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Outline

CONSTITUTIVE EQUATIONS FOR INELASTIC ANALYSES OF LIQUID-METAL FAST BREEDER REACTOR SYSTEM COMPONENTS*

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ABSTRACT

This paper describes constitutive relations selected for use in performing inelastic analyses of structures for use in early liquid-metal-cooled, fast breeder reactor (IMFBR) systems. Components of these systems are to operate at temperatures where creep is important and under time-varying temperature and loading conditions. The materials considered are types 304 and 316 stainless steels. Small deformation behavior is assumed, and it is postulated that the total strain tensor can be decomposed into time-independent (elastic and plastic) components and time-dependent (creep) components. Thus, constitutive equations for elastic-plastic and for creep behavior are discussed separately.

Results from short-time stress-strain tests at various temperatures are examined and essential features are discussed in the context of mathematical modeling requirements. Existing mathematical theories of plasticity are reviewed in terms of describing the observed behaviors, which include nonlinear kinematic hardening type characteristics. It is shown that extensions to the theories are required to model the features of interest, and promising approaches for modification are indicated. Finally, constitutive equations for interim use are outlined.

In the case of creep behavior, both primary and secondary stage creep deformations are of importance. An equation-of-state type formulation is adopted for interim use in this case. Various hardening rules are examined, and the strain-hardening rule is shown to approximate behavior observed from uniaxial

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tests. Modifications to this rule are required when stress reversals are encountered. Auxiliary rules for this type loading are developed and described for the uniaxial case. These latter rules are then generalized for combined stress conditions in which near-radial loadings are encountered.

Experimental studies for examining important unknowns related to creep response are outlined, and directions for research related directly to theory development are proposed. Interactions between essentially time-independent permanent (plastic) strains and time-dependent (creep) strains can be significant and represent an area in need of concentrated study. Manifestations of such interactions are discussed briefly.

PART I. ELASTIC-PLASTIC BEHAVIOR

I. Introduction

In this portion, we describe results from tests on stainless steel specimens and discuss these results in terms of phenomenological models. Mathematical representations appropriate to the observed behaviors are discussed. It is shown that extensions to existing mathematical theories are required, and a promising avenue for developmental pursuit is briefly explored. Finally, using the background information provided, the constitutive equations recommended for interim use by the IMFBR industry are outlined and discussed.

II. Material Response and Mathematical Representations

A. Material Response (Type 304 Stainless Steel)

1. Uniaxial test results

- a. Monotonic loading
- b. Cyclic loading curves
 - (1) Room temperature
 - (2) 1200°F

SLIDE NO. 1

SLIDE NO. 2

SLIDE NO. 3

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2. Combined stress

a. Radial loading

(1) Loading paths, stress and strain space

SLIDE NO. 4

(2) Effective stress-effective plastic strain response

SLIDE NO. 5

b. Nonradial loading

(1) Path number 1

(a) Loading path and loading surface behavior

SLIDE NO. 6

(b) Stress-strain response

SLIDE NO. 7

(2) Path number 2

(a) Loading path and loading surface behavior

SLIDE NO. 8

(b) Stress-strain response

SLIDE NO. 9B. Mathematical Formulations

1. Nonlinear kinematic hardening

SLIDE NO. 10

a. Basic equations

b. Translation tensor formulation for nonlinear case

c. Comparison of calculated and experimental results - cyclic loading, uniaxial case

SLIDE NO. 11

d. Conclusions

(1) Response for subsequent loading of prestrained specimen requires study

SLIDE NO. 12

(2) Subsequent yield surface behavior, including changes in configuration due to plastic deformation and temperature influences, need investigation

(3) Examination of basic postulates, that is, normality, proportionality of translation and strain tensors, etc., is needed.

2. Recommended constitutive equations

SLIDE NO. 13~~a. Classical kinematic hardening theory, modified for nonisothermal conditions~~

b. Effects of prior permanent deformations

SLIDE NO. 14

c. Procedures to account for prior deformation history

SLIDE NO. 15

SLIDE 1

ORNL-DWG 72-12325

SLIDE 2

ORNL-DWG 71-3413

Cyclic Stress-Strain Behavior Type 304 Stainless Steel
Room Temperature.

