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STIMULATED BRILLOUIN SCATTERING IN PLASMAS

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August 31, 1971

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~~This paper was prepared for submittal to~~
~~The Physics of Fluids.~~

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STIMULATED BRILLOUIN SCATTERING IN PLASMAS*

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ABSTRACT

Hydrodynamic equations are used to develop the theory of the decay of transverse radiation into acoustic waves and backscattered Stokes radiation in a nonmagnetized plasma. The power threshold for this instability can be the lowest of any of the known decay modes. For incident radiation with a frequency bandwidth large compared to the acoustic frequency, the Stokes radiation has half the bandwidth of the incident radiation.

I. INTRODUCTION

An intense electromagnetic wave traversing a plasma can excite a variety of parametric instabilities through the coupling of electron plasma waves, ion acoustic waves, and transverse electromagnetic waves. The purpose of this paper is to develop the theory of stimulated Brillouin scattering in a plasma, i.e., the unstable decay of a transverse wave with frequency and wave vector $(\omega_0, \underline{k}_0)$ into an ion-acoustic wave $(\omega, \underline{k})$ and a scattered transverse wave $(\omega_0 - \omega, \underline{k}_0 - \underline{k})$. This instability, well-known in liquids and solids, has received little analysis in plasmas.

Volkov¹ is apparently the first to have studied Brillouin and Raman scattering in a plasma. He described the coupled transverse and ion-acoustic waves as an electroacoustic oscillation and showed that the system was unstable. However, he found an infinity in the growth rate when the longitudinal propagation vector of the ion-acoustic wave approached twice the incident radiation vector $\underline{k}_0$. Gurovich and Karpman² derived a more accurate dispersion relation but considered only the limit $\underline{k} \ll \underline{k}_0$. Shen and Bloembergen³ and Comisar⁴ have examined Raman scattering in plasmas but also restricted their attention to the case $\underline{k} \ll \underline{k}_0$. Their restriction was necessary because they considered scattering from low-density plasmas where the Debye wave number k_D was comparable to the incident radiation wave number $|\underline{k}_0|$, so that the plasma waves were heavily Landau-damped in the case of large-angle scattering. However, as noted by Kidder⁵ and Goldman,⁶ for plasma densities high enough that $\underline{k}_D \gg |\underline{k}_0|$, large-angle scattering can occur, and the maximum growth rate for stimulated Raman scattering is obtained for backward-scattering, i.e., $\underline{k} \sim 2\underline{k}_0$. The requirement that k_D be much greater than $|\underline{k}_0|$ leads to the inequality.

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$$\frac{\omega_{pe}}{(\omega_o^2 - \omega_{pe}^2)^{1/2}} \gg \frac{v_{th}}{c} \quad (1)$$

easily satisfied in the underdense region of an expanding plasma created by laser irradiation of a solid target. The authors have recently learned that Palmer⁷ has also considered the case of stimulated Brillouin scattering in plasmas.

In Sec. II, the general dispersion relation for Brillouin scattering will be derived from the linearized hydrodynamic and Maxwell's equations. In Sec. III, the dispersion relation is analyzed to determine the thresholds and growth rates of the instability. For plasma densities satisfying inequality (1), the maximum growth rate is obtained for backward-scattering. In Sec. IV, we generalize our dispersion relation to include the frequency spread of the incident radiation.

II. DISPERSION RELATION

The hydrodynamic two-fluid equations describing the plasma are the continuity equation,

$$\frac{\partial N_\sigma}{\partial t} + \nabla \cdot (N_\sigma \underline{V}_\sigma) = 0, \quad (2)$$

and the momentum conservation equation,

$$\frac{\partial \underline{V}_\sigma}{\partial t} + \underline{V}_\sigma \cdot \nabla \underline{V}_\sigma = \frac{q_\sigma}{m_\sigma} \left(-\nabla \phi - \frac{1}{c} \frac{\partial \underline{A}}{\partial t} \right) + \frac{q_\sigma}{m_\sigma c} \underline{V}_\sigma \times (\nabla \times \underline{A}) - \frac{\gamma_\sigma \kappa T_\sigma}{m_\sigma N_\sigma} \nabla N_\sigma, \quad (3)$$

where σ refers to the electron or ion species and $\underline{A}$ and ϕ are the vector and scalar potentials.

We now make the substitutions

$$\underline{V}_\sigma = \frac{q_\sigma}{m_\sigma c} (\underline{A}_L + \underline{A}_1) + \underline{U}_\sigma \quad (4)$$

and

$$N_\sigma = N_O + n_\sigma, \quad (5)$$

where $\underline{A}_L$ is the vector potential of the incident radiation field and N_O is the unperturbed density.

