = \frac{8\pi r_0^2 \beta g(\beta) d\beta}{\alpha^2 \sqrt{1-\beta^2} [\beta^2 - g(\beta)]^2} \left[1 + \frac{2 g(\beta)}{\alpha [\alpha \beta^2 - (\alpha+2)g(\beta)]} \times \left(\frac{g(\beta)}{[\beta^2 - g(\beta)]} - \frac{[\alpha^2 \beta^2 - (\alpha+1)^2 g(\beta)]}{\alpha [\alpha \beta^2 - (\alpha+2)g(\beta)]} \right) \right], \quad (2.1.1)$$

where the incident gamma ray energy $h\nu = \alpha\mu$, μ = rest mass energy of an electron, $\beta = \frac{1}{c}v$, v = velocity of the scattered electron, r_0 is the classical radius of an electron, and $g(\beta) = [2(1-\sqrt{1-\beta^2})-\beta^2]$. By using the conservation of energy and momentum in the collision between the photon and the electron, the scattering angle of the electron, that is, the angle between the direction the photon is initially moving and the direction the scattered electron moves, is found to depend uniquely upon the velocity of the scattered electron. This scattering angle, θ , is

$$\theta = \cos^{-1} \left[\frac{\alpha+1}{\alpha} \frac{1}{\beta} \sqrt{g(\beta)} \right]. \quad (2.1.2)$$

Also the scattering angle of the electron is uniquely dependent upon the forward component of the scattered electron's velocity, $u = \beta \cos \theta$,

$$\theta = \cos^{-1} \left[\frac{(\alpha+1)^2 u}{2\alpha(\alpha+1) - \alpha^2 u} \right]^{1/2}. \quad (2.1.3)$$

To obtain the distribution of the forward components of the scattered electrons' velocities, it is necessary to obtain the corresponding differential cross section,

$$d\sigma(u) = \frac{d\sigma}{d\beta} \left(\frac{d\beta}{du} \right) du, \quad (2.1.4)$$

where

$$\frac{d\beta}{du} = \frac{\alpha(\alpha+1) - \alpha^2 u}{(\alpha+1)^2 u} \cos \theta. \quad (2.1.5)$$

Both (2.1.2) and (2.1.3) predict that the maximum velocity of the scattered electrons is given by

$$\beta_{\max} = u_{\max} = \frac{2\alpha(\alpha+1)}{\alpha^2 + (\alpha+1)^2}. \quad (2.1.6)$$

The total collision cross section per electron, σ , for the Compton scattering process is according to the Klein-Nishina theory,

$$\sigma = 2\pi r_0^2 \left\{ \frac{\alpha+1}{\alpha^2} \left[\frac{2(\alpha+1)}{1+2\alpha} - \frac{1}{\alpha} \ln(1+2\alpha) \right] + \frac{1}{2\alpha} \ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}. \quad (2.1.7)$$

With the use of the foregoing relations it is possible to predict not only the total number of electrons that are scattered within the object by a gamma flux but also their velocity distribution. To determine the velocity distribution of the electrons escaping from the object requires a knowledge of the energy losses suffered by the electrons in traversing to the object's surface. This velocity distribution allows, in principle, the determination of the motion of the escaping electrons.

2.2 Velocity Distribution of Electrons Scattered from the Slab

An accurate theoretical treatment of the penetration of electrons

through matter is complicated by the combination of multiple scattering and energy losses. The qualitative behavior of fast electrons is: initially the electrons move in an almost straight line path; as their energy decreases their scattering becomes more important and the angular distribution about the initial direction approaches the gaussian distribution characteristic of multiple scattering--in this region the most probable angle of scattering increases proportionally with the square root of the thickness of material traversed; and after sufficient scattering there is no longer a preferred direction of motion, and the motion of the electrons can be described as diffusion. An excellent review article by Birkhoff [7] gives more details concerning the behavior of fast electrons penetrating matter.

Using the Møller formula for the relativistic electron-electron scattering cross section, Bethe [8] derived an expression for the average energy loss per unit path length of an electron penetrating matter. This result is

$$\left\langle \frac{dT}{dx} \right\rangle = -2\pi r_0^2 N Z \frac{\mu}{\beta^2} \left\{ \ln \left[\frac{\mu}{2I^2} \frac{T\beta^2}{(1-\beta^2)} \right] - (2\sqrt{1-\beta^2} - 1 + \beta^2) \ln 2 \right. \\ \left. + 1 - \beta^2 + \frac{1}{8} (1 - \sqrt{1-\beta^2})^2 \right\}, \quad (2.2.1)$$

where T is the kinetic energy of the electron with velocity, $c\beta$; N is the density of target atoms with atomic number, Z ; and I is the average atomic ionization energy which is determined by experiment (some representative values are given in Table 2.1). The limitation on (2.2.1) is that the speed of the colliding electron must be large compared to the electronic speeds of the target atoms, that is $\beta \gg \frac{Z}{137}$.

TABLE 2.1

REPRESENTATIVE VALUES OF THE EXPERIMENTALLY DETERMINED
AVERAGE ATOMIC IONIZATION ENERGY, I

C	Air	Al	Cu	Ag	Sn	Pb	Au
77	90	160	320	540	560	920	960 ev

An expression for the average velocity loss per unit path length can be obtained from (2.2.1) by using,

$$\left\langle \frac{d\beta}{dx} \right\rangle = \frac{(1-\beta^2)^{\frac{3}{2}}}{\mu\beta} \left\langle \frac{dT}{dx} \right\rangle. \quad (2.2.2)$$

The average velocity loss, $\langle \Delta\beta \rangle$, suffered by electrons in traveling a path distance, s , is obtained from,

$$s = \int_{\beta_0}^{\beta_0 - \langle \Delta\beta \rangle} \left\langle \frac{d\beta}{dx} \right\rangle^{-1} d\beta \quad (2.2.3)$$

where β_0 is the initial velocity of the electrons and $\beta_0 - \langle \Delta\beta \rangle \gg \frac{Z}{137}$.*

It should be emphasized that the path of electrons that are traversing a material is not straight in general because of multiple scattering.

A collimated beam of monoenergetic electrons incident upon a foil will be dispersed and the electrons will have a distribution of paths through the foil. The average path length of these electrons through a thin foil†, as obtained in Appendix I, is

$$\langle S \rangle \approx \frac{d}{\cos \theta} \left(1 + \frac{d}{\lambda \cos \theta} \right) \quad (2.2.4)$$

$$\text{where } \frac{1}{\lambda} = 2\pi r_0^2 \frac{NZ^2(E_0+1)^2}{E_0^2(E_0+2)^2} \ln \left[\frac{274 \beta (E_0+1)}{\frac{1}{Z^3}} \right]. \quad (2.2.5)$$

* The range of validity of (2.2.3) can be extended by requiring that $\left\langle \frac{d\beta}{dx} \right\rangle^{-1} = 0$ for $\beta \leq \frac{Z}{137}$ if $\beta_0 \gg \frac{Z}{137}$. This is allowable since $\left\langle \frac{d\beta}{dx} \right\rangle^{-1}$ is very small for $\beta \sim \frac{Z}{137}$.

† A thin foil for a particular electron energy is one that produces very small average energy loss to those electrons traversing it.

E_0 is the kinetic energy of the penetrating electron expressed in units of electron rest mass energies, d is the foil thickness and θ is the angle of incidence measured from the normal to the slab. The limitation on the angle of incidence, θ , in (2.2.4) is that $\cos \theta$ must be much greater than $\frac{d}{\lambda}$. Table 2.2 gives the values of λ , in gm/cm², for various energies and a few materials.

By using (2.2.4) and (2.2.3) a relation can be obtained for finding the average velocity loss of monoenergetic electrons initially in a collimated beam penetrating a thin foil.

$$\frac{d}{\cos \theta} \left(1 + \frac{d}{\lambda \cos \theta} \right) = \int_{\beta_0}^{\beta_0 - \langle \Delta \beta \rangle} \left\langle \frac{d\beta}{dx} \right\rangle^{-1} d\beta, \quad (2.2.6)$$

where d is the foil thickness and θ is the angle of incidence. Since medium energy electrons on the average travel over 80 per cent of their range before losing one-half their initial energy (see Figure 2.2), (2.2.6) can be considered to be approximately valid over the whole range of the electrons by evaluating λ for the initial energy and considering $\left\langle \frac{d\beta}{dx} \right\rangle^{-1} = 0$ for $\beta < \frac{Z}{137}$. Electron ranges calculated by using (2.2.6) compare remarkably well with the experimental extrapolated ranges in Aluminum (Table 2.3).

The irradiation of a slab by a gamma flux produces energetic electrons throughout the slab. With the restriction that the thickness of the target slab be less than the mean free path of the incident gamma rays, there is no need to consider multiple collisions of the gamma rays within the slab. And by using the results of the foregoing section, it is possible to predict the velocity distribu-

FIGURE 2.2. Average velocity loss of electrons versus foil thickness penetrated for initial velocities .8 C and .923 C.

