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SOME LIFETIME MEASUREMENTS OF
ISOMERIC STATES IN THE RARE
EARTH REGION WITH MAGNETIC
COINCIDENCE BETA-RAY
SPECTROMETERS

by

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TABLE OF CONTENTS

	Page
ABSTRACT	v
I. INTRODUCTION	1
A. Historical Survey	3
B. Experimental Methods of Measuring Lifetimes of Isomeric States	6
II. THEORY	14
III. EXPERIMENTAL EQUIPMENT AND METHODS OF ANALYSIS	25
A. Experimental Equipment	25
B. Methods of Analyzing the Delayed Coincidence Distribution	43
IV. MEASUREMENTS AND RESULTS	50
A. Half-Lives of Three Isomeric Levels in Thulium 169	50
1. Half-life of the 8.40 keV level	51
2. Half-life of the 118-keV level	84
3. Half-life of the 139-keV level	85
4. Discussion of the Tm^{169} ground-state rotational band	86
B. Half-Lives of Two Isomeric Levels in Gadolinium 155	92
1. Half-life of the 87-keV level	96
2. Half-life of the 105-keV level	99
3. Comparison with theoretical predictions	102

C.	Measurements of Half-Lives of Isomeric Levels in Samarium 152, Gadolinium 156, Lutetium 175 and Praseodymium 144	108
1.	Half-life of the 122-keV level in Samarium 152	109
2.	Half-life of the 89-keV level in Gadolinium 156	114
3.	Half-life of the 396-keV level in Lutetium 175	120
4.	Half-life of the 80-keV level in Praseodymium 144	127
D.	Concluding Remarks	135
V.	LITERATURE CITED	137
VI.	APPENDIX A: CIRCUIT DIAGRAMS	143
VII.	APPENDIX B: CALIBRATION OF THE TIME-TO-PULSE- HEIGHT CONVERTER	147
VIII.	APPENDIX C: MEASUREMENTS ON THE TIME DELAY OF THE CONVERTER CALIBRATION CABLES AND THE SPIRAL DELAY LINE	159
IX.	APPENDIX D: COMPUTER PROGRAMS	162
X.	APPENDIX E: THEORETICAL CALCULATIONS FOR GADOLINIUM 155	172

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SOME LIFETIME MEASUREMENTS OF ISOMERIC STATES IN
THE RARE EARTH REGION WITH MAGNETIC COINCIDENCE
BETA-RAY SPECTROMETERS*

Robert E. McAdams and Eastman H. Hatch

ABSTRACT

The half-lives of nine isomeric states in the rare earth region were measured using magnetic beta-ray spectrometers and an associated fast-slow coincidence system utilizing a time-to-pulse-height converter. The half-lives of the 8.40-, 118- and 139-keV levels in Tm^{169} were determined to be (4.13 ± 0.12) nsec, (63 ± 7) psec and (289 ± 24) psec, respectively. The half-life of the 8.40-keV level was determined using both Er^{169} and Yb^{169} decay. Several different methods were used to measure the 8.40-keV level lifetime including applying various accelerating voltages to the radioactive sources. Comparisons are made with theory for the $K = 1/2$ rotational band in Tm^{169} . The half-life obtained for the 8.40-keV level in Tm^{169} corresponds to an energy level width of $(1.10 \pm 0.03) \times 10^{-7}$ eV. If there were no broadening of the individual line profiles in the Mössbauer effect pattern, the width at half-maximum of the resolved Mössbauer lines would be $(2.20 \pm 0.06) \times 10^{-7}$ eV compared to the narrowest experimental Mössbauer gross line width of 5.04×10^{-7} eV. The half-lives of the 87- and 105-keV levels in Gd^{155} were also determined and were found to be (6.27 ± 0.35) nsec and (1.20 ± 0.04) nsec, respectively. Comparisons were made with the theoretical predictions of Nilsson for these two levels. The theoretical-experimental comparison would assign a spin of $5/2$ to the 105-keV level and a spin of $3/2$ to the 87-keV level if the comparison could be taken at face value. The half-lives of the 122-keV level in Sm^{152} , 89-keV level in Gd^{156} , 396-keV level in Lu^{175} and 80-keV level in Pr^{144} were also determined. The half-lives of these four levels were found to be (1.36 ± 0.06) nsec, (2.22 ± 0.08) nsec, (3.31 ± 0.10) nsec and (143 ± 11) psec, respectively. These four half-life values agree well with values obtained by other investigators.

*This report is based on a Ph.D. thesis submitted by Robert E. McAdams November, 1964, to Iowa State University, Ames, Iowa.

I. INTRODUCTION

This investigation is concerned with the measurements of the lifetimes of several low-lying excited nuclear levels in nuclei in the rare earth region. The measurements of lifetimes of excited nuclear states have been an important contribution to the development of nuclear theory and to our general knowledge of nuclei. They are an especially important tool for checking the validity of nuclear models.

Lifetimes of excited nuclear states in Tm^{169} , Gd^{155} , Sm^{152} , Gd^{156} , Lu^{175} and Pr^{144} have been measured and these measurements will be presented in detail. A major portion of this work is concerned with the determination of the lifetime of the 8.4-keV level in Tm^{169} . Because of the low energies involved in the 8.4-keV transition, it was necessary to develop and apply new techniques to this measurement. The measurement of the 8.4-keV level lifetime makes possible, in conjunction with other experimental quantities, a check on the validity of the unified nuclear model as applied to the first three excited states in Tm^{169} . The 8.4-keV transition is also widely used in Mössbauer studies and it is desirable to have a reliable determination of the lifetime which can be used in the analysis of the Mössbauer results. In addition, the techniques involved in determining the lifetime of the 8.4-keV level in Tm^{169} can be applied to lifetime measurements of other low-energy levels in other nuclei.

In this section, after a few preliminary definitions and remarks, a brief review of the historical developments concerned with lifetime measurements will be presented. This will be followed by a brief review of experimental methods available for measuring lifetimes of excited

nuclear levels. Two general references, one by Alburger (1) and one by Bashandy (2), were used in the preparation of this section and may be referred to for a more detailed review.

An excited nuclear state which has a measurable lifetime is referred to as an isomeric state, and a nucleus which can exist in an isomeric state is called an isomer. The lifetime of an excited nuclear state is intimately associated with the energy E , the angular momentum I , and the parity π of the excited state and of other nuclear states having lower energy. These other nuclear states, of course, include the ground state of the nucleus.

Practically all excited levels in nuclei decay to levels of lower energy by emitting gamma radiation. Because of this, it is convenient to classify these transitions according to the angular momentum L carried off by the gamma ray. Electromagnetic radiation of angular momentum L corresponds, classically, to radiation arising from either an electric 2^L pole (EL) or a magnetic 2^L pole (ML). The multipolarity of the radiation is specified as EL or ML according to the class and the multipole order of the radiation. Consider a nucleus in an initial excited state, characterized by the quantities I and π , which decays to a final state of lower energy characterized by the quantities I' and π' . If the gamma radiation emitted during this transition has angular momentum L , the conservation of angular momentum requires L to obey the following selection rule:

$$|I - I'| \leq L \leq I + I' \quad . \quad (1)$$

In addition, the transverse nature of electromagnetic waves forbids L

being equal to zero. Furthermore, the conservation of parity requires:

$$\Delta \pi = \pi \pi' = (-1)^L \quad \text{for EL radiation} \quad (2)$$

and

$$\Delta \pi = \pi \pi' = (-1)^{L-1} \quad \text{for ML radiation.} \quad (3)$$

In these equations ($\Delta \pi = +1$) means the initial state and the final state have the same parity, and ($\Delta \pi = -1$) means the parity changes.

A. Historical Survey

The first case of nuclear isomerism was discovered in 1921 by Hahn (3). It was well known that Th^{234} decayed by beta-ray emission to Pa^{234} which has a half-life of 1.14 min. Hahn discovered that a beta-ray activity with a half-life of 6.7 hr also followed the decay of Th^{234} . Furthermore, this second substance had the same atomic number and the same atomic mass as Pa^{234} . No explanation of this behavior was given at the time.

The next well established case of nuclear isomerism did not occur until 1935. In that year Amaldi et al. (4) and Kourtchatow et al. (5) discovered an isomeric state in bromine. The two stable isotopes of bromine, Br^{79} and Br^{81} , were irradiated with slow neutrons and the resulting activities were studied. It was found that activities were present having half-lives of 18 min, 4.4 min, and 34 hr. Ruling out the possibility of a third isotope necessitated assigning two of the half-lives to either Br^{80} or Br^{82} . It was later shown by Bothe and Gentner (6) and by Snell (7) that the two half-lives of 18 min and 4.4 hr were associated with Br^{80} while the 34-hour half-life belonged to Br^{82} .

The first proposed explanation of isomeric states was given in 1936 by von Weizsäcker (8). He stated that nuclei could exist in excited states with different angular momenta. If the angular momentum of an excited nuclear state differed considerably from the angular momenta of other states of lower energy, the nucleus would be required to make a relatively difficult transition involving a large change in angular momentum. He showed that angular momentum changes of one or two units resulted in a lifetime of the order of 10^{-13} sec. He further showed that each additional unit of change in angular momentum could result in the lifetime being longer by a factor of about 10^6 .

Von Weizsäcker's explanation encouraged experimentalists to discover other isomeric states with an aim of establishing other properties of the states. It was soon realized, however, that the measured lifetime of a state might not be sufficient to determine the angular momenta and parities of the states involved, because lifetime calculations require detailed knowledge or assumptions about nuclear structure.

Early attempts to calculate lifetimes were made using various nuclear models such as a radiating proton moving in an average field, or the liquid drop model where it was assumed radiation arose from vibrations of the drop. These early calculations did not prove too successful since the models were not based on sufficient experimental evidence. The first significantly successful nuclear model, the shell model, was introduced in 1950 by Mayer (9) and by Haxel, Jensen and Suess (10). The data on isomeric transitions contributed significantly to the experimental evidence on which this model was based.

In 1951 Goldhaber and Sunyar (11) published an experimental classi-

fication of the then known isomeric transitions. They showed that the experimental results were in agreement with the shell model if the $j-j$ coupling rule is relaxed for an odd number of particles in the $1g_{9/2}$ shell. They also gave empirical K/L conversion electron ratio curves which could replace the less accurate non-relativistic theoretical curves. In addition, transitions which had been previously assigned the multipole orders of E3, E4 and E5 on the basis of the theoretical K/L ratios were reassigned the multipole orders of E2, E3 and E4, respectively. That is, the previous multipole assignments were one unit too large. Furthermore, all electric transitions, with the exception of some E2 transitions, had lifetimes which were longer than those obtained from the Weisskopf formula (12). These conclusions were found to be correct as more data were accumulated.

The main discrepancy between the experimental results and the single particle shell model was the short lifetime of some E2 transitions. An explanation of these transitions was given in 1953 by Bohr and Mottelson (13) with the introduction of the collective nuclear model.

In 1955 Nilsson extended the shell model to include deformed nuclei far from closed shells, i.e. far from the magic numbers. In many cases, comparisons between the magnitude of theoretical values of the lifetime and experimental values were considerably improved.

In this investigation comparisons will be made between the experimental values of the lifetimes of several isomeric states and the appropriate theoretical values.

B. Experimental Methods of Measuring Lifetimes of Isomeric States

A considerable number of methods have been developed for measuring the lifetimes of isomeric states. These methods can be broken up into two main categories, one composed of methods where the lifetime is measured directly and the other one composed of methods where the lifetime is obtained by an indirect measurement. In an indirect measurement another quantity, e.g. the Coulomb excitation cross-section or the level width, is measured and the lifetime is calculated. Several methods in each category will be mentioned or briefly outlined.

Lifetimes as small as a few seconds can be measured by using ordinary counting techniques. For example, a beta-ray spectrometer can be used to determine the counting rate of the internal conversion electrons arising from the isomeric transition. This counting rate is recorded as a function of time and can be used directly to determine the lifetime. This method requires a relatively simple decay scheme for the isotope being examined.

Another method is that of pulsed activation. This method is very similar to the one mentioned above. In this case the sample is left in the beam of an accelerator and the bombarding beam is interrupted using some type of shutter. The counting rate is then obtained only during the time the beam is interrupted. This method has been used by Alvarez (14). Bureau and Hammer (15) have also used this method, employing the Iowa State University synchrotron, to measure the (14.4 ± 0.3) - μ sec half-life of the 366-keV level in W^{181} . In addition, Hammer and Stewart (16) have used this method to measure several half-lives in the range of 10 to 10^{-4} sec.

Lifetimes can also be measured directly using a time channel analyzer. This instrument has an electronic gate which is opened by an initiating pulse and which remains open for a fixed period of time. While the gate is open the decay transition can be counted. By moving the gate in time with respect to the initiating pulse, which arises from a transition feeding the isomeric level, the delayed radiation intensity distribution is obtained. An instrument of this type has been used by von Dardel (17) to measure the 163.7- μ sec half-life of Po^{214} .

Alburger and Pryce (18) have used a triggered synchroscope method to obtain a half-life of 144 μ sec for the 2.20-MeV level in Pb^{206} . The pulse from a single detector was fed to the sweep trigger and to the vertical deflection circuits. A slotted mask was then placed over the face of the oscilloscope tube and a phototube used to count the number of pulses at successive positions along the horizontal axis. In this manner the delayed pulse distribution was obtained.

The nuclear recoil method may be used, in some cases, to measure lifetimes of isomeric states. This method consists of measuring the distance a recoiling nucleus travels before it emits a gamma ray. This method has been used by Thirion and Telegdi (19) to investigate the 0.87-MeV level in O^{17} .

A study of the Doppler effect on radiation arising from a recoiling nucleus can also lead to an upper limit for the half-life of an isomeric state. In some cases an actual value can be obtained for the half-life. Doppler broadening of the photoelectron line of the 478-keV gamma ray in Li^7 has been used by Elliott and Bell (20) to determine the lifetime of the 478-keV excited state. A more detailed account of this method as well

as the recoil method mentioned above can be found in reference (1).

A relatively new method has been used by Blaugrund et al. (21) to measure the lifetimes of the 118- and 139-keV levels in Tm^{169} . The source is inserted between two beta-ray spectrometers which are adjusted to transmit the conversion electrons of the cascade of interest. The electrons are then alternately accelerated and decelerated using high-frequency electric fields in two microwave cavities. The coincidences between the two conversion lines are then recorded as a function of the phase difference between the two electric fields. A "prompt" curve is obtained by using a low-frequency field and the half-life is obtained by folding different half-lives into the prompt curve until a fit of the experimental delay curve is obtained.

A method which is similar to the acceleration method is the deflection method. The main difference is that the high-frequency modulating field is applied in a way which deflects the electrons perpendicular to the beam. This method has been used by Johansson and Alväger (22) to measure half-lives of the order of 10^{-10} sec.

Perhaps the most widely used method for measuring lifetimes directly is the method using delayed coincidences. This method utilizes two detectors which we shall refer to as A and B. Detector A detects the radiation associated with a transition populating the isomeric state of interest, while detector B detects the radiation depopulating the state. A delayed coincidence distribution is then obtained by delaying the pulses from detector B, with respect to the pulses from detector A, by various amounts of time. In general it is necessary to have a prompt curve for comparison. A prompt curve is obtained from a cascade which has the same

radiation energies as the two radiations used in obtaining the delayed coincidence distribution. Furthermore, the lifetime of the intermediate level in the prompt radiations should be negligible with respect to the lifetime of the isomeric state. The lifetime of the isomeric state can then be determined by the shift of the delayed coincidence curve with respect to the prompt curve.

A significant extension of the delayed coincidence method was made by Sunyar (23) and by Green and Bell (24). In this method a time-to-pulse-height converter is used to convert the time overlap distribution of the pulses from the two detectors into a pulse-height distribution. The output of the time-to-pulse-height converter is fed into a multichannel pulse-height analyzer. In this manner the time distribution of the isomeric level can be obtained much more quickly and accurately than by using the delayed coincidence method mentioned above. Since this is the method employed in the present investigation, a detailed description of this method will be given in Section III.

The range of lifetimes that can be measured with delayed coincidences has a lower limit of the order of 10^{-12} sec. For a given instrument, the lower limit of measurable lifetimes is usually of the order of one tenth of the width of the prompt curve at half maximum. The width of the prompt curve is limited by the phosphor decay time of the scintillator and by the time spread in the phototube. A discussion of the time spread in phototubes is given by Post and Schiff (25) and by Gatti and Svelto (26). They also discuss the best methods for extracting the signal from the phototube.

For a review of later developments in delayed coincidence measurements

the reader is referred to a discussion by Schwarzschild (27).

There are, as mentioned previously, several indirect methods for obtaining the lifetime of an isomeric state. Two of these methods make use of the fact that the energy width Γ of a nuclear state is related to the mean life, τ , by the Heisenberg uncertainty relation:

$$\Gamma \tau = \hbar \quad (4)$$

where $\hbar$ is Planck's constant divided by 2π .

If gamma radiation having the same energy as a ground state transition is incident on a nucleus, the gamma radiation may be absorbed by the target nucleus. If the cross-section is measured as a function of the energy of the bombarding gamma ray, the level width Γ may be determined. A major difficulty with this method is in obtaining gamma radiations with the correct energies. Radiation from the same nucleus is not sufficient because of the recoil energy of the nucleus. Therefore, some method must be employed which will compensate for this recoil effect.

One method of compensating for this loss in energy due to recoil was studied experimentally by Pollard and Alburger (28). This method is to use a nucleus which emits a beta particle before emitting a gamma ray. When the beta particle is emitted the nucleus recoils; if the gamma ray is then emitted before the nucleus comes to rest, the energy loss (due to nuclear recoil in emitting the gamma ray) may be compensated by the energy shift due to the Doppler effect. Pollard and Alburger used Na^{24} in their experimental study but no resonant scattering was observed. A similar study by Metzger (29) also produced negative results.

The first positive result for a method of restoring the condition

of resonance was presented by Moon (30, 31). The method used by Moon consisted of using mechanical motion to compensate for the energy loss due to nuclear recoil. The mechanical motion is supplied by placing the source on a high-speed rotor operated at speeds up to about 7×10^4 cm/sec. He used a Hg^{198} source and a liquid-mercury (10% Hg^{198}) scatterer. He obtained a significant increase in the scattering and obtained a width Γ of about 3×10^{-5} eV, for the 411-keV level of Hg^{198} . This value for Γ corresponds to a half-life of the order of 10^{-11} sec.

Another method which depends on measuring the width of the nuclear level for the determination of the level lifetime is provided by the Mössbauer effect (32, 33, 34). In the Mössbauer effect, a low-energy gamma ray is emitted by a nucleus in a transition to the ground state. The gamma ray is then absorbed by an absorber of the same material as the nucleus which emitted the gamma ray. If neither the source nucleus nor the absorber nucleus experiences a recoil during emission or absorption, the gamma ray will not suffer a change in energy due to nuclear recoil, and the energy of the gamma ray will correspond exactly to the transition energy between the excited state and the ground state of the nucleus. Therefore, the probability for absorption of the gamma ray in the absorber will be high. In some cases, if the source and absorber are at the correct temperature, or if the gamma-ray energy is sufficiently low, the emission and absorption will be recoil-free. The momentum of the gamma ray is taken up by the crystals in which the source and absorber nuclei are bound. If either the source or absorber is moved, the gamma ray then suffers a Doppler shift, and the cross-section for absorption of the gamma ray as a function of the velocity can be obtained. The absorp-

tion line width, if there were no broadening effects due to absorber thickness or impurities, would correspond approximately to 2Γ , where Γ is again the width of the excited state.

The basic experimental setup of a Mössbauer experiment consists of a source mounted on a velocity drive, an absorber target and a gamma-ray detector. (In some cases the absorber is mounted on the velocity drive instead of the source.) The energy of the emitted gamma-ray is changed as a result of the Doppler effect, by an amount

$$\Delta E = E(v/c) \quad (5)$$

where E is the gamma-ray energy, v is the source velocity and c is the velocity of light. The gamma-ray transmission intensity is determined as a function of the source velocity. As indicated above, the resulting experimental absorption line has a width at half maximum of 2Γ . The width is 2Γ because the energy distributions of the emission line and absorption line each has a width of Γ , and the experimental absorption curve results from the overlapping of the emission line and the absorption line.

There are difficulties in this method. Extreme care must be used in source preparation and in the design of the velocity apparatus. Vibration in the equipment can cause considerable broadening of the experimental absorption line. In addition, solid-state effects such as internal fields due to impurities can cause broadening. In many cases, a width measured by this method is considerably broader than the same level width determined from a lifetime measurement using other methods. A more detailed discussion of Mössbauer work is given by Frauenfelder (35).

