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A STUDY OF THE $1/E$ SLOWING-DOWN NEUTRON SPECTRUM
USING 4π -RECOIL PROPORTIONAL COUNTERS

by

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ABSTRACT

The response to a $1/E$ neutron spectrum of a hydrogen-filled proportional counter in the energy region below 100 keV was studied using pulse-shape discrimination to reject gamma-induced background. The observed proton-recoil distributions permitted an evaluation of W (the energy loss per ion pair) for protons in hydrogen below 10 keV. Some problems associated with the use of 4π -recoil proportional counters as neutron spectrometers were considered including calibration, optimization of neutron-to-gamma ratios, and resolving power.

I. INTRODUCTION

A proportional counter will multiply ionization created by fast particles. This multiplication occurs in an avalanche of electrons initiated when electrons from the primary ionization, under the influence of the applied field, have drifted close to the anode wire. The multiplication process is almost noiseless, and this fact, together with the possibility of including radiation sources homogeneously throughout the gas volume, explains the continued interest in these devices. Solid-state detectors, by contract, apparently do not possess a usable, linear, avalanche mechanism, so that ionization equivalent to 10-20 keV or less is not seen above system noise.

Neutrons are detected by interacting with nuclei. If, for example, some He^3 is included in a counter-filling mixture, neutrons will interact to release energetic fragments of ~ 0.7 MeV, plus whatever energy the neutron itself had. A difficulty at low energy is still present, however, although it takes a different form. For one reason or another, the resolution of practical counters will not usually be less than about 5%. Add to this the hypersensitivity (as $1/\sqrt{E}$) of positive- Q charged-particle reactions to slow neutrons, and one finds that the resulting ionization spectrum does not lend itself at all well to providing neutron spectral information below about 100 keV.

Instead of using a positive- Q reaction, one may study proton recoils from hydrogen included in the counter-filling gas. A problem arises

immediately, however, for no longer are pulses large compared to gamma-ray-induced background. Gamma events are found over almost all of the ionization spectrum where interpretation in terms of neutron spectrum is easiest.

One may surmount this difficulty by pulse-shape discrimination against the gamma-induced events. Electrons from gammas are produced in the counter body and pass through the gas along chords. Because the filling is hydrogen, negligible conversion occurs in the gas itself. The pulse-height distribution of electron background increases as ionization increases, goes through a maximum, and decreases again. The total rate of electron events does not depend upon gas pressure. As pressure is increased, the ionization distribution moves to a higher average value with a corresponding decrease, on the average, in the number of events in any fixed interval. The number of proton-recoil events in any ionization interval is just proportional to pressure, however, so that increasing the gas pressure improves the relative neutron/gamma-induced event ratio. This is fortunate since high gas pressures are also desirable for limiting the range of the more energetic recoils, thus confining them to the counting region and reducing end- and wall-effect distortions.

The 4π -recoil proportional counter, with shape discrimination against gamma background, lends itself best to measurements below about 1 MeV. The low energy response cannot be extended indefinitely, however, since a proton having energy less than the ionization potential of the gas in which it stops (about 15 eV) will create no ion pair and hence will be undetected, let alone measured (a single ion pair is sufficient to detect an event, but many ions are needed to define the energy in a statistically meaningful way). A quantity $W(E)$ represents the average energy loss per ion pair for fast protons in the gas and without knowledge of the way W varies with energy, measured ionization cannot be related to proton energy. W is known for some gases above about 10 keV, but adequate data below this energy are lacking.

Another problem is in detector calibration over the range of its use. Proportional counters are not absolute ionization detectors; they must be calibrated relative to a source of known energy and ionization. Once this has been done, the counter will produce pulses proportional to ionization over a range of ionizations generally adequate for spectroscopy. A difficulty arises once the initial range has been studied and it is necessary to change to another range. The best thing would be to have a suitable standard for each multiplication range, but since this may be difficult, an accurate means of predicting changes in gas multiplication with counter voltage is required.

There is also the problem of space-charge saturation. If one tries to get large pulses, far from noise, by increasing the gas multiplication, the avalanche could saturate, making the induced pulse less than it should be. This type of nonlinear effect would introduce systematic errors which, at all costs, are to be avoided.

Even with clean, undistorted proton-recoil spectra, one is left with a rather peculiar distribution judged by spectroscopic standards accustomed to peaks and structural variation with energy. Because of the systematics of n-p scattering, a complicated neutron spectrum will lead to a proton-recoil spectrum increasing monotonically as energy decreases. The proton-recoil spectrum, $D(E)$, is a convolution over the neutron spectrum, $\phi(E)$, and the n-p scattering cross section $\sigma(E)$ according to the well-known result,

$$D(E) \sim \int_E^\infty \phi(E')\sigma(E')dE'/E'. \quad (1)$$

Ocular demonstration of neutron spectrum is lost, and one must resort to the derivative of the proton distribution to extract a neutron spectrum. This appears a worse state of affairs than it actually is, however. Even a modest computer will provide straight-line fits to input data, and the precision with which details of n-p scattering are known eliminates any significant error, other than a statistical one, from the analysis.

Any discussion of detector resolution or efficiency must draw in details of the differentiation as well as of the intrinsic resolving power. The peculiar differentiation requirement may not invariably be the one limiting resolution. There are situations in which a 4π -recoil counter, by virtue of its high rate of detection, produces spectra having resolution and statistical accuracy superior to telescopic arrangements that select only forward scatters.

In any event, the range of protons is too short to permit any kind of collimation at low energies. The gas pressure cannot be decreased indefinitely because a minimum amount of gas is required to quench the multiplication process.

These then are some of the problems to be faced in using the 4π -recoil counter. Most have been considered previously, but low-level work has been principally with noble gases for beta measurements rather than with hydrogen for proton recoils. There is a large difference, for example, between W for an electron in a noble gas and that for a proton of the same energy in hydrogen.

To study a few of these problems, measurements were made on the characteristic $1/E$ neutron slowing-down spectrum, a spectrum which can be reasonably anticipated under some rather general conditions. For an infinite moderator, such as graphite or heavy water (or mixtures of these), and for neutron energies below 100 keV where the scattering cross section is unchanging and isotropic, the neutron spectrum from a uniformly distributed source (of energy in excess of 100 keV) will assume a $1/E$ spectral shape. This general result also holds well for finite media and for point sources where source energies are again in excess of 100 keV and the

source and the detector are well separated. For the results presented here, a graphite-D₂O block (1.5 x 1.5 x 1.5 m) was used in two different ways in conjunction with a plutonium-beryllium neutron source of 1×10^7 neutrons/sec. The source could be immersed in the D₂O providing well-thermalized neutrons for counter calibration via the $N^{14}(n,p)C^{14}$ reaction on some nitrogen included in the filling gas. The source could then be placed in the graphite stack to get a suitably intense $1/E$ flux. The source and the detector were separated by about 20 cm of graphite.

Studying a $1/E$ spectrum is admittedly not interesting, except insofar as it reveals information concerning the counter. A $1/E$ spectrum is well suited for this purpose, however, and results obtained are sufficient to permit studies in other more interesting cases.

This report will consider problems of calibration, shape discrimination, and variation of W , as well as of the effects of differentiation on intrinsic counter resolution. The problem of wall- and end-effect distortion of long-range proton recoils has received considerable attention elsewhere and will be discussed only briefly here.