TABLE 2.2

TRANSPORT MEAN FREE PATH λ IN g/cm^2 (FROM BETHE [15])

Scattering Material	E_0	0.1	0.2	0.5	1.0	2.0	5.0	10	20 μ
Al		0.0063	0.0155	0.050	0.124	0.304	0.990	2.43	5.95
Cu		0.0047	0.0116	0.034	0.093	0.227	0.725	1.76	4.35
Pb		0.0031	0.0084	0.027	0.065	0.160	0.505	1.23	3.00

TABLE 2.3

CALCULATED VALUES FOR THE AVERAGE ELECTRON RANGES
IN ALUMINUM

Energy	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{2}$	$\pi\mu$
Range	0.0066	0.0177	0.0310	0.107	0.266 cm

tion of the electrons which are initially scattered at a given distance from the surface of the slab.

Consider the flux density incident upon the slab to be unity. The number of electrons, dn_0 , per unit area that are scattered in an increment slab section (Figure 2.3), dx , is

$$dn_0 = NZ\sigma dx, \quad (2.2.7)$$

and the number of electrons, d^2n_0 , per unit area that are scattered out of the increment slab with velocities between β and $\beta + d\beta$ is

$$d^2n_0 = NZ \frac{d\sigma}{d\beta} dx d\beta. \quad (2.2.8)$$

The velocity distribution, $F(\beta)$, of electrons after passing through a foil is related to incident velocity distribution, $f(\beta_0)$, through

$$F(\beta) = \int_0^1 d\beta_0 f(\beta_0) g(\beta - \beta_0, x),$$

where the scattering function $g(\beta - \beta_0, x)$ gives the transmitted velocity distribution for the case of a collimated beam of monoenergetic electrons with velocity β_0 incident upon a foil of thickness, x . Thus the velocity distribution of the electrons egressing from the slab after being scattered from the increment slab, dx , is

$$dF(\beta) = NZdx \int_0^1 d\beta_0 \frac{d\sigma}{d\beta_0} g(\beta - \beta_0, x), \quad (2.2.9)$$

By integrating (2.2.9) over x from zero to b (the slab thickness) the total velocity distribution of the egressing electrons is obtained.

FIGURE 2.3. Scattering of electrons within a slab by a uniform flux of gamma rays.

$$F(\beta) = NZ \int_0^b dx \int_0^1 d\beta_0 \frac{d\sigma}{d\beta_0} g(\beta - \beta_0, x). \quad (2.2.10)$$

The scattering function $g(\beta - \beta_0, x)$ approaches a Dirac delta as x approaches zero, that is,

$$g(\beta - \beta_0, x) \xrightarrow{x \rightarrow 0} \delta(\beta - \beta_0). \quad (2.2.11)$$

For small average energy losses of the electrons $g(\beta - \beta_0, x)$ remains a sharply peaked, almost gaussian function describing the velocity dispersion due to straggling. However, for larger x the scattering function broadens depending on the initial velocity of the electrons [9]. Accurate theory has been worked out predicting the scattering function for collimated beams of electrons normally incident upon thin foils [10, 11]. However, (2.2.10) requires the knowledge of the scattering function for arbitrary foil thickness and arbitrary angle of incidence. Because $g(\beta - \beta_0, x)$ is in the integrand of a double integral, a crude approximation of it could lead to good results for the integral.

A crude approximation to $g(\beta - \beta_0, x)$ is

$$g(\beta - \beta_0, x) = \psi(\beta_0, x) \delta(\beta - \beta_0 - \langle \Delta\beta_0 \rangle), \quad (2.2.12)$$

where $\langle \Delta\beta_0 \rangle$ is the average velocity loss of the electrons that are scattered with velocity β_0 from position x , and $\psi(\beta_0, x)$ gives the fraction of these electrons that egress from the slab. The average velocity loss of these electrons not only depends upon initial velocity and position x , but also upon the scattering angle.

Over one-half of the electrons that are scattered from the

incremental slab, dx , at x by 1 Mev gamma rays have a scattering angle less than 35° . Therefore (2.2.6) can be used with reasonable accuracy to obtain the average velocity losses of the electrons that are scattered by the gamma rays.

$$\frac{b-x}{\cos \theta_0} \left[1 + \frac{b-x}{\lambda_0 \cos \theta_0} \right] = \int_{\beta_0}^{\beta_0 - \langle \Delta \beta \rangle} \left\langle \frac{d\beta'}{dx} \right\rangle^{-1} d\beta', \quad (2.2.13)$$

where $\cos \theta_0 = \frac{\alpha+1}{\alpha} \frac{1}{\beta_0} \sqrt{g(\beta_0)}$ is the cosine of the scattering angle of the electrons that are scattered with velocity β_0 , from position x and λ_0 is λ evaluated for $\beta = \beta_0$.

Since no explicit functional relation can be obtained for $\langle \Delta \beta \rangle$, it is more convenient to express (2.2.12) in another form. Using the relation for Dirac delta functions,

$$\delta(h(x)) = \frac{1}{\left| \frac{dh}{dx} \right|} \delta(x-x_0), \quad (2.2.14)$$

and letting $h(x) = \beta - \beta_0 - \langle \Delta \beta \rangle$ the following expression is obtained:

$$\delta(\beta - \beta_0 - \langle \Delta \beta \rangle) = \frac{1}{\left| \frac{d\langle \Delta \beta \rangle}{d\beta} \right|} \delta(x-x_0). \quad (2.2.15)$$

From (2.2.13) it is found that

$$\left. \begin{aligned} \frac{d\langle \Delta \beta \rangle}{d\beta} &= \left(\frac{-\cos \theta_0}{1 + 2 \frac{b-x}{\lambda_0 \cos \theta_0}} \right)^{-1} \left\langle \frac{d\beta'}{dx} \right\rangle_{\beta'=\beta} \\ \text{and} \quad \frac{b-x_0}{\cos \theta_0} \left[1 - \frac{b-x_0}{\lambda_0 \cos \theta_0} \right] &= \int_{\beta_0}^{\beta} \left\langle \frac{d\beta'}{dx} \right\rangle^{-1} d\beta. \end{aligned} \right\} \quad (2.2.16)$$

Substituting (2.2.15) and (2.2.16) into (2.2.12) yields

$$g(\beta - \beta_0, x) = \psi(\beta_0, x) \left\langle \frac{d\beta}{dx} \right\rangle^{-1} \frac{\cos \theta_0 \delta(x - x_0)}{1 + 2 \frac{b - x}{\lambda_0 \cos \theta_0}}. \quad (2.2.17)$$

By using (2.2.17) in (2.2.10) and integrating over x , a first approximation of the velocity distribution, $F_1(\beta)$, of the egressing electrons is obtained.

$$F_1(\beta) = NZ \left\langle \frac{d\beta}{dx} \right\rangle^{-1} \int_{\beta}^{\beta_m} d\beta_0 \cos \theta_0 \frac{d\sigma}{d\beta_0} \frac{\psi(\beta_0, x_0)}{1 + 2 \frac{b - x_0}{\lambda_0 \cos \theta_0}} \eta(x_0), \quad (2.2.18)$$

where $\eta(x) = 0 \quad x < 0$

$= 1 \quad x > 1.$

x_0 is defined by (2.2.16), θ_0 is defined by (2.1.2), $\frac{d\sigma}{d\beta_0}$ is given by (2.2.1), and $\left\langle \frac{d\beta}{dx} \right\rangle$ is given by (2.2.2).

In order to evaluate (2.2.18) further it is necessary to determine the function form of $\psi(\beta_0, x_0)$. Accurate Monte Carlo calculations have been made by Berger [12] for the transmission coefficients of electron beams after passing through foils of various thicknesses. A function which roughly fits his results for electrons of medium energy is

$$J(\beta_0, d) = \exp[-3.25 \left(\frac{d}{R_e}\right)^2], \quad (2.2.19)$$

where R_e is the range of electrons with initial velocity β_0 . By using (2.2.19) an approximation can be obtained for $\psi(\beta_0, x)$,

$$\psi(\beta_0, x) = \exp \left\{ -3.25 \left[\frac{b - x}{R_e \cos \theta_0} \right]^2 \right\}, \quad (2.2.20)$$

$$\text{where } R_e = \frac{\lambda_0}{2} \left[\sqrt{1 + \frac{4S_0}{\lambda_0}} - 1 \right] \quad (2.2.21)$$

$$S_0 = \int_{\beta_0}^0 \left\langle \frac{d\beta}{dx} \right\rangle^{-1} d\beta .$$

With the use of (2.2.20) one is in a position to evaluate the first approximation to the velocity distribution of the egressing electrons. Unfortunately this must be done numerically because of the complexity of the functions that are to be integrated. Knowing the velocity distributions makes it possible to obtain the kinetic energy distribution, $\frac{dn}{dE}$,

$$\frac{dn}{dE} = \frac{1}{\mu} F(\beta) \frac{(1-\beta^2)^{3/2}}{\beta} . \quad (2.2.22)$$

A rough approximation to (2.2.18) can be obtained by examining the behavior of the integrand. The graph of $\frac{d\sigma}{d\beta_0}$ (Figure 2.4) reveals that it is a sharply peaked function for β_0 near β_m . In this region the other factors of the integrand are nearly unity. Therefore, setting these smoothly varying factors to one over the whole range of integration yields

$$F_1(\beta) \approx NZ \left\langle \frac{d\beta}{dx} \right\rangle^{-1} [\sigma(\beta_m) - \sigma(\beta)] . \quad (2.2.20)$$

From (2.2.20) it is seen that the most important factor determining the form of the velocity distribution is the average velocity loss of the electrons per unit path length.

