Numerical Study of DWBA in a Three-Body Model of Stripping

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ABSTRACT

Exact and DWBA wave functions, stripping amplitudes, and angular distributions have been calculated and compared in a separable-potential, three-body model of deuteron stripping. Detailed results are presented and analyzed for the case of deuterons incident with energy $E = 6.7$ MeV; these are similar to results also obtained for $E = 1.78, 11.2$ and 15.12 MeV. Despite good agreement between exact and DWBA angular distributions, the two sets of wave functions and partial wave stripping amplitudes themselves do not agree too well. This casts doubts on the validity of the DWBA approximation to the exact wave function, but does support use of the adiabatic wave function of Johnson and Soper in analyses of stripping data.

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We have recently shown^{1,2} that the assumption of a separable form for the neutron-proton interaction in the deuteron allows us to justify the approximation used in the DWBA description of deuteron stripping of replacing the three-body deuteron wave function by a product of two-body wave functions in the stripping matrix element. In particular, in the three-body model for stripping proposed by Mitra³ - a model in which the nucleons 1 and 2 in the deuteron interact with one another and with a structureless core via separable potentials - a modified DWBA transition matrix has been defined which is identical to the exact transition matrix.

In the model, the exact T matrix for the process wherein an incident deuteron with momentum $\vec{k}_d$ deposits nucleon j into a bound state with nucleon i continuing free with momentum $\vec{k}_i$ is*

$$T = \langle \chi_{\vec{k}_i}^{(-)} \phi_j | V_{12} | \psi \rangle. \quad (1)$$

Here, $|\phi_j\rangle$ is the bound state of nucleon j with the core, $|\chi_{\vec{k}_i}^{(-)}\rangle$ is an ingoing scattering state of the nucleon i interacting with the core, V_{12} is the interaction between the model nucleons, and $|\psi\rangle$ is the total wave function which describes the model system completely. The modified or no distortion DWBA is obtained from (1) by replacing $|\psi\rangle$ by the "no deuteron internal distortion state" $|\psi^{ND}\rangle$, where $|\psi^{ND}\rangle \equiv |\phi_d F\rangle$:

$$T^{ND} = \langle \chi_{\vec{k}_i}^{(-)} \phi_j | V_{12} | \phi_d F \rangle. \quad (2)$$

In this equation, $|\phi_d\rangle$ is the deuteron bound state and $|F\rangle$ the "spectator state" for the motion of the deuteron center of mass³ (cm). The standard DWBA in the model is obtained by replacing $|\psi\rangle$ by the product $|\phi_d u_d\rangle$:

*Momentum and energy dependence is (and will be) suppressed in the symbol(s) T.

$$T^{DW} = \langle \chi_{k_1}^{(-)} \phi_j | V_{12} | \phi_d u_d \rangle, \quad (3)$$

where $|u_d\rangle$ is a state which describes the elastic scattering of the deuteron center of mass, and is defined by $|u_d\rangle = \langle \phi_d | \psi \rangle$.

The assumption that V_{12} is separable leads to the result¹ that $T^{ND} = T$. Hence replacement of T by T^{DW} , a standard procedure encountered in the DWBA as used to analyze nuclear reaction data^{1,2}, here is equivalent to T^{DW} being a good approximation to T^{ND} . Thus, in the model, the validity of the replacement $T \rightarrow T^{DW}$ depends on $|u_d\rangle$ producing results similar to those of $|F\rangle$ in the stripping matrix element.

Use of the approximation $|\psi\rangle \approx |\phi_d u_d\rangle$ leading to (3) has been the subject of much criticism^{2,4,5}. The fact that exact numerical calculations can be carried out in the three-body model, plus the fact that $|\psi\rangle \approx |\phi_d F\rangle$ is an "exact" approximation in the model matrix element allows one to test the validity of $|\psi\rangle \approx |\phi_d u_d\rangle$ by comparing $|F\rangle$ and $|u_d\rangle$. Since it is already known⁷ that the model predicts stripping angular distributions like those observed experimentally, and further, since the three-body reaction problem contains the rearrangement and breakup channels of a many-body reaction problem, comparison of $|F\rangle$ and $|u_d\rangle$ should provide a better understanding of DWBA as used in actual nuclear reaction analyses. In addition, such a comparison will also help to test the adiabatic model recently proposed by Johnson and Soper.⁶

The previously reported² qualitative comparison of $|F\rangle$ and $|u_d\rangle$ indicated that these states obey similar two-body scattering equations and that they approach identical values asymptotically. We have begun a series of calculations to determine quantitatively how these states differ. We shall present a limited set of preliminary results here; more complete and detailed results will be reported elsewhere. A critical discussion of other three-body model, stripping calculations is given in ref. 2.