The method which has been used for determining more lifetimes of excited nuclear levels than any other method is Coulomb excitation. This process involves the interaction of the electric field of the target nucleus and of the electric field of a closely-passing bombarding particle. This interaction excites the target nucleus into a higher excited nuclear state. By measuring the absolute cross-section one can calculate the lifetime of the inverse radiative transition. Practically all of the excitations observed so far have been of the E2 character.

As mentioned previously, the method that was used in this investigation is the delayed coincidence method utilizing a time-to-pulse-height converter. A detailed discussion of the method and the equipment used in this investigation will be presented in a later section. The section immediately following will present a pertinent portion of the theory of transition probabilities.

II. THEORY

In this section a brief outline of the theory of gamma-decay transition probabilities will be presented. The discussion will be restricted to those portions of the theory which are directly applicable to the present investigation.

The selection rules for gamma radiation have been given in Section I (see Equations 1, 2 and 3). The same notation used in Section I will be used in this section, i.e. the unprimed quantities will refer to the initial state involved in the transition and the primed quantities will refer to the final state. The half-life $\tau_{1/2}$, the mean life τ and the transition probability T are related by the following equation:

$$\tau_{1/2} = 0.693\tau = 0.693/T \quad (6)$$

The transition probability from an initial state to a final state is given by the well-known formula from time-dependent perturbation theory:

$$T = (2\pi/\hbar) |\langle \psi' | H' | \psi \rangle|^2 (dN/dE) \quad (7)$$

where $\hbar$ is Planck's constant divided by 2π , dN/dE is the density of final states, H' is the perturbing Hamiltonian and ψ and ψ' are the wavefunctions of the initial and final state, respectively.

Weisskopf (12) and Moszkowski (36) have calculated the transition probability for electromagnetic radiation. For gamma radiation of multipole order σL ($\sigma L = ML$ or EL), Equation 7 gives the transition probability as:

$$T(\sigma L) = \frac{8\pi}{\hbar} \cdot \frac{L+1}{L[(2L+1)!!]^2} \left(\frac{\omega}{c}\right)^{2L+1} B(\sigma L) \quad (8)$$

where $(2L+1)!! = 1 \cdot 3 \cdot 5 \cdot 7 \cdots (2L+1)$, c is the velocity of light, $\hbar\omega$ is the gamma-ray energy and $B(\sigma L) = |\langle \psi' | \mathcal{M}_{LM}^\sigma | \psi \rangle|^2$ is the reduced transition probability. The operator $\mathcal{M}_{LM}^\sigma$ is the appropriate electric or magnetic multipole operator for the Hamiltonian interaction H' .

Equation 8 is arrived at by confining the radiation field, represented by the vector potential $\vec{A}$, in a large sphere of radius R_0 . The tangential component of $\vec{A}$ is required to vanish at the surface of the sphere and $\vec{A}$ must remain finite inside the sphere. The vector potential $\vec{A}$ is then expanded into a series of standing spherical waves. Each standing wave has a definite angular momentum LM and corresponds to radiation emitted by either a magnetic 2^L pole or an electric 2^L pole located at $r = 0$. In addition, it is necessary to quantize the radiation field. A more detailed discussion of the derivation of Equation 8 can be found in reference (36).

The reduced transition probability $B(\sigma L)$ depends on the particular nuclear model being considered. Two models will be briefly discussed. The single particle model using an isotropic potential will be discussed first, followed by a discussion of the unified model.

As mentioned in the introduction, the nuclear shell model was introduced by Mayer (9) and by Haxel, Jensen and Suess (10). In the single-particle shell model the essential assumption is that the nucleons in an odd-A nucleus fill the single-particle levels in such a manner that only the last odd particle contributes to the angular momentum or electromagnetic moments of the nucleus. It is also assumed that the odd nucleon experiences a central potential which results from the average action of

the rest of the nucleons. The potential V experienced by the last nucleon is assumed to be nonsingular at $r = 0$ and $\partial V/\partial r = 0$ at $r = 0$, where r is the radius from the nuclear center of mass. Furthermore, the potential is assumed to have spherical symmetry and goes to zero rapidly at the boundary of the nucleus. This potential is approximated by using either a harmonic oscillator potential or a square-well potential. In practice, an interpolation between these two potentials is used to obtain an energy level diagram. The spherical symmetry of the potential leads to conservation of angular momentum; hence, the angular momentum quantum number ℓ is a good quantum number.

Finally, it is found that when a spin-orbit coupling term $C(\vec{\ell} \cdot \vec{s})$ is added to the potential the magic numbers can be obtained. The individual energy levels are characterized by a specific orbital angular momentum quantum number ℓ and by a specific total angular momentum quantum number $j = \ell \pm 1/2$. For an odd- A nucleus the single-particle levels are then filled by nucleon pairs in such a manner that only the last odd particle contributes to the angular momentum or the electromagnetic moments of the nucleus. A more detailed discussion of this model may be found in references (37, 38).

Weisskopf (12) has used the single particle model to calculate an estimate of the transition probability for gamma radiation. He considers the case of a single proton transition and obtains, for E1 and M1 transitions:

$$T(E1) = 1.0 \times 10^{14} A^{2/3} E^3 \quad (9)$$

$$T(M1) = 3.1 \times 10^{13} E^3 \quad (10)$$

where E is the gamma-ray energy in MeV and A is the mass number. These equations were obtained by assuming the radial wavefunctions of both the initial and final states to be constant inside the nucleus and to vanish outside the nucleus. In addition, the nuclear radius has been taken as $1.2 \times 10^{-13} A^{1/3}$ cm.

Moszkowski (36) has performed a similar calculation and has obtained, for E1 and M1 transitions:

$$T(E1) = 1.5 \times 10^{14} A^{2/3} E^3 S \quad (11)$$

$$T(M1) = 3.1 \times 10^{13} E^3 S \quad (12)$$

where the nuclear radius has been used as $1.45 \times 10^{-13} A^{1/3}$ cm, and where S is a statistical factor.

The unified nuclear model combines the shell model and the collective model. In the single particle shell model described above, the potential is assumed to be spherical, but it has been found that many nuclei far from closed shells have large deformations, as shown by their large quadrupole moments. In the unified model a deformed nonspherical nuclear potential is assumed for calculating the particle motion. The collective model, which was introduced by Bohr and Mottelson (13), is concerned with nuclear motion in which all particles in the nucleus participate. This collective motion is made up of vibrational and rotational motion. For large nuclear deformations the nuclear wavefunction ψ can be expressed as a product:

$$\psi = \chi \cdot \phi \cdot \mathcal{D} \quad (13)$$

where χ represents the intrinsic motion (independent particle motion), q represents vibrational motion and $\mathcal{D}$ represents the rotational motion.

For this discussion the nucleus will be assumed to be in the vibrational ground state. For this case, the vibrational wavefunctions do not enter into the calculations.

In the case of axial symmetry, which is assumed for this model, the intrinsic motion is characterized by the quantum number Ω , which is the projection of the intrinsic angular momentum on the nuclear symmetry axis. The total Ω is just equal to $\sum \Omega_p$ summed over all the particles, where Ω_p is the symmetry axis projection of the particle intrinsic angular momentum.

The rotational motion is characterized by the quantum numbers I , M , and K , where I is the total angular momentum, and M and K are the projections of the total angular momentum on the space-fixed axis and on the nuclear symmetry axis, respectively. The angular momentum $\vec{I}$ results from the intrinsic angular momentum $\vec{j}$ coupling to the rotational angular momentum $\vec{R}$. In the ground state K equals I . A group of rotational states which all have the same intrinsic wavefunction are referred to as a rotational band.

The energy states of a rotational band in odd- A nuclei with $K \neq 1/2$ are given to first order by:

$$E_K(I) = E_K^0 + \frac{\hbar^2}{2\mathcal{I}} I(I+1) \quad (14)$$

where E_K^0 is a constant, and $\mathcal{I}$ is the moment of inertia of the nucleus.

For $K = 1/2$ the energies of the states are given to first order by:

$$E_{1/2}(I) = E_{1/2}^0 + \frac{\hbar^2}{2J} [I(I+1) + a(-1)^{I+1/2}(I+1/2)] \quad (15)$$

where a is the decoupling factor and arises from the partial decoupling of the intrinsic motion from the rotational motion.

The intrinsic wavefunctions which are used in Equation 13 are the Nilsson wavefunctions (39) for the anisotropic single particle model. Nilsson (39) has extended the single particle model to the case of an anisotropic potential. He uses an anisotropic harmonic oscillator potential with axial symmetry. To this potential is added the spin-orbit term $C(\vec{l} \cdot \vec{s})$ as was done in the spherical shell model. An additional term $D\vec{l}^2$ is added to the potential to depress the high angular momentum states. In effect, this term interpolates between the oscillator potential and the square-well potential. The single-particle Hamiltonian interaction is then split up into two terms. One term H_0 is spherically symmetric and the other term H_δ is a function of the deformation parameter δ . The constants C and D are then chosen in such a manner as to reproduce the isotropic shell model levels when the deformation is zero.

A representation is chosen such that H_0 is diagonal. In addition, $\vec{l}^2$, l_z , and s_z are diagonal and commute with H_0 . These three operators are the orbital angular momentum and the projections of the orbital angular momentum and the spin on the nuclear symmetry axis. Their corresponding quantum numbers are l , Λ and Σ . None of these operators commute with the total Hamiltonian H . However, $l_z + s_z$ does commute with H and its corresponding quantum number is Ω . For states having a given Ω the basic vectors $|N\Lambda\Sigma\rangle$ are used, where $\Lambda + \Sigma = \Omega$. The quantum number

N represents the total number of nodes in the harmonic oscillator wave-function.

For large deformations the $\vec{l} \cdot \vec{s}$ and $\vec{l}^2$ terms can be treated as perturbations. In this case the states are identified with N, n_z , Λ and Σ , where n_z is the number of nodal planes perpendicular to the symmetry axis.

Nilsson (39) has used this model to calculate the transition probability for a gamma transition in which only one proton changes levels. For an E1 transition, he obtains:

$$T_N(E1) = 2.93(1 - \frac{Z}{A})^2 A^{1/3} \left(\frac{\hbar\omega}{197\text{MeV}} \right)^3 10^{21} | \langle I1KK' - K | I1I'K' \rangle |^2$$

$$\times \left\{ \sum_{\ell\ell'} [\langle N'\ell' | r | N\ell \rangle \sqrt{\frac{2\ell+1}{2\ell'+1}} \langle \ell 100 | \ell 1\ell'0 \rangle \sum_{\Lambda\Lambda'} \delta_{\Sigma\Sigma'} a_{\ell'\Lambda'} \right.$$

$$\left. \times a_{\ell\Lambda} \langle \ell 1\Lambda K' - K | \ell 1\ell'\Lambda' \rangle \right\}^2 \text{sec}^{-1} \quad (16)$$

where $\hbar\omega$ is the gamma-ray energy in MeV, K is the projection of the total angular momentum I on the nuclear symmetry axis and $\delta_{\Sigma\Sigma'}$ is the Kronecker delta, which is defined as being equal to one if $\Sigma = \Sigma'$ and equal to zero if $\Sigma \neq \Sigma'$. In addition, $\langle I1KK' - K | I1I'K' \rangle$ is the Clebsch-Gordon coefficient in the Condon-Shortley (40) notation, and the coefficients $a_{\ell\Lambda}$ are defined by:

$$|N\Omega\alpha\rangle = \sum_{\ell\Lambda\Sigma} a_{\ell\Lambda} |N\ell\Lambda\Sigma\rangle \quad (17)$$

where $\sum_{\ell\Lambda} a_{\ell\Lambda}^2 = 1$ and $|N\Omega\alpha\rangle$ is the eigenvector whose configuration space representation is denoted χ . In Equation 17, α represents an additional quantum number which is not important to the present discussion but which

can be identified by referring to Nilsson (39). If Equation 16 is to be used for a neutron transition, the quantity $(1-Z/A)^2$ is replaced by $(Z/A)^2$. This is the result of replacing the proton effective charge $(1-Z/A)e$ by the neutron effective charge $(Z/A)e$.

The radial matrix element $\langle N'l' | r | Nl \rangle$ can be easily calculated using the formula given by Nilsson (39). The radial matrix element for E1 transitions is different from zero only if the following conditions are fulfilled:

$$l + 1 \geq l' \geq l - 1 \quad (18)$$

$$N + 1 \geq N' \geq N - 1 \quad (19)$$

In practice $l' \neq l$ and $N' \neq N$. For $N' = N - 1$ and E1 transitions, the radial matrix element reduces to two terms:

$$\langle N-1 \ l-1 | r | Nl \rangle = [(N+l+1)/2]^{1/2} \quad (20)$$

$$\langle N-1 \ l+1 | r | Nl \rangle = [(N-l)/2]^{1/2} \quad (21)$$

In the Tm^{169} investigation, the interest is in M1 transitions in a $K = 1/2$ rotational band. In this case, there is a partial decoupling of the intrinsic motion from the rotational motion and the intrinsic structure appreciably affects the transition probability. Nilsson (39) has also calculated the transition probability for this case. For a transition from a level with spin $I'+1$ to a level with spin I' (both levels belong to the same $K = 1/2$ rotational band) he obtains:

$$T(M1) = (2.63 \times 10^{11}) \frac{2I'+1}{I'+1} G_o^2 |1+b_o(-1)^{I'-1/2}|^2 E^3 \quad (22)$$

where G_0 and b_0 are constants which can be obtained from other experimental measurements. The following equations give relations involving G_0 and b_0 , in addition to other quantities which are measurable, for the case of $I' = \Omega = K' = 1/2$:

$$G_0 = 3\mu - \alpha(g_I - g_R) - 1/2g_S + g_I - 2g_R \quad (23)$$

$$b_0 = -(2G_0)^{-1} [3\mu + \alpha(g_I - g_R) + 1/2g_S - g_I - g_R] \quad (24)$$

In these equations μ is the magnetic moment of the ground state, α is the decoupling factor and g_R , g_I and g_S are the gyromagnetic ratios. For free nucleons $g_S = (5.585, -3.826)$ and $g_I = (1, 0)$ for protons and neutrons. Of course, the values of the magnetic ratios for nucleons bound in a nucleus are not necessarily the same as for free nucleons. The gyromagnetic ratio g_R is approximately equal to Z/A for irrotational flow of uniformly charged nuclear matter. Again, this value is not necessarily the value for a nucleus but is used in some cases as an approximate value.

The value of the magnetic moment for a level of spin I in a $K = 1/2$ rotational band is given by the following equation:

$$\mu_I = \frac{1}{4(I+1)} (g_K - g_R) [1 - (2I+1)(-1)^{I-1/2} b_0] + Ig_R \quad (25)$$

where $g_K = G_0 - g_R$.

Branching ratios of transitions to different members of a rotational band obey a rather simple relation and are sometimes helpful in assigning spins to single particle levels. For electromagnetic transitions of multipole order L from a state characterized by I and $K = \Omega$ to two members of a rotational band characterized by I' , I'' (I'' refers to the lower of

the two rotational states) and $K' = \Omega'$, the theoretical gamma-ray intensities $\gamma(I \rightarrow I'')$ and $\gamma(I \rightarrow I')$ satisfy the relation:

$$\frac{\gamma(I \rightarrow I'')}{\gamma(I \rightarrow I')} = \left(\frac{E''}{E'} \right)^{2L+1} \left(\frac{\langle ILKK'-K | ILI''K' \rangle}{\langle ILKK'-K | ILI'K' \rangle} \right)^2 \quad (26)$$

where E'' is the gamma-ray energy for the transition $I \rightarrow I''$, E' is the gamma-ray energy for the transition $I \rightarrow I'$, and the quantities in the brackets are Clebsch-Gordan coefficients in the Condon-Shortley notation (40). For a more detailed discussion of intensity rules for beta- and gamma-ray transitions to rotational levels, cf. Alaga et al. (41).

In almost all cases the mean life of an isomeric state which is measured experimentally is not the gamma-ray mean life. This is true because of the presence of internal conversion and branching to different lower levels. For this reason, it is necessary to correct the experimental mean life for the presence of any of these other possible means of decay.

When internal conversion is present the gamma-ray mean life is given by:

$$\tau(\gamma) = \tau(\text{exp})(1+\alpha) \quad (27)$$

where α is the total internal conversion coefficient. If, in addition, branching to other levels is also present, the gamma-ray mean life of the i^{th} branch is given by:

$$\tau_i(\gamma) = \tau(\text{exp})(1+\alpha)I_i^{-1} \quad (28)$$

where I_i is the relative intensity of the i^{th} branch depopulating the level, i.e. $I_i < 1$ and $\sum I_i = 1$. If the radiation is also an admixture,

(for example M1 and E2), the mean life for the electric multipole radiation is:

$$\tau_1^E(\gamma) = \tau(\text{exp}) (1+\alpha) I_1^{-1} \left[\frac{I(M)}{I(E)} + 1 \right] \quad (29)$$

where $I(M)$ is the intensity of the magnetic multipole radiation and $I(E)$ is the intensity of the electric multipole radiation.

The theory and the equations given in this section will be used extensively in the discussions of the lifetime measurements of isomeric levels in Tm^{169} and Gd^{155} . These discussions are presented in Section IV.

III. EXPERIMENTAL EQUIPMENT AND METHODS OF ANALYSIS

The delayed coincidence method is used in this investigation for the determination of all isomeric lifetimes. Two intermediate-image beta-ray spectrometers are used for the energy discrimination. The use of the spectrometers allows excellent energy resolution and in many cases eliminates the possibility of detecting other contaminating transitions which could affect the experimental delayed coincidence distribution obtained for the level of interest. One spectrometer is adjusted to transmit either beta particles or conversion electrons of the desired energy from a transition populating the level of interest. The other spectrometer is adjusted to transmit conversion electrons arising from a transition depopulating the level. An associated fast-slow coincidence system utilizing a time-to-pulse-height converter is then used to obtain the beta-electron or electron-electron delayed coincidence distribution associated with the electrons transmitted by the two spectrometers. Finally, the delayed coincidence distribution which is obtained with the aid of the fast-slow coincidence system is analyzed to determine the half-life of the intermediate level.

A. Experimental Equipment

The equipment used in this investigation consists mainly of two magnetic beta-ray spectrometers placed back-to-back for coincidence measurements and an associated fast-slow coincidence system. The fast-slow coincidence system is composed of a time-to-pulse-height converter, a four hundred channel pulse-height analyzer and a slow coincidence circuit which is used to gate the analyzer. A block diagram of the equipment used in this investigation is shown in Figure 1.

Figure 1. Block diagram of the equipment used for lifetime measurements

In Figure 1 the radioactive source is placed between the two spectrometers and the spectrometer currents adjusted to transmit the electrons from the two transitions of interest. The electrons impinge on scintillation crystals and the scintillations are detected by the photomultiplier tubes. The phototube pulses, taken off the anode, are fed into the limiters in order to obtain constant amplitude pulses. The clipping cables are used to obtain pulses 40 nsec in length which are then fed into the time-to-pulse-height converter. The converter converts the time overlap of the two pulses into an output pulse. The amplitude of the output pulse is proportional to the time overlap of the two pulses. The converter output is fed into the multichannel analyzer via the analyzer amplifier and the analyzer gate. If the gate is open the output pulse is recorded by the analyzer.

The analyzer gate is controlled by the slow coincidence output. The signals for the slow coincidence circuit are taken off the 11th dynodes and fed through cathode follower circuits. The signals are then amplified and are fed through trigger circuits to the slow coincidence circuit. The delays in the slow channels are for the purpose of controlling the timing of the two pulses at the slow coincidence circuit as well as the timing of the gating pulse. The output of the slow coincidence circuit is fed into a trigger circuit and finally through an amplifier to the gate of the analyzer. The amplifier in the coincidence output is only for the purpose of obtaining a pulse of the correct shape and length necessary to operate the analyzer gate. The scalers are used to record the slow coincidence rate as well as both single counting rates.

The purpose of the slow coincidence circuit is to discriminate against

small pulses which do not cut the limiters off long enough to allow a 40-nsec pulse to be generated. In order for a coincidence to be recorded in the analyzer, both pulses contributing to the coincidence must have their associated pulses from the 11th dynodes large enough to get through their respective trigger circuits in the slow coincidence arrangement.

The spiral delay line and the pulse generator shown schematically in Figure 1 were used in calibrating the converter and will be discussed later in this section. A more complete description of the critical pieces of equipment used in this investigation will be given in the following portion of this section.