It is also of interest to obtain the distribution of the forward components, $u = \beta \cos \theta$, of the velocities of the electrons egressing from the slab. To obtain a crude approximation, the electrons which

FIGURE 2.4. Differential cross section for the velocities of the recoil electrons.

are Compton scattered are considered to traverse a linear path.

Otherwise following the same development that leads to the velocity distribution, the distribution of forward components of the velocities, $G(u)$, can be obtained.

$$G(u) = NZ \int_u^{u_m} du_0 \left\langle \frac{d\beta}{dx} \right\rangle_{\beta=\frac{u}{\cos \theta_0}}^{-1} \frac{d\sigma}{du_0} \psi(u_0, x_0) n(x_0) \quad (2.2.21)$$

$$\left. \begin{aligned} \text{where } x_0 &= b - \cos \theta_0 \int_{\frac{u_0}{\cos \theta_0}}^{\frac{u}{\cos \theta_0}} \left\langle \frac{d\beta}{dx} \right\rangle^{-1} d\beta \\ \cos \theta_0 &= \left[\frac{(\alpha+1)^2 u_0}{2\alpha(\alpha+1) - \alpha^2 u_0} \right]^{1/2} \\ \frac{d\sigma}{du_0} &= \frac{\alpha(\alpha+1) - \alpha^2 u_0}{(\alpha+1)^2 u_0} \cos \theta_0 \left[\frac{d\sigma}{d\beta} \right]_{\beta=\frac{u_0}{\cos \theta_0}} \\ \psi(u_0, x_0) &= \exp \left[-3.25 \left(\frac{b - x_0}{R_e \cos \theta_0} \right)^2 \right] \\ R_e &= \frac{\lambda_0}{2} \left[\sqrt{1 + \frac{4S_0}{\lambda_0}} - 1 \right] \\ S_0 &= \int_{\frac{u_0}{\cos \theta_0}}^0 \left\langle \frac{d\beta}{dx} \right\rangle^{-1} d\beta \\ \lambda_0 &= \lambda \left(\frac{u_0}{\cos \theta_0} \right) \end{aligned} \right\} \quad (2.2.22)$$

Only in the region $u \sim u_m$ is $G(u)$ expected to be accurate because of the assumption of linear paths. However, since every electron is accounted for in the derivation of (2.2.21)--also (2.2.18)--the integral of this distribution should be quite accurate. A graph of (2.2.22) is compared with a graph of (2.2.20) in Figure 2.5.

2.3 Equation of Motion of the Electrons after being Scattered from the Slab

The energy losses of the electrons after emerging from an object depend upon both the nature of the medium about the slab and the resulting buildup of electromagnetic fields outside the object. Suppose that a dielectric medium surrounds the object. A good approximation to the average rate of energy loss due to inelastic collisions of the electrons can be obtained from (2.2.1) by taking an average value for the bracketed quantity, which is slowly varying, and letting $A = 2Z$,

$$\left\langle \frac{dT}{dx} \right\rangle \approx - \frac{1130}{\beta^2} e \rho, \quad \frac{Z}{137} < \beta < 1, \quad (2.3.1)$$

where $\left\langle \frac{dT}{dx} \right\rangle$ is expressed in units of joules per meter; if e , the charge of an electron, is expressed in coulombs; and ρ , the density of the medium, is expressed in kilograms per cubic meter. If a velocity dependent force is associated with this rate of energy loss, the force would be,

$$F_c \equiv \left\langle \frac{dT}{dx} \right\rangle \approx - \frac{1130}{\beta^2} e \rho. \quad (2.3.2)$$

FIGURE 2.5. Comparison of the velocity distribution, $F(\beta)$, to the distribution of the forward components of the velocities, $G(u)$.

The electric field, E_c , that would exert the same force on the electron with velocity β is

$$E_c = -\frac{1130}{\beta^2} \rho \frac{\text{volts}}{\text{meter}}, \quad (2.3.2)$$

where ρ is expressed in units of kilograms per cubic meter. The density of air at STP is 1.29 kg/m^3 , then

$$E_c = \frac{1460}{\beta^2} \frac{\text{volts}}{\text{meter}} \quad (2.3.3)$$

for $0.06 < \beta < 1$.

From (2.3.2) and (2.3.3) it is clear that the electric fields required to dominate the motion of the electrons in a dielectric medium must be $\approx 10^5$ volts/meter. However, electric fields of this magnitude can produce the dielectric breakdown of the medium. To avoid the complexities of dielectric breakdown and inelastic scattering, the medium about the irradiated object is considered to be a vacuum.

The electric fields produced by the electrons scattered from an object will depend upon the geometrical configuration of the object. For simplicity the object is chosen to be an infinite aluminum slab that is irradiated uniformly over one surface by monoenergetic gamma rays propagating normal to the surface. Consider the temporal variation of the incident gamma flux density to be given by $T(\tau)$. Then the flux density of the electrons that are scattered from the slab is $T(\tau) \int_0^1 du G(u)$, where $G(u)$ is given by (2.2.21).

Consider that the number of electrons scattered from the slab

per unit area with forward velocity components between u and $u+du$ and at times between τ and $\tau+d\tau$ is given by $G(u)T(\tau)du d\tau$. The space charge density $d^2\rho(x,t)$ with only these electrons present is

$$d^2\rho(x,t) = e G(u) T(\tau) du d\tau \{ \delta(x) - \delta(x-\xi) \} , \quad (2.3.4)$$

where the first Dirac delta gives the residual charge density on the slab surface $x = 0$ (the surface from which the electrons emerge is considered to be at $x = 0$ rather than $x = b$ as before); the second Dirac delta is the contribution made by the electrons that were scattered from the slab; and $\xi = \xi(u, \tau, t)$ is the position at time t of the electron that is scattered from the slab with initial velocity component along the normal to the slab u at time $t = \tau$. The integration of (2.3.4) yields

$$\rho(x,t) = e \int_0^t d\tau \int_0^1 du T(\tau) G(u) \{ \delta(x) - \delta[x - \xi(u, \tau; t)] \}. \quad (2.3.5)$$

The electric field produced by this charge density is obtained by the use of Maxwell's equation

$$\text{div } \underline{D} = \rho(x', t). \quad (2.3.6)$$

Because of symmetry there is only one component of the electric field. Integrating (2.3.6) over x' from 0 to x and considering the field to be zero inside the metal slab, the following expression is found for the electric field:

$$E(x,t) = \frac{e}{\epsilon} \int_0^t d\tau T(\tau) \int_0^1 du G(u) \eta[\xi(u, \tau, t) - x], \quad (2.3.7)$$

where the electric field is directed normal to and away from the slab.

For the electron energies of interest relativistic corrections amount to only a few percent in the motion of the electrons; therefore the formulation presented here will be non-relativistic for the sake of simplicity. Starting with force equation of the electron and integrating it twice over time, the following equation is obtained:

$$\xi(u, \tau; t) = cu(t-\tau) - \frac{e}{m} \int_{\tau}^t dt' (t-t') E(\xi', t') \quad (2.3.8)$$

(2.3.7) can be combined with (2.3.8) to obtain the nonlinear integral equation of motion:

$$\xi(u, \tau; t) = \xi_0 - A \int_{\tau}^t dt' K(\xi', t') \quad (2.3.9)$$

where $A = \frac{e^2}{m\epsilon}$, $\xi_0 = cu(t-\tau)$, and

$$K(\xi', t') = (t-t') \int_0^{t'} dt'' \int_0^1 du' G(u') T(\tau') n[\xi'(u', \tau; t') - \xi'(u, \tau; t')]. \quad (2.3.10)$$

In Appendix II it is proved that (2.3.9) possesses a unique solution which can be obtained, in principle, by a method of successive approximations where the zeroth solution is ξ_0 . However, the solution of (2.3.9) may become a tedious process depending upon the functional forms of $G(u)$ and $T(\tau)$.

Consider the special case: $G_1(u) = G_0 \delta(u-u_0)$

$$\text{and } T_1(\tau) = \begin{cases} \alpha e^{\alpha\tau} & \tau \leq \tau_0 \\ 0 & \tau \geq \tau_0 \end{cases}, \text{ where } \alpha = 10^8, G_0 = \int_0^{u_m} G(u) du$$

$$\text{and } u_0 = \frac{1}{G_0} \int_0^{u_m} G(u) u du \quad (2.3.11)$$

with $G(u)$ given in (2.2.21). This time dependence of the incident flux corresponds to the temporal production of gamma rays by a nuclear explosion. The first order approximation for the equation of motion is

$$\xi_1(u, \tau; t) = \begin{cases} cu_0(t-\tau) - \frac{A}{2} G_0 e^{\alpha\tau} (t-\tau)^2 & \tau \leq \tau_0 \\ cu_0(t-\tau) - \frac{A}{2} G_0 e^{\alpha\tau_0} (t-\tau)^2 & \tau > \tau_0 \end{cases} \quad (2.3.12)$$

The second order approximation for the equation of motion cannot be obtained without numerical computations. The first order approximation to the electric field produced by the scattered electrons is

$$E(x, t) = \frac{e}{\epsilon} G_0 \int_0^t d\tau T(\tau) \eta[\xi_1(u, \tau; t) - x], \quad (2.3.13)$$

which also cannot be evaluated without numerical computation.