Linearizing Eqs. (2) and (3) with respect to the small perturbations $\underline{A}_1$, ϕ , $\underline{U}_\sigma$, and n_σ , we obtain

$$\frac{\partial n_\sigma}{\partial t} + N_O (\underline{\nabla} \cdot \underline{U}_\sigma) = 0 \quad (6)$$

and

$$\frac{\partial \underline{U}_\sigma}{\partial t} + \frac{q_\sigma}{m_\sigma} \underline{\nabla} \phi + \frac{\gamma_\sigma \kappa T_\sigma}{m_\sigma N_O} \underline{\nabla} n_\sigma = - \left(\frac{q_\sigma}{m_\sigma c} \right)^2 \underline{\nabla} (\underline{A}_L \cdot \underline{A}_1), \quad (7)$$

where we have used the transverse gage, $\underline{\nabla} \cdot \underline{A} = 0$, made the simplifying assumptions,

$$\underline{U}_\sigma \cdot \underline{A}_L = 0 \quad \text{and} \quad \underline{\nabla} \times \underline{U}_\sigma = 0, \quad (8)$$

and used the vector identity,

$$\underline{A} \times (\underline{\nabla} \times \underline{A}) + (\underline{A} \cdot \underline{\nabla}) \underline{A} = \frac{1}{2} \underline{\nabla} (\underline{A} \cdot \underline{A}).$$

The Maxwell's equations of interest

$$\nabla^2 \phi = -4\pi \sum_{\sigma} q_{\sigma} N_{\sigma} \quad (9)$$

and

$$-\nabla^2 \underline{A} + \frac{1}{c} \frac{\partial^2 \underline{A}}{\partial t^2} = \frac{4\pi}{c} \sum_{\sigma} q_{\sigma} N_{\sigma} \underline{V}_{\sigma} - \frac{1}{c} \frac{\partial}{\partial t} \nabla \phi, \quad (10)$$

become, after linearization,

$$\nabla^2 \phi = -4\pi \sum_{\sigma} q_{\sigma} n_{\sigma} \quad (11)$$

and

$$-\nabla^2 \underline{A}_1 + \frac{1}{c} \frac{\partial^2 \underline{A}_1}{\partial t^2} = -\frac{4\pi}{c} \sum_{\sigma} \frac{q_{\sigma}^2}{m_{\sigma} c} (N_o \underline{A}_1 + \underline{A}_L n_{\sigma}). \quad (12)$$

Combining Eqs. (6), (7), (11), and (12) we have,

$$\frac{\partial^2 n_e}{\partial t^2} + (\omega_{pe}^2 - S_e^2 \nabla^2) n_e = \omega_{pe}^2 n_i + N_o \left(\frac{e}{mc} \right)^2 \nabla^2 (\underline{A}_L \cdot \underline{A}_1), \quad (13)$$

$$\frac{\partial^2 n_i}{\partial t^2} + (\omega_{pi}^2 - S_i^2 \nabla^2) n_i = \omega_{pi}^2 n_e + N_o \left(\frac{e}{Mc} \right)^2 \nabla^2 (\underline{A}_L \cdot \underline{A}_1), \quad (14)$$

and

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_{pe}^2 \right) \underline{A}_1 = -\frac{\omega_{pe}^2}{N_o} \underline{A}_L n_e; S_{\sigma}^2 \equiv \frac{\gamma_{\sigma} k T_{\sigma}}{m_{\sigma}}. \quad (15)$$

Considering plane wave perturbations of the form

$$n_{e,i} = n_{e,i}(\omega, \underline{k}) \exp(i \underline{k} \cdot \underline{x} - i \omega t) + \text{c.c.}$$

and

(16)

$$\underline{A}_1 = \underline{A}_1(\omega - \omega_o, \underline{k} - \underline{k}_o) \exp[i(\underline{k} - \underline{k}_o) \cdot \underline{x} - i(\omega - \omega_o)t] + \text{c.c.},$$

where

$$\underline{A}_L = \frac{1}{2} \underline{A}_L(\omega_o, \underline{k}_o) \exp(i \underline{k}_o \cdot \underline{x} - i \omega_o t) + \text{c.c.} \quad (17)$$

and substituting into Eqs. (13), (14), and (15), we obtain the following set of coupled equations:

$$\begin{aligned} & \left[(\omega^2 - \omega_{pe}^2 - S_e^2 k^2) - \frac{\omega_{pi}^2 \omega_{pe}^2}{(\omega^2 - \omega_{pi}^2 - S_i^2 k^2)} \right] n_e(\omega, \underline{k}) \\ & = \frac{1}{2} N_o \left(\frac{e}{mc} \right)^2 k^2 \underline{A}_L(\omega_o, \underline{k}_o) \cdot \underline{A}_1(\omega - \omega_o, \underline{k} - \underline{k}_o) \end{aligned} \quad (18)$$