II. COUNTER OPERATION

The counter configuration considered here is the simplest possible, consisting of a fine wire anode (0.0254-mm diameter) at high potential on the axis of a grounded, cylindrical cathode. The field near the ends of the counter is defined by hypodermic needles extending into the body a distance of a counter diameter from the end seals.⁽¹⁾ Since both counter body and anode wire were made of stainless steel, no springs, etc., were needed to maintain wire tension under differential temperature expansion of wire and body. Results reported here were taken with a counter having a 2.54-cm radius, 0.8-mm wall thickness, and a 15-cm effective length.

The mechanism of pulse formation in the proportional counter is different from that in an ion chamber. An electron created in the gas will (if not captured) drift to the anode wire and there create an avalanche of secondary electrons which are immediately swept up by the anode. The observed pulse is the consequence of motion of the residual positive ion sheath away from the anode.

Any gas that does not capture electrons will work in a proportional counter. Unfortunately, many potential contaminants, including oxygen and water vapor, are electropositive and their presence, even to the extent of a few parts per million, will have a deleterious effect upon counter resolution. It has also been observed⁽¹⁾ that counters operated at substantial values of gas multiplication, A , require a quenching additive to stabilize the multiplication process. The quencher absorbs ultraviolet generated in the avalanche and prevents after-pulsing from a photoelectric effect at the cathode. In addition, the quenching gas will neutralize hydrogen ions generated in the avalanche and prevent electron emission that could occur if the hydrogen ions were permitted to reach the cathode. A small amount (10 cm Hg) of a complex molecular gas, such as methane, will suffice for quenching counters 2 cm or more in diameter.

The most appropriate filling gas to use will depend upon the energy interval over which measurements are made. For the energy region 0.1 to 1 MeV, one wants as much stopping power as can be obtained to minimize end- and wall-effect distortion. A paraffin series gas at as high a pressure as can be obtained without sacrificing resolution is a logical choice. Methane and propane exhibit excellent pulse-shape gamma discrimination characteristics, and others of the series are expected to do equally well in this respect. Members of the paraffin series can be expected to show a variation of W , the energy loss per ion pair, in the region 0.1 to 1 MeV,⁽²⁾ and a correction is necessary for this effect.

The paraffin series are ill-suited for proton recoils below 100 keV since recoils from carbon in the molecular structure of the series introduce a distortion of the proton spectrum. Hydrogen gas has no effect of this kind

and possesses, as well, a W constant to about 10 keV. As has been noted, a small quenching additive of methane must be added to the hydrogen, but this will not be sufficient to change very much the basic electronics from that of hydrogen. Hydrogen pressure in the counter used for the $1/E$ neutron-spectrum measurements was 3 atm. Methane to a pressure of 10 cm Hg was added for quenching, and an equal amount of nitrogen was added for calibration (as is discussed later).

The pressure of hydrogen gas should be high for several reasons, one of which involves gamma background (and is discussed later). Unless the gas is dense, there may be distortion of the shape of the spectrum due to wall and end effects in which a recoil starts in the gas but ends outside the effective counting volume. An interesting Monte Carlo study of this finite-range distortion effect has been made by Benjamin,⁽³⁾ and a result of his is reproduced in Fig. 1. The counter had a 3.81-cm length and diameter and was filled with 1 atm of hydrogen and subjected to an isotropic neutron flux with a spectrum characteristic of a fast reactor (17% U^{235} , 34% U^{238} , 49% carbon by volume). The upper curve is the total recoil-proton spectrum, and the lower curve is for that part of the spectrum from 0 to 500 keV that arises from recoils in excess of 500 keV. In the absence of distortion by recoils greater than 500 keV, the lower curve would be "flat" below 100 keV. Therefore the derivative, which is proportional to the neutron spectrum, would not be sensitive to the high-energy component. By restricting interest to the interval 0-100 keV, one may differentiate the

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Fig. 1. Wall- and End-effect Distortion of a Proton-recoil Distribution

observed spectrum without introducing a serious error. As gas pressure increases, for comparable geometry, distortion in the 0- to 100-keV region will diminish. For counters of interest here (3-4 atm), the distortion problem for energies less than 100 keV was considered to be too slight to require a correction. This conclusion could be otherwise if the spectrum was markedly different from the one assumed in the Monte Carlo study. If, for example, the spectrum consisted of an intense monoenergetic neutron line at 1 MeV, together with a very weak line at 50 keV, measurements of the weak line could be significantly distorted. Generally, however, the rapid falloff of the n-p cross section with energy in the 1-MeV region, together with systematics of the proton-recoil distribution, will suppress the end- and wall-effect distortion to low-energy data from high-energy recoils.

III. CALIBRATION AND RESOLUTION

The proportional counter must be calibrated against a standard of known ionization. Any source of known ionization will do, but since recoil protons will originate uniformly over the gas volume, a calibration source consisting of monoenergetic protons uniformly dispersed over the volume will furnish not only calibration, but a realistic measure of line width as well. For this reason, a small amount of nitrogen gas was included in the counter, providing ionization equivalent to a 615-keV proton from the reaction $N^{14}(n,p)C^{14}$ when the counter was exposed to a thermal neutron flux. The full width at half maximum of the observed line is the intrinsic counter "width" at 615 keV, and its value checks the purity of the filling gas as well as the quality of the anode wire or any other effect that could contribute to line broadening. When 0.0254-mm diameter anodes were used, the observed line width was never less than 4%, and for modest gas pressures of a few atmospheres, this width would increase to 10% or more. Theoretical width, based on statistics in ionization and multiplication, is less than 2%, so that resolution at the higher energies was mostly influenced by "mechanical" effects involving counter design and filling gas.

Statistical broadening has two independent sources. The first involves the fact that the ionization mechanism is a random process so that monoenergetic protons will actually produce a distribution of ion pairs. Details of this distribution are, however, not very clear.⁽⁴⁾ The second source of line broadening is inherent in the statistical nature of the avalanche created by the initial ionization near the wire. The average number of final electrons per initiating electron is defined to be A , but the distribution of final number is broad⁽⁵⁾ (nearly exponential), and the relative variance is near unity. An approximate idea of statistical broadening may be obtained by assuming that the multiplication process has unit relative variance and the ionization process has a normal distribution. If it is assumed that W is 36 eV/ion pair, the full width at half maximum will be about $20/\sqrt{E}$ of E in eV at low energies. The true intrinsic line width versus energy is estimated by adding the statistical and mechanical contributions together according to

$$(\text{Intrinsic Fractional Width})^2 = (\text{Mechanical Fractional Width})^2 + 400/E \quad (2)$$

All other considerations aside, line-width estimates indicate that proportional counters are not likely to provide much information about spectra for energies below one kilovolt, where line width is about half of line value. Reaction fragments from the $N^{14}(n,p)C^{14}$ reaction will release approximately $615,000 \div 36 = 1.71 \times 10^4$ electrons in hydrogen. Having calibrated the counter with this reaction, one may study a spectrum over a range of ionization near 1×10^4 , but to extend the calibration to fewer initial ion pairs, an accurate dependence of multiplication on voltage is needed.