Consider a second special case: $T_2(\tau) = T_0 \delta(\tau)$

$$\text{and } G_2(u) = \begin{cases} \frac{2G_0}{u_m^2} u & 0 \leq u \leq u_m, \text{ where } G_0 = \int_0^{u_m} G(u) du \\ 0 & u > u_m \end{cases}$$

with $G(u)$ given in (2.2.21). This time dependence of the incident flux is a rough approximation to the temporal production of gamma rays by a nuclear explosion, and the distribution of forward components of the egressing electron velocities is a crude approximation to $G(u)$. The first order approximation for the equation turns out to be the exact solution,

$$\xi(u, t) = cut - \frac{A}{2} T_0 G_0 \left[1 - \left(\frac{u}{u_m} \right)^2 \right] t^2. \quad (2.3.14)$$

The electric field produced by the scattered electrons is

$$E(x,t) = + \frac{e}{\epsilon} T_0 G_0 \left[1 - \left(\frac{u}{u_m} \right)^2 \right] n(u_m - u_1) , \quad (2.3.15)$$

$$\text{where } u_1(x,t) = \frac{-cu + \sqrt{c^2 u^2 + A(G_0/u_m^2)t^2[2x + AG_0 t^2]}}{A \frac{G_0}{u_m^2} t} .$$

Both of the foregoing special cases have definite merits in predicting the behavior of the electrons that are scattered from a slab by the gamma flux emitted from a nuclear explosion. The first case is closer to the physical situation; whereas the second case is more easily handled. Also, the time dependence of the egressing electron flux is found to be more important in determining the radiated field than the velocity distribution of the electrons; whereas, the velocity distribution of the egressing electrons is more important in determining the form of the resulting electric field.

CHAPTER III

THE RF RADIATION FROM A HIGH ALTITUDE
NUCLEAR EXPLOSION

In the theoretical studies of the RF radiation from a nuclear explosion numerous mechanisms have been considered to produce the radiation. The most notable studies have been made by Karzas and Latter [5, 6, 13], who present two possible mechanisms: (1) gamma rays that are emitted by a nuclear explosion scatter electrons from the device shell or surrounding shield; these electrons are accelerated by the resulting large electric fields and radiate provided they are asymmetrically distributed and (2) gamma rays and X-rays that are emitted by a nuclear explosion scatter electrons in the atmosphere, thereby building up electric currents; these currents in turn are acted upon by the geomagnetic field causing them to radiate. By assuming that an RF signal which is a factor of ten greater than cosmic RF noise can be detected, Karzas and Latter [5] estimate from mechanism (1) that an Earth-based observer can detect a nuclear explosion 10^6 kilometers away from the earth's surface. By using mechanism (2) they estimate the maximum detectable range as $10^7 \sqrt{Y}$ kilometers, where Y is the yield of the explosion in kiloton units; however, if the nuclear device has an X-ray shield, this range is reduced to $0.8 \times 10^6 \sqrt{Y}$ kilometers. For nuclear explosions within the atmosphere, mechanism (1) does not produce an appreciable RF signal because the ionization of the atmosphere by the gamma rays and X-rays is sufficient to absorb the radiation

be absorbed.

If the nuclear explosion occurs in outer space, then only mechanism (1) is responsible for the signal recorded by an artificial satellite outside the Earth's atmosphere. The RF radiation resulting from mechanism (2) is produced in the atmosphere below the ionosphere. That part of this radiation which is directed away from the Earth is absorbed because of the ionization of the upper atmosphere produced by the gamma rays and X-rays and because of the naturally occurring ionization in the ionosphere. In their treatment of the RF radiation due to mechanism (1)--the so-called case signal--Karzas and Latter make very crude approximations to obtain only order of magnitude results. Because of the importance of these results, a more accurate treatment for obtaining this case signal is formulated and presented here.

3.1 Karzas-Latter Model of a Nuclear Explosion

A nuclear explosion emits a very large number of gamma rays ($\sim 7.5 \times 10^{21}$ per kiloton of yield) in a short time duration ($\sim 10^{-7}$ sec). Karzas and Latter [5] consider for this temporal production of gamma rays the following time dependence:

$$\begin{aligned} T(\tau) &= \alpha e^{\alpha\tau} && \text{for } \tau \leq \tau_0 \\ &= \sigma\beta e^{-\beta(\tau-\tau_0)} + \alpha\tau_0 && \text{for } \tau > \tau_0 \end{aligned} \tag{3.1.1}$$

where $\alpha\tau_0 = \ln [(1+\sigma) 7.5 \times 10^{21}Y]$, $\sigma < 1$, $\alpha = 10^8$, $\beta \ll \alpha$, and $T(\tau)d\tau$ is the number of gamma rays produced between time, τ , and time, $\tau+d\tau$. For convenience they chose as a model of the nuclear

explosion a point source of gamma rays at the center of a hemispherical shell. This hemispherical shell could represent either the device shell or an X-ray shield about the device. It has been suggested that a hemispherical shell about a nuclear device could shield the explosion from possible X-ray sensors.

A qualitative description of the process producing the case signal is: the emitted gamma rays scatter electrons from the device shell or shield building up a large positive charge density which accelerates the electrons back to the shell or shield. There is a sufficiently large number of electrons accelerated to radiate a relatively large RF signal. If the gamma rays are emitted from a point source at the center of the hemispherical shell, they will propagate radially, and electrons will be scattered predominantly in the radial direction. If the flux density of gammas, $\frac{1}{4\pi a^2} T(\tau)$, where a = the outside radius of the nuclear device shell or shield, is considered to be incident upon an infinite slab, an upper limit on the number of electrons, N_e , traveling further than 10^{-4} meters from the shell surface can be obtained from (2.3.14),

$$N_e < \frac{1}{2} u_m^2 \left(\frac{a}{r_0}\right)^2 \frac{1}{G_0} \ll \frac{G_0}{4\pi a^2} \int_0^\infty T(\tau) d\tau \quad (3.1.2)$$

where u_m is the maximum velocity of the scattered electrons and $G_0 = \int_0^1 G(u) du$ with $G(u)$ given by (2.2.21). Under these conditions, the equation of motion of the scattered electrons is approximately the same as for the irradiated slab case, (2.3.9), where $\xi(u, \tau; t)$ becomes the radial distance from the shell. The collective motion of the

scattered electrons produces a temporal dipole density, $z(t)$, on the surface of the shell,

$$z(t) = \frac{-e}{4\pi a^2} \int_0^1 du \int_0^t d\tau G(u) T(\tau) \xi(u, \tau; t) \eta[\xi(u, \tau; t)] \quad (3.1.3)$$

where $\xi(u, \tau; t)$ is the position at time t of the electrons egressing from the shell at time τ with forward velocity component u . From Figure 2.5 it is seen that $G(u)$ is peaked near the high velocity end of the spectrum. This occurs for two principal reasons: (1) the higher velocity electrons lose less energy in traversing the slab material and (2) more Compton electrons recoil with higher velocities. Thus a reasonable approximation to $G(u)$ is

$$G(u) \approx G_0 \delta(u - u_0) \quad (3.1.4)$$

where

$$u_0 = \frac{1}{G_0} \int_0^1 G(u) u \, du.$$

By using this approximation (3.1.3) yields

$$z(t) \approx - \frac{e G_0}{4\pi a^2} \int_0^t d\tau T(\tau) \xi(u_0, \tau; t) \eta[\xi(u_0, \tau; t)]. \quad (3.1.5)$$

The first order solution to the equation of motion for the $G(u)$ given by (3.1.4) can be obtained from the foregoing chapter. It is

$$\xi_1(u_0, \tau; t) = cu_0(t-\tau) - \frac{AG_0}{8\pi a^2} (t-\tau)^2 \int_0^\tau d\tau' T(\tau'). \quad (3.1.6)$$

The second order solution is

$$\xi_2(u_0, \tau; t) = cu_0(t-\tau) -$$

$$\frac{A G_0}{4\pi a^2} \int_{\tau}^t dt' (t-t') \int_0^{t'} d\tau' T(\tau') n[\xi_1(u_0, \tau'; t') - \xi_1(u_0, \tau; t')].$$

(3.1.7)

According to (3.1.6) the electrons egressing from the shell at time τ return to the shell at time t_r given by

$$t_r(\tau) = \tau + \frac{1}{\alpha} e^{\alpha(t_0 - \tau)} \quad \text{for } \tau \leq t_0 \quad (3.1.8)$$

$$\text{where } T(\tau) = \alpha e^{\alpha\tau}$$

$$e^{\alpha t_0} = 8\pi a^2 \frac{cu_0 \alpha}{AG_0}$$

(3.1.9)

If $\frac{d}{d\tau} t_r(\tau) \leq 0$, none of the electron paths cross. Thus (3.1.8) predicts that no electron paths cross until $\tau > t_0$ (physically t_0 represents the emission time of the first returning electrons). The first order approximation to the equation of motion, (3.1.6), would represent the exact equation of motion, provided none of the electron paths cross. Therefore, the first order approximation to the equation of motion is exact for times $t \leq t_0$. This conclusion can also be obtained from (3.1.7).