TYP 304 STAINLESS STEEL, ANNEALED (972796)
TEMPERATURE: 1100° F
STRAIN RATE: 0.005 /min.

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SLIDE 3

SLIDE 4

ORNL-DWG-72-10042

ORNL-DWG-72-10041

TYPE 304 STAINLESS STEEL, ANNEALED (9T2796)
TEMPERATURE: ROOM TEMPERATURE
TEST CONDITION: BIAXIAL

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TYPE 304 S/S ANNEALED (9T2796)
ROOM TEMPERATURE
TEST CONDITION: BIAXIAL

SLIDE 7

2

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TYPE 304 S/S, ANNEALED (9T2796)
 ROOM TEMPERATURE
 TEST CONDITION: BIAXIAL

TYPE 304
 ROOM TEM
 TEST CONTA

ORNL-DWG. 72-12965

TYPE 304 S/S, ANNEALED (9T2796)
ROOM TEMPERATURE
TEST CONDITION: BIAXIAL

SLIDE 9

PLASTIC BEHAVIOR

$$1. f^*(\sigma'_{ij}, \epsilon_{ij}^P, T) = f(\sigma'_{ij} - \alpha_{ij}) - \kappa(T) = 0$$

$$2. d\epsilon_{ij}^P = \Lambda \frac{\partial f^*}{\partial \sigma'_{ij}}$$

$$3. d\alpha_{ij} = g \left(\epsilon_{kl}^P, T, H \right) d\epsilon_{ij}^P$$

$$4. d\epsilon_{ij}^P = g_{ij} \neq 0 \text{ (during loading)}$$

where

$$g_{ij} = \frac{1}{2\kappa g} \frac{\partial f^*}{\partial \sigma'_{ij}} \left(\frac{\partial f^*}{\partial \sigma'_{kl}} d\sigma'_{kl} + \frac{\partial f^*}{\partial T} dT \right)$$

when the initial yield condition (formulation for f)
used is that of von Mises.

SLIDE 11

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(1) ——— SUBSEQUENT LOADING PATH
----- α_{ij} PATH

PLASTIC BEHAVIOR

$$1. f^*(\sigma'_{ij}, \epsilon_{ij}^P, T) = f(\sigma'_{ij} - \alpha_{ij}) - \kappa(T) = 0$$

$$2. d\epsilon_{ij}^P = \Lambda \frac{\partial f^*}{\partial \sigma'_{ij}}$$

$$3. d\alpha_{ij} = c d\epsilon_{ij}^P$$

$$4. d\epsilon_{ij}^P = g_{ij} \neq 0 \text{ (during loading)}$$

where

$$g_{ij} = \frac{1}{2\kappa c} \frac{\partial f^*}{\partial \sigma'_{ij}} \left(\frac{\partial f^*}{\partial \sigma'_{kl}} d\sigma'_{kl} + \frac{\partial f^*}{\partial T} dT \right)$$

when the initial yield condition (formulation for f)
used is that of von Mises.

SLIDE 14

SLIDE 15

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PART II. EQUATIONS FOR CREEP ANALYSES

I. Introduction

Constitutive equations for time-dependent behavior of structural materials are discussed here from a nuclear industry point-of-view, particularly that of the Liquid-Metal Fast Breeder Reactor (LMFBR) systems. As system temperatures, design lives, and loading complexities increase, design by analysis becomes an increasingly necessary approach. Therefore, specific attention must be given to time-dependent (creep) behavior of structures under sustained and variable mechanical and thermal conditions. In this presentation, the materials of interest are stainless steels, especially type 304.