The conventional picture of multiplication is that the initiating electron drifts, in approximate equilibrium with the gas, to within a diameter or so of the anode wire. Here the field is strong enough to allow the electron to gain energy sufficient to ionize molecules of the gas. Secondary electrons so produced will contribute further to the process, and an avalanche with an increase in number of 10^5 or more may result. It is customary to introduce the quantity α , defined as the mean number of secondary electrons produced by an electron per centimeter of path along field lines. The reciprocal of α is the mean free path for ionizing collisions. Alpha depends on the electric field and increases as the field increases. If A is the number of electrons at the wire surface ($r = a$), then, according to the definition of α ,

$$\log A = \int_a^{r_c} \alpha(r) dr, \quad (3)$$

where r_c is the distance at which the electron is first able to multiply. In principle, then, from knowledge of α and its dependence on electric field strength, Eq. (3) can be integrated to give A in terms of counter parameters. The principal difficulty is that data on α are not extensive, and the form that the results would take would not be convenient for quickly determining the effects of voltage changes.

To facilitate the use of proportional counters, various theoretical approximations to α have been made. Rose and Korff⁽⁶⁾ have analyzed the problem and introduced an effective α with an $X^{1/2}$ field dependence. The result obtained does not predict the variation of A with voltage well, as has been reported in numerous independent measurements.^(7,8) W. Diethorn⁽⁹⁾ postulates that α should be directly proportional to field, and the resulting formula, which predicts $(\log A)/V$ to be linear with $\log V$, is in good agreement with experiment for all the gases and mixtures so far tested. This formula makes possible a reliable gas gain determination at any counter voltage after calibrating with a single standard ionization source. Diethorn's formula follows directly from his assumption:

$$\begin{aligned} \alpha &= \frac{\log 2}{\Delta V} X & r < r_c, \\ \alpha &= 0 & r > r_c. \end{aligned} \quad (4)$$

Here ΔV is a constant, the average potential drop suffered by an electron between ionizing collisions. Electric field X depends upon wire radius a , cathode radius b , anode voltage V , and distance r , as

$$X = V/[r \log(b/a)]. \quad (5)$$

Let p be the gas pressure (mm of Hg), and K be the critical field-to-pressure ratio at r_c ; then,

$$K = V/[pr_c \log(b/a)]. \quad (6)$$

Therefore, from Eq. (3),

$$\log A = \int_a^{r_c} \alpha(r) dr = \frac{\log 2}{\Delta V} \frac{V}{\log(b/a)} \log \frac{r_c}{a}; \quad (7)$$

or, if the constant, K , is introduced,

$$\log A = \frac{\log 2}{\Delta V} \frac{V}{\log(b/a)} \log \frac{V}{Kpa \log(b/a)}; \quad (8)$$

so that $(\log A)/V$ is proportional to $\log V$. Extensive plots of $(\log A)/V$ versus $\log V$ are in References 7 to 9, and the accuracy of this prediction is quite evident.

Because most of the previous work was with noble gas mixtures, it was felt that an additional test using a predominantly hydrogen filling would be useful, especially one in which the largest possible range of A was covered. The $N^{14}(n,p)C^{14}$ reaction with 1.7×10^4 initial electrons was used, and it was found to be impossible to increase multiplication in excess of about 100 without introducing an effect that caused the proton line to undergo a rapid broadening with increase in voltage. The reason for this is apparently a space-charge saturation phenomenon⁽¹⁰⁾ caused by the large ($\sim 2 \times 10^6$) number of collected electrons. To study A versus V for larger A , one must start with fewer initial electrons. Ultraviolet light was introduced into the counter, causing single electrons to be ejected randomly from the cathode by the method of Curran, *et al.*⁽⁵⁾ The pulse-height distribution is quite broad because of statistical effects, but it was easy to determine the mean value as a function of voltage, which now had to be set quite high. The counter used for these tests had an atmosphere of hydrogen, along with 20 cm of methane and 10 cm of nitrogen. It was not convenient to operate at higher pressures because of limitations on high-tension operation of the counter. Results of a calibration are plotted in Fig. 2 for A greater than 10. The Diethorn formula which predicts $(\log A)/V$ linear with $\log V$ appears to do quite well over the whole range of gas multiplication. If one fits the $N^{14}(n,p)C^{14}$ results at low A with the Rose-Korff formula, the extrapolated line falls far enough below the observed high A values to miss the proper energy calibration by 50% or more.

Fig. 2

Counter Ionization Calibration Plotted as $(\text{Log } A)/V$ Versus $\text{Log } V$

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Equation (8) involves absolute multiplication and requires a measurement of amplifier gain, as well as of input capacity. For fast proportional counting (clipping time constants of a few microseconds), the peak of the induced pulse is $-0.5 \text{ eA}/C$ related to gas multiplication A , anode capacitance C , and electron charge e . The factor 0.5 is only approximate, but should not be left out, as is sometimes done for fast counting.

The problem of space-charge saturation is of greatest importance to the use of proportional counters. The larger the gas multiplication that can be achieved, the more are events raised above electronics noise; but if multiplication is excessive, the discharge may saturate. For example, if electronics noise is 2 V from an amplifier that is linear to 100 V (full-scale output without saturation), then events below 20 V will be broadened 10%, and one may have to settle for a factor of 5 in range. The strictly proportional region of counter operation, with which this report is concerned, is between two unpleasant extremes - amplifier noise at the low end, and space-charge distortion at the upper end. Only events which lie in the window between these extremes are suitable for study, and one would like this window to be wide. Unfortunately, the saturation process seems to involve several factors, none of which is well understood.

Curran⁽¹¹⁾ has observed, for a given anode, that saturation occurs at lower multiplication as pressure increases. He also has observed saturation to be reduced by reducing the anode diameter for a given pressure. The problem is compounded by the distribution of initial ionization since, if this ionization is distributed uniformly along the anode, avalanches from individual electrons are not likely to overlap as they would if the initial ionization were all at a point. Even a point initial-ionization distribution will be spatially broadened by Brownian movement of the electron swarm drifting to the anode. A space-time overlap of avalanches from individual electrons in the swarm may not be likely if this broadening is large, but this effect is sensitive to the kind of gas used. Saturation can be studied without involving factors other than the avalanche itself by using single electrons from an ultraviolet leak. The resulting pulse-height distribution departs from its normal shape if multiplication is too high. When this was done with the ~ 1 atm hydrogen counter, distortion effects were observed for about 2×10^5 collected electrons. About ten times more electrons were required to produce measurable saturation where the source was the $N^{14}(n,p)C^{14}$ reaction, due presumably to the sizable extension of proton tracks.

The upshot of this question of space-charge is that experimental recoil spectral shapes should be tested against multiplication to determine the limiting A obtainable without distortion. The electronics system noise should be optimum, of course. Assuming this is so, one can reasonably expect an adequate working range. The use of shape analysis to reject gamma background requires additional electronic bandwidth. More noise therefore will be present than is the case for ordinary pulse-height analysis of counter events.

IV. GAMMA REJECTION

Measurements of proton recoils are, unfortunately, obscured by the background of electrons from gamma rays which will invariably accompany the neutron flux. There is a region of ionization in a counter of given size and gas pressure beyond which electron events are ordinarily negligible relative to proton events. This region is, however, that in which proton tracks are a significant fraction of counter dimensions so that the recoil spectrum is distorted by wall and end effects. A Monte Carlo type calculation is then necessary for interpretation of data. Because of the sensitivity of the Monte Carlo calculation to flux anisotropy, one must use spherical proportional counters⁽³⁾ to make the calculation generally useful. Gas pressure can be minimized, but cannot be reduced below the point where quenching is no longer adequate.