and

$$\begin{aligned} & \left[(\omega - \omega_L)^2 - \omega_{pe}^2 - c^2 |\underline{k} - \underline{k}_o|^2 \right] \underline{A}_1(\omega - \omega_o, \underline{k} - \underline{k}_o) \\ & = \frac{1}{2} \frac{\omega_{pe}^2}{N_o} \underline{A}_L^*(\omega_o, \underline{k}_o) n_e(\omega, \underline{k}). \end{aligned} \quad (19)$$

We have ignored the last term in Eq. (14) because of the small mass ratio.

The coupled set of Eqs. (18) and (20) are combined to yield a dispersion relation,

$$\left[(\omega^2 - \omega_{pe}^2 - S_e^2 k^2) - \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega^2 - \omega_{pi}^2 - S_i^2 k^2} \right] \left[(\omega - \omega_o)^2 - \omega_{pe}^2 - c^2 |\underline{k} - \underline{k}_o|^2 \right] - \frac{1}{4} \omega_{pe}^2 k^2 |\underline{v}_L|^2, \quad (20)$$

where $\underline{v}_L \equiv \frac{e}{mc} \underline{A}_L$ is the maximum electron quivering velocity in the incident radiation field.

Equation (20) can be further simplified if we restrict our attention to solutions with $\text{Re} \omega \ll \omega_{pi}$, the region of interest for stimulated Brillouin scattering. In this case, we have the approximate dispersion relation,

$$(\omega_a^2 - \omega^2)(\omega - \delta) = \Omega^3, \quad (21)$$

with the definitions

$$\omega_a^2 \equiv \left(S_i^2 + \frac{m}{M} S_e^2 \right) k^2 \equiv S_a^2 k^2, \quad (22)$$

$$\delta \equiv \frac{1}{2\omega_o} \left[\omega_o^2 - \omega_{pe}^2 - c^2 |\underline{k} - \underline{k}_o|^2 \right] = \frac{c^2}{2\omega_o} [2\underline{k} \cdot \underline{k}_o - k^2], \quad (23)$$

and

$$\Omega^3 = \frac{1}{8} \frac{\omega_{pi}^2}{\omega_o} k^2 |\underline{v}_L|^2. \quad (24)$$

The solutions of Eq. (20) with $\text{Re} \omega \approx \omega_{pe}$ yield stimulated Raman scattering (i.e., stimulated scattering of light from high-frequency electron plasma oscillations).

III. ANALYSIS

The dispersion relation, Eq. (21), is a cubic in ω . The largest growth rate occurs when Ω is largest. For fixed incident radiation intensity, Ω is largest when k^2 is largest. For weak incident radiation intensity, $\Omega \ll \omega_a$, we must have $\delta \approx \omega_a$ for instability. Since $\omega_a \ll \omega_o$, we thus obtain the maximum growth rate when $\underline{k} \approx 2\underline{k}_o$. The scattered "Stokes" light is backscattered with wave vector $\underline{k}_o - \underline{k} \approx -\underline{k}_o$. For weak incident radiation we then have

$$\begin{aligned} \omega &\approx \omega_a + i \left(\frac{\Omega^3}{2\omega_a} \right)^{1/2} \quad \text{if } \Omega \ll \omega_a \\ &\approx \omega_a + i \frac{\omega_{pi}}{2} \frac{|\underline{v}_L|}{c} \left(\frac{\omega_o}{\omega_a} \right)^{1/2}. \end{aligned} \tag{25}$$

As the incident intensity increases, $\Omega \gg \omega_a$, the ponderomotive pressure of the incident and Stokes light dominates over the electron pressure. The ions then no longer respond at the ion acoustic frequency. For this strong incident light we still have $\underline{k} \approx 2\underline{k}_o$, but now

$$\begin{aligned} \omega &\approx \Omega \exp(i\pi/3) \quad \text{if } \Omega \gg \omega_a \\ &\approx \left(\omega_o \omega_{pi}^2 \frac{|\underline{v}_L|^2}{2c^2} \right)^{1/3} \exp(i\pi/3). \end{aligned} \tag{26}$$

Thus far we have ignored damping. To accurately calculate the effects of electron-ion collisions and Landau damping we would have to use the Vlasov equation plus some collisional term appropriate to our instability. Instead,

to determine roughly a threshold for the Brillouin instability, we insert phenomenological damping coefficients $\nu_{\sigma-V}$ in Eq. (7).⁸ We obtain

$$|v_L^2| = \frac{\gamma_e \kappa T_e + \gamma_i \kappa T_i}{m_e} \left(\frac{\nu_a}{\omega_a} \right) \left(\frac{\nu_{ei}}{\omega_o} \right). \quad (27)$$

The phenomenological damping coefficient ν_a ⁸ is the effective damping rate on the ion acoustic waves due to Landau damping and ion-electron collisions; the damping rate ν_{ei} is the electron-ion collision frequency and $\nu_{ei} \omega_{pe}^2 / \omega_o^2$ is the damping rate of the Stokes wave.