Another form of expressing (3.2.6) for $\tau \leq t_0$ and $e^{\alpha\tau} \gg 1$ is

$$\xi_1(u_0, \tau; t) = \frac{cu_0}{\alpha} \{ [\alpha(t-\tau)] - [\alpha(t-\tau)]^2 e^{\alpha(\tau-t_0)} \}. \quad (3.1.10)$$

By using (3.1.6) in (3.1.5) an exact evaluation of the integral can be obtained. It yields for $e^{\alpha t} \gg 1$ and $t \leq t_0$

$$z(t) = -2\left(\frac{E}{e}\right) \mu u_0^2 e^{\alpha(t-t_0)} \left[1 - \frac{1}{4} e^{\alpha(t-t_0)} \right]. \quad (3.1.11)$$

For $t > t_0$ (3.1.5) must be evaluated numerically to obtain $z(t)$. It is expected that a reasonable approximation to $z(t)$ for $t < t_0$ can be obtained by using the second order approximation to the equation of motion of the electrons; that is, for $t > t_0$

$$z(t) \approx - \frac{e G_0}{4\pi a^2} \int_0^t d\tau T(\tau) \xi_2(u_0, \tau; t) \eta[\xi_2(u_0, \tau; t)] . \quad (3.1.12)$$

A graph of $z(t)$, which is obtained from (3.1.11), is presented in Figure 3.1. The Fourier transform of $z(t)$, $\hat{z}(\omega)$, can be obtained by numerical techniques.

By considering that (3.1.12) is the temporal dipole density on the surface of the nuclear device or surrounding shield, if one is present, the resulting coherent RF radiation can be obtained. As derived in Appendix III, the frequency content of the electromagnetic field radiated by a uniform dipole density, $z(t)$, on a hemisphere is for $r \gg a$,

$$\hat{E}(\omega) = \frac{3a}{2r} \omega^2 \hat{z}(\omega) \frac{J_1\left(\frac{\omega}{c} a \sin \theta\right)}{\omega} e^{-j\frac{\omega}{c} r} , \quad (3.1.13)$$

where

a = outside radius of the hemispherical shell

r = radial distance from the center of the shell

θ = angle between the axis of symmetry of the shell and the direction to the observer

ζ = characteristic impedance of free space

$$\hat{z}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt z(t) e^{-j\omega t} . \quad (3.1.14)$$

FIGURE 3.1. Surface dipole density as a function of time.

The electric field is linearly polarized toward the axis of symmetry of the shell and perpendicular to the direction to the observer. The factor $\frac{1}{\omega} J_1(\frac{\omega}{c} a \sin \theta)$ tends to decrease sharply the frequency content of the electric field for frequencies greater than $\omega_0 = c(a \sin \theta)^{-1}$. The observed magnitude of the radiated RF signal depends upon the frequency response of the receiving device; however, the maximum signal that can be observed is approximately

$$E(t) = \frac{\zeta a^2 \sin \theta}{4rc} \int_{-\omega_0}^{\omega_0} \omega^2 \hat{z}(\omega) e^{j\omega(t - \frac{r}{c})} d\omega, \quad (3.1.15)$$

which yields

$$E(t) \sim \frac{\zeta a^2}{4rc} \sin \theta \ddot{z}(t - \frac{r}{c}), \quad (3.1.16)$$

if $\omega_0 > \alpha$, since $\hat{z}(\omega)$ is found to sharply decrease for $\omega > \alpha$.

It should be pointed out that (3.1.15) neglects the dispersion of the RF signal by the low density ionized plasma occupying interplanetary space. For a treatment of this dispersion the reader is referred to a paper by Karzas and Latter [4], who also present a detailed discussion of the detection of the signal at great distances. These particular topics are not treated here since this work is concerned only with the generation of the RF signal.

For $t \leq t_0 + \frac{r}{c}$, (3.1.11) predicts that the radiated signal increases to a peak value at $\alpha(t - t_0 - \frac{r}{c}) = -\ln 2$ and decreases to zero at $t = t_0 + \frac{r}{c}$. From the numerical results for $t > t_0 + \frac{r}{c}$ it is found that the radiated signal is small as compared to the signal at times $t < t_0 + \frac{r}{c}$ [4]. The peak electric field is

$$E_{\max} = \frac{\zeta a^2}{4c} \left[\frac{3}{4} \left(\frac{\epsilon}{e} \right) \mu u_0^2 \right] \alpha^2 \frac{\sin \theta}{r} . \quad (3.1.17)$$

For $a = 10$ m and $u_0 = .75$ (3.1.17) yields,

$$E_{\max} = 0.60 \times 10^3 \frac{\sin \theta}{R} \frac{\text{volts}}{\text{meter}} , \quad (3.1.18)$$

where R is the radial distance in kilometers.

3.2 Comparison with Karzas and Latter's Results

The treatment presented here is very similar to the treatment of the case signal by Karzas and Latter. The major difference is in the determination of the temporal dipole density produced on the device shell. They choose a functional form which gives correctly certain asymptotic behaviors that are obtained by using quantitative and qualitative arguments. The treatment presented here can be used to obtain the temporal dipole density as accurately as desired.

The expression for the temporal dipole density obtained by Karzas and Latter is

$$z(t) = -\frac{1}{4} \left(\frac{\epsilon}{e} \right) \mu u_0^2 \left[1 + \tanh [\alpha(t-t_0')] + 8 \operatorname{sech} [\alpha(t-t_0')] \right] , \quad (3.2.1)$$

where $\alpha t_0' = \alpha t_0 + \ln 2$. A comparison of (3.2.1) with the results of the foregoing section is made in Figure 3.1. However, it is $\ddot{z}(t)$ that determines the radiation field. For $t = t_0 - \ln 2$, $\ddot{z}(t)$ obtained from (3.2.1) differs by only about 20% from the $\ddot{z}(t)$ obtained from (3.1.11); whereas the values of $z(t)$ differ by only about 10%. Also (3.1.11) predicts $\ddot{z}(t_0) = 0$, but (3.2.1) gives $\ddot{z}(t_0) = \frac{1}{4} \ddot{z}(t_0 - \ln 2)$ where $\ddot{z}(t_0 - \ln 2)$ is the maximum value of $\ddot{z}(t)$. Therefore, the temporal

dipole density obtained by Karzas and Latter can be expected to give only order of magnitude results or less.

By using (3.2.1) Karzas and Latter estimate the signal duration to be about 10^{-7} seconds, but from (3.1.11) and (3.1.16) the signal duration is seen to be equal to $t_0 \sim .4 \times 10^{-6}$. To make a more accurate determination of the RF signal than the one presented here would require a knowledge of both the device geometry and the angular distribution of the emitted gamma rays.

3.3 Factors Determining the RF Radiation

The highly idealized model of the nuclear explosion is chosen for convenience. It was stated earlier that the hemispherical shell that was chosen could represent either the device shell or an X-ray shield about the device. However, the distribution of electrons scattered from the hemispherical shell model would also be equivalent to the distribution of electrons scattered from a spherical shell if the gamma rays were emitted only within a hemisphere. In general this angular distribution of electrons is equivalent--at least to first order--to any asymmetric angular distribution approximately given by $Y_0(\theta, \phi) + Y_1^0(\theta, \phi)$. However, some of the essential quantities determining the radiation depend on the actual shape of the device or shell.

Besides the hemispherical X-ray shield, a flat X-ray shield has also been suggested. By considering a uniform dipole density, $z(t)$, on the surface of a disk the radiated field is obtained in Appendix III. The frequency content of the electric field at a

large distance from the disk is

$$\underline{E}(\omega) = \frac{\zeta}{2} \frac{a}{r} \omega^2 \hat{z}(\omega) \frac{J_1\left(\frac{\omega}{c} a \sin \theta\right)}{\omega} e^{-j\frac{\omega r}{c}} \underline{n}_t, \quad (3.3.1)$$

where a is the radius of the disk, $\underline{n}_t$ is a unit vector perpendicular to the direction to the observer and pointing toward the axis of the disk, and θ is the angle between the direction to the observer and the axis of the disk. The electric field as a function of time is obtained by taking the Fourier transform of (3.3.1). An approximation to the electric field is obtained by using the same arguments as in the treatment of the hemispherical shell:

$$E(r,t) = \frac{\zeta a^2}{4rc} \sin \theta \int_{-\omega_0}^{+\omega_0} \omega^2 \hat{z}(\omega) e^{j\omega(t-\frac{r}{c})} d\omega, \quad (3.3.2)$$

where $\omega_0 = c(a \sin \theta)^{-1}$. If $\hat{z}(\omega)$ is very small for $\omega > \omega_0$ then

$$E(r,t) \sim \frac{\zeta a^2}{4rc} \sin \theta \ddot{z}(t-\frac{r}{c}), \quad (3.3.3)$$

which is of the same form as the field radiated from the hemispherical shell.