In the presence of intense gamma fields, the low-pressure Monte Carlo approach appears to be the only feasible one. If, however, the total rate of occurrence of all events is not greater than about 2000/sec, each event can be "examined" individually and a decision made as to its origin.^(12,13) The gas pressure problem then reverses; best results are achieved by keeping it as high as possible. The reason for this is that electrons from gamma rays originate predominantly in the counter body and their total number does not depend on pressure. As ionization increases from zero, the ionization distribution of electron-induced events rises sharply, goes through a broad maximum depending on counter size and pressure, and falls off to zero again. Increasing pressure shifts the distribution to a higher mean ionization, but does not change the total rate of events. Recoil proton rates, on the other hand, increase with pressure so that the overall neutron-to-gamma ratio improves as pressure rises. In addition to good neutron-to-gamma ratios, one would like as low an absolute gamma rate as possible so that the counter surface area should be small. The combination of small size and high pressure promises best overall performance, and this is also the configuration desired for a probe in general. With high gas pressures, an energy range where end- and wall-effect distortion is negligible can be used, and one need only differentiate proton spectra to derive neutron spectra. Limitations exist, of course, on counter size and pressure involving gas purity, high voltage, space charge saturation, and electron drift velocity.

In regard to gamma background, techniques exist that provide possible alternatives to shape discrimination. The anticoincidence cylindrical shell configuration⁽¹⁴⁾ offers good potential gamma-rejection features. One serious difficulty is the greater size required for a workable arrangement. This is not merely a detail of construction, because gamma rates increase as size increases and would be considerably in excess of rates possible with a single tube.

One could also imagine using two counters, one containing hydrogen, and one containing another gas at pressures adjusted for equal stopping power. The difference between spectra would constitute a direct background subtraction. This approach could be useful if gamma rates were substantially less than proton rates, but if gamma rates were comparable to or in excess of proton rate, statistics on the difference spectra would deteriorate. When the bulk of gamma background has been removed by shape discrimination, it becomes feasible to make a subtraction correction for any residual component of gamma-induced origin if this component is small.

Shape rejection of electron events, which is the technique used in this report, is based upon the large difference in specific ionization between a recoil proton and an electron of the same ionization. The electron crosses the counter along a chord so that the projection on the radial of the ionization created is, on the average, a substantial fraction of counter radius. The proton recoil, on the other hand, has negligible range so that its ionization is highly localized spatially. Electrons arriving at the anode over a short time interval, as for a proton recoil, induce a pulse that rises much more rapidly than does the pulse from a broadly-distributed, gamma-induced electron event of low specific ionization. The distribution of the radial projection of chord lengths for electrons is zero for zero projection and increases as projection increases. Short projections occur with correspondingly low probability causing a fraction of gamma-induced events to demonstrate high specific ionization. This leads to a "gray" area between the two extremes of specific ionization that can be unambiguously recognized as belonging to neutron- or gamma-induced events. Figure 3 shows an experimental spectrum of specific ionization of events in a recoil detector containing hydrogen gas and exposed to a mixed gamma-neutron field. Spectra of this type

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Fig. 3. Distribution of Specific Ionization Showing Both Neutron and Gamma-induced Events

can be observed for any ionization level from ~ 500 eV on up. The relative amplitude of the neutron and gamma components changes as a function of ionization, but the general shape of the curves stays the same. The abscissa, which is actually the ratio of rate of rise over total ionization, is a measure of specific ionization of the event. Events up to about a specific ionization of 10 are gamma-induced, while those in excess of about 13 are proton-induced. The gray area between 10 and 13 may contain events of either kind and represents the limit of the shape-rejection technique. The dotted

line is an extrapolation made on the basis of spectra obtained with gamma sources only. Adding gamma events and neutron events indicates that they are in the ratio of seven to one. The number of gammas in the tail of the gamma distribution ($\geq$ channel 11) is one in 40. This ratio is representative of hydrogen and is not much different for paraffin-series gases. It is not in general sufficiently low so that one can use a yes or no approach to discrimination. By accumulating two parameter spectra, one parameter being ionization and the other specific ionization, one can make a correction for the tail that is generally adequate, unless gamma-to-neutron ratios are very unfavorable.

Fortunately for shape discrimination, the "pine tree" of protons sits well out on the "hillside" of electrons. The reason that protons are not further out originates in the Brownian movement of electrons from the point-proton-recoil distributions as they move to the anode. Consider that electrons are formed at a point in the counter at time zero. Before a pulse is induced, these electrons must drift to the anode under the influence of the applied electric field. The total drift time depends upon the drift velocity of electrons in the gas as well as the distance to the anode, and while the electron swarm is drifting, as a whole, random motion of electrons broadens the distribution spatially. If D is the diffusion coefficient for electrons and is assumed to be fixed with a value appropriate to the average anode-cathode electric field, then the distribution in space of the electron cloud t sec after formation at a point is

$$n,(r,t) = e^{-r^2/4Dt}(4\pi Dt)^{3/2}. \quad (9)$$

A measure of the distance, δ , across the cloud is twice the rms value of r , that is,

$$\delta = 2 \sqrt{6} \sqrt{Dt}. \quad (10)$$

From kinetic theory, one may estimate D as

$$D = (1/3)\lambda C, \quad (11)$$

where λ is the scattering free path and C the electron agitation velocity (not to be confused with v , the drift velocity of electrons under an applied electric field). In the absence of a field, C has the value 1.15×10^7 cm/sec, equivalent to thermal motion at room temperature. The value of C under influence of a field is determined by the means by which the electron can lose the energy it acquires. Elastic collisions with the gas provide one means for losing energy, but if it were the only one, C would be quite large, as is observed for noble gases. With polyatomic gases, such as hydrogen and methane, inelastic collisions may occur with the low-lying vibrational and rotational level structure of the molecule. Electron agitation velocity will, as a consequence, remain at a value not too much in excess of thermal value for the useful range of field-to-pressure ratios.

For example, the average value of electric field is

$$\langle X \rangle = \frac{\int_a^b X \, dr}{\int_a^b dr} = \frac{V}{b}. \quad (12)$$

For b equal to 2.5 cm and a pressure of 3 atm with 4000 V potential, the average field-to-pressure is about 0.7 V/cm · mm-Hg. Data on λ and C for this range of X/p can be found in Reference 15, for example, and using Eq. (11), one finds D to be about 130 cm²/sec. Drift velocities can, in addition, be found in the above-cited reference and lead to a drift time in hydrogen gas of several microseconds (the drift time in methane, or any paraffin gas for that matter, is substantially less than this). Equation (10), together with these rough estimates of gas parameters, indicates that diffusion broadening will be of the order of one millimeter. The exact value of broadening will depend upon the distance of the event from the anode, but the large uncertainties in the electronic parameters of gases make a more refined calculation of questionable significance. Because of the greater electron drift velocity in paraffin gases relative to hydrogen, together with the shorter mean free path of electrons in these gases, the diffusion broadening is substantially reduced and the gamma discrimination is improved.

Whether the broadening discussed above is good or bad for counter operation is not altogether obvious. It does set a limit as to the quality of gamma-neutron discrimination, but it may permit higher values of gas multiplication without space charge saturation, as was mentioned earlier. In any event, shape discrimination offers advantages over other techniques for background removal. The information needed to reject background is part of the signal, and one need use only the simplest tube-plus-wire configuration, of minimum size, in the probe end; all complications occur in the pulse-processing stage, which is well-removed from the radiation field. The shape-recognition electronics can be made to work in microsecond time intervals, so that recognition does not involve a rate limitation. Shape recognition is effective over the entire ionization range that is of practical interest. Admittedly, more than one initial electron is needed to determine specific ionization, but statistics require some ten or more pairs for reasonable energy definition, and this is quite sufficient to recognize the gamma component.