Comparison of the threshold conditions shows that the threshold for stimulated Brillouin scattering is lower than that for the well known electron plasma wave-ion acoustic wave decay instability. The threshold formula is identical to the latter with the substitution of ν_{ei} for the electron plasma wave damping rate. Because the Brillouin scattering can occur in the underdense region whereas the above longitudinal decay mode can only occur near the critical density for reflection, the Brillouin scattering threshold is lower by the ratio $\omega_{pe}^2 / \omega_o^2$.

For small $k\lambda_{De}$, the Landau damping ratio ν_a / ω_a is of order $(m_e / m_i)^{1/2}$. For $k\lambda_{De}$ near one, ν_a / ω_a is of order one. For $k\lambda_{De}$ large compared to one, ν_a / ω_a is very much larger than one. We can therefore obtain a rough density threshold for the Brillouin scattering by setting $k_o \lambda_{De} \leq 1$. This can be written as $1 > N/N_c \geq v_e / c$, with N the local plasma density, v_e the electron thermal velocity, and $4\pi N_c e / m_e \equiv \omega_o^2$.

For weak incident intensities on a weak density gradient, the first two instabilities to occur are the Brillouin mode and the above longitudinal

mode. The Brillouin mode has a smaller growth rate but a larger spatial domain. The mode that dominates the radiation-plasma coupling is unknown at present. As the incident radiation intensity increases, there are many modes of interaction, including the Raman and Brillouin Stokes backscattering. The problem of radiation-plasma coupling is then even more obscure.

IV. FREQUENCY SPREAD

There are several applications for the Brillouin scattering: radio waves on the ionosphere, microwaves on a plasma, laser radiation on a high density plasma. The authors are particularly interested in the last interaction. A high power neodymium glass laser with wavelength 1.06μ focused on a solid can produce electron temperatures of order 1 keV. Then, for an aluminum target using Brillouin scattering, we obtain $\omega_a/\omega_o \sim 4 \times 10^{-4}$. But nanosecond-type neodymium lasers generally have a frequency width $\Delta\omega/\omega_o$ in the ballpark of 5×10^{-3} . Clearly the single-frequency theory used in the earlier sections has to be modified.

We start with Eqs. (13), (14), and (15). If we restrict ourselves to low-frequency density fluctuations, then we obtain

$$\frac{\partial^2 n_e(\underline{k}, t)}{\partial t^2} + k^2 S_a^2 n_e(\underline{k}, t) = - \frac{N_o e^2 k^2}{mMc^2} \sum_{\underline{k}'} \underline{A}_L(\underline{k} - \underline{k}', t) \cdot \underline{A}_1(\underline{k}', t) \quad (28)$$

and

$$\frac{\partial^2 \underline{A}_1(\underline{k}, t)}{\partial t^2} + (k^2 c^2 + \omega_{pe}^2) \underline{A}_1(\underline{k}, t) = - \frac{\omega_{pe}^2}{N_o} \sum_{\underline{k}'} \underline{A}_L(\underline{k} - \underline{k}', t) n_e(\underline{k}', t). \quad (29)$$

We solve Eq. (28) in terms of a Green's function and substitute into Eq. (29)

$$\frac{\partial^2 \underline{A}_1(\underline{k}, t)}{\partial t^2} + (k^2 c^2 + \omega_{pe}^2) \underline{A}_1(\underline{k}, t) = \frac{e^2 \omega_{pe}^2}{mMc^2} \sum_{\underline{k}'} \sum_{\underline{k}''} \underline{A}_L(\underline{k} - \underline{k}', t) k'^2 \cdot \int_{-\infty}^t dt' \underline{A}_L(\underline{k}' - \underline{k}'', t') \cdot \underline{A}_1(\underline{k}'', t') \frac{\sin k'S_a(t - t')}{k'S_a} \quad (30)$$