Although the dipole density produced on the surface of a disk shield is not uniform, it should vary only a small amount over the surface of the disk since the electrons would travel only a very short distance from the surface. Therefore the RF radiation from the disk shield is approximately equal to the RF radiation from a hemispherical shield of the same radius, if the axis of the disk passes through the source point and the disk is far enough away from the source so

that most of the electrons are scattered normal to the disk.

The gamma source is considered to be at the center of the hemispherical shell to eliminate the occurrence of surface currents which would contribute to the RF signal. But including surface currents would complicate the situation to the point of making it virtually impossible to obtain the radiated field. If the gamma source is displaced only a small distance from the center of the hemispherical shell, it is expected that the resulting surface currents would not contribute substantially to the radiated field.

The magnitude of the RF signal is also strongly dependent upon the time dependence of the resulting temporal dipole density as is indicated by (3.1.16) and (3.3.3). It is found that only the early time behavior $t < t_0$ of the dipole density determines the RF signal. Because of this the RF signal from a nuclear explosion is independent of the yield of the explosion.

CHAPTER IV

THE SIGNAL PRODUCED IN A MONOPOLE ANTENNA
BY GAMMA RAY IRRADIATION

According to experimental observations, a transient signal is induced in a receiving antenna, or probe, by the high intensity gamma flux emitted from a nuclear explosion. It has been established that an object when irradiated by this gamma flux becomes positively charged because of electrons being scattered from it by virtue of the Compton effect. An attempt is made to explain the transient signal produced in an irradiated antenna by considering the temporal charging of the antenna to be exciting the receiving circuit.

An exact treatment of the interaction of gamma rays with an antenna is not feasible. The electromagnetic field generated by the interaction is composed of two parts; that attributable to the antenna and that generated by the space current of scattered electrons. However, the motion of the scattered electrons depends upon the electromagnetic field. Thus the general problem is a rather complicated one involving coupled electro-mechanical equations. There is an additional complication in that the total field must satisfy appropriate boundary conditions at the surface of the antenna.

In an attempt to obtain a useful simplification of the general problem the space current of scattered electrons is neglected in this paper; it being assumed that the primary effect as far as the induced signals are concerned is the redistribution of the

charge on the antenna through the receiving circuit. Qualitatively, one expects the neglect of the space current to be a fair approximation if most of the electrons egress from the antenna with "escape velocity" normal to the antenna axis so that they are removed from the scene of action in such a way as to produce a very small electric field tangential to the antenna surface. For irradiation by low gamma flux intensities nearly all the electrons scattered from an object are scattered with velocities greater than the "escape velocity." However, for gamma flux intensities as great as those within a few meters of a nuclear explosion the resulting space current cannot be neglected. In this case nearly all the scattered electrons are attracted back to the irradiated object producing intense RF fields as obtained in the foregoing chapter. An estimate is obtained in this paper for the upper limit of the gamma flux intensity that will allow the neglect of the space current of scattered electrons.

With the foregoing assumption it is possible to obtain the signal produced in the receiving circuit of the antenna by solving a simple circuit problem. The temporal charge production on the antenna is treated as the charging of a capacitance at a time dependent rate by an isolated source. This circuit approach is admittedly approximate. However, it avoids the rather formidable boundary value problem of an antenna in the presence of a moving charge distribution (the Compton electrons).

4.1 Time Variation of Charge Production on the Antenna

A coaxially fed monopole antenna above an infinite ground

plane is considered to be bombarded by gamma rays propagating normal to the antenna axis. This incident gamma flux scatters electrons from the antenna surface through the Compton effect. The electrons are predominantly scattered into the direction of propagation of the incident gamma rays, but only those electrons sufficiently close to the antenna surface can be scattered from the antenna. In order to obtain the number of electrons scattered from the antenna, the following conditions are made: the electrons are considered to be scattered into the direction of propagation of the incident gamma rays; the scattered electrons are considered to be monoenergetic with the average energy of the recoil electrons from the Compton interaction; and the bombarding gamma rays are considered to be monoenergetic with an energy corresponding to the average energy of the gamma rays produced by a nuclear explosion.

The region of the antenna containing electrons that may be scattered from the antenna can be obtained by using the foregoing conditions. This region shown in Figure 4.1 is outlined by $r = a$, the antenna radius, and $r = r_1(\theta) = [a^2 - R_e \sin^2 \theta]^{\frac{1}{2}} - R_e \cos \theta$ for $-\theta_1 \leq \theta \leq \theta_1$, where $\theta_1 = \cos^{-1}(-\frac{R_e}{2a})$ and R_e is the range of the Compton recoil electrons in the antenna material. The direction of the incident gamma rays is given by $\theta = 0$. The outlined area, A , is given by the double integral

$$A = 2 \int_0^{\theta_1} d\theta \int_{r_1(\theta)}^a dr r,$$

which yields

FIGURE 4.1. Monopole antenna cross section with the incident gamma flux. The shaded region designates the region containing electrons that can be scattered from the antenna.

$$A = R_e \sin \theta_1 \sqrt{a^2 - R_e^2} \sin \theta_1 + a^2 \sin^{-1} \left(\frac{R_e}{a} \sin \theta_1 \right) - \frac{1}{2} R_e^2 \sin 2\theta_1. \quad (4.1.1)$$

The Compton recoil electrons have an average kinetic energy of .43 Mev when the interacting gamma rays have 1 Mev energy. And for electrons with this energy their range is

$$R_e = \frac{1}{\rho} (.15) \text{ cm}, \quad (4.1.2)$$

where ρ is the density of the stopping material in units of grams per cubic centimeter. In general $R_e \ll 2a$ which yields $\theta_1 \approx \frac{\pi}{2}$ and

$$A \approx 2aR_e. \quad (4.1.3)$$

If the antenna height is h and σ is the Compton interaction cross-section per electron, the total number of electrons that are scattered from the antenna is

$$n = \frac{1}{2} INZA h \sigma, \quad (4.1.4)$$

where I = number of incident gamma ray photons passing through a unit area of a plane perpendicular to the direction of propagation of the gamma rays.

The factor of $\frac{1}{2}$ accounts for the number of scattered electrons initially in area A which do not escape the antenna. This estimate is used by Karzas and Latter [4]. Substituting (4.1.3) into (4.1.4) yields

$$n = (2ah)I \frac{R_e}{2\lambda_v}, \quad (4.1.5)$$

where the mean free path of the gamma ray photons, λ_v , is given by

$$\frac{1}{\lambda_v} = NZ\sigma . \quad (4.1.6)$$

It should be pointed out that (4.1.4) requires that $2a \ll \lambda_v$; otherwise multiple scattering of the gamma ray photons must be considered.

As mentioned earlier, the buildup of charge on the antenna can increase to the point that the scattered electrons are attracted back to the antenna. It is necessary to estimate the maximum charge on the antenna that will allow the scattered electrons to escape the potential field of the charged antenna. The potential energy of an electron on the surface of the charged antenna is

$$PE \approx e \frac{Q}{C_0} , \quad (4.1.7)$$

where C_0 is the d.c. capacitance of the antenna and Q is the total charge on the antenna.* In order for an electron to escape the potential field of the antenna, its kinetic energy must be greater than its potential energy. If the average kinetic energy of the electrons scattered from the antenna is $\frac{1}{4}$ Mev when the antenna is bombarded by 1 Mev gammas (as it is for electrons that are scattered from a slab [4]), the electrons will escape the potential field provided

$$Q \ll \frac{1}{4} C_0 \times 10^6 \text{ Coulombs}, \quad (4.1.8)$$

*Since the capacitance of an antenna is frequency dependent, its potential depends upon the time dependence of the charging process. Later it is seen that this potential is approximately given by $\frac{Q}{C_0}$ when the antenna is charged by the interaction with the gamma flux from a nuclear explosion.

when C_0 is expressed in units of Farads. Thus, in order for most of the scattered electrons to escape from the potential field of the antenna, the incident gamma ray intensity, I , must be restricted by

$$I \ll \frac{2}{(2ah)} \frac{\lambda_\gamma}{R_e} \frac{C_0}{4e} \times 10^6 \frac{\text{photons}}{\text{m}^2} . \quad (4.1.9)$$

For thin antennas ($\frac{2h}{a} > 60$) the antenna d.c. capacitance can be obtained from King's [14] theory of short antennas.