The major difficulty with shape discrimination is the tail of the gamma distribution in Fig. 3. The shape of this component is somewhat sensitive to the gamma spectrum, and this will inevitably be different for the measured radiation field and for the pure gamma sources used in estimating the correction. When gamma distributions are compared over the region between six and eleven units in Fig. 3, the tail can be predicted to

within $\pm 25\%$, at worst, with sources of cesium or cobalt and, if the tail contribution is less than 10% of the total proton count, then residual errors are about $\pm 2\frac{1}{2}\%$. This correction will, in the last analysis, limit the precision of spectrum measurements, but it is in general less than statistical errors resulting from the differentiation requirement to be discussed in Appendix A.

The desirability of applying a correction for the gamma tail means, among other things, that the electronics should present a complete spectrum of specific ionization instead of a simple "yes" or "no." The shape-discrimination circuitry is, consequently, more elaborate than for scintillators.

V. ELECTRONICS

The counter shape-discrimination circuit consists of two amplifiers, a stretcher, and a divider. The preamplifier signal is split into two parts; one is integrated in an appropriately constructed integrating amplifier, and the other is differentiated in a suitable differentiating amplifier. The output of the differentiating amplifier is narrow (25 nsec or more) and is stretched to the peak value for a time of the order of a microsecond, after which the ratio of peak pulses (differentiated over integrated) is determined by the divider. This ratio is a measure of specific ionization and passes, in coincidence with the integrated pulse, to a two-parameter pulse-height analyzer.* The divider may take different forms.⁽¹⁶⁾ A modification of an earlier version⁽¹⁷⁾ was used here, but an improved system under development, which converts pulses to a logarithmic input-output relationship and then subtracts, is expected to give overall performance improvement.

One could avoid the divider altogether and analyze, in coincidence, differentiated and integrated pulse heights. When this is done, however, the valley between gamma events and proton-recoil events runs diagonally across the two-parameter spectrum, and the result is poorly suited to make corrections for the gamma-tail effect. A topological picture of this kind is shown in Fig. 4. The two-parameter analyzer has the integrating amplifier pulse height running vertically and the differentiating amplifier

Fig. 4. Two-parameter Topology without Division (Protons are along diagonal, gamma events are to left of diagonal.)

pulse height running horizontally. The intensified dots indicate that the memory content at that location exceeds a prescribed number of counts. In Fig. 5, the vertical axis is again the integrating amplifier pulse height, but the horizontal axis is specific ionization from the divider. For this arrangement one can split the two-parameter spectrum by a vertical line and, apart from a correction for gamma "tail," the right plane can be attributed to proton recoils.

The proton-recoil events in Fig. 5 at low ionization are broadened by amplifier noise. Consider that the "quotient" A/B of two pulses is derived from pulses having initially

*Nuclear Data Model ND 160-F, 150 M, Nuclear Data Incorporated, 100 West Golf Road, Palatine, Illinois.

Fig. 5. Two-parameter Topology of Ionization Versus Specific Ionization

noise and space-charge saturation. Any improvement in the signal-to-noise ratio through preamplifier design will increase this range where the full-scale output is kept short of saturation-induced nonlinearity. The problem is somewhat different for shape discrimination than for conventional charge integration. The preamplifier must possess good high-frequency response because the information as to specific ionization is associated with differences in the rates of rise of pulses, and the time scale for these observations is much shorter than that needed for charge integration of the event alone. A thermionic input device is indicated because of the low input capacity (15-20 pF) and the desirability of a clipping time of approximately 100 μ sec. When the counter is to be used in a small probe, it is preferable that only a single tube be attached to the counter, with additional bulkier components removed by distances of the order of a meter. These requirements indicate that a cascode connection is perhaps the best all-around compromise. A direct triode connection will produce limited gain because of plate-to-grid feedback (Miller effect), and noise from subsequent stages may add in if first-stage gain is too small. Cascode connection also permits a sizable length of cable to be connected from the plate of the input tube to the body of the preamplifier without much sacrifice in rise time. The input tube needs high g_m to reduce shot noise (which is the chief source of noise in the differentiating amplifier) as well as to maintain a short rise time. Since tubes having high g_m collect appreciable grid current, and since grid-current noise is the principal noise source in the integrating amplifier, noise levels can be expected to exceed those ordinarily attainable without shape discrimination.

Formulas⁽¹⁸⁾ may be readily deduced that predict the mean-square noise voltage referred to the grid of the input tube (triode-connected). These are

a noise component σ of full-scale output for both. If the range of pulse heights of interest is over a factor of five, then it may be verified that the quotient has a noise width of 10σ at the low limit. A 2% amplifier noise here gives a 20% noise width to the quotient, and the desirability of limiting noise to the least permissible level is evident.

Noise originates in the pre-amplifier and, as in most detector devices, its minimization is important. The useful range of counter operation lies in a window sandwiched between electronics

of T_1 . Feedback, as shown, is advantageous for several reasons. It can be shown that if the gain of T_3 is large, preamplifier gain is the product of the g_m of T_1 and the cathode resistance (2000 ohms) of T_4 . Gain is entirely independent of input capacitance. With an ordinary cascode amplifier, gain can be changed by as much as 30% by changing the input C. In addition, feedback improves the rise time significantly, relative to an ordinary cascode connection. With lengths of RG-71/U of approximately one meter between T_1 and the main body, the rise time was less than 20 nsec.

Assume an input C of 14 pF, $\tau_C = 10 \mu\text{sec}$, $\tau_R = 20 \text{ nsec}$, and $\Sigma I_g = 10^{-8}$ Amp; the rms noise voltage referred to the grid of T_1 calculated from Eqs. (13) and (14) is 6.5 and 8.0 μV for the integrating and differentiating amplifiers, respectively. These noise inputs are about the same for both, but gain of the differentiating amplifier has to be made about a factor of two larger to account for the fact that differentiation is done before the pulse profile has reached a value comparable to that sampled by the integrating amplifier. So it appears that the fast-rise amplifier following differentiation will set noise levels, and these will be several times greater than one could obtain under ideal conditions without shape discrimination.

The cascode preamplifier does not include the input tube in a feedback loop so that gain will vary with the g_m of T_1 . This is not a serious problem for proportional counters since counting times need not be long and the resolving power is not very great.

In the absence of the diffusion-broadening limitation on electron bunches, gamma discrimination would improve as the differentiation is done more sharply. In practice, however, broadening indicates an amplifier rise time of about 25 nsec; shorter rise times will generally not produce much improvement in discrimination, and for hydrogen gas, differentiation done with an R-C slipping time constant of 50-75 nsec suffices, although much depends upon gas pressure and geometry.

Because of the broad range of ionization (10 to 10^4 pairs), over which measurements are desirable, the electronics are subjected to severe frequent overload when interest is on the lower energies. Overloading is not serious for the differentiating amplifier because of the short input time constant, but care is needed in the design of the integrating amplifier to avoid distortion and still maintain count rates at the desired level of approximately 2000/sec.