We have calculated a special case of the Mitra³ model for the stripping of deuterons incident with kinetic energies of 1.78, 6.7, 11.2 and 15.12 MeV. The nucleons of the deuteron interact with one another via a Yamaguchi separable potential⁸ with range parameter $\beta = 1.45 \text{ F}^{-1}$; this yields a bound state of 2.225 Mev binding energy. Each of the nucleons interacts with the core via the same Yamaguchi potential with range parameter $\beta = 1.06 \text{ F}^{-1}$, giving a bound state of 3.300 MeV binding energy. Thus, the two stripping channels lead to identical, though distinguishable, results in our model.

The two intermediate values of the energy were chosen so as to be able to compare our results with those of Reiner and Jaffe⁷, who used different nucleon-core interactions V_1 ; ie, $V_1 + V_2$ in their calculations. The high and low energy values were selected to extend the range over which exact and DWBA could be compared. Comparison of the exact and DWBA results leads to similar conclusions in all four cases and we present detailed results here for the 6.7 MeV case only. This particular case is the most interesting in that the exact and DWBA angular distributions are very close; for the 11.2 MeV case, the DWBA is not as good. This is in contrast to the results reported by Reiner and Jaffe⁷ for these two energies.

In Figure 1, the exact, the DWBA, and the plane wave stripping cross sections are presented. These are computed from, respectively, T^{ND} , T^{DW} , and Eq. (3) with $\langle \chi_{k_1}^{(-)} |$ and $|u_d\rangle$ replaced by their plane wave parts. The agreement between the DWBA and the exact results seems to confirm the results of Reiner and Jaffe⁷ that for the separable-potential three-body model, DWBA is a good approximation. However, examination of the partial wave amplitudes T_l^{ND} and T_l^{DW} of Table 1, where T_l is in general defined by $\langle \hat{k} = \hat{k}_d / |\hat{k}| \rangle$

$$T = \sum_l (2l+1) T_l P_l(\hat{k}_d \cdot \hat{k}_1),$$

shows that this closeness of fit is somewhat fortuitous: $T_2^{DW} \neq T_2^{ND}$ except for $l \geq 2$. Similar results hold for the other three energies as well, although in these cases, the magnitudes of the exact and DWBA cross sections are in worse agreement. This is striking evidence that agreement between exact and approximate cross sections need not imply agreement between the corresponding partial wave amplitudes. From Table 1, we can conclude that the main reason for such good agreement between the angular distributions is that the $l=2$ and $l=3$ partial wave amplitudes are the most important in that their multiplicative factors of $(2l+1)$ are much greater than the similar factor for $l=0$, even though $|T_0^{ND}|$ and $|T_0^{DW}|$ are reasonably large.

The cm states $|F\rangle$ and $|u_d\rangle$ are the only quantities that differ in T^{ND} and T^{DW} , and we examine typical behavior in Figures 2 and 3. We plot the $l=0$ and $l=3$ partial wave components of $RF(R) \equiv R\langle R|F\rangle$ $Ru_d(R) \equiv R\langle R|u_d\rangle$ where for example we have expanded $F(R)$ via

$$F(R) = \sum_l F_l(R) P_l(\hat{k}_d \cdot \hat{R}).$$

A similar expansion is used to define $u_{d,l}$. Here, R is the deuteron cm coordinate.

Two points are evident from these figures. First, the major differences between the elastic and spectator functions occur in the region $R \lesssim 1.5 F$. Second, the maximum differences range between 50% and 100%, with $|\text{Re}(F_0)| < |\text{Re}(u_{d,0})|$ and $|\text{Im}(F_0)| > |\text{Im}(u_{d,0})|$ in this region. Now, as we have shown¹, T^{ND} and T^{DW} are given by

$$T^{ND} = -\frac{\pi^2}{2M} \langle \chi_{k_1}^{(-)} \phi_j | fF \rangle$$

and

$$T^{DW} = -\frac{\pi^2}{2M} \langle \chi_{k_1}^{(-)} \phi_j | f u_d \rangle,$$

where $|f\rangle$ is the form factor^{1,3} for the nucleon-nucleon interaction.

These matrix elements could be computed in a space representation either by a $d^3r d^3R$ integration or a $d^3r_1 d^3r_2$ integration,

where $\underline{r}_1$ is the coordinate of particle 1 and $\underline{r} = \underline{r}_1 - \underline{r}_2$. Since stripping is a rearrangement process, both sets of coordinates enter into the matrix elements, and it is not obvious which of the factors, ϕ_j and f , is controlling the range of integration. Let us consider $d^3r d^3R$. Then $\phi_j(\underline{r}_j)$ and $f(\underline{r})$ have ranges of about 1.0 and 0.7 F, respectively, implying that only those regions where $F_\ell(R)$ and $u_{d,\ell}(R)$ differ ($R \lesssim 1.5F$) are important in their matrix elements. However, the effect of the weighting factors $\chi_{k_1} \phi_j$ as given in a $\underline{r}, R$ representation is not easy to assess. What one can say is that in such a representation even $\langle \phi_j |$ will have an infinite series, oscillatory, partial wave expansion, each term of which will be complex. Consequently, differences in $u_{d,\ell}$ and F_ℓ may not produce important differences in partial wave stripping amplitudes, as is evident on comparison of, for example, the real parts of the $k=0$ and $k=3$ amplitudes of Table 1 with the behavior exhibited in Figs. 2 and 3. Results similar to these hold for the other energies.