$$C_0 = \frac{4\pi}{c\zeta_0} \frac{h}{\Omega - 3.39} , \quad (4.1.10)$$

where c = speed of light

ζ_0 = characteristic impedance of free space

$$\Omega = 2\ell n \left(\frac{2h}{a} \right) .$$

Choosing $h = 1$ m, $2a = 1$ cm, and an aluminum antenna for a typical example, the limit on the incident gamma ray intensity is

$$I \ll 10^{17} \frac{\text{photons}}{\text{m}^2} . \quad (4.1.11)$$

Based on the temporal variation given by Karzas and Latter [4] for the production of gammas by a nuclear explosion, the incident gamma ray intensity is

$$I(r,t) = D(r)T(t) \frac{\text{photons}}{\text{m}^2} , \quad (4.1.12)$$

where $D(r)$ expresses the decreases in intensity as a function of the distance (4), from the explosion and

$$\left. \begin{aligned} T(t) &\approx e^{at} - 1 & 0 \leq t \leq t_0 \\ &\approx e^{at_0} (1 + \sigma - \sigma e^{-\beta(t-t_0)}) & t > t_0 \end{aligned} \right\} \quad (4.1.13)$$

Typical values for the explosion parameters are

$$\left. \begin{aligned} \alpha &\approx 10^8 \text{ sec}^{-1} \\ \sigma &\approx 1 \\ \beta &\ll \alpha \\ t_0 &\approx \frac{1}{\alpha} \ln \left[\frac{1}{1+\sigma} 7.5 \times 10^{21} Y \right], \end{aligned} \right\} \quad (4.1.14)$$

where Y is the total yield of the explosion in kilotons. Provided that the incident gamma ray intensity is limited by (4.1.9), the time history of the charge buildup on the antenna is obtained by substituting (4.1.12) into (4.1.5), yielding

$$Q(t) \approx (2ah)e \frac{R_e}{2\lambda_v} D(r) T(t). \quad (4.1.15)$$

The function $D(r)$ must be obtained from experimental data to which the author does not have access. If there is only free space about the nuclear detonation, $D(r) = \frac{1}{4\pi r^2}$.

4.2 Excitation of the Receiving Antenna

The excitation of the receiving circuit of the monopole antenna is provided by the lumped capacitance of the antenna receiving charge from an external source. For convenience the receiving circuit is considered to be only a load capacitance connected to the antenna, although a more refined circuit may be treated following the technique presented. This load capacitor could represent the input impedance of a wide band amplifier.

It is shown in Appendix III that a frequency dependent capacitance receiving charge from an external source has the same circuit properties

as an initially uncharged capacitance in series with the idealized voltage source,

$$v(t) = \frac{1}{2\pi j} \mathcal{L}^{-1} \left\{ \frac{\hat{Q}(s)}{C(s)} \right\}, \quad (4.2.1)$$

where $\mathcal{L} \rightarrow$ Laplace transform

$\mathcal{L}^{-1} \rightarrow$ inverse Laplace transform

$$\hat{Q}(s) = \mathcal{L}\{Q(t)\}$$

$C(s)$ is the transform capacitance

$Q(t)$ is the amount of charge deposited on the capacitor by the external source.

Therefore the receiving circuit of the antenna can be treated as an idealized voltage source, $v_i(t)$, in series with the lumped antenna impedance, Z_0 , and the load impedance, Z_L . The transform current, $\hat{i}(s)$, in this circuit is

$$\hat{i}(s) = \frac{\hat{v}_i(s)}{Z_0(s) + Z_L(s)}, \quad (4.2.2)$$

where $Z_0(s)$ is the transform impedance of the antenna (transform impedance is obtained by letting $j\omega = s$ in the ordinary impedance expression) and $Z_L(s)$ is the transform impedance of the load. The current in the receiving circuit as a function of t is

$$i(t) = \mathcal{L}^{-1}\{\hat{i}(s)\}, \quad (4.2.3)$$

and the voltage across the load impedance is

$$v_L(t) = \mathcal{L}^{-1}\{\hat{i}(s)Z_L(s)\}. \quad (4.2.4)$$

Substitution of (4.2.2) into (4.2.4) yields

$$v_L(t) = \frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \frac{Z_L(s) \hat{v}_i(s)}{Z_L(s) + Z_O(s)} e^{st} ds, \quad (4.2.5)$$

where

$$\hat{v}_i(s) = \frac{Q(s)}{C(s)} \quad (4.2.6)$$

with $\hat{Q}(s)$ as the transform of (4.1.15) and $C(s)$ as the transform capacitance of the antenna. Unfortunately, (4.2.5) cannot be evaluated exactly because of the very complicated frequency dependence of the antenna impedance and because the frequency dependence of the antenna capacitance is known only in the low frequency range.

By considering $\beta = \sigma = 0$ in (4.1.13), a rather simple expression can be obtained for $\hat{Q}(s)$. It is

$$\hat{Q}(s) = Q_0 \left\{ \frac{1}{\alpha-s} [e^{(\alpha-s)t_0} - 1] + \frac{1}{s} [e^{-st_0} - 1] + \frac{1}{s} e^{(\alpha-s)t_0} \right\}, \quad (4.2.7)$$

where $Q_0 = (2ah)e^{\frac{Re}{2\lambda_v}} D(r)$. The following behavior is obtained from (4.2.7):

for $|\omega| \ll \alpha$

$$\frac{1}{Q_0} \hat{Q}(j\omega) \sim -\frac{j}{\omega} e^{\alpha t_0} e^{-j\omega t_0} \quad (4.2.7a)$$

for $|\omega| \gg \alpha$

$$\frac{1}{Q_0} \hat{Q}(j\omega) \sim e^{-j\omega t_0} \left[\frac{\alpha}{\omega^2} e^{\alpha t_0} - j \frac{1}{\omega} \right] - \frac{\alpha}{\omega^2}. \quad (4.2.7b)$$

This limiting behavior indicates that a fair approximation to $\hat{Q}(s)$ is

$$\hat{Q}(s) \approx 0 \quad \text{for } |s| > \alpha. \quad (4.2.8)$$

Using (4.2.8) in (4.2.6) yields

$$\hat{V}_i(s) \approx 0 \quad \text{for } |s| > \alpha. \quad (4.2.9)$$

The foregoing expression indicates that most of the contribution to the integral in (4.2.5) occurs for low frequencies or small $|s|$. If the height of the monopole antenna is restricted so that $h \ll \frac{c}{\omega}$, then the short antenna theory of King [14] can be used to predict the analytical behavior of the antenna impedance and capacitance for this frequency range. It is for $\beta h \ll 1$ or $h \ll \frac{c}{\omega}$

$$\left. \begin{aligned} C(s) &\approx C_0 \\ Z_0(s) &\approx \frac{1}{sC_0} \end{aligned} \right\} \quad (4.2.10)$$

with C_0 given by (4.1.10). By using (4.2.9) and (4.2.10) in (4.2.5), the following is obtained:

$$v_L(t) \approx \frac{1}{C_0 + C_L} \int_0^{t-1} \{\hat{Q}(s)\} \quad (4.2.11)$$

which reduces to

$$v_L(t) \approx \frac{Q(t)}{C_0 + C_L}. \quad (4.2.12)$$

A more accurate determination of this voltage can be obtained from (4.2.5) but it would require numerical integration. The relation (4.2.12)

indicates that the charge produced on the antenna instantaneously distributes itself in the receiving circuit of the antenna.

In order to make a sample calculation for the signal induced in a monopole antenna, a convenient geometry is selected. Consider that there is only free space about the explosion. Then if an aluminum monopole antenna, $h = 1$ m and $2a = 1$ cm, is 30 km from the point of detonation and has a load capacitance $C_L = C_0$, the voltage across the load is

$$v_L(t_0) \sim \frac{1}{16} Y \text{ volts} , \quad (4.2.13)$$

where Y is the yield in kilotons and

$$t_0 = \frac{1}{\alpha} \ln (3.75 \times 10^{21} Y) .$$

Although this antenna height does not quite satisfy the condition that $h \ll \frac{c}{\alpha}$, the results obtained for this height are expected to be correct within an order of magnitude.

CHAPTER V

SUMMARY AND CONCLUSIONS

It has been the aim of this dissertation to present a study of the interaction of a high density gamma flux with matter by considering some of the resulting electromagnetic effects. A charge separation is found to be the direct effect of the interaction. It in turn produces electromagnetic fields which are found to be quite large.

In treatment of the irradiation of a slab by a gamma flux an approximate expression is obtained for the velocity distribution of the electrons that are scattered from the slab. It shows that the scattered electrons are quite energetic. The only other existing theoretical method of obtaining the velocity distribution is a Monte Carlo calculation. However, Monte Carlo calculations are extremely tedious and time-consuming even with the modern computers, and to obtain great accuracy may require too much time.

The treatment of the RF radiation from a high altitude nuclear explosion is found to be a great improvement in accuracy over the theory by Karzas and Latter [4], whose results were accurate only to an order of magnitude. Because of the importance of their results, the present treatment is formulated to improve the accuracy. A more accurate treatment requires a detailed knowledge of the nuclear device.