VI. COUNTER RESPONSE TO A $1/E$ NEUTRON FLUX

Proton recoil spectra were recorded with a counter (5.08-cm diameter, 3 atm of hydrogen) exposed to the characteristic $1/E$ slowing-down spectrum obtained by placing a plutonium-beryllium α -n source ($10^7/\text{sec}$) in a mixed graphite and D_2O block. Gamma background, which was everywhere comparable to or in excess of proton-recoil rates, removed by shape discrimination. A correction for "tail effect," as shown in Fig. 3, was made and this correction did not exceed 10% of the proton count.

The proton-recoil spectrum, which one would expect to observe in response to a $1/E$ neutron flux, can be calculated from the neutron-proton scattering cross section. This cross section is one of the most accurately analyzed that exist.⁽¹⁹⁾ The value of $\sigma(n-p)$ decreases slowly from ~ 20 barns at low energies to ~ 4 barns near 1 MeV. An analytical expression representing σ to 1-2% for energies below approximately 20 MeV is

$$\sigma = \frac{3\pi}{1.206E + (-18.6 + 0.09415E + 0.0001306E^2)^2} + \frac{\pi}{1.206E + (0.4223 + 0.13E)^2} \quad (15)$$

For energies $E \lesssim 1$ MeV, one may neglect terms in E^2 and get the following simplified version, which is still quite accurate:

$$\sigma(E) = \frac{11.01}{E + 4.041} + \frac{2.387}{E + 0.1355}, \quad E \leq 1 \text{ MeV}. \quad (16)$$

In the limit $E \rightarrow 0$, σ is constant, and Eq. (1) will become [with $\phi(E) \sim 1/E$]

$$D(E) \sim \int_E^\infty \phi \sigma \frac{dE'}{E'} \sim \int_E^\infty \frac{1}{E'} \frac{dE'}{E'} = 1/E. \quad (17)$$

The result is interesting; at low energies, the shapes of the proton-recoil distribution and the neutron spectral shape are both $1/E$. The true variation of the proton-recoil spectrum with E will be obtained by using Eq. (16) for $\sigma(E)$, along with the $\phi(E) = 1/E$ assumption in Eq. (1), and the result may be written as

$$ED(E) \sim 1 - E \left\{ 0.03315 \log \frac{E + 4.041}{E} + 6.39 \log \frac{E + 0.1355}{E} \right\}. \quad (18)$$

Since $W(E)$, the energy loss per ion pair, is not expected to remain fixed, there is no reason to expect a measured proton-recoil distribution to agree

with Eq. (18). If there is a disagreement, however, it may be used directly to determine W . In any case, measurements⁽²⁾ indicate that W is nearly constant down to about 10 keV (~ 300 ion pairs), and one would expect agreement of the observed ionization spectrum with Eq. (18) in excess of this energy.

Figure 7 is the experimentally observed proton-recoil ionization distribution. The ordinate is actually the product of ionization and the ionization spectrum. The predicted shape is also shown where the value 36.0 eV/ion pair has been assumed for the region in excess of 300 pairs.⁽²⁰⁾ Agreement for $I > 350$ pairs is good, as would be expected, but below this a complicated dependence of W on E is apparent.

112-4377

Fig. 7. Experimental Distribution of Ionization from a Counter Exposed to a $1/E$ Neutron Flux

To calculate $W(E)$, one may integrate the equality

$$N(I)dI = D(E)dE, \quad (19)$$

where $N(I)$ and $D(E)$ are distributions in ionization and energy, and obtain

$$\int_I^{E_0/W_0} N(I)dI = \int_E^{E_0} D(E)dE, \quad (20)$$

where E_0 is any energy (≥ 10 keV) beyond which W can be assumed to have its asymptotic value W_0 . An analytic result for the integral over $D(E)$ can be derived using Eq. (18). This can be compared with numerical integrals over $N(I)$ using the observed spectrum, and the value of E for which integrals are equal is the energy corresponding to ionization I . Having evaluated I versus E , one may determine $W(E)$ from the definition

$$W(E) = \frac{dE}{dI} = \frac{N(I)}{D(E)}. \quad (21)$$

The data shown in Fig. 7 are suitable for analysis in this manner down to an ionization of about 40 pairs (~ 1.5 keV), but below this a rather complicated dependence upon energy is apparent. The explanation for the group at 1 keV is probably deuteron recoils, following thermal neutron capture on hydrogen; the group did not appear when a cadmium liner was placed over the detector, and the observed count rates and ionization are consistent with known parameters for the $n(p,d)\gamma$ reaction.

The proton-recoil results are approximately consistent with the following assumption regarding $W(E)$ (see Appendix C).

$$\begin{aligned} W &= W_0, & E &> 290 \text{ eV;} \\ W &= \infty & E &< 290 \text{ eV.} \end{aligned} \quad (22)$$

The effect of statistics on the measurement will be important ultimately, and some additional remarks are included in Appendix B. Whatever the behavior of W below one kilovolt, its behavior above this energy is such as to require a correction to be made to the observed proton-recoil distribution. Fortunately a simple approach to the evaluation of W over the region 1 to 10 keV exists for without it a serious error could be introduced into spectrum measurements in this decade. The $1/E$ observation, which can be done using a conventional "hand" neutron source, not only supplies W , but provides a convenient means of testing counting procedures and techniques generally for systematic errors that would be difficult to discover in the low-energy region by other means.

VII. DIFFERENTIATION AND RESOLUTION

After a differential proton recoil histogram is accumulated and all needed corrections are applied, it is necessary to "differentiate." Differentiation is done by means of straight-line fits to the data over preselected channel intervals. Each datum point has an associated error, and the derivative will have an error that is also determined along with the slope. The slope, from a least-squares fit to $2k + 1$ channels (k is an integer greater than or equal to one) containing counts C_j per channel is

$$M = \frac{\sum_{j=-k}^{+k} jC_j}{\sum_{j=-k}^{+k} j^2}, \quad (23)$$

and the error in M on the basis of statistics in the C_j is

$$\text{error in } M = \frac{\sqrt{\sum_{j=-k}^{+k} j^2 C_j}}{\sum_{j=-k}^{+k} j^2}. \quad (24)$$

The input to the computer consists of the spectrum of counts per channel and an instruction dependent on channel number as to the interval over which slopes are to be taken. The computer then provides a sequence of slopes and errors in the prescribed interval around each channel; the minimum interval is three channels.

The smoothing interval must be chosen, and the choice is relative to the intrinsic width of the counter, which may be taken as the full width at half maximum of the response to monoenergetic protons. Clearly, structure in the neutron spectrum narrower than the intrinsic width is lost and cannot be retrieved regardless of restrictions placed on the smoothing interval. On the other hand, a smoothing interval in excess of the intrinsic width diffuses details that are potentially observable in the spectrum. The requirement that the smoothing interval is much less than the intrinsic width ensures that retrievable information about neutron spectrum is retrieved from the counter, but if the interval is excessively narrow, statistical definition is sacrificed. The situation is the same for any detector. Data more closely spaced than the intrinsic width are not generally justified unless details of the resolution function are so well known that they can be unfolded from the data.