The beta-ray spectrometers which were used are of the Slatis-Siegbahn intermediate-image type (42). Of the two spectrometers in this laboratory, one was originally constructed by Nichols *et al.* (43, 44) in 1953. During 1956 the other spectrometer was constructed by Schupp as described in reference (45). In addition, Schupp modified the first spectrometer in such a manner as to make it possible to use the two spectrometers for coincidence measurements. A description of the construction and operating characteristics of the spectrometers can be found in references (43, 44, 45). A photograph of the spectrometers is shown in Figure 2 and a schematic cross-section of the coincidence spectrometers is shown in Figure 3. In the schematic diagram C_1 , C_2 and C_3 are the water-cooled current coils; B_c and B_r are the center baffle and the resolving baffle, respectively; furthermore, S and D are the source and the detector, respectively. The lines originating at the source and ending at the detector show the approximate paths over which the electrons travel.

The detection system is composed of scintillation crystals placed,

Figure 2. Photograph of the coincidence spectrometer and variable delay line

Figure 3. Schematic cross-section of the coincidence spectrometer

as is shown in Figure 3, on the ends of lucite light pipes which transmit the light back to the fourteen-stage photomultiplier tubes. The light pipes are used in order to keep the phototubes outside of the magnetic fields of the spectrometers.

Anthracene scintillation crystals were used in channel B for detecting the electrons with the higher energy of the two electron energies associated with each half-life measurement. The anthracene crystals were 1.3 cm in diameter and 2 mm in thickness. Both plastic-coated and uncoated anthracene crystals were used for different measurements. The plastic-coated crystals were used whenever the electron energy was greater than the order of 100 keV. If the electron energy was much less than 100 keV, the counting efficiency decreased as electrons lost energy or were stopped in the plastic coating. For energies below 100 keV uncoated anthracene crystals were used for better counting efficiency. The uncoated crystals had to be replaced every ten days because anthracene sublimates rapidly in a vacuum.

For most of the measurements, Nash-Thompson Naton 136 scintillating plastic was used in channel A for detecting the electrons with the lower energy of the two energies associated with each measurement. The plastic scintillator was 1.3 cm in diameter and 0.68 mm in thickness.

The plastic scintillator was used for the low-energy detection because the phosphor decay time for the plastic scintillator is considerably smaller than for anthracene. The use of the plastic scintillator improved the half-width of the prompt curve considerably, especially for electron energies below 40 keV. For example, the half-width of the prompt curve obtained using anthracene in both channels was 5.9 nsec for the 20.8-keV

L-conversion electrons in coincidence with the 110-keV L-conversion electrons in Yb^{169} decay. The half-width of the prompt curve obtained using the plastic scintillator in the low-energy channel was 2.9 nsec for the same cascade. Prompt curves obtained using anthracene and using Naton 136 for detecting the low-energy electrons are shown in Figure 4 for comparison.

The disadvantage of using the plastic scintillator instead of anthracene is the much weaker light signal obtained from the Naton 136. A comparison of the pulse-height distribution for 70-keV electrons for the two different scintillators is shown in Figure 5. In order to obtain the same size pulse, at a given energy, using Naton 136 as is obtained using anthracene, the voltage applied to the phototube must be increased by several hundred Volts. This means the signal-to-noise ratio is much smaller for the plastic scintillator than it is for anthracene.

In summary, the anthracene crystal was used for the high-energy electrons because of the large signal-to-noise ratio obtained and because the phosphor decay time was not too important at the higher energies. The Naton 136 was used for detecting the low-energy electrons because the most important characteristic at these energies was the phosphor decay time of the scintillators. The lifetime of the 8.4-keV level in Tm^{169} could not have been determined from the slope of the delayed coincidence distribution without using the Naton 136 plastic scintillator for the low-energy detection.

The photomultiplier tubes which were used were fourteen-stage Amperex 56 AVP phototubes. These were used because of their high gain and their excellent time-spread characteristics. The phototubes were cooled, using

Figure 4. A comparison of the prompt curves obtained using Naton 136 plastic scintillator and anthracene in the low-energy channel. Both curves were obtained using anthracene in the high-energy channel. Coincidences for both curves were obtained between the 20.8-keV L-conversion electrons and the 110-keV L-conversion electrons in the Yb^{169} decay

Figure 5. A comparison of the pulse-height distributions of Naton 136 plastic scintillator and anthracene for 70-keV electrons

ordinary refrigeration units, to a temperature of about -45° C. As can be seen in Figure 6, the signal-to-noise ratio is improved considerably by cooling.

In Figure 6 the curves show the pulse-height distribution of approximately 100-keV electrons using an anthracene crystal. The distributions were recorded on the multichannel pulse-height analyzer. The only difference between the two curves is that the curve with the circle points was taken with the phototube at room temperature while the other curve was obtained with the phototube cooled as mentioned. The peak in both distributions arises from the 100-keV electrons while the sharply rising portion at low channel numbers is due to noise in the phototubes. As can be easily observed, the gain increased and the noise decreased when the phototube was cooled. It is quite possible that the increase in gain was due to a change in the temperature of the voltage-divider resistors, since cooling the phototube could also result in slight changes in the temperature of the voltage divider. The decrease in the noise is more noticeable in Figure 7. In this figure the amplifier gain was decreased for the cooled case until the peaks of the electron distributions occurred at the same pulse height. Again the curve with the circles was obtained with the phototubes at room temperature and the other curve was obtained with the phototubes at -45° C.

The voltage dividers for the 56 AVP phototubes were constructed using the suggestions in the Philips pamphlet included with the 56 AVP photomultipliers. A circuit diagram of the voltage divider is given in Appendix A (Figure 43). The potentiometers shown in the circuit diagram were adjusted to give maximum gain and minimum noise. The phototube used

Figure 6. A comparison of the pulse-height distributions obtained with the phototube at room temperature and with the phototube cooled to approximately -45°C . Both curves were obtained using an anthracene crystal and 100-keV electrons

Figure 7. A comparison of the pulse-height distributions obtained with the phototube at room temperature and with the phototube cooled to approximately -45°C . The gain of the amplifier was decreased in obtaining the distribution for the cooled phototube until the distribution peak occurred at the same position as for the uncooled phototube distribution. Both curves were obtained using an anthracene crystal and 100-keV electrons

in the low-energy channel (channel A in Figure 1) was usually operated with an applied voltage of approximately 2,450 Volts while the phototube in the high-energy channel (channel B in Figure 1) was operated at approximately 2,150 Volts.

The limiter circuits, which were used to give a constant amplitude output pulse, were constructed by the Ames Laboratory Instrumentation Group. The limiters were modeled after one described by Simms (46). Two modifications were made in Simms' circuit, however. One modification was to use two 404A limiter tubes in parallel in place of the one limiter tube used by Simms. This was done in order to obtain output pulses 40 nsec long with small input pulses. This was desirable since the investigation of the lifetime of the 8.4-keV level in Tm^{169} necessitated working with low-energy electrons and, hence, with small output pulses from the photomultiplier tube. The second modification involved using a 1.35-Volt mercury battery in biasing the grids of the 404A limiter tubes. A circuit diagram (Figure 44) of the limiter circuit is given in Appendix A.

The time-to-pulse-height converter was also constructed by the Ames Laboratory Instrumentation Group and was modeled after one described by Simms (46). Again some slight modifications were made in building the converter. A circuit diagram (Figure 45) of the converter is also included in Appendix A. The ability of the converter to operate properly is strongly dependent on the two constant-current transistors being balanced (the 2N643 transistors in the circuit diagram in Appendix A). It was necessary to try a large number of transistors in order to find two which give an appreciable linear range of time-to-pulse-height conversion. The methods used in adjusting the converter for use are described by Simms

(46).

The converter is also very temperature-dependent; it was therefore necessary to enclose the converter circuit in an insulated box. This eliminated appreciable short-term drifting due to short-term temperature fluctuations. In addition, the laboratory was air conditioned and this aided in minimizing long-term temperature drifts.

The spiral delay line, shown schematically in Figure 1 and visible in Figure 2, is similar to one described by Graham et al. (47). The delay line was designed to have a characteristic impedance of 125 Ohms. A continuous layer of plexiglass was used to support the center wire, instead of lucite posts used by Graham et al., which decreased the velocity of propagation considerably, but the plexiglass layer was more quickly constructed than the posts. The spiral delay line was constructed by the Iowa State University Instrument Shop.

The pulse generator and the 400 channel pulse-height analyzer are both Radiation Instrument Development Laboratory instruments. The pulse generator is RIDL Model 47-8 and has a rise time less than 0.5 nsec. The analyzer is RIDL Model 34-12. For all lifetime measurements the 400 channels were broken up into four 100-channel sections. The rest of the equipment associated with the fast-slow coincidence system is more or less standard and will not be described.

Several of the half-life measurements, which will be described in the next section, were obtained using various voltages applied to the radioactive sources. Three advantages can be obtained from the application of a voltage to the radioactive source. All three advantages arise

from the fact that when a voltage is applied to the source the electrons are either accelerated (negative voltage applied) or decelerated (positive voltage applied). The energy gained or lost by the electrons is proportional to the applied voltage, e.g. an applied voltage of -10,000 Volts results in an increase in the electron energy of 10 keV. This ability to change the energy of the electrons aids considerably in making it possible to obtain a prompt distribution using the same energy electrons that are used in the delayed coincidence distribution.

Another advantage is related to low-energy electrons. The intensity of the scintillations arising from electrons losing their energy in the scintillator is strongly dependent upon the energy of the electrons. By increasing the electron energy using the applied voltage, a more intense light signal is obtained from the scintillator. The more intense light signal results in a larger pulse being detected at the anode of the photomultiplier. The net result is a much better signal-to-noise ratio for the detecting system. The ability to apply -25,000 Volts to a source means that electrons with an energy of only a few eV of energy can be detected as easily as electrons having 25-keV energy without an applied voltage.

The third advantage is closely related to the one just mentioned. This advantage arises from the fact that the width of the prompt distribution is also dependent upon the electron energies involved. This is true because of the statistical nature of the scintillation process. On the average more photons are emitted for the higher energy electrons. A detailed discussion of this statistical process in connection with the use of photomultiplier tubes is given by Post and Schiff (25) and by Gatti and Svelto (26).

Two high-voltage power supplies were used to apply a voltage to the various radioactive sources. One power supply is an NJE Corporation Model HSV-5-20, 500-5,000 Volts, DC supply and the other is a Spellman Model LAB 30-PN, 1-30 kiloVolts, DC supply. The NJE model was used in most of the experimental studies involving an applied source voltage. The Spellman model was used primarily for a study of the effect of the source voltage on the measured half-life of the 8.4-keV level of Tm^{169} .

A source holder was constructed to allow a voltage of approximately 25,000 Volts to be applied to the source. The source holder was constructed using lucite as the primary insulating material as well as for holding the source in position. (A portion of the source holder with the high-voltage cable attached is visible in the upper left hand corner of the photograph in Figure 2.) The spectrometers, and hence the spectrometer end plates, were at ground potential while the voltage was applied directly to the source. The schematic of the source and its surroundings can be viewed in Figure 3. An attempt was made to place grounded screens on the faces of the end plates to aid in a more uniform acceleration of the electrons emanating from the source, but the screens decreased the spectrometer electron transmission by orders of magnitude. The best operation was obtained by leaving the source surroundings unchanged as is shown in Figure 3.

For voltages greater in magnitude than approximately 15,000 Volts, some difficulty was encountered in maintaining an applied source voltage for an indefinite period of time. A discharge would occur between the source and the end plates after a period of time depending upon the magnitude of the applied voltage. At voltages above 27,000 Volts it was

impossible to avoid a discharge for a long enough period to obtain a set of half-life data. (A discharge would burn out the protective fuse in the high-voltage power supply.) With 25,000 Volts applied, a discharge could be avoided for several hours but eventually a discharge would occur.

The radioactive sources used in this investigation were vaporized onto aluminum backings and were 0.5 mm in diameter. Two different aluminum backings were used; one backing was $180 \mu\text{g}/\text{cm}^2$ in thickness while the other backing was $1.8 \text{ mg}/\text{cm}^2$ in thickness. All but two or three sources were prepared on the thin aluminum since this allowed much better electron transmission through the backing. In all cases the front side of a source (the side with the radioactive material) was used for the low-energy channel. This means the electrons detected in the high-energy channel were required to penetrate the aluminum backing before being detected.

The calibration of the time-to-pulse-height converter is an important part of the experimental work involved in measuring the lifetimes of the isomeric states reported in this investigation. An extensive description of the methods of calibrating the time-to-pulse-height converter is given in Appendix B. In addition, a discussion of several checks of the reliability and accuracy of the converter is presented in the same Appendix. Since the converter calibration is discussed extensively elsewhere, only a brief discussion will be presented here.

Two methods were used to calibrate the converter. One method involves obtaining prompt curves with different amounts of cable between the limiter and the converter. The other method is a method described by Graham et al. (47) and utilizes a spiral delay line and a pulse generator

as shown in Figure 1. Both methods are described in detail in Appendix B and are shown to give the same calibration constant in nsec/channel.

The remainder of the results discussed in Appendix B can be summarized briefly. The converter calibration is found to be linear over a range of approximately 20 nsec. The accuracy of the calibration constant is found to be 1.7%. In addition, attenuation effects are shown to be negligible in the cable calibration of the converter. In the case of the pulse generator calibration of the converter, the amplitude of the pulse generator pulses is shown to have no noticeable effect on the calibration constant for pulses greater than 2.5 Volts in amplitude.

B. Methods of Analyzing the Delayed Coincidence Distribution

There are three methods of analyzing the delayed coincidence distribution in order to obtain the half-life of the isomeric level associated with the coincidence distribution. The three methods are the center of gravity method, the slope method and the folding method. The method which is chosen in a given analysis will depend upon the value of the half-life of the isomeric level and on the sharpness of the associated prompt distribution.

The center of gravity method was used as the primary means of determining the half-lives which are less than one nanosecond in this investigation. In this method, the delayed coincidence distribution was obtained for the level under investigation. The spectrometers and all the other pieces of equipment were left untouched and the prompt source (in most cases ThB) was inserted into the spectrometer in place of the source of interest. A prompt distribution was then obtained for comparison

using the beta particles feeding the 239-keV level in Bi^{212} in coincidence with the conversion electrons of the 239-keV ground state transition. The appropriate accidental background was then subtracted from the two curves and the center of gravity of each distribution was calculated. It has been shown (48) that the mean life can then be obtained from the following equation:

$$C - C_0 = \tau - \tau_0 \quad , \quad (29)$$

where C is the center of gravity of the coincidence distribution of interest, C_0 is the center of gravity of the comparison distribution, τ is the mean life of interest and τ_0 is the mean life of the level associated with the comparison distribution. In most cases τ_0 is small enough to be neglected.

When a set of data was taken for determining a half-life using the center of gravity method, the coincidence distributions were obtained alternately. That is to say, a prompt comparison curve was obtained first and then the source was changed and a delay distribution obtained; the source was changed again and a prompt distribution was obtained. The process was then repeated until the desired data were obtained. This alternation procedure was used in order to minimize errors due to possible drifts in the calibration of the time-to-pulse-height converter.

The slope method was used as the primary method for measuring the half-lives greater than one nanosecond. Newton (49) has shown that the mean life can be obtained, in certain cases, from the slope of the $\log N(t)$ versus t plot, where $N(t)$ is the number of coincidences obtained at time delay t . Expressed in terms of an equation:

$$\frac{d}{dt} \log N(t) = - \frac{1}{\tau} \left[1 - \frac{P(t)}{N(t)} \right] \quad (30)$$

where τ is the mean life, $P(t)$ is the prompt coincidence distribution and $N(t)$ is the delay coincidence distribution associated with the mean life τ . As can be seen in Equation 30, when $P(t) \ll N(t)$ the slope of the $\log N(t)$ plot is $-1/\tau$. One advantage to this method is that the comparison prompt curve does not have to be obtained using exactly the same electron energies, used for the delay curve. The prompt curve is used only to show which portion of the delay curve can validly be used in determining the slope.

In general, when this method was used for determining a half-life, a prompt curve was not obtained for every delay distribution. Prompt curves were obtained only periodically in order to verify that the correct portion of the delay curve was being used in the half-life determination. The data outside of the prompt region were then analyzed using the Iowa State University IBM 7074 computer. An iterated weighted least-squares fit to the data was performed by the computer. A discussion of the computer program is given in Appendix D.

The folding method of Newton (49) was used twice for determining the half-life of the 8.4-keV level in Tm^{169} . This method requires a set of prompt coincidence curves obtained at various time delays. A half-life is then assumed for the level of interest and these prompt curves are folded into the assumed half-life until a calculated coincidence distribution curve is obtained. This calculated curve is then compared to the experimentally determined coincidence distribution. Various half-lives

are assumed until the half-life which gives the best comparison with the experimental coincidence curve is determined.

The accidental coincidence distributions, which must be subtracted from both the prompt and delay coincidence distributions, were obtained in two different ways. If the coincidence distribution was sufficiently narrow, the accidental distribution was obtained from the coincidence distribution data by using the accidental background on either side of the coincidence distribution. The accidental background within the region of the coincidence distribution was obtained by linear interpolation.

When the coincidence distribution was not sufficiently narrow to use the above-mentioned method, it was necessary to make a separate determination of the accidental distribution. This was done by adding approximately forty nanoseconds of cable to channel A and recording the accidental distribution on the multichannel analyzer. When forty nanoseconds of cable were inserted, the coincidence pulses arising from the nuclear transitions no longer overlapped at the converter.

For most delay coincidence distributions that required a separate determination of the accidental background, a determination was made for each delay distribution. However, in some cases the same accidental distribution was used for two delayed coincidence distributions. When the accidental distribution was used for two sets of lifetime data, it was taken between the two sets of delayed coincidence data. In all cases the accidental distribution was corrected to the same number of single counts and to the same counting time using:

$$N_a = (2\tau_c N_A N_B)/t \quad (31)$$

where N_a is the number of accidental coincidence counts recorded in time t , τ_c is the coincidence resolving time, N_A and N_B are the number of single counts recorded in channel A and B in time t . It was not necessary to measure τ_c in this equation since

$$N_a = (N_A N_B t' N'_a) / (N'_A N'_B t) \quad (32)$$

where the unprimed quantities refer to the coincidence values and the primed quantities refer to the accidental background values. All the quantities on the right hand side of Equation 32 were readily available in each measurement.

After the background distribution was obtained, the distribution was plotted on graph paper and a smooth curve was drawn through the set of points. The values for the accidental counts were obtained from this curve and subtracted from the delay coincidence distribution.

Since it was necessary to add at least forty nanoseconds of cable to channel A in order to obtain a separate determination of the accidental distribution, there could possibly be an attenuation effect in the accidental background. This possibility was checked by using two transitions which could not be in coincidence; an accidental distribution was obtained with the usual amount of cable inserted and with an additional 45 nsec of cable added to channel A. The two distributions were normalized and the counts every ten channels summed to give a total of ten points for each distribution. In Figure 8 the ordinate scale is the relative difference between N_1 and N_2 multiplied by one hundred, where N_1 is the

Figure 8. Comparison between the accidental coincidence distribution obtained with the usual amount of cable inserted between the limiters and the converter and the accidental distribution obtained with an additional 45 nsec of cable added to channel A. N_1 and N_2 refer to the sum of the counts in ten channels from these respective accidental distributions

sum of ten channels from the accidental distributions taken with the usual amount of cable in channel A; and N_2 is the sum of counts in ten channels of the accidental distribution obtained with 45 nanoseconds of cable added to channel A. The points are plotted every ten channels starting at channel 5. The first point arises from the sums of counts in the first ten channels, the second point from the second ten channels, etc. The top three sets of points show the results of three separate runs with the flags showing the errors. The bottom set of points is the result of adding the first three runs together before obtaining the relative differences. Since the majority of the error flags (eight out of ten in the bottom set) overlap with the horizontal line at zero, there is no evidence of any attenuation effect within the accuracy of the data.

All three of the above-mentioned methods of analyzing the delayed coincidence distributions were used in obtaining the results of the lifetime measurements of this investigation. These measurements and results will be discussed next.

IV. MEASUREMENTS AND RESULTS

In this section the measurements and the results of the lifetime investigations will be presented. Lifetimes of isomeric levels were measured for levels in Tm^{169} , Gd^{155} , Sm^{152} , Gd^{156} , Lu^{175} and Pr^{144} . Each of these isotopes will be discussed and the results of the measurements presented. The experimental results for Tm^{169} and Gd^{155} will be compared with the appropriate theoretical values. All the experimental lifetimes will be compared with the experimental values obtained by other investigators. The first portion of this section will cover the half-life measurements of levels in Tm^{169} with special emphasis on the measurement of the half-life of the 8.4-keV level. The second portion will be concerned with measurements in Gd^{155} while the third portion will include the measurements of one isomeric lifetime in each of the four isotopes: Gd^{156} , Lu^{175} , Sm^{152} and Pr^{144} .