The gamma ray irradiation of an object charges the object positively. This charging effect is used to explain the signal produced

in a receiving antenna by gamma ray irradiation, and an approximate expression is obtained for the voltage across the load impedance of a monopole antenna that is bombarded by gamma rays propagating normal to the antenna axis. The signal is found to be relatively large at great distances from the explosion. A more accurate treatment to obtain the produced signal would be virtually impossible to formulate.

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APPENDIX I

ON THE AVERAGE PATH LENGTH OF ELECTRONS
PENETRATING A THIN FOIL

Consider a collimated beam of monoenergetic electrons penetrating matter. In their classic paper on the multiple scattering of electrons Bethe, Rose and Smith [15] derive an expression for the average cosine of the angles between the direction of motion of the electrons and the primary beam. For a small average energy loss by the electrons this expression is

$$\langle \cos \phi \rangle = \exp \left[-2 \frac{s}{\lambda} \right], \quad (\text{A1.1})$$

where s is the length of path traversed and λ is the transport mean free path of the electrons (see page 12). If the beam of electrons is normally incident upon a thin foil, (A1.1) can be used to obtain the average path length through the foil. If d is the foil thickness, then

$$d = \int_0^{\langle s \rangle} \langle \cos \phi \rangle ds, \quad (\text{A1.2})$$

where $\langle s \rangle$ is the average path length. (A1.1) substituted into (A1.2) yields after integration

$$\langle s \rangle = - \frac{\lambda}{2} \ln \left(1 - \frac{2}{\lambda} d \right),$$

but since $\frac{2d}{\lambda} \ll 1$,

$$\langle s \rangle \approx d \left(1 + \frac{d}{\lambda} \right). \quad (\text{A1.3})$$

This result can also be obtained from the formulation of Rose [16] in his paper correcting the theory developed by Goudsmit and Saunderson [17].

The average lateral displacement of the electrons from the primary beam can be obtained from (A1.1). It is

$$\langle y \rangle = \int_0^{\langle s \rangle} ds \sqrt{1 - \langle \cos \phi \rangle^2}, \quad (\text{A1.4})$$

which yields after integration and using (A1.3)

$$\langle y \rangle \approx \frac{4}{3} \frac{d^{\frac{3}{2}}}{\lambda^{\frac{1}{2}}} \left(1 + \frac{9}{10} \frac{d}{\lambda} \right). \quad (\text{A1.5})$$

If a beam of electrons is incident upon a foil at some angle θ measured from the normal to the foil, then the average path length through the foil would be approximately

$$\langle s \rangle \approx \frac{d}{\cos \theta} \left(1 + \frac{d}{\lambda \cos \theta} \right), \quad (\text{A1.6})$$

provided that $d/\cos \theta \ll \lambda$. (A1.6) holds since the spread of the electron beam predicted by (A1.5) is very small for thin foils.

APPENDIX II

ON THE EXISTENCE OF A SOLUTION TO THE
EQUATION OF MOTION

The equation of motion of electrons after egressing from a slab is the nonlinear integral equation,

$$\xi(u, \tau; t) = \xi_0 - A \int_{\tau}^t dt' K[\xi(u, \tau; t'), t'], \quad (A2.1)$$

where $A = \frac{e^2}{mc}$, $\xi_0 = cu(t-\tau)$, and

$$K[\xi(u, \tau; t'), t'] = (t-t') \int_0^{t'} d\tau' \int_0^1 du' G(u') T(\tau') \\ \times n[\xi(u', \tau'; t') - \xi(u, \tau; t')]. \quad (A2.2)$$

The kernel, $K[\xi(u, \tau; t'), t']$, is proportional to the number of electrons further from the slab than $x = \xi(u, \tau; t')$ and thus is continuous and bounded for all t' . Suppose that $c_1 K[\xi(u, \tau; t'), t']$ gives the total number of electrons further from the slab than $\xi(u, \tau; t')$.

Then

$$|K(\xi, t) - K(\xi', t)| < \frac{1}{c_1} M |\xi - \xi'|, \quad (A2.3)$$

where $M = 1 +$ the maximum density of electrons occurring outside the slab.

It is now established that the kernel, K , satisfies the Lipschitz condition, (A2.3), and is continuous and bounded for all t' . Therefore according to a theorem given by Davis [18] on integral equations

of the foregoing type, (A2.1) has one and only one solution which can be obtained by the Method of Picard, a method of successive approximations which takes ξ_0 as the zeroth solution.

APPENDIX III

THE RADIATION FROM SURFACE DISTRIBUTIONS
OF DIPOLES

Consider a uniform density of dipoles, $z(t)$, on the surface of a hemisphere with the dipoles radially oriented. The Fourier transform of the radiation field from an increment area of dipoles is

$$\underline{\hat{E}}(\underline{r}, \omega) = - \frac{\omega^2 e^{-j\frac{\omega}{c}r_1}}{4\pi\epsilon c^2 r_1^3 r_0} [\underline{r}_1 \times (\underline{r}_1 \times \underline{r}_0)] \hat{z}(\omega) dA, \quad (A3.1)$$

where $\underline{r}_1 = \underline{r} - \underline{r}_0$, $\hat{z}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} z(t) e^{-j\omega t} dt$, $\underline{r}_0$ is the radius vector locating the dA area, and $\underline{r}$ is the radius vector locating the field point. For $r \gg r_0$ (A3.1) yields upon integration

$$\underline{\hat{E}}(\underline{r}, \omega) = - \frac{\omega^2 \hat{z}(\omega) e^{-j\frac{\omega}{c}r}}{4\pi\epsilon c^2 r^3} \int_S dA e^{+j\frac{\omega}{c} \frac{\underline{r} \cdot \underline{r}_0}{r}} [\underline{r} \times (\underline{r} \times \underline{r}_0)], \quad (A3.2)$$

where $|\underline{r}_0| = a$ is the radius of the hemisphere. The evaluation of the integral in (A3.2) yields

$$\underline{\hat{E}}(\underline{r}, \omega) = + \frac{\omega^2 \hat{z}(\omega)}{2r} \frac{J_1\left(\frac{\omega}{c} a \sin \theta\right)}{\omega} e^{-j\frac{\omega}{c}r} \underline{t}, \quad (A3.3)$$

where $\underline{t}$ is a unit vector perpendicular to $\underline{r}$ and pointing toward the axis of symmetry of the hemisphere, and θ is the angle between $\underline{r}$ and the axis of symmetry of the hemisphere.

Consider now a uniform density of dipoles, $z(t)$, on the surface of a disk with dipoles oriented perpendicular to the disk. For $r \gg r_0 = \rho_0$ (A3.2) yields

$$\underline{\hat{E}}(\underline{r}, \omega) = + \frac{\omega^2 \hat{z}(\omega) e^{-j \frac{\omega r}{c}}}{4\pi \epsilon c^2 r} \int_0^a d\rho_0 \int_{-\pi}^{+\pi} d\phi_0 e^{+j \frac{\omega}{c} \rho_0 \sin \theta \cos \phi_0} \underline{t}, \quad (\text{A3.4})$$

where a is the radius of the disk, The evaluation of the integral in (A3.4) yields

$$\underline{\hat{E}}(\underline{r}, \omega) = + \left(\frac{\epsilon a}{2r} \right) \omega^2 \hat{z}(\omega) \frac{J_1\left(\frac{\omega}{c} a \sin \theta\right)}{\omega} e^{-j \frac{\omega r}{c}} \underline{t}. \quad (\text{A3.5})$$

where θ and $\underline{t}$ are interpreted the same as in (A3.3).

APPENDIX IV

AN EQUIVALENCE PROPERTY OF A CIRCUIT CONTAINING
A CAPACITOR THAT IS BEING CHARGED

Consider a frequency dependent capacitance, $C_0(s)$, that is being charged at the rate $R(t)$. The voltage across the capacitor as a function of time is

$$v_1(t) = \mathcal{L}^{-1} \left\{ \frac{1}{C_0(s)} \mathcal{L} \left\{ \int_0^t R(t) dt \right\} \right\}, \quad (\text{A4.1})$$

where $\mathcal{L}^{-1} \rightarrow$ inverse Laplace transform and $\mathcal{L} \rightarrow$ Laplace transform. If this capacitor is connected to some impedance, Z , and is receiving the charge for a source external to the circuit, the equation for the transform current, $\hat{i}(s) = \mathcal{L}\{i(t)\}$, is

$$\hat{v}_1(s) - \frac{\hat{i}(s)}{sC_0(s)} = \hat{i}(s) Z(s). \quad (\text{A4.2})$$

If an idealized voltage source, $v(t)$, were put in series with the same capacitor, now not receiving charge, the circuit equation for the transform current is

$$\hat{v}(s) = \hat{i}(s) Z(s) + \frac{\hat{i}(s)}{sC_0(s)}. \quad (\text{A4.3})$$

Note that (A4.2) and (A4.3) are equivalent when

$$\hat{v}(s) = \hat{v}_1(s) \quad (\text{A4.4})$$

or

$$v(t) = v_1(t). \quad (\text{A4.5})$$

Therefore it is proved that a circuit containing a capacitor that is being charged from an external source is equivalent to a circuit with the capacitor in series with an idealized voltage source where the capacitor is not receiving charge from an external source.