To include the effects of a finite slope-taking interval on the spectrometer, it is necessary to introduce a resultant width, W_R , which is appropriate for the measured neutron spectrum and depends upon both the intrinsic width, W_I , and the slope-taking interval, Δ . One might expect the relationship to be like

$$W_R^2 = W_I^2 + \Delta^2,$$

but this would overemphasize the broadening by slope taking. The reason is that slope taking is done only after the recoil spectrum has already been partially smoothed by effects of the intrinsic width. As a consequence, additional broadening by slope taking is less effective than initial broadening by intrinsic proton response. A correct expression for resultant width may be shown (see Appendix A) to be related to W_I and Δ by

$$W_R = W_I \sqrt{1 + 0.277(\Delta/W_I)^2 + \dots} \quad (25)$$

For example, if the slope-taking interval is chosen equal to the intrinsic width, then the resultant width is 1.13 times greater than the intrinsic width. A counter with a measured full width at half maximum of 10% to monoenergetic protons will have a line width of 11.3% appropriate for a measured neutron spectrum, if differentiation is done over 10% intervals of the proton-recoil spectrum. The relatively small loss of resolution due to differentiation is a consequence of the fact that measurements of neutron spectra really involve two consecutive smoothing operations; the first is done "automatically" by the intrinsic response to protons, and the second by the computer when it differentiates by line fits over finite intervals. Most information is lost in the first smoothing; subsequent smoothing has little additional effect where intervals are chosen to be the same.

These remarks on resolution may appear to belabor the subject unnecessarily. However, statements can be found in the literature that express, although vaguely, an uneasiness regarding the use of 4π -recoil counters as neutron spectrometers. The source of this uneasiness is apparently the feeling that statistical precision and resolution, so essential to an understanding and proper use of any spectroscopic device, can only be expressed where neutron energy and counter response bear a one-to-one correspondence as for measurements using positive-Q reactions or collimated proton recoils. If the response function to monoenergetic neutrons were more complicated than the simple rectangular shape of Eq. (1), the approach to spectroscopy would be different. Unfolding a complex response function from an experimental histogram apparently involves some subtle mathematical problems, and it is not obvious in general how one should proceed to extract spectrum, resolution, and statistics. Because of the simple "unfolding" operation of single differentiation, the 4π -recoil technique provides directly and unambiguously these spectroscopic

essentials from the raw proton-recoil data. It may be argued that differentiation is "wasteful" of statistics, which it is, because the statistical error in the slope of a histogram is much greater than the error in total counts over the slope-taking interval. This is an irrelevant argument, however, since it is also true that high 4π -recoil rates can be achieved in low-level fluxes because of the considerable quantity of hydrogen target material that can be placed in the counter, together with lack of any restriction on the angle of the recoil proton or on the symmetry of the incident flux. The statistical loss in differentiation may be more than compensated by the higher detection rate. Fractional statistical errors in neutron spectra are proportional to the reciprocal square root of the total number of counts in the slope-taking interval, so that changes in the interval affect statistical precision accordingly.

These problems have, of course, been confronted elsewhere; they also apply to analysis of proton-recoil tracks in emulsions, although the concept of intrinsic width is more complicated than for a proportional counter. Also, because of difficulty in acquiring more than $\sim 10^4$ counts by hand-scanning emulsions, one must compromise resolution for statistics; the decision is ordinarily to make the statistical width and the slope-taking interval equal. Emulsions become convenient to use at about

the upper energy limit of gas counter operation, so that the two 4π -recoil techniques complement each other.

112-4340

Fig. 8. Neutron Spectrum Measurement of a $1/E$ Slowing-down Spectrum

There exists for the 4π counter a rather complicated interrelationship between efficiency, statistics, and resolving power that is more obscure than for most detector types. To demonstrate some numerical results, a measurement of a $1/E$ neutron spectrum is shown in Fig. 8 over the interval 10 to 100 keV, an interval not requiring a correction for ion pairs versus energy. The corresponding proton-recoil distribution was accumulated in an hour at an average proton-recoil rate of 250/sec. The average rate of rejected gamma background was 500/sec. Differentiation of the proton-recoil distribution was over a slope-taking interval of 16% of energy. The intrinsic width was about 13%, so that from Eq. (25)

the resultant width was about 16%, and this width has been included on the figure. Statistical accuracy ranged from $\pm 4\%$ to $\pm 8\%$ and, although the ordinate [neutrons/(cm²)(sec)(keV)] was arbitrarily marked, all information exists to determine fluxes absolutely if desired. A $1/E$ line has been drawn for reference.

VIII. CONCLUSION

The proportional counter will respond linearly to a broad range of initial ionization from a single electron up to 10^5 electrons or more. A good part of this range can be used for studies of recoiling protons, although the complication of measurement in the presence of an associated gamma field requires a more detailed experimental analysis than is conventionally done, at least for low ionization levels.

Low-energy limitations are rather firmly set at about 1 keV by the increase in W and by statistics in the ionization and multiplication processes. An upper energy bound will depend upon confinement of the recoil proton to the counter proper, and if spherically symmetrical geometries are used to facilitate numerical treatment of distortion effects,⁽³⁾ one may obtain results at energies of a million volts, or perhaps somewhat higher.

Resolution ranges from 5% or more at high ionization levels to 50% or more at low ionization levels. Where no limit exists on the number of counts collected, neutron spectra will be primarily influenced by the intrinsic response to protons, and to a lesser extent by the differentiation requirement inherent in processing 4π -recoil results to extract a neutron spectrum.

The 4π -recoil technique places no restrictions on the angular symmetry of the incident neutron flux. Adequate count rates can be achieved with modest source intensities; recoil counters are actually much too sensitive for many applications. The full range of response can be utilized only if the rate of occurrence of all events in the detector can be kept below about 2000/sec so that broad-range studies require special consideration for the intensity of the radiation field.

Essential calibration and testing of spectrometer probes, together with a determination of W , may be done in a $1/E$ slowing-down spectrum using α -n sources.

APPENDIX A

Resolving Power Versus Smoothing Interval

The square of the intrinsic width of a detector is here defined as

$$W_I^2 = 8 \log 2 \int_0^\infty (E - E_0)^2 P(E, E_0) dE, \quad (26)$$

where $P(E, E_0)$ is the response function of the detector to proton-recoil events of energy E_0 . For a Gaussian response function, W_I is the full width at half maximum. The measured proton-recoil distribution, when $P(E, E_0)$ is taken into account is given by

$$D(E) \sim \int_0^\infty P(E, E') dE' \int_{E'}^\infty \phi(E'') \sigma(E'') dE'' / E'', \quad (27)$$

rather than Eq. (1). The derivative of $D(E)$ is then

$$\frac{-dD(E)}{dE} \sim \int_0^\infty \phi(E') \sigma(E') P(E, E') dE' / E', \quad (28)$$

which is the average of $\phi \sigma / E$ over the response function. If $P(E, E_0)$ were precisely known, it would be possible in principle to unfold it from the integral and improve the accuracy of the measurement. If $P(E, E_0)$ is only approximately known, as is more customarily the case, one must settle for $\phi \sigma / E$ at the average energy of the response function, along with the width given by Eq. (26). This width estimate is now of limited value, but it is nonetheless essential to a spectrum since it conveys an idea of the kind of information that cannot be observed.