A. Half-Lives of Three Isomeric Levels in Thulium 169.

The half-lives of the first three excited levels in Tm^{169} which were measured in this investigation belong to levels in the ground state rotational band with $K = 1/2$. This rotational band is one of the most well-known $K = 1/2$ rotational bands and has been extensively studied. At the time the present experiments were undertaken there was wide disagreement between the experimental values of the half-life of the 8.4-keV level. In addition, it was thought that confirming values of the half-lives of the 118- and 139-keV levels would be desirable. Accurate and reliable values for these half-lives, in conjunction with other experimental values, could

then be used for a test of the theory for a $K = 1/2$ rotational band in a deformed nucleus. The 8.4-keV transition is also widely used in Mössbauer studies and it would be extremely useful to these investigators to have a reliable value for the half-life of the 8.4-keV level. For these reasons, a large part of this investigation is devoted to an extensive study of the half-life of the 8.4-keV level in Tm^{169} .

1. Half-life of the 8.40-keV level

The half-life of the 8.4-keV level was investigated using both the beta decay of Er^{169} and the electron-capture decay of Yb^{169} . The Yb^{169} was obtained by neutron capture of 14% enriched Yb^{168} irradiated in the Argonne National Laboratory reactor. The Er^{169} was obtained by irradiating 77% enriched Er^{168} either in the Argonne reactor or in the Materials Testing Reactor operated by Phillips Petroleum Co. at Arco, Idaho. In addition, an Er^{169} sample was kindly made available by Professor Felix Boehm of the California Institute of Technology.

The decay schemes of Er^{169} and Yb^{169} are shown in Figure 9. References for the decay scheme may be found in the Nuclear Data Sheets (50) and in a recent paper by Alexander and Boehm (51). Both of these references were used in the preparation of Figure 9. The Tm^{169} internal conversion spectrum, as recorded with the magnetic spectrometer, is shown in Figure 10. The main portion of the figure is the internal conversion spectrum obtained with a Yb^{169} source, and the inset is the spectrum obtained with an Er^{169} source. Both sets of data were obtained using one of the beta-ray spectrometers described in Section III and an anthracene detecting crystal.

Figure 9. Decay scheme of Er^{169} and Yb^{169} (50, 51). Energies are given in MeV

Figure 10. The main figure shows the major portion of the internal conversion spectrum of Tm^{169} following the Yb^{169} decay. The inset shows the internal conversion spectrum of Tm^{169} following the Er^{169} decay

The first measurements of the half-life of the 8.4-keV level were made using anthracene detectors in both channels. The width of the prompt curve using anthracene detectors made it impossible to use the slope method for determining the half-life of the 8.4-keV level. For this reason, the folding method was used in conjunction with an Yb^{169} source.

In this method a set of prompt curves was obtained using the 20.8-keV L-conversion electrons in coincidence with the 110-keV K-conversion electrons with a voltage of -1,600 Volts applied to the Yb^{169} source. The delay curve was obtained using -4,000 Volts applied to the source; this voltage moved the 8.4-keV N-conversion line up to the position of the 20.8-keV L-conversion line without a current change in the A-channel spectrometer. A small change of 1.7% in the current was necessary in the B-channel spectrometer in order to refocus on the 110-keV K-conversion electrons. It was felt this small change would not significantly affect the results of the folding method.

The prompt curves were obtained at one nanosecond intervals. The prompt curves obtained for 0, 5, 10, 15, 20, 25 and 30 nanoseconds of delay are shown in Figure 11. (The experimental points were omitted in Figure 11 to avoid the confusion of a large number of overlapping points.) The prompt curves were normalized by determining the total counts in each prompt curve and correcting them according to the counts recorded in the slow coincidence scaler. These corrected counts were then plotted versus the delay associated with each curve. This plot is shown in Figure 12. A smooth curve was then drawn through the points and the curve used to complete the normalization. The curve decreases at large time delay due

Figure 11. Prompt curves obtained by detecting 20.8-keV L-conversion electrons in coincidence with 110-keV K-conversion electrons in the Yb^{169} decay. These curves were used in the folding method to determine the half-life of the 8.4-keV level in Tm^{169} . The experimental points were omitted to avoid the confusion of a large number of overlapping points

Figure 12. This figure shows the total corrected counts in each prompt curve obtained for the folding method as a function of the time delay associated with each prompt curve

to the fact that some of the coincidence pulses no longer overlap at the converter and are therefore not recorded on the analyzer. The difference between the straight line and the curve gives the number of coincidences not recorded on the analyzer.

After the prompt curves were normalized, different half-lives were assumed and a calculated delay curve obtained for each assumed half-life. Before a comparison could be made with the experimental delay curve, one additional correction had to be made. There are some prompt coincidences present in the experimental delay curve. These arise from coincidences between the Auger electrons under the 8.4-keV N-conversion electron peak and the 110-keV K-conversion electrons. Most of these coincidences result from the Auger electrons emitted following the K-conversion of the 110-keV transition or from the internal conversion of the 198- or 20.8-keV transitions which are in coincidence with the 110-keV transition. The contribution to the prompt distribution from the half-life of the 118-keV level can be considered prompt also since the half-life of the 118-keV level is short compared to the half-life of the 8.4-keV level.

Because of this prompt contribution, a correction factor must be applied to correct the calculated curve for the presence of these prompt coincidences. This correction was made by matching the calculated delay curve and the experimental delay curve at the twenty-four nanosecond point. The calculated and the experimental delay curves were then made to match again at time delay zero by adding a prompt curve (the time delay zero curve in Figure 11) of the correct size. Figure 13 shows the experimental delay curve and points of the calculated delay curve for assumed half-lives of 4.0 and 4.4 nanoseconds. The value obtained for the half-

Figure 13. A comparison between the experimental delay curve for the 8.4-keV level in Tm^{169} and the calculated points of the delay curve, obtained using the folding method, for different assumed half-lives. The experimental delay curve was obtained by detecting 8.4-keV N-conversion electrons in coincidence with 110-keV K-conversion electrons using the Yb^{169} decay. Anthracene detecting crystals were used in both coincidence channels

life in this case is 4.25 nsec. The best fit is determined by minimizing the sum:

$$\sum_{i=1}^n [X(t_i) - X'(t_i)]^2, \quad (34)$$

where $X(t_i)$ is a point on the calculated delay curve at time delay t_i and $X'(t_i)$ is a point on the experimental delay curve. A second determination, using the same method, gave a half-life of 4.00 nsec. The average of these two values gives a final value, using the folding method, of (4.12 ± 0.20) nsec, where the error is an estimated error limit.

After obtaining the results mentioned above, the Naton 136 plastic scintillation detectors were obtained. The use of the Naton 136 made possible the determination of the half-life using the slope method. Several sets of data were obtained and analyzed using the slope method and Yb^{169} decay. For these measurements, the Naton 136 plastic scintillation crystal was used in the low-energy channel. Voltages in the range of -2,000 to -4,500 Volts were applied to the source in order to accelerate the 8.4-keV internal conversion electrons up to the same energy as the 20.8-keV L-conversion electrons. In obtaining the delay curves, coincidences were obtained between the 8.4-keV conversion electrons and the conversion electrons of a transition feeding the 8.4-keV level. Figure 14 shows a delay curve and a prompt curve obtained using 8.4-keV N-conversion electrons in coincidence with the 110-keV K-conversion electrons. Figure 15 shows three other curves obtained using different conversion electrons in coincidence with either 8.4-keV M-conversion electrons or 8.4-keV N-conversion electrons. (The half-lives given in these figures

Figure 14. Delay curve for the 8.4-keV level in Tm¹⁶⁹ and a comparative prompt curve using the Yb¹⁶⁹ decay. The points between the arrows were used to obtain the half-life given in the figure

Figure 15. Three delay curves for the 8.4-keV level in Tm^{169} obtained by detecting conversion electrons from various transitions in coincidence with the 8.4-keV conversion electrons. All three curves were obtained using the Yb^{169} decay. The points between the arrows were used to obtain the half-life values in the figure

and in all subsequent figures are the half-lives obtained from the curve shown; furthermore, they are obtained only from the data between the arrows shown in the figures.) The data using different conversion electrons in coincidence with the M- and N-conversion electrons of the 8.4-keV transition were obtained for two reasons. One reason was to detect any possible contributions to the delay curve arising from some other level in Tm^{169} or arising from a possible contaminant in the Yb^{169} activity. The other reason was to determine if there was any change in the measured half-life as the energy of the conversion electrons in coincidence with the 8.4-keV conversion electrons was varied. No significant change in the measured half-life was found in the results of these measurements which are given in Table 1.

The weights given in Table 1 are proportional to the reciprocal of the square of the relative standard deviation of the half-life as obtained from the computer. This value of the standard deviation is therefore dependent only on the statistical scatter of the individual points used in the least squares fit of a given set of data.

It should be mentioned that the result of an individual determination was very sensitive to the accidental background which was subtracted from the delay curve. In some cases the background was much larger than the true coincidences, especially for large time delay points. The effect of a possible error in the background was investigated by changing the background by 5% for several sets of data. In general, a change of 5% in the background resulted in a change of at least 5% in the determined half-life. In one or two cases a change of 10% in the half-life resulted from a 5% change in the background. This probably accounts for the large

Table 1. Half-life values of the 8.4-keV level in Tm^{169} obtained using the slope method, cable calibration and Yb^{169} decay

Coincidences obtained between conversion electrons	Half-life in 10^{-9} sec	Weight
8.4N vs 110L	4.24	60
	4.38	24
	4.20	389
	4.46	1
	4.20	7
	4.09	36
	4.23	36
8.4M vs 110L	4.76	5
8.4N vs 110K	4.57	117
	4.40	46
	4.56	81
	4.48	26
	4.57	117
8.4M vs 110K	4.75	2
	4.63	1
	4.05	22
	4.19	16
8.4N vs 130K	3.84	117
	4.34	19
	4.48	2
8.4M vs 130K	4.23	1666
	4.84	2
	4.79	3
Weighted average	4.26	

fluctuations in the half-life values in Table 1.

The weighted average of the half-life values in Table 1 results in a half-life of (4.26 ± 0.15) nsec. The error arises from a 1% error due to the statistical spread of the data, a 3% error assigned to account for

differences due to weighting (equal weighting gives a values of 4.41 nsec) and a 1.7% error in calibration of the converter (mentioned in the previous section). The square root of the sum of the squares of the errors gives a final determinate error of 3.6%.

A check was made on the reliability of the experimental method and equipment using low-energy electrons by measuring the half-life of the 396-keV level in Lu^{175} . The 396-keV level in Lu^{175} is fed by a beta group (see Figure 36), and coincidences can be obtained by setting at low energies on this group. The results of this check are discussed later in this section, but they indicate that the method and equipment are reliable at low energies.

At this point in the investigation, the results of the half-life measurement of the 8.4-keV level were published by McAdams et al. (52). The value which was reported was (4.36 ± 0.17) nsec. This previously reported value should be reduced by approximately 3% due to a more accurate determination of the time delay of the calibrating cables which was made after the half-life value was reported.

After these results were published, a value of (3.45 ± 0.25) nsec was reported for the half-life of the 8.4-keV level in Tm^{169} by Sundström et al. (53). Their equipment and experimental methods were very similar to the equipment and methods used in this investigation with one exception. This exception was in the acceleration voltage applied to their source. They applied an acceleration voltage of -35,000 Volts to their source compared to a maximum acceleration voltage of -4,500 Volts applied in the investigation described above. In order to determine if this could be the cause of the discrepancy between their reported value of the half-life

and the value determined as described above, experiments were undertaken to determine the half-life of the 8.4-keV level as a function of the applied source voltage.

Figure 16 shows the internal conversion spectrum of Tm^{169} as obtained using a Yb^{169} source with a voltage of -25,000 Volts applied to the source. This spectrum was obtained using the Naton 136 plastic scintillation crystal. A comparison with the internal conversion spectrum in Figure 10 will show the large gain obtained in the detection efficiency for the 8.4-keV conversion electrons.

In this portion of the investigation the spiral delay line and the pulse generator were used to calibrate the time-to-pulse-height converter. Figure 17 shows a delay curve obtained for the 8.4-keV level with -25,000 Volts applied to the source. The prompt curve in this figure was obtained by reversing the polarity of the high-voltage supply and applying +24,000 Volts to the source. This moved the 110-keV K-conversion electron peak down to the same current position as the 8.4-keV M- and N-conversion electron peak with -25,000 Volts applied. Thus, the current in the low-energy spectrometer was unchanged for the two curves. It was necessary to make a slight change in current in the high-energy spectrometer in order to focus on the 198-keV K-conversion electrons.

Figure 18 shows six different delay curves for the 8.4-keV level obtained with different voltages applied to the source. These curves were obtained by detecting coincidences between the 8.4-keV M- and N-conversion electrons and the 198-keV K-conversion electrons. The 198-keV K-conversion electron peak was used in order to obtain a large coincidence counting rate using electrons with as high an energy as possible. These

Figure 16. A portion of the internal conversion spectrum of Tm^{169} obtained from the decay of Yb^{169} with a voltage of -25,000 Volts applied to the source

Figure 17. Delay curve for the 8.4-keV level in Tm¹⁶⁹ and a comparative prompt curve using source voltages given in the Figure. The Yb¹⁶⁹ decay was used to obtain both distributions. The half-life given was obtained from the points between the arrows

Figure 18. Six delay curves for the 8.4-keV level in Tm^{169} obtained with different voltages applied to the Yb^{169} source. These curves were obtained by detecting coincidences between the 8.4-keV M- and N-conversion electrons and the 198-keV K-conversion electrons. The half-life values given were obtained from the points between the arrows

Figure 18. (Continued)

electrons give a sharper prompt peak because they have a higher electron energy and because the width of the prompt distribution is a function of the energy of the electrons involved in obtaining the prompt distribution. This effect was mentioned in the discussion of the advantages of using an applied source voltage in Section III and is discussed extensively by Post and Schiff (25) and by Gatti and Svelto (26).

In using the 198-keV K-conversion electrons, a contribution to the delay curve is obtained from the half-life of the 118-keV level. This is not significant however, since the half-life of the 118-keV level is 63×10^{-12} sec as is shown later in this investigation.

The values of the half-life obtained are given in Table 2. The different voltages applied to the source for each half-life value are also given in Table 2. Again, the weights are proportional to the reciprocal of the square of the relative standard deviation obtained from the least squares fit of the data. Table 3 gives weighted averages of the half-life as a function of the high voltage applied to the source. The third column gives the number of sets of data that contributed to the half-life given in the second column. The error in column two is the determinate error only. On the basis of the values in Table 3, it is concluded that there is no significant change in the half-life as a function of the source voltage over the voltage range of this investigation.

A weighted average of all the values in Table 2 gives a value of (4.13 ± 0.08) nsec for the half-life of the 8.4-keV level of Tm^{169} . The error in this value arises from a 0.3% error in the spread of the data, a 1% error due to possible differences in weighting (equal weighting gives a value of 4.10 nsec), and a 1.7% error in calibrating the converter.

Table 2. Half-life values of the 8.4-keV level in Tm^{169} obtained as a function of the applied source voltage

Source voltage	Half-life ^a in 10^{-9} seconds	Weight
3 - 5 kV	4.20	1
	4.27	1
	4.32	9
	4.33	100
	3.79	6
	3.95	20
	4.37	4
	4.33	100
	4.06	8
	4.34	400
	4.07	5
	3.89	5
	4.41	32
	4.47	4
	4.45	20
	4.01	32
	4.22	1600
	4.06	174
	4.13	64
	4.38	100
	4.11	100
	4.03	64
	4.17	20
	4.15	100
	3.83	32
	4.02	11
	4.18	100
	4.06	32
	4.15	8000
7 - 12 kV	4.04	400
	3.89	174
	4.26	174
	4.07	400
	4.00	174
	4.05	174
	4.16	174
	4.05	64

^aThese values of the half-life were obtained by detecting coincidences between the 8.4-keV conversion electrons and either 110-keV L-conversion electrons or 198-keV K-conversion electrons in Yb^{169} decay.

Table 2. (Continued)

Source voltage	Half-life ^a in 10 ⁻⁹ seconds	Weight
7 - 12 kV	3.77	25
	4.00	4
15 kV	4.13	552
	4.05	800
	4.03	1143
	4.11	727
	4.13	174
	4.17	174
20 kV	4.02	16
	3.84	20
	3.97	20
	3.89	44
21 kV	3.95	400
25 kV	4.15	16
	4.18	25
	4.02	100
	4.06	400
27 kV	3.98	9
Weighted average	4.13	

Table 3. The weighted average of the half-life of the 8.4-keV level in Tm^{169} as a function of applied source voltage

Source voltage	Half-life in 10 ⁻⁹ seconds	Number of sets of data con- tributing to $\tau_{1/2}$
3 - 5 kV	4.17 ± 0.08	29
7 - 12 kV	4.06 ± 0.09	10
15 kV	4.08 ± 0.09	6
20 kV	3.94 ± 0.09	5
25 - 27 kV	4.06 ± 0.09	5

These errors combine to give a determinate error of 2.0%.

This half-life measurement of the 8.4-keV level as a function of the applied source voltage, as described above, has already been reported by McAdams et al. (54). The value reported was (4.11 ± 0.12) nsec. The difference between the value given above and the value previously reported is the result of a more detailed evaluation of the weights used in obtaining the weighted average.

The center of gravity method cannot be used for obtaining the half-life of the 8.4-keV level using Yb^{169} decay. This is because of the prompt coincidences present in the delay curve. As mentioned previously, these arise from coincidences between the Auger electrons under the 8.4-keV conversion peak and the conversion electrons used in the high-energy channel. As mentioned previously, these coincidences result mainly from the Auger electrons which are emitted following internal conversion. A center of gravity calculation would result in a value for the half-life that was too short. Indeed, when this calculation was performed a value of approximately 3.4 nsec was obtained. This value is considerably shorter than the other values obtained for the half-life.

The decay of Er^{169} was also used to determine the half-life of the 8.4-keV level in Tm^{169} . In this case, coincidences were obtained between the 8.4-keV M-conversion electrons and the beta particles of the beta transition feeding the 8.4-keV level. See Figure 9 for the decay scheme and Figure 10 for the conversion spectrum.

Figure 19 shows a delay curve using Er^{169} and a prompt curve using the 20.8-keV L-conversion electrons in coincidence with the 110-keV K-conversion electrons. The current settings were unchanged for these two

Figure 19. Delay curve for the 8.4-keV level in Tm¹⁶⁹ using an Er¹⁶⁹ source. The comparative prompt curve was obtained using the Yb¹⁶⁹ decay. The half-life value given was obtained from the points between the arrows

curves. A voltage of approximately -4,500 Volts was applied to the Er^{169} source in order to move the 8.4-keV M-conversion line up to the same current position as the 20.8-keV L-conversion line.

The curves in Figure 19 are situated, with respect to the calibration curve, such that there is some folding back of the curve in the negative time delay region. This is because the converter is nonlinear in the region to the left of the time delay equals zero position. This accounts for what appears to be a slight prompt contribution in the delay curve.

Table 4 gives the results of the six half-life determinations using the decay of Er^{169} . A weighted average of these six values gives a value of (4.19 ± 0.12) nsec for the half-life. The error arises from a 1.2% error due to the statistical spread of the values, a 2% error due to possible differences in weighting (an unweighted average gives a value of 4.27 nsec), and a 1.7% error in the calibration of the converter. These errors combine to give a resultant determinate error of 2.9% or 0.12 nsec.

It is possible to determine the half-life of the 8.4 keV level using a center of gravity calculation and Er^{169} decay. The prompt curve, in this calculation, was obtained using 20.8-keV L-conversion electrons in coincidence with 110-keV K-conversion electrons in Yb^{169} decay. Two determinations of the half-life were made using this method. One set of data is shown in Figure 20. The crosses in the figure indicate the positions of the centers of gravity of the two curves. The differences between the two centers of gravity is 5.66 nsec for this set of data. It is, however, necessary to subtract 0.06 nsec from the half-life of 3.92 nsec obtained from the difference between the two centers of gravity.

Table 4. The half-life values of the 8.4-keV level in Tm^{169} obtained using Er^{169} decay

Half-life in 10^{-9} seconds	Weight
4.41	5
4.14	19
4.68	1
4.26	4
4.18	29
3.96	3
4.19 -- Weighted average	

This is necessary because the 118-keV level in Tm^{169} has a half-life of 0.06 nsec and the prompt curve in Figure 20 is therefore displaced to the left by 0.09 nsec. This results in a corrected value of 3.86 nsec for the half-life.