Current system designs and those in the immediate future must be based on the use of existing technology. This is based upon limited information concerning the overall mechanical response of structural materials in the creep regime, with the greatest concentration on results from tests under sustained conditions. Mathematical foundations have been developed throughout recent years for different constitutive formulations for creep; some of which differ in basic premises and some of which differ in more restrictive details. Inherent in these formulations are different capabilities and limitations. The choice of constitutive equations for use in design must incorporate the ability to predict as many of the known features of material behavior as possible. The choice must also be compatible with the computational capabilities available to the designer.

The current position, therefore, is a further application of old, but ~~somewhat extended theory.~~ In all situations, such approaches are not expected to yield precise quantitative results, but rather give at least a qualitative statement of structural response. Future endeavors must be aimed at filling the differences.

II. Application of Theory to Design

A. Choice of theory

1. On the basis of the framework within which creep constitutive equations must be selected, currently the only real candidates are equation-of-state approaches.
2. The most favorable candidates possess some limitations that may be eliminated by specific alterations.
3. The theory chosen must be made usable by rank and file design analysts.

B. Equation-of-state theory

1. Main ingredients.

SLIDE NO. 1

2. Uniaxial constant stress creep data and creep equations.

- a. Primary and secondary creep must be considered. The primary creep magnitude and duration are stress-dependent.

SLIDE NO. 2

b. Available data.

- (i) Generally available data are for relatively high stresses and temperatures and for short times. Extrapolations are required for design use and the extrapolations are sensitive.
- (ii) Limited data exist and possess limited resolution in low strain regimes.
- (iii) General data inaccuracies are associated with load application, strain measurement, and temperature measurement and control.

c. Mathematical representations.

- (i) Various mathematical relations have been used for different materials and parameter ranges. Choices exist for type 304 stainless steel data.
- (ii) Equations currently considered by design analysts include one exponential primary creep term and two exponential primary creep terms. Steady-state creep is added to the primary creep expression.
- (iii) Qualitative comparisons between the creep equa-

SLIDE NO. 3

3. Multiaxial stress condition.

- a. Mathematical form (Slide No. 1) postulates colinearity of creep strain rates and deviatoric stress.
- b. Sustained combined tension-torsion loads result in predicted axial and torsional strains which are proportional. SLIDE NO. 4
SLIDE NO. 5
- c. Multiaxial creep strain-time data compared with predictions which utilize a creep law developed for the same heat of material. SLIDE NO. 6
SLIDE NO. 7

4. Hardening rules for variable loads.

- a. Creep response to stepwise increasing loads. SLIDE NO. 8
- b. Physical interpretation of strain hardening model for variable loads. SLIDE NO. 9
- c. Comparison of strain-hardening predictions with experimental data for increasing load test. Note the extended time until the first step in stress. SLIDE NO. 10
- d. Auxiliary rules for loading reversals.
 - (i) Needed to prevent anomalous predictions from classical strain-hardening models.
 - (ii) Physical interpretation of auxiliary rules. SLIDE NO. 11
 - (iii) Comparison between predictions of strain-hardening model with auxiliary rules and experimental data. SLIDE NO. 12

5. Constant strain relaxation.

- a. Constitutive models give inherent predictions to general loading conditions such as relaxation tests.
- b. Design analyses of relaxation situations decompose the time interval into small, but discrete, time increments and assume creep occurs at constant stress over these increments. The stress for the next increment is established by elastically adjusting the total strain back to the specified constant value.
- c. Comparisons between predicted relaxation behavior and experimental results. SLIDE NO. 13
SLIDE NO. 14
SLIDE NO. 15

III. Summary

Constitutive equations currently used in creep analysis of nuclear components have been discussed. These equations are based on the current state-of-the-art and, along with the auxiliary rules established, will model essential features of structural material behavior. The choice of these relations take into account both known features of material behavior and the current computational capabilities available to the designer.

Further examinations of the creep response of structural materials are needed. This includes both uniaxial and multiaxial studies. In addition to systematic studies of the creep response to mechanical loading histories, variable temperature conditions are important. The discussions given here were based on the premise that total strains can be decomposed into time-independent (elastic and plastic) components and time-dependent (creep) components. A systematic examination of possible interactions between these strains will form a basis for establishing the need for coupling between the two constitutive theories.