In addition to the intrinsic width of Eq. (26), the differentiation requirement has to be included in estimating a line width appropriate to a measured neutron spectrum. These estimates will be made relative to a Gaussian response function. Assume, for algebraic convenience, that

$$D(E) = \frac{\sqrt{8 \log 2}}{W_I \sqrt{2\pi}} \int_E^\infty \exp \left[-4 \log 2 \frac{(E' - E_0)^2}{W_I^2} \right] dE' \quad (29)$$

represents the proton-recoil distribution from a monoenergetic neutron source of energy E_0 . The true derivative is

$$-D'(E) = \frac{\sqrt{8 \log 2}}{W_I \sqrt{2\pi}} \exp \left[-4 \log 2 \frac{(E - E_0)^2}{W_I^2} \right], \quad (30)$$

and W_I , the intrinsic width, is given by

$$W_I^2 = 8 \log 2 \int_0^\infty -D(E)(E - E_0)^2 dE. \quad (31)$$

$D'(E)$ is also the derivative obtained by line fits to $D(E)$ over intervals much less than W_I .

As the slope-taking interval increases, however, the resulting numerical derivative will depart from the Gaussian shape of Eq. (30). Figure 9 demonstrates the broadening effect of a finite smoothing interval upon some data generated by Eq. (29) and differentiated by line fits, using a computer.

112-4382

Fig. 9. Effect of Finite Slope-taking Interval on Neutron Spectrum Resolution

For slope-taking intervals not in excess of the intrinsic line width, the numerical derivative, $M(E)$, departs but slightly from the true derivative. The resultant width, W_R , is given by

$$W_R^2 = 8 \log 2 \int_0^\infty -M(E)(E - E_0)^2 dE \quad (32)$$

and is the width appropriate to the measured neutron spectrum.

An estimate of W_R can be made where Δ , the slope-taking interval, is comparable with W_I . Expand $D(E')$ about some energy E to get

$$D(E') = \sum_{\ell=0} \frac{D^{(\ell)}(E)}{\ell!} (E' - E)^\ell. \quad (33)$$

Let $M(E)$ be the slope from a linear fit to Eq. (33) over an interval Δ of E' , consisting of $2k + 1$ equally spaced points. If only the first two terms in the expansion (33) are retained, $M(E)$ will be given by

$$M(E) = D'(E) + \frac{D^3(E)}{6} \frac{\Delta^2}{(2k+1)^2} \frac{\sum_0^k j^4}{\sum_0^k j^2} + \dots \quad (34)$$

With the definitions (32) and (31) of W_R and W_I , respectively, the second moment of $E - E_0$ over Eq. (34) gives

$$W_R^2 = W_I^2 + \frac{8 \log 2}{6} \frac{\Delta^2}{(2k+1)^2} \frac{\sum j^4}{\sum j^2} \int_0^\infty -(E - E_0)^2 D^3(E) dE + \dots \quad (35)$$

Equation (35) establishes the dependence of resultant width upon intrinsic width and smoothing interval when Δ is comparable to or less than W_I . If moments of the third derivative of $D(E)$ are introduced from Eq. (30) and k is assumed large (k must always be one or more, and the exact value makes little difference), then the result is

$$W_R^2 = W_I^2 + 0.277 \Delta^2 + \dots \quad (36)$$

Equation (36) was derived from assumptions made for numerical conveniences, but moments, which are all that appear, are not expected to be very sensitive to details of the proton response function.

APPENDIX B

Effect of Statistics on the Measurement of W in a $1/E$ Flux

Equations (19) through (22) involve ionization efficiency W , ionization spectrum $N(I)$, and proton-recoil spectrum $D(E)$. It was tacitly assumed that statistics in the ionization and multiplication processes were absent, but since this is not so, additional comment is required as to the effect statistics may have on the measurability of W .

All information about statistical precision is contained in the function $P(V,E)$ which relates proton energy E to induced pulse height V . A definition of W is required, other than Eq. (22), which holds only for the "zero statistics" case. This generalization may be taken as

$$N(V) = W(V) \cdot \int_0^{\infty} D(E)P(V,E)dE. \quad (37)$$

Since the following remarks are only intended to include very crudely the statistics, W is assumed constant. When $P(V,E)$ is the delta function, $\delta(V - E)$, Eqs. (37) and (22) are identical. Division of a measured pulse-height distribution by W converts the distribution to one in proton energy averaged over P , thereby including the statistical broadening effect.

The integral that occurs in Eq. (37) with $D(E) = 1/E$ (only the low-energy regions are involved) must be examined for some realistic distribution $P(V,E)$. Insofar as it is correct to write

$$\int_0^{\infty} \frac{1}{E} P(V,E)dE = \frac{1}{V}, \quad (38)$$

as would be the case if P were a delta function, one may ignore statistics and use Eqs. (19) through (22) as though statistics were absent. One simple model is to neglect statistics in the creation of initial ionization and assume that the anode avalanche in response to each primary electron has an exponential frequency-versus-number distribution. $P(V,E)$ is, for this model, Poisson's function

$$P(V,E) = \frac{(E)^{V-1} e^{-E}}{\Gamma(V)}, \quad (39)$$

with E and V now in units of ion pairs (the average E is $V + 1$).

If Eq. (39) is introduced into Eq. (38), one finds that

$$\int_0^{\infty} \frac{1}{E} \frac{E^{V-1} e^{-E}}{\Gamma(V)} dE = \frac{1}{V},$$

which is the same result as if P were a delta function and so statistics are ignored. Unfortunately a detailed description of the statistics is not available and the form of $P(V,E)$ can only be guessed at. It is probably safe to assume, however, that statistics will not cause difficulty in the interpretation of data, at least for ionization levels in excess of 10 pairs. Lack of a detailed statistical model will pose a problem in W measurements where the number of pairs approach unity.

APPENDIX C

Cut-off Model for $W(E)$

Figure 10 is a plot of the ionization distribution for a cadmium-covered hydrogen counter exposed to a $1/E$ neutron flux (W_0 , the asymptotic eV per ion pair, can be set equal to unity in what follows). A small correction for the departure from linearity of the product of energy by recoil-proton spectrum $[E D(E)]$ has been made so that $E D(E)$ can be given unit value over the energy range below 10 keV. From Eq. (20) it may be verified that

$$\int_I^{I_0} N(I) dI = \int_E^{E_0} D(E) dE = \log (E_0/E),$$

so that

$$E = E_0 \exp \left(- \int_I^{I_0} IN(I) dI/I \right) \quad (40)$$

and, in addition,

$$W(E) = dE/dI = [E/I] IN(I), \quad (41)$$

where I_0 is the ionization at energy E_0 .

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Fig. 10. Ionization Distribution of Cadmium-filtered Neutrons, Compared with an Energy Cut-off Model for $W(E)$

As energy decreases, $W(E)$ will increase; it will be infinite when protons are below the ionization potential (15 eV) of the gas and will be large for energies considerably above this. About the simplest model one can think of is that W remains at a fixed value for energies in excess of some cut-off value below which it is infinite. This model will permit a rough estimate at least of an effective energy barrier to ionization when the calculated distribution is compared with the measured one. Assume

$$W(E) = 1, \quad E > E_c;$$

$$W(E) = \infty, \quad E < E_c.$$

By definition,

$$I = \int_0^E dE' / W(E') = \int_{E_c}^E dE' = E - E_c, \quad (42)$$

so that, from Eq. (41),

$$IN(I) = \frac{I}{E} = \frac{I}{I + E_c}. \quad (43)$$

For the "cut-off" assumption, $IN(I)$ gradually rolls off as I decreases.

Figure 10 also includes a curve of Eq. (43) for an energy cut-off E_c of 290 eV. Within estimated errors, the "cut-off" assumption is sufficient to explain the observed proton-recoil distribution, although better data will undoubtedly reveal a variation of W with energy more complicated than this.

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