The two sets of data give values of (3.86 ± 0.27) nsec and (4.17 ± 0.21) nsec for the half-life of the 8.4-keV level. These two values combine to give a weighted average value of (4.06 ± 0.25) nsec for the half-life. The large errors in these values results largely from the uncertainty in extending the delay curve into the region beyond 16 nsec in Figure 20.

Table 5 brings together the five values of the half-life of the 8.4-keV level which were obtained using the different methods and the different sources (Yb^{169} and Er^{169}). The most accurate value, and the value which is used as the final determined value of the half-life, is the value

Figure 20. Delay curve for the 8.4-keV level in Tm^{169} using an Er^{169} source and a comparative prompt curve using an Yb^{169} source. The crosses indicate the centers of gravity for the two distributions. The difference between the centers of gravity does not give the mean life in this case, since a correction must be made for the mean life of the 118-keV level in Tm^{169} which is associated with the prompt distribution in the figure

Table 5. Values of the half-life of the 8.4-keV level in Tm^{169} obtained using different methods and different sources

Source	Method	Half-life in 10^{-9} seconds
Yb^{169}	Folding	4.12 ± 0.20
Yb^{169}	Slope & cable calibration	4.26 ± 0.15
Yb^{169}	Slope, pulse generator calibration and source voltage varied	4.13 ± 0.08
Er^{169}	Slope	4.19 ± 0.12
Er^{169}	Center of gravity	4.06 ± 0.25

obtained using the variation of the source voltage. The determination of this value is also the least dependent on the weights chosen in obtaining a weighted average value. If a weighted average of all the original values is obtained, using the same weighting procedure as in obtaining the five separate values, the final value would change by approximately 0.01 nsec.

The final value for the half-life of the 8.4-keV level in Tm^{169} is therefore (4.13 ± 0.12) nsec, where the error is obtained by multiplying the determinate error by 1.4 to include a possible systematic error.

A comparison between the value of the half-life of the 8.4-keV level in Tm^{169} determined in this investigation and the values obtained by other investigators is shown in Figure 21. In addition, the values are tabulated in Table 6. Figure 21 includes only the latest value reported by each investigator or group of investigators listed in Table 6.

Table 6. Half-life of the 8.4-keV level in Tm^{169} as reported by different investigators

Measurement	Half-life in 10^{-9} seconds	Investigator
1	4.13 ± 0.12	Present investigation
2	3.7 ± 0.5	Beekhuis and de Waard (55)
3	6.6 ± 1.0	Hauser <u>et al.</u> (56)
4	7 ± 2	Schneider <u>et al.</u> (57)
5	3.45 ± 0.25	Sundström <u>et al.</u> (53)
6	3.24 ± 0.17	Blechschmidt <u>et al.</u> (58)
7	3.53 ± 0.20	Bloess and Münnich (59)

Of the values listed in Table 6, only the second value listed is in agreement with the value determined in this investigation. This value was the first value reported for the half-life of the 8.4-keV level and it was reported by Beekhuis and de Waard (55). They used a parallel plate acceleration device using accelerating voltages of 21, 30 and 40 kV. They obtained a delayed coincidence distribution and a prompt distribution, and determined the half-life of the level using the slope method.

The third value obtained by Hauser et al. (56) and the fourth value by Schneider et al. (57) were obtained by the same group. They used Er^{169} decay and a NaI crystal for detecting the 8.4-keV gamma rays. It was necessary for them to use the folding method in order to make a determination of the half-life. Although no details of the experiment are published it is thought that they could possibly have an error in their background

Figure 21. Half-life values of the 8.4-keV level in Tm^{169} reported by different investigators: a. Present investigation, b. Beekhuis and de Waard (55), c. Sundström et al. (53), d. Bloess and Münnich (59), e. Blechschmidt et al. (58) and f. Hauser et al. (56). The horizontal line represents the value of the present investigation

subtraction. If too small a background was subtracted this could account for the large value of the half-life which they obtained.

The fifth value in Table 6 was obtained by Sundström et al. (53) using a technique which was very similar to the one used in this investigation. No explanation can be given for the discrepancy between the value determined in this investigation and the value determined by Sundström et al. It is possible that their source was contaminated by some other isotope since their source was prepared on an aluminum backing and then irradiated in a flux of 2×10^{14} neutrons/cm²-sec. However, it should be mentioned that no definite indication of this was found upon examination of the internal conversion spectrum given in their publication.

Value six by Blechschmidt et al. (58) was determined by using a spark chamber. This reference does not give any details of the experiment. However, the last value in Table 6 by Bloess and Münnich (59) was also determined using a spark chamber. The delay curve given in reference (59) appears to be almost symmetrical and introduces the possibility that their prompt curve at this low energy is not as sharp as they indicate. The prompt curve which is given was obtained using Cs¹³⁴.

The value of (4.13 ± 0.12) nsec for the half-life of the 8.4-keV level in Tm¹⁶⁹ determined in this investigation corresponds to an energy level width of $(1.10 \pm 0.03) \times 10^{-7}$ eV. If there were no effects which could broaden the individual line profiles in the Mössbauer effect pattern, the width at half-maximum of the resolved Mössbauer lines would correspond to twice the level width or $(2.20 \pm 0.06) \times 10^{-7}$ eV.

Several investigators have recently reported recoil free resonant absorption measurements using the 8.4-keV transition in Tm¹⁶⁹ (60-71).

Two of the narrowest gross line widths have been obtained by Barnes et al. (70) and Hüfner et al. (67). Barnes et al. (70) have obtained a gross line width at half maximum of 1.8 cm/sec, and Hüfner et al. have obtained a value of 2.2 cm/sec. Using Equation 5 these values correspond to energy widths of 5.04×10^{-7} eV and 6.17×10^{-7} eV, respectively. Thus the narrowest experimental gross width at half-maximum of the Mössbauer line profiles is approximately 2.3 times broader than the width of the unbroadened lines predicted from the half-life measurement determined in this investigation.

Poindexter (71) reports an experimental gross line width at half maximum of 1.8 cm/sec for the Mössbauer line profiles. [This measurement is the same measurement as reported by Barnes et al. (70).] He applies a correction factor of 1.45 to the line width in order to correct for the finite thickness of the absorber. When this correction is applied, the width of the Mössbauer line is 1.24 cm/sec or 3.47×10^{-7} eV. This width is still larger than the width predicted by the present lifetime measurement of the 8.4-keV level by a factor of 1.6. Poindexter states that the origin of this line broadening is uncertain.

A cursory comparison of Mössbauer line widths with the true line widths obtained by the lifetime measurements for other isomeric levels in the rare earth region, indicates the Mössbauer line widths are approximately a factor of two broader than the widths predicted by the lifetime measurements. This comparison was made using the compilations of Muir and Ando (72) and the Nuclear Data Sheets (50).

This cursory comparison indicates that it is not unusual in the rare earth region to obtain experimental Mössbauer line widths that are

broadener than the true line widths predicted by the lifetime measurements. Thus, the factor of 1.6 mentioned above for the 8.4-keV level in Tm^{169} is not surprising.

2. Half-life of the 118-keV level

The half-life of the 118-keV level in Tm^{169} was measured using the shift of the center of gravity. Coincidences were obtained between the 110-keV L-conversion electrons and the 198-keV K-conversion electrons. A comparative prompt source was obtained by using a ThB source on 1.75 mg/cm² aluminum backing. Coincidences were then obtained between the beta particles and the ThB F-line (239-keV K-conversion electrons from the 239-keV level in Bi^{212}). The mean life of this Bi^{212} level has been shown to be less than 2×10^{-13} sec (73). The ThB F-line electrons were attenuated in energy in passing through the aluminum foil and this allowed a considerable number to be detected at the current position corresponding to the 198-keV K-conversion electrons. The currents in both spectrometers were left unchanged during the process of obtaining a set of data. The prompt and delayed distributions were obtained using the alternation procedure described in Section III. The mean life was then obtained from the difference between the centers of gravity.

The thirty-four values of the half-life that were obtained are listed in Table 7. An unweighted average of these values gives a final value for the half-life of (63 ± 7) psec. The determinate error of 5 psec in this value arises almost entirely from the statistical spread of the values. The determinate error is increased by a factor of 1.4 to include a possible systematic error.

Table 7. Half-life values of the 118-keV level in Tm^{169}

Half-life in 10^{-12} seconds	Half-life in 10^{-12} seconds	Half-life in 10^{-12} seconds
66	29	55
119	43	78
66	56	49
91	60	31
84	58	92
47	53	69
92	53	59
118	92	69
105	81	46
15	26	33
114	46	Average 63
13	12	

The half-life of the 118-keV level has been measured by Blaugrund (74) and Blaugrund et al. (21). The most accurate value is reported in reference (21) and is reported as (62 ± 2.8) psec. The value of (63 ± 7) psec determined in this investigation is in good agreement with this value.

3. Half-life of the 139-keV level

The half-life of the 139-keV level in Tm^{169} was also measured using the center of gravity method. Coincidences were obtained between the 130-keV K-conversion electrons and the 177-keV K-conversion electrons in

obtaining a delay curve. A delay curve and a prompt curve are shown in Figure 22. The prompt curve was obtained using the ThB F-line and the beta particles. It was possible to obtain coincidences for the prompt curve at the same spectrometer current setting as for the delay coincidence distribution because of the large attenuation in energy of some of the F-line electrons as they penetrated the aluminum source backing.

Six values for the half-life of the 139-keV level were obtained in this investigation: 281, 327, 278, 242, 302, and 306 psec. An unweighted average of these six values results in a value of (289 ± 24) psec. The determinate error of ± 15 psec arises largely from the spread of the six half-life values. The determinate error is multiplied by 1.4 to include a possible systematic error.

The half-life of the 139-keV level has been measured by two other investigators. Blaugrund (74) has obtained a value of (291 ± 25) psec for the half-life and Sundström *et al.* (75) have recently obtained (321 ± 14) psec. The value for the half-life determined in this investigation agrees well with the value determined by Blaugrund but not as well with the value determined by Sundström *et al.* The present value does agree with the value determined by Sundström *et al.* within the overlapping errors, however.

4. Discussion of the Tm^{169} ground-state rotational band

The experimental data on the ground state rotational band in Tm^{169} have been recently reviewed and discussed in terms of the unified nuclear model by Sundström *et al.* (76) and by Bowman *et al.* (77). This band is composed of the ground state and the first three excited states shown in

Figure 22. Delay curve for the 139-keV level in Tm^{169} obtained using the Yb^{169} decay and a comparative prompt curve obtained from the ThB decay

Figure 9. The spin assignments are $1/2$, $3/2$, $5/2$ and $7/2$ for the ground state, and the first three excited states, respectively.

Sundström et al. (76) used the seven possible equations obtained from Equations 22 and 25 and the experimental values of $T(M1, 7/2 \rightarrow 5/2)$, $T(M1, 5/2 \rightarrow 3/2)$, $T(M1, 3/2 \rightarrow 1/2)$, $\mu_{1/2}$, $\mu_{3/2}$, $\mu_{5/2}$, and $\mu_{7/2}$, which were available at the time of their comparison to obtain a best fit for the model parameters $g_K = G_O - g_R$, g_R and b_O . They then used the three parameters obtained from the best fit to recalculate the seven values mentioned above. These calculated values were then compared to the experimental values. Their conclusion was that the magnetic properties were not consistent with the model predictions.

A more recent discussion is given by Bowman et al. (77) using more recent experimental values for some of the quantities used by Sundström et al. (76) in their discussions. Bowman et al. have redetermined $\mu_{5/2}$ and $\mu_{7/2}$ and have obtained values which differ considerably from the values of these magnetic moments which had been used by Sundström et al. Bowman et al. use the experimental values for $\mu_{1/2}$, $(\mu_{3/2})/(\mu_{1/2})$, $\mu_{7/2}$, $\tau_{5/2}$ and $B(E2, 1/2 \rightarrow 5/2)$ to determine the set of parameters Q_O , g_K , g_R , and b_O where Q_O is the intrinsic quadrupole moment. They then use these parameters to calculate the mean lives, the magnetic moments, the mixing ratios and the branching ratios for the ground-state rotational band. They obtain excellent agreement between these calculated values and the experimental values, indicating that the disagreement obtained by Sundström et al. (76) was most probably due to errors in some of the experimental values available at that time, especially in $\mu_{5/2}$ and $\mu_{7/2}$.

Since these two studies are rather extensive, a general study of the

rotational band properties will not be repeated here. Rather, a comparison will be made of the agreement between the values of $|G_0|$ and b_0 as obtained from the three half-lives, $\tau_{3/2}$, $\tau_{5/2}$ and $\tau_{7/2}$ measured in this investigation and these will be compared to G_0 and b_0 obtained by Bowman et al. (77).

The internal conversion coefficient of the 8.4-keV transition has been determined by two different investigators. Charpak and Suzor (78) have obtained a value of 106 ± 6 for the total conversion coefficient while Kankeleit et al. (79) have obtained a value of 325 ± 35 . These two values are in wide disagreement and for this reason separate calculations are made using both values of the total conversion coefficient. Using Equations 27 and 22 and the half-life determined in this investigation, it is found that:

$$|G_0| (1+b_0) = 2.75 \pm 0.11 \quad \text{using } \alpha = 106 \pm 6 \quad (35)$$

and $|G_0| (1+b_0) = 1.57 \pm 0.08 \quad \text{using } \alpha = 325 \pm 35, \quad (36)$

where it has been assumed that the 8.4-keV transition is pure M1. The energy of the transition has been used as (8.40 ± 0.01) keV (80).

The internal conversion coefficient of the 110-keV transition has been measured to be 2.58 ± 0.21 by Grabowski et al. (80). They have also measured the total conversion coefficient for the 118-keV transition. They obtained $\alpha = 1.69 \pm 0.12$ for this transition.

The gamma-ray intensities have been measured accurately by three different experimental groups. Grabowski et al. (80) obtained 51 ± 2.5 and 4.8 ± 0.4 for the gamma intensities of the 110- and 118-keV transitions

respectively. Alexander and Boehm (51) have obtained 50 ± 2.5 and 5.2 ± 0.4 while J. E. Brown (81) has obtained 47 ± 1 and 5.4 ± 0.4 for the gamma intensities of the same transitions. A weighted average of these gamma intensities results in final values of 48 ± 1 and 5.1 ± 0.2 for the gamma intensities of the 110- and 118-keV transitions. These gamma intensities and the conversion coefficients of Grabowski et al. mentioned above combine to give total relative intensities of 172 ± 11 and 13.7 ± 0.8 for the 110- and 118-keV transitions respectively. These values combine to give a final value of 0.925 ± 0.007 for the ratio of the total 110-keV transition to the total number of transitions depopulating the 118-keV level.

When the above values are used in Equation 28 along with the half-life for the 118-keV level determined in this investigation, a gamma-ray mean life of (0.360 ± 0.045) nsec is obtained. Using this value in Equation 22 leads to:

$$|G_0| (1-b_0) = 2.23 \pm 0.14 \quad (37)$$

The internal conversion coefficient for the 20.8-keV transition has not been measured. In addition, the tables by Rose (82) and by Sliv and Band (83) do not cover this energy. The L_I conversion coefficient can be obtained by extrapolation, however. If this is done, a value of 25 is obtained from the tables of Rose and a value of 38 is obtained from the tables of Sliv and Band.

The subshell ratios have been measured by Shliagin and Samoilov (84) and by Hatch et al. (85). The results obtained are $L_I:L_{II}:L_{III}:MNO = 356 \pm 18:50 \pm 11:11 \pm 3:44 \pm 7.5$ and $L_I:L_{II}:L_{III} = 200:25:10$ respectively.

Using these subshell ratios and the theoretical L_I conversion coefficients, the total conversion coefficient is calculated to be 32.4 ± 2.8 from the tables of Rose and 49.4 ± 4.2 from the tables of Sliv and Band. The errors arise from a 7% error in the sum of the subshell ratios and an assumed error of 5% in the extrapolated theoretical L_I conversion coefficients.

Sundström et al. (76) have used the experimental ratios to determine the total transition intensity ratio of the 130.5-keV transition to the 20.8-keV transition. They obtained a value for this ratio of 2.75 ± 0.42 . This leads to a value of 0.267 ± 0.030 for the ratio of the 20.8-keV transition to the total number of transitions depopulating the 139-keV level. For the purpose of calculating the gamma-ray mean life, the 20.8-keV transition will be assumed to be pure $M1$. Using the above values and the half-life of the 139-keV level as determined in this investigation, the gamma-ray mean life is found to be $(8.16 \pm 1.31) \times 10^{-8}$ sec using the internal conversion coefficient of Sliv and Band (83) and $(5.39 \pm 0.86) \times 10^{-8}$ sec using the conversion coefficient of Rose. If these values are used in Equation 22, the following values are obtained:

$$|G_0| (1+b_0) = 1.61 \pm 0.13 \quad \text{using Sliv and Band} \quad (38)$$

$$\text{and} \quad |G_0| (1+b_0) = 1.98 \pm 0.16 \quad \text{using Rose.} \quad (39)$$

If these two values of $|G_0| (1+b_0)$ are compared with the two values obtained from the 8.4-keV level measurement, it is easily seen that the best agreement is obtained using $\alpha = 325 \pm 35$ for the 8.4-keV transition

and the internal conversion coefficient of Sliv and Band for the 20.8-keV transition. If these two values are combined, a final weighted average value is obtained:

$$|G_0| (1+b_0) = 1.58 \pm 0.07 \quad . \quad (40)$$

Using Equations 37 and 40, it is found that $b_0 = -(0.17 \pm 0.04)$ and $|G_0| = (1.90 \pm 0.16)$. Bowman et al. (77) obtained $b_0 = -0.16$ and $G_0 = -1.98$. As can be easily seen, the values of $|G_0|$ and b_0 obtained using the measurements of this investigation agree very well with $|G_0|$ and b_0 obtained by Bowman et al.

It does appear evident that there is a definite need for an experimental determination of the internal conversion coefficient for the 20.8-keV transition. In addition, additional measurements are needed to clear up the discrepancy in the experimental values of the 8.4-keV internal conversion coefficient before the agreement mentioned above can be definitely established.

B. Half-Lives of Two Isomeric Levels in Gadolinium 155

The half-lives of two excited levels in Gd^{155} were determined using the decay of Eu^{155} . The two levels studied were the 87- and 105-keV levels. A decay scheme for Eu^{155} (50) is shown in Figure 23. In addition, the electron spectrum of Eu^{155} decay is shown in Figure 24. The electron spectrum shown in Figure 24 was obtained using an anthracene detecting crystal and one of the beta-ray spectrometers.

The Eu^{155} source materials were obtained by irradiating 99.2% enriched Sm^{154} in the Argonne reactor or in the Materials Testing Reactor

Figure 23. Decay scheme of ^{155}Eu (50). Energy values are in MeV

Figure 24. Electron spectrum of the decay of Eu^{155}

Figure 25. Delay curve for the 87-keV level in Gd¹⁵⁵ obtained using Eu¹⁵⁵ decay and a comparative prompt curve obtained using the Yb¹⁶⁹ decay. The half-life given was obtained from the points between the arrows

operated by Phillips Petroleum Co. at Arco, Idaho. The Sm^{155} decays to Eu^{155} with a half-life of 10 hours. The source material was set aside for 6 months before it was used to allow the Eu^{156} activity to dissipate. Eu^{156} has a half-life of 15 days, and is present as a contaminant as a result of neutron capture by the Eu^{155} .

1. Half-life of the 87-keV level

The half-life of the 87-keV level in Gd^{155} was determined using the slope method. One delay curve which was obtained using a Naton 136 plastic detecting crystal is shown in Figure 25. This curve was obtained by detecting coincidences between the 87-keV K-conversion electrons and the beta particles at 130 keV. The prompt curve was obtained using coincidences between the K-Auger electrons and the 198-keV K-conversion electrons in Yb^{169} decay. The half-life values obtained for the 87-keV level are summarized in Table 8. The weights in this table are proportional to the square of the reciprocal of the relative error obtained from the least squares fit of the data.

The weighted average of the half-life values in Table 8 results in a final value of (6.27 ± 0.35) nsec for the half-life of the 87-keV level in Gd^{155} . The error results from a 2% error in the spread of the values, a possible 3% error due to different methods in weighting, and a 1.7% error arising from calibration and stability of the converter.