TIME-DEPENDENT (CREEP) BEHAVIOR

1. Constant-uniaxial-stress creep equation:

$$\epsilon^c(\sigma, T, t) = \epsilon_t(\sigma, T) \left[1 - e^{-r(\sigma, T)t} \right] + \dot{\epsilon}_m(\sigma, T)t.$$

2. Multiaxial constitutive equations:

$$\dot{\epsilon}_{ij}^c = \lambda \sigma'_{ij}.$$

Define

$$\bar{\sigma}^2 = 3J'_2 = \frac{3}{2} \sigma'_{ij} \sigma'_{ij},$$

$$\bar{\epsilon}^2 = \frac{4}{3} I_2 = \frac{2}{3} \epsilon_{ij}^c \epsilon_{ij}^c.$$

Write:

$$\dot{\epsilon}_{ij}^c = \frac{3}{2} \frac{\dot{\bar{\epsilon}}(\bar{\sigma}, t, T)}{\bar{\sigma}} \sigma'_{ij}.$$

3. For strain-hardening law:

$$\dot{\epsilon}_{ij}^c = \frac{3}{2} \frac{\dot{\bar{\epsilon}}(\bar{\sigma}, \bar{\epsilon}, T)}{\bar{\sigma}} \sigma'_{ij}.$$

1100°F TYPE 304 STAINLESS STEEL PROJECT AR-2

Least Squares Fit to Constant-Load Creep Curves for 304 Stainless Steel (Heat 8043813) at 1200°F.

SLIDE 4

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**Creep Strain Ratios for Combined Tension-Torsion
Loadings -- Type 304 Stainless Steel (Ht. 9T2796), 1100°F**

Test No.	Axial Stress, σ (ksi)	Torsional Stress, τ (ksi)	Effective Stress, $\bar{\sigma}$ (ksi)	Stress Ratio, τ/σ	Creep Strain Ratio: Torsion/Axial $\epsilon_{12}/\epsilon_{11}$	
					Observed	Predicted
1	10.628	6.111	15.00	0.575	0.92	0.862
2	13.975	3.407	15.17	0.244	0.37	0.366
3	5.570	7.934	14.83	1.424	2.18	2.137

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SLIDE 10

SLIDE NO. 11

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TEST RP 53 TEMP. = 1100 F

12000.

10000.

8000.

6000.

4000.

2000.

0.

10.

20.

30.

40.

TI

STRESS (PSI)
STRAIN (IN./IN.)

Thin

up by 2,000

~~100 F - TOTAL STRAIN = 0.5 PERCENT~~

This is to be a dashed line as in other drawings

TYPE 304 STAINLESS STEEL (Ht. 9T2796)

TEST RP 53

TEMPERATURE = 1100°F

TOTAL STRAIN = 0.5%

INITIAL STRESS = 12,000 PSI

2

40.
TIME (HOURS)

50.

60.

70.

80.

90.

Extend up by motion 2,000 psi

TEST RP 68 TEMP. = 1100 F

STRESS (PSI)
~~STRAIN (IN./IN.)~~

TIME

F TOTAL STRAIN = 1.0 PERCENT

✓
Slide 14

This is to be a dashed line as in the other drawings

TYPE 304 STAINLESS STEEL (HE.9T2796)

TEST DD 68

TEMPERATURE = 1100°F

TOTAL STRAIN = 1.0 %

INITIAL STRESS = 13,800 PSI

2

TEST BP 67 TEMP. = 1100

STRESS (PSI)
- STRAIN (IN./IN.)

OF TOTAL STRAIN = 1.8 PERCENT

slide 15

TYPE 304 STAINLESS STEEL (HL. 9T2796)

TEST RP 67

TEMPERATURE = 1100°F

TOTAL STRAIN = 1.8 %

INITIAL STRESS = 16,500 PSI

2

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TIME (HOURS)

40.

50.

60.

70.

80.