We now restrict our attention to an incident wave homogeneous in space and time with a spectrum that is broad compared to the acoustic frequency ω_a , and a correlation time that is short compared to ω_a^{-1} . If this noise spectrum has phases with Gaussian statistics, we can restrict the sum on $\underline{k}''$ to the single value $\underline{k}$, to lowest order in $\underline{A}_L$

$$\frac{\partial^2 \underline{A}_1(\underline{k}, t)}{\partial t^2} + (k^2 c^2 + \omega_{pe}^2) \underline{A}_1(\underline{k}, t) = \frac{e^2 \omega_{pe}^2}{mMc^2} \sum_{\underline{k}'} \int_{-\infty}^t dt' \underline{A}_1(\underline{k}, t') k'^2 \cdot |\underline{A}_L(\underline{k} - \underline{k}')|^2 \exp [+i\omega_L(\underline{k} - \underline{k}')(t' - t)] \frac{\sin k'S_a(t - t')}{k'S_a} \quad (31)$$

Solutions to this equation exist with $\underline{A}_1(\underline{k}, t)$ varying as $\exp(-i\omega t)$. We then obtain the dispersion relation

$$-\omega^2 + k^2 c^2 + \omega_{pe}^2 = \frac{e^2 \omega_{pe}^2}{mMc^2} \sum_{\underline{k}'} \frac{k'^2 |\underline{A}_L(\underline{k} - \underline{k}')|^2}{-[\omega - \omega_L(\underline{k} - \underline{k}')]^2 + k'^2 S_a^2} \quad (32)$$

Now, let the spatial box size go to infinity, and change variables from wave number to frequency, assuming $\underline{A}_L$ is one-dimensional.

$$-\omega^2 + k^2 c^2 + \omega_{pe}^2 = \frac{e^2 \omega_{pe}^2}{m M c^2} \int d\omega' \frac{|\underline{k}' + \underline{k}|^2 |\underline{A}_L(\omega')|^2}{-(\omega - \omega')^2 + |\underline{k}' + \underline{k}|^2 S_a^2} \quad (33)$$

$$k'^2 c^2 = \omega'^2 - \omega_{pe}^2.$$

This is the dispersion relation for Stokes radiation with a noisy incident spectrum. Assuming an unstable solution of Eq. (33) with $\omega = \omega_R + i\gamma$, we obtain, for the imaginary part of the equation,

$$-2\omega_R = \frac{e^2 \omega_{pe}^2}{m M c^2} \int d\omega' \frac{|\underline{k}' + \underline{k}|^2 (\omega_R - \omega') |\underline{A}_L(\omega')|^2}{\left[-(\omega_R - \omega')^2 + \gamma^2 + |\underline{k}' + \underline{k}|^2 S_a^2 \right]^2 + 4\gamma^2 (\omega_R - \omega')^2}. \quad (34)$$

The denominator is even in the variable $(\omega_R - \omega')$ and has a resonant width equal to the acoustic frequency. If $|\underline{A}_L(\omega')|^2$ has a much broader spectrum, we can Taylor expand inside this integral. Then, if $\omega_R > 0$, we can conclude that backscattered Stokes light will only occur at frequencies ω_R such that

$$\frac{\partial}{\partial \omega_R} |\underline{A}_L(\omega_R)|^2 > 0. \quad (35)$$

This result is simple to understand. Conservation of energy requires that if an incident photon with frequency ω_o decays into an acoustic wave with frequency ω_a and another photon with frequency ω_R then $\omega_o = \omega_a + \omega_R$ and $\omega_R < \omega_o$. If one attempted to excite an instability with $\omega_R > \omega_o$, Eq. (21) gives damping not growth. With a broad incident spectrum laser frequency, components with $\omega_o > \omega_R$ drive the instability while components with $\omega_o < \omega_R$ tend to damp it. Thus, instability requires a positive slope, Eq. (39).

All of the above analysis has been in a frame of reference at rest with respect to the plasma. If one hits a solid target with a high-power laser

the plasma expands toward the laser source. This introduces two Doppler shifts: first, when going into the plasma frame, and second, when returning to the laboratory frame. Thus, an experimentalist would see the reflected light upshifted by approximately $2 \omega_o v_p / c$, where v_p is the plasma velocity and ω_o is the average laser frequency. If the plasma is expanding at the acoustic velocity, then, the upshifted frequency is identically the Brillouin acoustic frequency. Also, if v_p is the acoustic velocity then the acoustic wave has zero velocity in the laboratory frame. We then have an absolute instability, not a convective instability. That is why we have described stimulated Brillouin scattering in terms of a growth rate, not a spatial gain coefficient.

FOOTNOTES

*This work was performed under the auspices of the U.S. Atomic Energy Commission.

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