Bozek et al. (86) have recently reported a value of (6.68 ± 0.12) nsec for the half-life of the 87-keV level. They used gamma-gamma coincidences from Tb^{155} decay and a time-to-pulse-height converter. A half-life of (5 ± 1) nsec has been obtained for the 87-keV level by Vergnes (87)

Table 8. Half-life values obtained for the 87-keV level in Gd^{155} using coincidence between 87-keV K-conversion electrons and beta particles

Energy of beta particles used in measurement	Half-life in 10^{-9} seconds	Weight
100 keV	6.00	16
	6.78	1
130 keV	6.30	711
	5.33	400
	5.62	256
	6.52	64
	6.33	256
	6.12	25
	6.29	256
	6.94 ^a	79
	7.01 ^a	130
	6.77 ^a	400
	7.26	53
	7.68	64
140 keV	6.47 ^a	13
Weighted average	6.27	

^aThese four values were obtained using the left side of the calibration curve in Figure 46.

using a fast-slow coincidence system. A value of (5.04 ± 0.25) nsec was reported for the half-life at a 1961 conference by Hauser et al. (56) and a value of (4.3 ± 0.20) nsec has been reported by Runge et al. (88). The last two values mentioned were reported by the same group. Their latest reported value is the value reported by Runge et al. and this value will be used as the reported value of this group in subsequent discussions.

Bozek et al. (86) state in their report that a least squares fit to the data results in a value of (6.68 ± 0.12) nsec for the half-life of the 87-keV level. There is an implication in their statement that the reported

error arises only from the spread of the data. The experimental study reported by Bozek et al. was not aimed primarily at a determination of the half-life, but a value of the half-life was necessary for their calculation of the nuclear g-factor. From the report it seems possible that the total error in their value could be larger than reported, since, in addition, they mention only one set of half-life data. If the total error was larger, the value of the half-life determined in this investigation would be in agreement with their value. If their reported error is correct, the agreement is only within overlapping errors.

Because of the large error reported by Vergnes (87), and because of the fact that the present measurement and Vergnes' measurement agree within overlapping errors, it is thought that these two values are not in disagreement.

There is a definite discrepancy between the value of the half-life determined in this investigation and the value determined by Runge et al. (88). This discrepancy could possibly result from a contribution to their delay curve from the 105-keV level half-life. They obtained coincidences between the beta particles and the 87-keV gamma-ray photo peak using a NaI crystal. By using the 87-keV gamma-ray photo peak they could possibly be obtaining a contribution from the 105-keV gamma-ray photo peak. Also, they would be getting a contribution from the Compton distribution of the 105-keV gamma ray. These effects would depend on what energy beta particles they used in obtaining coincidences.

2. Half-life of the 105-keV level

The half-life of the 105-keV level in Gd^{155} was also measured using the slope method. A typical set of data is shown in Figure 26. In this figure the delay curve was obtained by using 105-keV K-conversion electrons in coincidence with 125-keV beta particles. The prompt curve was obtained by using 110-keV K-conversion electrons in coincidence with 198-keV K-conversion electrons in Yb^{169} decay. It was necessary to change the current slightly in both spectrometers in order to obtain the prompt curve in this case. This is not extremely important however, because the slope method is used to determine the half-life.

Table 9 gives the results of twelve separate determinations of the half-life of the 105-keV level. A weighted average of the twelve values in Table 9 results in a value of (1.20 ± 0.04) nsec. The error results from a 1% error in the spread of the data, a 1% error due to the possibility of different weights and a 1.7% error arising from the calibration and stability of the time-to-pulse-height converter. This gives a determinate error of 2.2% which is multiplied by 1.4 in order to allow for possible systematic errors.

The value of the half-life of the 105-keV level obtained in this investigation is in agreement with most of the values of the half-life obtained by other investigators. Vergnes (87) has reported an upper limit of 1.2 nsec for the half-life of the 105-keV level. Hauser et al. (56) reported a value for the half-life of (1.05 ± 0.05) nsec at a 1961 conference and Runge et al. (88) have reported a value of (1.07 ± 0.10) nsec. Deutch et al. (89) have measured the half-life of the 105-keV level using nuclear resonance fluorescence. They report a value of (0.6 ± 0.2) nsec

Figure 26. Delay curve for the 105-keV level in Gd¹⁵⁵ obtained using the Eu¹⁵⁵ decay and a comparative prompt curve obtained using the Yb¹⁶⁹ decay. The half-life given was determined from the points between the arrows

Table 9. Half-life values of the 105-keV level in Gd^{155} obtained using Eu^{155} decay and 105-keV K-conversion electrons in coincidence with beta particles

Energy of beta particles used in measurement	Half-life in 10^{-9} seconds	Weight
70 keV	1.22	7
	1.20	2
	1.44	1
100 keV	1.18	7
	1.14	2
	1.16	7
125 keV	1.15	19
	1.17	38
	1.17	378
	1.21 ^a	676
	1.22	11
	1.22	169
Weighted average	1.20	

^aThis value was obtained using the left side of the calibration curve shown in Figure 46.

for the gamma mean life. This corresponds to a measured half-life of (0.3 ± 0.3) nsec. They use three points with rather large errors to determine the gamma mean life. From observing the curve, it appears that their errors could be larger than reported if the rather large error in the background is taken into consideration. The values by Hauser et al. and by Runge et al. were obtained by the same experimental group; the latest value by this group is (1.07 ± 0.10) nsec. The agreement between this value and the present measurement is within overlapping errors.

3. Comparison with theoretical predictions

In order to make a comparison between the experimentally determined mean lives of the 87- and 105-keV levels of Gd^{155} and the theoretical predictions of Nilsson (39), it is necessary to correlate the Gd^{155} levels with the Nilsson levels.

The decay scheme shown in Figure 23 is well established [see Nuclear Data Sheets (50) for references] with the exception of the spin assignments of the 87- and 105-keV levels. The 60- and 145-keV levels in Gd^{155} are the first two excited rotational states of the ground state rotational band with $K = 3/2$. The 105- and 87-keV levels are both single particle levels and correspond to the [642] 5/2 and [651] 3/2 Nilsson orbitals (90).

In Figure 23 both possible spin assignments and Nilsson orbital assignments for the 87- and 105-keV levels are given. The two possible spin assignments for these two levels are: 5/2 and 3/2, respectively, or 3/2 and 5/2, respectively. The Nilsson (39) orbital [642] is associated with the spin 5/2, and the orbital [651] is associated with the spin 3/2, as indicated above.

An accurate comparison of the branching ratios from the 87- and 105-keV levels to the 60-keV and ground state levels using Equation 26 could lead to an assignment of the spins. However, a comparison of the present measurement for these levels (91, 92, 93) leads to inconclusive assignments.

Deutch et al. (89) have compared the experimental gamma mean lives of the 87- and 105-keV level ground state transitions with the theoretical values. In their comparison with Nilsson's predictions the agreement is much better when the 5/2 spin is assigned to the 105-keV level. They

state that if the theoretical values of the mean lives are taken at face values they unambiguously assign the $5/2$ spin to the 105-keV level and the $3/2$ spin to the 87-keV level.

The results of the present investigation are also compared to Nils-son's predictions later in this section; this comparison agrees with the statement of Deutch et al. There is considerable doubt however, that this method is sufficiently accurate to allow an accurate assignment of the spins. For example, assigning the $3/2$ spin to the 105-keV level would only result in a disagreement of factors of 2 and 5 between the experimental values and the theoretical values for the 105- and 87-keV levels, respectively. In most cases of mean life comparison between experiment and theory, a factor of 2-5 is not considered significant.

In addition, there is a possibility that band mixing is present. A possibility of this exists for the 105-keV level in Gd^{155} . Harmatz et al. (93) indicate that the first excited rotational level based on the 87-keV level is at 118 keV. If the 87-keV level is a spin $3/2$ level, the 118 keV level has a spin of $5/2$ and there could be mixing between this rotational level and the [642] $5/2$ single particle level.

An additional point should be remembered. It is also possible that the deformation parameter δ of a nucleus is not necessarily the same in an excited state as in the ground state. In theoretical-experimental comparisons it has usually been assumed that the deformation parameter of the nucleus is the same in an excited state as in the ground state. However, small changes in δ in an excited state can lead to relatively large changes in the lifetime of an excited state. Calculations of this effect have been made by Dutt and Mukherjee (94) and by Vergnes (95).

Subba Rao (92) has performed an angular correlation experiment in which he obtains coincidences between the 45-keV gamma ray and the 60-keV gamma ray. On the basis of the angular correlation, he assigns the $3/2$ spin to the 105-keV level. He also reports a measured branching ratio for the 87-keV level which agrees with this assignment.

The experimental evidence mentioned above seems to be inconclusive in giving a definite assignment of the spins to these two single particle states. It seems evident that additional experiments will be necessary before a definite spin assignment can be made.

In order to compare the experimental values with the theoretical values, the experimental gamma mean life must be calculated using Equation 28. In the case of the 87- and 105-keV levels in Gd^{155} , it is necessary to have the experimental conversion coefficients and the transition intensities for the ground state transitions. The multipolarity of the two ground state transitions will be taken as pure E1 for the purpose of calculating the experimental gamma mean lives.

The K-shell conversion coefficient for the 105-keV transition has been measured by Subba Rao (92, 96) to be (0.29 ± 0.05) , reference (96), and (0.22 ± 0.05) , reference (92). The later value of (0.22 ± 0.05) will be used in this calculation. This value along with the K:L:M ratios given by Harmatz et al. (93) gives $\alpha = (0.27 \pm 0.06)$. The transition intensity for the 105-keV transition has been reported as 0.63 by Hatch and Boehm (91) and as 0.77 by Harmatz et al. (93). An average value of (0.70 ± 0.07) will be used in calculating the gamma mean life. Using the present value of (1.20 ± 0.04) nsec for the half-life of the 105-keV level, the gamma mean life is calculated to be (3.13 ± 0.38) nsec using Equation 28.

The K-shell conversion coefficient for the 87-keV transition has been measured to be (0.4 ± 0.1) by Vergnes (87). In addition, Subba Rao (96) has reported a value of (0.49 ± 0.08) . These two values combine to give an average value of (0.45 ± 0.07) for the K-shell conversion coefficient. If the K:L:M:N conversion ratios of Harmatz et al. (93) are used, a final value of (0.57 ± 0.09) is obtained for the total conversion coefficient.

The 87-keV transition intensity is reported as 0.94 by Hatch and Boehm (91) and as 0.98 by Harmatz et al. (93). These two values combine to give a value of 0.96 ± 0.02 for the 87-keV transition intensity. Using the present value of the half-life as (6.27 ± 0.35) nsec Equation 28 gives the gamma mean life as (14.8 ± 1.4) nsec.

The gamma mean lives for the two levels are compared with the theoretical values in Figure 27. The horizontal scale in the figure is drawn assuming $\delta = 0.057$. The solid curves in Figure 27 are obtained by assuming the 105-keV level to be spin 5/2 and the 87-keV level spin 3/2, while the dashed curves are obtained assuming the spins to be reversed.

The left side of Figure 27 shows the experimental-theoretical comparison for the experimental values of this investigation. The theoretical values of the gamma mean lives in the left side of the figure were calculated as a function of δ using Equation 16 corrected for a neutron transition. This correction was made because Gd^{155} has an odd number of neutrons and a neutron is involved in the transition rather than a proton. A detailed discussion of these calculations is presented in Appendix E.

The right side of Figure 27 is an approximate copy of the comparison made by Deutch et al. (89) using their experimental value of the mean

Figure 27. A comparison of the experimental gamma mean lives of the 87- and 105-keV levels of Gd^{155} with the theoretical mean lives as a function of δ . The solid lines represent the theoretical values of Nilsson for the orbital assignment of the 87- and 105-keV levels as [651] $3/2$ and [642] $5/2$, respectively. The dotted curves represent the opposite assignment. The left side of the figure is a comparison between the experimental gamma mean lives determined in this investigation with the theoretical mean lives for a neutron transition. The right side is an approximate copy of a similar figure given by Deutch et al. (89). The experimental mean lives in the right side of the figure are those of Vergnes (87) and of Deutch et al. (89). The theoretical curves are for a proton transition and are a factor of two smaller than the theoretical curves for a neutron transition

life of the 105-keV level and Vergnes' (87) experimental value of the mean life of the 87-keV level. Their theoretical curves were obtained using Equation 16 for a proton transition. This results in their theoretical values being a factor of two smaller than those calculated assuming a neutron transition. Their value for the mean life of the 105-keV level is not in as good agreement with the theoretical values calculated assuming a neutron transition.

As can be easily seen in Figure 27, the solid set of curves gives the best agreement with the experimental values. In the left portion of the figure both shaded areas intersect the solid curves in the region of $\delta = 0.25$. As stated previously by Deutch et al. (89), if the theoretical values could be taken at face value they would assign the spins of $3/2$ and $5/2$ to the 87- and 105-keV levels, respectively.

The Weisskopf estimate can be calculated using Equation 9. It gives values of 5.38×10^{-13} sec and 3.0×10^{-13} sec for the 87- and 105-keV levels, respectively. The experimental mean life is longer than the Weisskopf estimate by a factor of 2.8×10^4 in the case of the 87-keV level and by a factor of 1.0×10^4 in the case of the 105-keV level. If these factors are compared with the good experimental-theoretical agreement obtained in Figure 27, the success of the unified model compared to the spherical single-particle model is apparent.

C. Measurements of Half-Lives of Isomeric Levels in Samarium 152, Gadolinium 156, Lutetium 175 and Praseodymium 144

A discussion will be presented in this section of half-life measurements of the 122-keV level in Sm^{152} , the 89-keV level in Gd^{156} , the 396-keV

level in Lu^{175} and the 80-keV level in Pr^{144} . The half-life measurements of the first three levels mentioned above were undertaken primarily as a check on the accuracy and reliability of the experimental method and equipment used in this investigation. These three half-lives have been measured by several different investigators and there is, in general, good agreement among the measured values.

1. Half-life of the 122-keV level in Samarium 152

The first measurement which will be discussed is the measurement of the half-life of the 122-keV level in Sm^{152} . The source which was used in this measurement was a Eu^{152} source made by Schupp and Hatch (45) and used by them in their investigation of the K-shell internal conversion coefficient of the 344-keV transition in Gd^{152} . A decay scheme of Eu^{152} (50) is shown in Figure 28. A portion of the electron spectrum is shown in Figure 29.

In this measurement, coincidences were obtained between the 244-keV L-conversion electrons and the 122-keV L-conversion electrons. A delay curve and a prompt curve are shown in Figure 30. The prompt curve was obtained by using the ThB I-line in coincidence with the 115-keV beta particles. In order to obtain the prompt curve a slight change in current in the high-energy spectrometer was necessary to focus on the ThB I-line. The slope of the delay curve was analyzed in order to determine the half-life. Two values were obtained for the half-life: 1.36 and 1.35 nsec. These combine to give a resultant value of (1.36 ± 0.06) nsec. This error results from a determinate error of 3% which was increased by a factor of 1.4 to include a possible systematic error.

Figure 28. Decay scheme of Eu^{152} (50). Energy values are in MeV

Figure 29. A partial internal conversion spectrum of Sm^{152} and Gd^{152} obtained from the decay of Eu^{152}

Figure 30. Delay curve for the 122-keV level in Sm^{152} obtained using Eu^{152} decay and a comparative prompt curve obtained using ThB decay. The half-life value given was obtained from the points between the arrows

Figure 31. Experimental half-life values obtained by various investigators for the 122-keV level in Sm^{152} : a. Present investigation, b. Fossan and Herskind (97), c. Birk et al. (98), and d. Sunyar (99). The horizontal line represents a weighted average of the four values

The values for the half-life of the 122-keV level of Sm^{152} , as reported by various investigators, are given in Table 10. Figure 31 shows a comparison of the various experimental values. The horizontal line in Figure 31 gives the weighted average of the four reported experimental values of the half-life. As can be seen in Figure 31, the four experimental values are in good agreement.

There is one reported measurement of the half-life of the 122-keV level which was not included in the table or the figure because an error was not reported for the value. This value is 1.43 nsec and was reported by Abou-Leila *et al.* (100). The present value is in agreement with the value of Abou-Leila *et al.* if their error is assumed to be at least 3%.

2. Half-life of the 89-keV level in Gadolinium 156

The half-life of the 89-keV level in Gd^{156} was studied using Eu^{156} decay. A decay scheme of Eu^{156} (50) is shown in Figure 32 and the low-energy electron spectrum of Eu^{156} decay is shown in Figure 33. The Eu^{156} source was obtained from the irradiation of 99.2% enriched Sm^{154} at the Materials Testing Reactor operated by the Phillips Petroleum Co. at Arco, Idaho. The Eu^{156} is the result of double neutron capture. The material used for the Eu^{156} sources was also used for the study of Gd^{155} after the Eu^{156} activity had dissipated. Eu^{155} decay was therefore present but can be considered negligible since the Gd^{155} internal conversion lines are seen to be very weak in Figure 33.

The delay curves for measuring the half-life of the 89-keV level were obtained by detecting coincidences between the 89-keV L-conversion electrons and the beta particles at approximately 140-keV in energy. This

Table 10. Experimental values of the half-life of the 122-keV level in Sm^{152}

Half-life in 10^{-9} seconds	Reference
1.36 ± 0.06	Present investigation
1.37 ± 0.04	Fossan and Herskind (97)
1.45 ± 0.06	Birk <u>et al.</u> (98)
1.40 ± 0.10	Sunyar (99)

means a contribution from the higher energy levels in Gd^{156} is present but the half-lives of these levels should be very small and can therefore be neglected. A typical set of data is shown in Figure 34. The prompt curve was obtained by using the 110-keV L-conversion electrons in coincidence with the 198-keV K-conversion electrons from the Yb^{169} decay. It was necessary to change the current slightly in the low-energy spectrometer in order to focus on the 110-keV L-conversion electrons.

Four values were obtained for the half-life of the 89-keV level in Gd^{156} . These four values are listed in Table 11. The first three values in Table 11 were obtained over a period of twenty-four hours and were obtained in such a manner that a different portion of the converter calibration curve was used for each curve. These three half-life values are in good agreement and give further indications that the converter is linear over the range of approximately twenty nanoseconds. The weights in Table 11 were obtained in the same manner as the weights for previously mentioned measurements.

Figure 32. Decay scheme of Eu^{156} (50). Energy values are in MeV

Figure 33. A portion of the internal conversion spectrum of Gd^{156} obtained from Eu^{156} decay

Figure 34. Delay curve for the 89-keV level in Gd^{156} obtained using the Eu^{156} decay and a comparative prompt curve obtained using the Yb^{169} decay. The half-life value given was obtained from the points between the arrows

Table 11. Half-life values of the 89-keV level in Gd^{156} using 89-keV L-conversion electrons in coincidence with beta particles from Eu^{156} decay

Half-life in 10^{-9} seconds	Weight
2.17 ^a	4
2.29 ^a	2
2.27 ^a	4
2.11	1
2.22 -- Weighted average	

^aThese three values were obtained in a 24-hr period using different portions of the calibration curve

A weighted average of the four values in Table 11 gives a final value of (2.22 ± 0.08) nsec. The error in this value arises from a determinate error of 2.6% which is increased by a factor of 1.4 to account for possible systematic errors. The determinate error arises from the same sources that were mentioned in previous measurements.

The reported experimental values of the half-life of the 89-keV level are given in Table 12. A comparison of the experimental values is given in Figure 35. The horizontal line in Figure 35 gives the value of a weighted average of the first four values in Table 12. The value obtained by Nathan (101) was omitted from the average since it seems probable that his value is low due to a possible prompt contribution in his delay curve. The first three values given in Table 12 are in excellent agreement. The fourth value by Birk (98) is slightly low and does not agree

Table 12. Experimental values of the half-life of the 89-keV level in Gd^{156}

Half-life in 10^{-9} seconds	Reference
2.22 ± 0.08	Present investigation
2.16 ± 0.06	Fossan and Herskind (97)
2.19 ± 0.07	Bell and Jorgensen (102)
2.05 ± 0.10	Birk <u>et al.</u> (98)
1.9 ± 0.1	Nathan (101)

too well with the present measurement; the fifth value by Nathan is in definite disagreement with the present measurement.

3. Half-life of the 396-keV level in Lutetium 175

The half-life of the 396-keV level in Lu^{175} was determined using the decay of Yb^{175} . A decay scheme of Yb^{175} (50) is given in Figure 36, and an electron spectrum is shown in Figure 37. The Yb^{175} source material was obtained by irradiating 99.0% enriched Yb^{174} in the reactor at the Argonne National Laboratory.