Not only do the exact and DWBA partial wave amplitudes differ for the low partial waves, we also see that real and imaginary parts of F_ℓ and $u_{d,\ell}$ may differ by nearly an order of magnitude (Fig. 3). However, as indicated above, such differences need not lead to any obvious conclusions concerning the sizes of the corresponding matrix elements. This behavior of F_ℓ (or $u_{d,\ell}$) occurs for other energies as well, as does the fact that for both the real and the imaginary parts of F_ℓ and $u_{d,\ell}$, their maxima and minima occur at roughly the same values of R , which indicates that F_ℓ and $u_{d,\ell}$ essentially differ only in scale in the region $R \lesssim 1.5F$.

Since we have calculated $d\sigma/d\Omega$ for energies ranging from 1.78 MeV to 15.12 MeV, we are able to examine, over this range, the convergence properties of DWBA as a function of energy. The results of Reiner and Jaffe, in which DWBA more closely approximated the exact cross-section at the higher of the two

energies, suggested that DWBA might converge to the exact result as the bombarding energy increased. Our results do not support such a suggestion, since the DWBA angular distributions (forward directions) are in slightly worse agreement with the exact ones as we go from 6.7 MeV to 15.12 MeV. This behavior is not seen in comparisons of integrated cross sections (Table 2), mainly because the $\sin^2\theta$ factor gives the least weight to the forward angles where the cross sections are largest in both magnitude and in their differences.

We may draw several conclusions from these model results. First, $F(R)$ and $u_d(R)$ are equal except in a region inside of about 1.5 to 2.0 F around the origin, $R=0$. Second, the differences in F and u_d occur where these functions make their main contributions to the stripping matrix elements. Third, obvious differences in $F_l(R)$ and $u_{d,l}(R)$ cannot be used to predict that the corresponding T_l^{ND} and T_l^{DW} will differ. Fourth, one cannot predict from near equality of angular distributions that partial wave matrix elements must be nearly equal. Hence, we cannot conclude that in the model, DWBA is a good approximation in detail, although it does produce reasonable angular distributions. Furthermore, we also cannot conclude that DWBA converges towards the exact result as the energy increases.

In one sense, these model calculations have tested the ability of DWBA to fit "data" where "data" is the exact result obtained from T^{ND} . We have found that DWBA in the model works about as well as DWBA when it is used to analyze actual stripping data. In another sense, however, we are also comparing DWBA with the "adiabatic" approximation method of Johnson and Soper⁶, since in the model, the adiabatic wave function reduces to F , i.e., in the three-body model the adiabatic wave function is "exact". Our results clearly indicate that the DWBA is not as accurate in detail as the adiabatic approximation.

This confirms theoretically the conclusion obtained from fitting nuclear reaction data⁹, viz, that use of the adiabatic wave function rather than u_d is called for in DWBA analyses of stripping reactions.

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Table 1. Partial Wave Stripping Matrix Elements, $E = 6.7$ MeV

l	$\text{Re}T_l^{\text{ND}}$	$\text{Im}T_l^{\text{ND}}$	$\text{Re}T_l^{\text{DW}}$	$\text{Im}T_l^{\text{DW}}$	T_l^{BORN}
0	1.403	0.450	0.792	-1.923	-10.595
1	0.471	0.017	0.173	-0.450	- 5.868
2	-1.947	-0.108	-1.709	-0.425	- 2.963
3	-1.352	0.006	-1.300	-0.072	- 1.461
4	-0.709	0.005	-0.699	-0.009	- 0.720
5	-0.356	0.001	-0.354	-0.001	- 0.357
6	-0.178	-	-0.178	-	- 0.178
7	-0.089	-	-0.089	-	- 0.089
8	-0.044	-	-0.044	-	- 0.044
9	-0.021	-	-0.021	-	- 0.021

Table 2. Exact and DWBA Integrated Cross Sections in mb

	$E=1.78$ MeV	$E=6.7$ MeV	$E=11.2$ MeV	$E=15.12$ MeV
Exact	352.46	231.70	147.17	125.84
DWBA	418.23	218.23	135.30	117.43

Figure Captions

- Figure 1: Comparison of exact, DWBA, and plane wave stripping cross sections for $E = 6.7$ MeV.
- Figure 2: Comparison of a) the real and b) the imaginary parts of $RF_0(---)$ and $Ru_{d,0}(-)$.
- Figure 3: Comparison of a) the real and b) the imaginary parts of $RF_3(---)$ and $Ru_{d,3}(-)$.

FIG. 1

FIG. 2a

FIG. 2b

FIG. 3a

FIG. 3b