The half-life of the 396-keV level was determined by obtaining coincidences between the 396-keV K-conversion electrons and the beta particles feeding the 396-keV level. Sets of data were obtained using 15-keV beta particles and using 35-keV beta particles. A delay curve obtained using 35-keV beta particles is shown in Figure 38. The delay curve in Figure 38 was obtained using the side of the converter calibration curve corresponding to the left side in Figure 46. The prompt curve

Figure 35. Experimental half-life values of the 89-keV level in Gd^{156} obtained by different investigators: a. Present investigation, b. Fossan and Herskind (97), c. Bell and Jorgensen (102), d. Birk *et al.* (98), and e. Nathan (101). The horizontal line represents a weighted average of values a, b, c and d

Figure 36. Decay scheme of Yb^{175} (50). Energy values are in MeV

Figure 37. Electron spectrum of the decay of Yb^{175} showing the internal conversion spectrum of Lu^{175}

Figure 38. Delay curve for the 396-keV level in Lu¹⁷⁵ using the Yb¹⁷⁵ decay and an approximate prompt curve using the 114-keV transition in Lu¹⁷⁵. The half-life given in the figure was obtained from the points between the arrows

in Figure 38 is an approximate prompt curve only, because it was necessary to change the current in both spectrometers to obtain this prompt curve. Since the prompt curve is used, in this case, only to define the region outside the prompt region, it is felt this prompt curve is sufficient. When the 15-keV beta particles were used, a more exact prompt curve was obtained using the Yb^{169} decay.

Table 13 gives a tabulation of the half-life values obtained using both 15-keV and 35-keV beta particles. Six of the values listed in Table 13 were obtained using the side of the calibration curve corresponding to the left side of the curve in Figure 46. A weighted average of the values determined using 15-keV beta particles gives a half-life value of (3.44 ± 0.19) nsec. A weighted average of the values determined using 35-keV beta particles and the usual side of the calibration curve (the right side of the curve in Figure 46) gives a resultant value for the half-life of (3.34 ± 0.09) nsec. A weighted average of the six values obtained using the left side of the calibration curve gives a value of (3.29 ± 0.07) nsec. The errors in these values are determinate errors only. The agreement between these three values indicates that the converter operates reliably at low electron energies (15 keV). They also indicate that either side of the converter calibration curve can be reliably used in the half-life measurements.

A weighted average of all the values listed in Table 13 gives a value of (3.31 ± 0.10) nsec. The error in the final average includes increasing the determinate error by a factor of 1.4 to include a possible systematic error.

The reported experimental values for the half-life of the 396-keV

Table 13. Half-life values of the 396-keV level in Lu^{175} using Yb^{175} decay

Energy of beta particles used in coincidences	Half-life in 10^{-9} seconds	Weight
15 keV	3.26	37
	3.28	2
	3.62	7
	3.16	25
	2.35	8
	3.92	6
	3.99	15
	4.11	21
	4.54	2
	2.90	10
	4.00	1
	3.53	15
	3.23	9
35 keV	3.30 ^a	121
	3.14 ^a	336
	3.35 ^a	37
	3.54 ^a	84
	3.24 ^a	336
	3.31 ^a	3025
	3.20	25
	3.35	756
	3.50	47
	3.29	37
	3.76	84
	3.14	121
	3.13	121
	3.15	189
	3.39	84
	3.51	84
	3.57	62
	3.96	25
Weighted average	3.31	

^aThese values were obtained by using the left side of the converter calibration curve as shown in Figure 46.

Table 14. Half-life of the 396-keV level in Lu^{175} as determined by different investigators

Half-life in 10^{-9} seconds	Reference
3.31 ± 0.10	Present investigation
3.25 ± 0.10	Berlovich <u>et al.</u> (103)
3.1 ± 0.3	Hauser <u>et al.</u> (104)
3.4 ± 0.3	Vartapetian (105)

level in Lu^{175} are tabulated in Table 14. A comparison of the values is shown in Figure 39. The horizontal line indicates the value of the weighted average of the four values in Table 14. All four experimental values are in good agreement.

4. Half-life of the 80-keV level in Praseodymium 144

The half-life of the 80-keV level in Pr^{144} was determined using Ce^{144} decay. The $\text{Ce}^{144} - \text{Pr}^{144}$ source material processed carrier-free was obtained from the Oak Ridge National Laboratory. A decay scheme of $\text{Ce}^{144} - \text{Pr}^{144}$ decay (50) is presented in Figure 40 and an electron spectrum is shown in Figure 41.

The half-life of the 80-keV level was determined using the center-of-gravity method. The delay curve was obtained by detecting 80-keV K-conversion electrons in coincidence with the beta particles. Two separate prompt sources were used in the half-life determination.

One set of half-life determinations was obtained by using 149-keV beta particles in obtaining the delay curve for the 80-keV level in Pr^{144} . In

Figure 39. Experimental half-life values obtained by various investigators for the 396-keV level in Lu^{175} : a. Present investigation, b. Berlovich *et al.* (103), c. Hauser *et al.* (104), and d. Vartapetian (105). The horizontal line represents the weighted average of the four values

Figure 40. Decay scheme of ^{144}Ce - ^{144}Pr (50). Energy values are in MeV

Figure 41. Electron spectrum of $\text{Ce}^{144}\text{-Pr}^{144}$ decay showing the internal conversion spectrum of Pr^{144}

this case ThB was used as a prompt source. The prompt distribution was obtained by detecting beta particles at the same current setting as the 80-keV K-conversion line of Pr^{144} in coincidence with the ThB F-line. Again the current settings were left unchanged in both spectrometers in obtaining the prompt curve and the delay curve.

A prompt curve and a delay curve for this case are shown in Figure 42. The two curves are normalized to the same total number of counts. The error flags are shown only for the prompt curve in order to avoid confusion. The statistics are much better for the delay curve than for the prompt curve.

The experimental values of the half-life determined using ThB for a comparison source are shown in the left column of Table 15. An unweighted average of (150 ± 10) psec is obtained for the half-life of the 80-keV level from these data. The error in this value arises mainly from the statistical spread of the values in the first column of Table 15.

There is a contribution to this measurement from the 134-keV level which also feeds the 80-keV level via the 54-keV transition (see Figure 40). This half-life has been measured by Burde et al. (106) and is reported as (6.6 ± 3.5) psec. Since this is small compared to the half-life of the 80-keV level, and since less than twenty per cent of coincidences recorded in the analyzer arise from coincidences between the 186-keV beta group and the 80-keV transition, the contribution to measurement of the half-life of the 80-keV level is less than one picosecond.

There could also be a small contribution due to the 41-keV M- and N-conversion electrons being detected with the 80-keV K-conversion electrons. This would give a contribution to the measured half-life from

Figure 42. Delay curve for the 80-keV level in Pr^{144} using the Ce^{144} - Pr^{144} decay and a comparative prompt curve obtained using the ThB decay. The crosses indicate the centers of gravity of the two distributions

Table 15. Experimental half-life values of the 80-keV level in Pr^{144}

Half-life using ThB betas vs F-line for prompt curve in 10^{-12} seconds	Half-life using Yb^{169} 110K vs 198L for prompt curve in 10^{-12} seconds
139	108 ^a
101	183
167	101
135	121
174	142
174	193
171	202
139	68
150 -- Average	126
	145
	127
	137 -- Average

^aIn obtaining the half-life values in this column the half-life of the 118-keV level in Tm^{169} was used as 63×10^{-12} sec as determined in this investigation.

the 100-keV level. The half-life of this level has been measured to be: (1.0 ± 0.2) nsec by Burde et al. (106). It is difficult to estimate this contribution.

A third contribution to the measured half-life above could come from the K-Auger electrons emitted after the conversion of K-shell electrons for the 134-keV transition. Again this contribution should be small and

would amount to the presence of a small portion of prompt coincidence in the delay distribution.

These three possible sources of error can be eliminated by setting the beta spectrum at a current position at or above the end-point energy of the 186-keV beta group.

A second set of data for this half-life was obtained by using the Yb^{169} decay for a prompt source. In this case the delay curve was obtained by detecting 185-keV beta particles in coincidence with the 80-keV K-conversion electrons. A voltage of -3,800 Volts was applied to the Ce^{144} - Pr^{144} source in obtaining the delay curve. For the prompt curve, a voltage of +3,800 Volts was applied to the Yb^{169} source and coincidence was obtained between the 110-keV K-conversion electrons and the 198-keV L-conversion electrons.

The 110-keV K-conversion line peak did not occur at the same current setting as the 80-keV K-conversion peak of Pr^{144} even with the voltages applied as mentioned. (These were the maximum possible voltages that could be applied for long periods of time at the time of this measurement.) Therefore, if the currents in the spectrometers were to be unchanged, it was necessary to retain the setting on the lower edge of the 110-keV K-conversion line. Previous checks indicated that this would not appreciably affect the measurements.

Since in this case the half-life of the 118-keV level in Tm^{169} is appreciable with respect to the half-life of the 80-keV level in Pr^{144} , the half-life was determined using Equation 30 where $\tau_0 = (62 \pm 2.8)$ psec (21). The values obtained using Yb^{169} as a reference source are given in column two of Table 15.

An unweighted average of the values in column two of Table 15 gives a value for the half-life of (137 ± 12) psec. The error in this value also arises primarily from the statistical spread of the data.

An unweighted average of all the values in Table 15 results in a final value of (143 ± 11) psec for the half-life of the 80-keV level. Again, the statistical error of 8 psec results mainly from the spread of the data. The statistical error is increased by a factor of 1.4 in order to include possible systematic errors arising from the possible contributions mentioned in the discussion above.

The half-life of the 80-keV level in Pr^{144} has also been measured by Burde et al. (106). They obtain a value of $(1.8 \pm 0.2) \times 10^{-10}$ sec for the mean life, which corresponds to a value of (125 ± 14) psec for the half-life. This value is in agreement with the value determined in the present investigation.

D. Concluding Remarks

The half-lives of nine different isomeric levels were measured and reported in this investigation. The half-lives of the isomeric levels in Sm^{152} , Gd^{156} and Lu^{175} were measured primarily to demonstrate the reliability of the experimental method and equipment used in this investigation. The good agreement between the values of the lifetimes of these three levels determined in the present experiments and those determined by other investigators is evident.

The present result for the half-life of the 80-keV level in Pr^{144} also agrees with the value determined by Burde et al. (106) and can be considered a confirming measurement.

The half-lives of the 87-keV and 105-keV levels determined in this investigation are in general agreement with most of the reported half-life values reported for these two levels. The measurements reported here can again be considered confirming measurements, since the present measurements confirm the measurement of the lifetime of the 87-keV level reported by Bozek et al. (86) and, less strongly, the value reported by Vergnes (87). The lifetime of the 105-keV level reported by Runge et al. (88) is also confirmed by the value of the lifetime reported for this level in this investigation.

The measurement of the Tm^{169} 118-keV level lifetime confirms the value reported by Blaugrund et al. (21) and the measurement of the lifetime of the 139-keV level in Tm^{169} gives further confirmation to the other reported values of the lifetime of this level (74, 75).

The half-life of the 8.4-keV level in Tm^{169} is the only measured lifetime reported in this investigation which is in general disagreement with other reported values. On the bases of the extensive investigation of the lifetime of this level and on the bases of the good agreement obtained with other investigators for the other measured lifetimes, it is thought that the value reported here is correct. It is not possible, however, to explain completely the disagreement with other investigators, especially the disagreement with Sundström et al. (53). It appears that measurements by other investigators using the general methods used in this investigation and used by Sundström et al. would be especially desirable.

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VI. APPENDIX A: CIRCUIT DIAGRAMS

This appendix includes circuit diagrams of the photomultiplier voltage divider (Figure 43), the limiter circuit (Figure 44) and the time-to-pulse-height converter (Figure 45). The limiter circuits and the time-to-pulse-height converter were constructed by the Ames Laboratory Instrumentation Group and were patterned after circuits described by Simms (46).

Figure 43. Circuit diagram of the phototube voltage divider

Figure 44. Circuit diagram of the limiter circuit

Figure 45. Circuit diagram of the time-to-pulse-height converter

VII. APPENDIX B: CALIBRATION OF THE TIME-TO-PULSE-HEIGHT CONVERTER

The time-to-pulse-height converter was calibrated using two different methods at different periods in this investigation. During the early stages of the measurements the converter was calibrated by obtaining prompt curves with different amounts of cable between the limiter and the converter. The second method, which was used in the latter part of the investigation, is the method described by Graham et al. (47). This method utilized a spiral delay line and a pulse generator as shown in Figure 1. Both methods will be described in detail and will be shown to have given the same calibration constant.

In the first part of this investigation, the method which was used to calibrate the converter was the method using a coincidence cascade and various lengths of cable. The cable that was used between the limiters and the converter was RG 63/U cable. The calibrating cables were cut to have an integral number of nanoseconds of delay in the range from one to twelve nanoseconds. A propagation constant of $0.84c$, where c is the velocity of light, was used for calculating the length of the cables corresponding to the desired time delays. In most cases the coincidence distribution which was used to calibrate was one obtained from the decay of Yb^{169} . Specifically, coincidences were usually obtained between the 198-keV K-conversion electrons and the 110-keV K-conversion electrons (see the Yb^{169} decay scheme, Figure 9).

The calibration curve of the converter has two sides as can be seen in Figure 46. The left side in Figure 46 corresponds to coincidence pulses from channel A arriving at the converter before the coincidence

Figure 46. A calibration curve for the time-to-pulse-height converter. This curve was obtained using a pulse generator and a variable spiral delay line

pulses from channel B. The right side corresponds to the opposite case. In most of the lifetime measurements the coincidence pulses in channel B arrived at the converter first; therefore only half of the calibration curve, corresponding to the right half in Figure 46, was ordinarily obtained in calibrating the converter.

In calibrating the converter using the 198-keV K- and 110-keV K-conversion electrons the delays were adjusted until the two coincidence pulses were almost completely overlapping. This position was then chosen as the zero time position for calibrating the converter. A coincidence distribution was then obtained at this position. Successive amounts of cable were then added and a coincidence curve obtained at each position until the linear range of the calibration curve was obtained. In general, calibration points were obtained every two nanoseconds. Points at 0, 4, 8, etc. nanoseconds were obtained first, and then the points at 2, 6, 10, etc. nanoseconds. This was done in order to minimize any drifting in the converter calibration due to short term effects.

After the calibration data were obtained, the accidental counts were subtracted and the center of gravity calculated for each coincidence distribution. The centers of gravity of the curves were then plotted versus the amount of cable delay added to channel A. The linear range of the curve was determined by inspection; the data in this range was analyzed using an unweighted least squares fit of the data. The analysis of the data was performed using the Iowa State University IBM 7074 computer. The inverse of the slope of the straight line gives the calibration constant in nsec/channel.

At a later time in this investigation it was decided to measure the

time delay of each cable used in calibrating the converter. The time delays were measured by the Ames Laboratory Instrumentation Group using a method devised by W. A. Rhinehart. A discussion of these measurements is given in Appendix C.

In all later converter calibrations the measured values of the cables were used. In order to obtain a correction factor for the data obtained using the calculated delay values for the cables, three sets of calibration data were analyzed using both the measured delay values and the calculated delay values. The ratio of the calibration constant obtained using the measured delay values to the calibration constant obtained using the calculated delay values was then determined for the three sets of data. The three values obtained for the ratio were: 1.003, 1.004, and 1.005. These values give an average correction factor of 1.004. This factor was applied to any half-life that was obtained using the calibration constant determined by using the calculated delay values in the converter calibration.

At a still later time, the pulse generator mentioned above was obtained and the method described by Graham et al. (47) was used for calibrating the converter. In this method a pulse generator is used to feed pulses to the movable tap on the spiral delay line. The pulse then travels to the inputs of the two limiters (see Figure 1), and a pulse-height distribution is recorded in the analyzer. The movable tap is then moved a distance corresponding to a time delay t , and the pulse-height distribution is again recorded. The difference in time between the two distributions, as recorded on the analyzer, is $2t$ since the pulse going to one limiter is delayed a time t while the pulse going to the other

limiter arrives a time t earlier than before. Thus, the relative time shift is $2t$. As the movable tap is moved successive amounts, a set of calibration points is obtained. This method is a considerable improvement over the cable method since it greatly decreased the time necessary to calibrate, and since it also greatly decreases any attenuation effects due to adding more cable between the limiter and the converter.

The time delay of the spiral delay line was also measured by the Ames Laboratory Instrumentation Group. This measurement is also discussed in Appendix C.

The effect of the amplitude of the pulse-generator pulse on the calibration constant was investigated. No change was noted for pulses greater than about 2.5 Volts amplitude. Two calibration curves obtained with different amplitude pulses are shown in Figure 47. The curves are displaced in the vertical direction in order to separate them for comparison. The top curve was obtained using 3.0-Volt pulse-generator pulses and produces a calibration constant of $-(0.380 \pm 0.002)$ nsec/channel. The lower curve was obtained using 5.0 Volt pulses and gives a calibration constant of $-(0.381 \pm 0.002)$ nsec/channel. The errors in these values are statistical errors only.

As a check on the equivalence of the two methods of calibrating, two comparisons were made between the two methods. The first comparison is shown in Figure 48 where the curves are again displaced for comparison. The top calibration curve was obtained using the pulse generator and the spiral delay line while the lower curve was obtained using 110-keV K-conversion electrons in coincidence with 198-keV K-conversion electrons in the decay of Yb^{169} . The calibration constants obtained for the pulse

Figure 47. A comparison of two converter calibration curves using the spiral delay line and different amplitude pulse-generator pulses. The calibration constants are $-(0.380 \pm 0.002)$ nsec/channel for the top curve and $-(0.381 \pm 0.002)$ nsec/channel for the bottom curve

Figure 48. A comparison of the pulse generator method and the cable method of calibrating the time-to-pulse-height converter. The calibration constants are $-(0.397 \pm 0.004)$ nsec/channel for the pulse generator method and $-(0.390 \pm 0.005)$ nsec/channel for the cable method

generator method and for the cascade method were $-(0.397 \pm 0.004)$ nsec/channel and $-(0.390 \pm 0.005)$ nsec/channel, respectively. The errors arise mainly from a 1% possible error in the cable measurements. As can be seen, the two values agree within the overlapping errors.

The curves for the second comparison are shown in Figure 49. Again the upper curve was obtained using the pulse generator. The lower curve was obtained using 20.8-keV L-conversion electrons in coincidence with 110-keV L-conversion electrons. In this comparison the calibration constants obtained for the pulse generator method and for the cascade method were $-(0.398 \pm 0.004)$ nsec/channel and $-(0.393 \pm 0.005)$ nsec/channel respectively. Again the two values obtained for the calibration constant are in agreement within the assigned errors.

The combination of the two comparisons mentioned above shows that the calibration of the converter is not significantly dependent on the electron energy within the range examined. The energy of almost all of the transitions examined in this investigation fall within the energy ranges used in the above mentioned comparisons.

A check was also made of the possibility of attenuation effects being present in the cable method of calibrating the converter. The presence of attenuation effects would tend to give a smaller calibration constant than the true time calibration constant. An examination of the two comparisons mentioned above might lead one to believe that there could be a small attenuation effect present in the cascade method. In order to check this, two sets of calibration curves were obtained using the cable method. In Figure 50 the upper curve was obtained in the usual manner when calibrating the converter with Yb^{169} . The lower curve was

Figure 49. A second comparison of the pulse generator method and the cable method of calibrating the time-to-pulse-height converter using a different cascade. The calibration constants are $-(0.398 \pm 0.004)$ nsec/channel for the pulse generator method and $-(0.393 \pm 0.005)$ nsec/channel for the cable method

Figure 50. A comparison between the regular method of cable calibration and the cable calibration using 20 nsec of cable delay added to each channel as a check on attenuation effects. Both curves give a calibration constant of $-(0.379 \pm 0.004)$ nsec/channel

obtained by adding an additional 20 nsec of cable to the low-energy channel and calibrating by adding cable to the high-energy channel. This means that for the point at time equals zero on the lower curve there would be an additional 20 nsec of cable added to each side compared to the same point on the upper curve. Both curves in this figure give the same calibration constant of $-(0.379 \pm 0.004)$ nsec/channel, where the error in this case arises only from the statistical scatter of the points. A second set of data for the regular method and for the method using more cable on each side gave values for the calibration constant of $-(0.380 \pm 0.002)$ nsec/channel and $-(0.384 \pm 0.004)$ nsec/channel respectively. Since these values agree within the statistical errors, it is concluded that the attenuation effects in the cable method are negligible and no correction need be applied to the calibration constant.

The complete calibration curve showing both sides is given in Figure 46. The calibration curve in Figure 46 was obtained using the pulse generator and the spiral delay line. At time delay zero the pulses from channel A and channel B completely overlap at the converter. The right side of the curve is obtained by delaying pulses in channel A with respect to pulses in channel B while the left side of the curve is obtained by delaying pulses from channel B with respect to pulses in channel A. It is quite obvious in Figure 46 that the two linear portions of the curve on either side of the zero position do not have the same slope. The calibration constant obtained from the left side of the curve is $-(0.253 \pm 0.001)$ nsec/channel, while the calibration constant obtained from the other side of the curve is $-(0.383 \pm 0.002)$ nsec/channel where the errors are statistical errors. This nonsymmetry arises from the fact

that the two 2N643 transistors do not have exactly the same characteristics. When the two transistors were transposed the calibration curve was reflected about the vertical line at time equals zero. The half-life of two of the levels measured in this investigation were measured using both sides of the calibration curve. The same half-life was obtained in these cases, independent of which side of the calibration curve was used. These measurements were mentioned in Section IV.

From the checks mentioned above, it is concluded that the two methods of calibrating give essentially identical results. It is further concluded that the converter was operating reliably over a linear range of approximately 20 nsec.

It is also concluded that, in general, the converter calibration is good to an accuracy of 1.7%. In obtaining this value, a 1% error is assigned to the cable measurements, a 1% statistical error is assigned to the determination of the slope in the linear range and an error of 1% is assigned to a possible time drift in the calibration of the converter. The square root of the sum of the squares of these values gives a resultant error of 1.7%.

VIII. APPENDIX C: MEASUREMENTS OF THE TIME DELAY OF THE CONVERTER
CALIBRATION CABLES AND THE SPIRAL DELAY LINE

The following method was used by W. A. Rinehart of the Ames Laboratory Instrumentation Group to measure the time delay of the cables and spiral delay line used in the present lifetime measurements.

The equipment used in the measurements consisted of a Tektronix-type 661 oscilloscope, an E. H. Research Laboratory fast-rise pulse generator, a Hewlett Packard variable-frequency oscillator and a Hewlett Packard high-frequency counter. The rise time of the E. H. pulse generator pulses is less than 1 nsec.

In this method the pulse generator output is fed to the external sweep trigger and through a reference cable to the vertical deflection input. The piece of cable to be measured is then inserted between the reference cable and the input to the vertical deflection plates. The displacement of the pulse is determined using the half amplitude point on the rise of the pulse as the fiducial point. Various time scales were used depending on the length of the cable being measured. Each time scale used was calibrated using the variable-frequency oscillator and the high-frequency electronic counter. Using this method the time delay of each cable was measured to an accuracy of 1%.

In this method to measure the RG 63/U cables used in calibrating the converter, a cable connector was left on each cable. In effect this means that the delay that was determined for each cable was a combination of the cable delay plus the delay introduced by the connector. This was necessary because in practice it was necessary to insert a cable connector

in either channel whenever a length of cable was inserted.

The time delays of several cables were also measured using the standing wave method. The agreement with the results obtained using the method described above was well within the 1% accuracy mentioned.

A plot was constructed of the measured delay of each cable plus connector versus the calculated delay using the velocity of propagation as $0.84c$. This curve is shown in Figure 51. The slope and y-intercept were determined to be (0.987 ± 0.002) and (0.185 ± 0.017) nsec respectively by using the IBM 7074 computer. The intercept gives the values of the delay for the cable connector as (0.19 ± 0.02) nsec while the slope gives the velocity of propagation as $(0.851 \pm 0.002)c$. The errors in these values are the standard deviation as determined by the computer and arise from the scatter of the points.

The propagation time of the spiral delay line was measured in a similar manner. The delay of each turn was measured individually. The time delays of several turns near each end of the spiral delay line were found to differ by several per cent from the delay of the other turns. Except for these external turns, the values of the delay per turn was found to be (1.24 ± 0.01) sec/turn. The external turns were not used in the converter calibration.

Figure 51. Curve showing the relation between the measured time delay of the RG 63/U calibrating cables plus connector and the calculated delay values assuming the velocity of propagation of the cables to be $0.84c$, where c is the speed of light

IX. APPENDIX D: COMPUTER PROGRAMS

Two computer programs for the Iowa State University IBM 7074 computer were used in this investigation. Both programs utilized a least squares fit to a linear relationship between the data analyzed. The determination of the calibration constant was accomplished using an unweighted least squares fit, and a weighted least squares fit was used for the determination of the half-life in the slope method described in Section III.

The least squares method is discussed by many authors, including Worthing and Geffner (107). In the least squares fit to a straight line, it is desired to find two constants, A and B, for a set of points (x_i, y_i) which satisfy the relationship:

$$y_i = Ax_i + B .$$

If we let ω_i equal the assigned weight to the i^{th} point, A and B are determined by finding the two constants that minimize the sum:

$$Q = \sum_i \omega_i (y_i - Ax_i - B)^2 . \quad (41)$$

(It is assumed that the error in x_i is small compared to the error in y_i in this method.) The values of A and B which make Q a minimum are given by the following equations:

$$A = [(\sum \omega_i)(\sum \omega_i x_i y_i) - (\sum \omega_i x_i)(\sum \omega_i y_i)]/D \quad (42)$$

$$B = [(\sum \omega_i x_i^2)(\sum \omega_i y_i) - (\sum \omega_i x_i)(\sum \omega_i x_i y_i)]/D \quad (43)$$

where

$$D = (\sum \omega_i)(\sum \omega_i x_i^2) - (\sum \omega_i x_i)^2 . \quad (44)$$

In addition the errors, ΔA and ΔB , in A and B are given by:

$$\Delta A = \sigma [(\sum \omega_i)/D]^{\frac{1}{2}} \quad (45)$$

$$\Delta B = \sigma [(\sum \omega_i x_i^2)/D]^{\frac{1}{2}} \quad (46)$$

where

$$\sigma = \left[\frac{N \sum \omega_i (y_i - Ax_i - B)^2}{(N-2) \sum \omega_i} \right]^{\frac{1}{2}} \quad (47)$$

As was mentioned previously, a computer program for an unweighted least squares fit to a straight line ($\omega_i = 1$ for all ω_i in Equations 41 through 47) was used to evaluate the converter calibration constant. The time delay and the channel number position of the center of gravity associated with the prompt coincidence distribution recorded on the pulse-height analyzer were used as the x and y coordinates, respectively, in the equations given above.

Since this program is a relatively simple one and is included in most computer program libraries, the details of this program will not be discussed.

The computer program for evaluating the half-life of an isomeric state using the slope method of analysis is more involved and will be discussed in detail. The computer program was written by the Ames Laboratory Computer Group under the supervision of Mr. Charles Turner.

In the slope method, Equation 30 is used as the basis of the method. In analyzing the data, only the data in the region R where $P(t) \ll N(t)$ is used. $P(t)$ is the prompt distribution as a function of the time delay t and $N(t)$ is the delayed distribution associated with the half-life τ of interest. We will restrict ourselves to the region R in the discussion that follows.

In the region R Equation 30 becomes:

$$\frac{d \log N(t)}{dt} = - \frac{1}{\tau} \quad (48)$$

Upon integrating this equation and changing to logarithms to the base ten, we obtain:

$$\log_{10} N(t) = At + B \quad (49)$$

where $A = -(\log_{10} e)/\tau$ and B is a constant. If we now let $y = \log_{10} N(t)$, we obtain the linear relationship:

$$y = At + B, \quad (50)$$

which allows the use of the least squares fit to a straight line.

The time delay, t_i , is calculated from the equation:

$$t_i = Ez_i + F \quad (51)$$

where z is the channel number, and the constants E and F are obtained from the computer analysis of the converter calibration data. Also, in the calculations, y_i is determined from the equation:

$$y_i = \log_{10}(N_i - G_i) \quad (52)$$

where N_i is the number of counts in the delayed coincidence distribution for the i^{th} point and G_i is the accidental coincidence background for the i^{th} point.

The computer program was constructed to perform an iterated weighted least squares fit using different weights for each iteration. In practice five iterations were performed for each set of data although the program is written to allow any number of iterations. Therefore, in practice,

we obtained five values for the constants A and B: $(A_1, A_2, \dots, A_5)$ and $(B_1, B_2, \dots, B_5)$.

In obtaining A_1 and B_1 , the weights ω_{1i} are calculated by the following equation:

$$\omega_{1i} = (N_i - G_i)^2 / (N_i + G_i) \quad (53)$$

Since the ω_{1i} are very erratic for points having poor statistics, i.e. a low number of counts and hence a large relative error, it was thought that a smoother weighting function could be obtained by using the values of N obtained from A_1 and B_1 for calculating the weights for another determination of the half-life which would be more accurate. Therefore, ω_{2i} for the second interaction is calculated using

$$\omega_{2i} = [\log_{10}(1 + \Delta N_{1i}/N_{1i})]^{-2} \quad \text{if } y_i - y_{1i} \geq 0, \quad (54)$$

where

$$N_{1i} = 10^{y_{1i}} \quad (55)$$

$$y_{1i} = A_1 t_i + B_1 \quad (56)$$

$$\Delta N_{1i} = (N_{1i} + 2G_i)^{\frac{1}{2}} \quad (57)$$

If $y_i - y_{1i} < 0$, and if $1 - (\Delta N_{1i}/N_{1i}) > 0$, ω_{2i} is calculated using

$$\omega_{2i} = [\log_{10}(1 - \Delta N_{1i}/N_{1i})]^{-2} \quad (58)$$

If $1 - (\Delta N_{1i}/N_{1i}) \leq 0$ for some value of i, say j, the program stops calculating the ω_{2i} and calculates A_2 and B_2 using the points up to point j.

The program then uses A_2 and B_2 to calculate the weights ω_{3i} for the next iteration using the same formulas as for ω_{2i} with the subscripts numbers increased by one. This process is repeated until the desired

number of iterations is completed. The final value of A is then used to determine $\tau_{1/2}$ using:

$$\tau_{1/2} = (-\log_{10} 2)/A. \quad (59)$$

The choice of the two different values of ω_i for the second and succeeding iterations is to help correct for the fact that the experimental points above the calculated curve are in general closer to the curve than the experimental points below the curve. The condition for $1 - (\Delta N_{ji}/N_{ji}) \leq 0$ is to avoid having the logarithm of a negative number or zero.

The errors in each A_j and B_j are also determined and printed out by the computer program.

A flow chart of the program is shown in Figure 52 and the Fortran program is given in Figure 53.

Two checks on the program were made, one concerning the relative change between A_1 and A_5 , and the other concerning the spread of the experimental points about the final calculated curve.

The relative change between A_1 and A_5 depends very much on the statistics involved. For the earlier measurements of the half-life of the 8.4-keV level in Tm^{169} , the relative difference averaged about 2% with A_5 producing the smaller value for the half-life. The relative change between A_1 and A_5 was negligible (less than 1% in the cases checked) for the measurements of the effect of the high voltage on the half-life value for the 8.4-keV level.

In addition, a visual check of 14 sets of data consisting of approximately 400 points gave 51% of the experimental points greater than the

Figure 52. Flow chart of the computer program used to determine the half-lives of isomeric states by the slope method

Figure 52. (Continued)

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C      S(I)=TOTAL COUNTS,G(I)=BACKGROUND,Z(I)=CHANNEL NUMBER
C      W(I)=INITIAL WEIGHTS,W1(I)=NEW WEIGHTS
C      N=NUMBER OF POINTS,M=NUMBER OF ITERATIONS
C      E=CALIBRATION CONSTANT,F=CALIBRATION INTERCEPT
C      DIMENSION Z(100),S(100),G(100),W(100),W1(100),Y(100),R(100),YC(100
1),P1(100),DP1(100),T(100),A(2,2),B(2),V(2)
COMMON Z,S,G,W,W1,Y,R,YC,P1,DP1,T
18 READ 100,N,M
IF(N) 10,10,11
10 STOP 89
11 READ 101,(Z(I),I=1,N)
READ 101,(S(I),I=1,N)
READ 101,(G(I),I=1,N)
READ 101,CONSE,CONSF
J=0.0
DO 12 I=1,N
Y(I)=S(I)-G(I)
W(I)=S(I)+G(I)
W(I)=Y(I)*Y(I)/W(I)
Y(I)=LOGKF(Y(I))
12 T(I)=CONSE*Z(I)+CONSF
C
PRINT 110
PRINT 111,(I,Z(I),S(I),G(I),Y(I),T(I),W(I),I=1,N)
PRINT 112,CONSE,CONSF
C
1000 FN=N
J=J+1
IF(J-M) 190,190,200
190 V(1)=0.
V(2)=0.
A(1,1)=0.
A(1,2)=0.
A(2,2)=0.
C
DO 13 I=1,N
A(1,1)=A(1,1)+W(I)
A(1,2)=A(1,2)+W(I)*T(I)
A(2,2)=A(2,2)+W(I)*T(I)*T(I)
V(1)=V(1)+W(I)*Y(I)
13 V(2)=V(2)+W(I)*Y(I)*T(I)
C
DET=A(1,1)*A(2,2)-A(1,2)*A(1,2)
IF(DET) 14,14,15
15 B(1)=(A(2,2)*V(1)-A(1,2)*V(2))/DET
B(2)=(A(1,1)*V(2)-A(1,2)*V(1))/DET
ERROR=0.
C
DO 16 I=1,N
16 R(I)=Y(I)-B(2)*T(I)-B(1)
DO 17 I=1,N
17 ERROR=ERROR+W(I)*R(I)*R(I)
DENU=(FN-2.0)*A(1,1)/FN
ERROR=ERROR/DENU
SIB=ERROR*A(2,2)/DET
SIB=SQRTF(SIB)
SIA=ERROR*A(1,1)/DET
SIA=SQRTF(SIA)
C
PRINT 113,B(1),SIB,B(2),SIA
DO 20 I=1,N
20 YC(I)=B(2)*T(I)+B(1)
DO 21 I=1,N

```

Figure 53. Fortran program of the computer program used to determine the half-lives of isomeric states by the slope method

```

      P1(I)=(10.0)**YC(I)
21 DP1(I)=SQRTF(P1(I)+2.*G(I))
      DO 22 I=1,N
      IF(Y(I)-YC(I)) 120,121,121
121 W1(I)=1.0/(LOGXF(1.0+DP1(I)/P1(I)))**2
      GO TO 22

C
120 IF (1.0-DP1(I)/P1(I)) 102,102,103
103 W1(I)=1.0/(LOGXF(1.0-DP1(I)/P1(I)))**2
      22 CONTINUE
      GO TO 25
102 NCUTOF=I-1
      PRINT 150,NCUTOF
      N=NCUTOF
      25 DO 77 I=1,N
      77 W(I)=W1(I)
      GO TO 1000
      14 PRINT 114,DET
200 GO TO 18
100 FORMAT (2I5)
101 FORMAT (7F10.0)
110 FORMAT (22H1PROGRAM FOR DR. HATCH//6HJ      18X,4HZ(I)13X,4HN(I)13X,
      14HB(I)13X,4HY(I)13X,4HT(I)13X,4HW(I)//)
111 FORMAT (3X,13,6E17.8)
112 FORMAT (4H E =E17.8,5X,4H F =E17.8//)
113 FORMAT (4H B =E17.8,3X,8HDELTAB =E17.8,3X,3HA =E17.8,3X,8HDELTAA =
      1E17.8)
114 FORMAT (30H ERROR MESSAGE , DETERMINANT =E17.8)
150 FORMAT (8HNCUTOF =I3)
      END

```

Figure 53. (Continued)

points on the computer determined curve and 49% of the experimental points less than the points on the determined curve.

The first check indicates that an iterated program is probably not necessary when the relative errors of the individual experimental points are small, but that a significant improvement is obtained when the relative errors of the points are large.

The second check indicates that the calculated curve is reliable since, on the average, 50% of the experimental points should lie above the calculated curve and 50% should lie below the curve.

X. APPENDIX E: THEORETICAL CALCULATIONS FOR GADOLINIUM 155

In this appendix a more detailed discussion will be given of the theoretical calculations of the lifetime of the two single particle levels in Gd^{155} . The odd particle in Gd^{155} is a neutron and, therefore, Equation 16 must be used as corrected for a neutron transition. For a neutron transition in ${}_{64}Gd_{91}^{155}$ Equation 13 becomes:

$$T = (2.70) \times 10^{21} \left(\frac{\hbar\omega}{197 \text{ Mev}} \right)^3 | \langle I \ 1 \ K \ K' - K \mid I \ 1 \ I' \ K' \rangle |^2 G_{E1}^2 \text{ sec}^{-1}$$

$$\text{where } G_{E1} = \sum_{\ell \ell'} \langle N' \ \ell' \mid r \mid N \ \ell \rangle \sqrt{\frac{2+1}{2-1}} \langle \ell \ 1 \ 0 \ 0 \mid \ell \ 1 \ \ell' \ 0 \rangle \\ \times \sum_{\substack{\Lambda \Lambda' \\ \Sigma \Sigma'}} \delta_{\Sigma \Sigma'} a'_{\ell' \Lambda'} a_{\ell \Lambda} \langle \ell \ 1 \ \Lambda \ K' - K \mid \ell \ 1 \ \ell' \ \Lambda' \rangle .$$

The first case considered will be the case of the transition from the $[651]3/2$ excited state to the $[521]3/2$ ground state. For this case $N' = N-1 = 5$, $K' = K = 3/2$, $I' = I = 3/2$ and:

$$| \langle I \ 1 \ K \ K' - K \mid I \ 1 \ I' \ K' \rangle | \\ = | \langle (3/2) \ 1 \ (3/2) \ 0 \mid (3/2) \ 1 \ (3/2) \ (3/2) \rangle | = 0.7746 .$$

If G_{E1} above is expanded we obtain:

$$G_{E1} = \frac{13}{\sqrt{22}} \langle 6100 \mid 6150 \rangle (a'_{51} a_{61} \langle 6110 \mid 6151 \rangle + a'_{52} a_{62} \langle 6120 \mid 6152 \rangle) \\ + 3 \sqrt{\frac{11}{14}} \langle 4100 \mid 4130 \rangle (a'_{31} a_{41} \langle 4110 \mid 4131 \rangle + a'_{32} a_{42} \langle 4120 \mid 4132 \rangle) \\ + \sqrt{\frac{15}{2}} \langle 2100 \mid 2110 \rangle a'_{11} a_{21} \langle 2110 \mid 2111 \rangle \\ + \frac{3}{\sqrt{11}} \langle 4100 \mid 4150 \rangle (a'_{51} a_{41} \langle 4110 \mid 4151 \rangle + a'_{52} a_{42} \langle 4120 \mid 4152 \rangle)$$

$$+\sqrt{\frac{10}{7}} < 2100 | 2130 > (a'_{31}a_{21} < 2110 | 2131 > + a'_{32}a_{22} < 2120 | 2131 >) .$$

The Clebsch-Gordon coefficients were obtained from tables of Ch1 (108) and Simon (109). When the numerical values of the Clebsch-Gordon coefficients are substituted, G_{E1} becomes:

$$\begin{aligned} G_{E1} = & -1.883(-0.6699a'_{51}a_{61} - 0.6405a'_{52}a_{62}) \\ & -1.773(-0.6455a'_{31}a_{41} - 0.5774a'_{32}a_{42}) \\ & +0.9487a'_{11}a_{21} + 0.6742(0.7303a'_{51}a_{41} + 0.6831a'_{52}a_{42}) \\ & +0.9258(0.7303a'_{31}a_{21} + 0.5774a'_{32}a_{22}) . \end{aligned}$$

The constants $a'_{\lambda, \Lambda}$ and $a_{\lambda \Lambda}$ were obtained from the tables given by Nilsson (39). The constants which are given by Nilsson must be normalized such that $\sum (a_{\lambda \Lambda})^2 = 1$ and $\sum (a'_{\lambda, \Lambda})^2 = 1$. The tables of Nilsson are tabulated as a function of η and are given only for integral values of η . In order to obtain intermediate points for the calculation, it was necessary to interpolate between the tabulated values. This interpolation was done using graphic interpolation. The constants in Nilsson's tables were plotted on linear graph paper in the case of the $[651]3/2$ orbital and on log-log paper in the case of the $[521]3/2$ orbital.

In the second case of a neutron transition from the $[642]5/2$ level ($I = K = 5/2$) to the $[521]3/2$ ground state ($I' = K' = 3/2$), G_{E1} was found to be:

$$\begin{aligned} G_{E1} = & -1.88(0.599a'_{51}a_{62} + 0.679a'_{52}a_{63}) \\ & -1.77(0.645a'_{31}a_{42} + 0.764a'_{32}a_{43}) -1.34a'_{11}a_{22} \\ & +0.674(0.365a'_{51}a_{42} + 0.258a'_{52}a_{43}) + 0.663a'_{31}a_{22} . \end{aligned}$$

In addition the quantity $\langle I1KK' | K I1I'K' \rangle$ is found to be:

$$| \langle (5/2)1(5/2)-1 | (5/2)1(3/2)(3/2) \rangle | = 0.816$$

The constants a_{Λ} from Nilsson's tables were plotted on linear graph paper for the $[642]5/2$ level, and the radial matrix elements given by Equations 20 and 21 were used in calculating both values of G_{E1} given above.