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MEASUREMENT OF ELECTRON g FACTOR

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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ABSTRACT

If polarized electrons trapped in a magnetic bottle can be produced by spin exchange collisions, it may be possible to use such electrons to measure the g factor of the electron to very high accuracy. This would be done by finding the radio frequencies which depolarize the electrons. The nature of the radio frequency fields required is analyzed and some problems associated with the experiment are examined.

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MEASUREMENT OF ELECTRON g FACTOR

If polarized electrons trapped in a magnetic bottle can be produced by spin exchange collisions as described in CEAL-1016 and CEAL-TM-150, it may be possible to use such electrons to measure the g factor of the electron to very high accuracy. This would be done by applying an electromagnetic field, at the g-2 frequency of the electrons in the magnetic bottle field to depolarize the electrons.

If detection of polarization is done by accelerating the electrons to a high voltage and using Mott scattering, a small number of electrons is necessary, and they may be trapped in a very small volume of magnetic field.

The type of electromagnetic field necessary to depolarize the electrons is a uniform electric field parallel to the magnetic trapping field. The principle depolarizing effect is due to the magnetic field $H_1 = v/c E_1$ seen by the electrons due to their orbital motion. This produces a rate of rotation of the spin axis of the electron

$$\frac{d\theta}{dt} = \left[\frac{g-1}{2} \right] \frac{veE_1}{mc^2} \quad (\text{including Thomas precession})$$

After one half cycle of the g-2 frequency, the relation between the direction of the rotation of the spin axis and the orbital position will have reversed, but then the g-2 electromagnetic field will also have reversed so as to continue the depolarization.

The angular cyclotron frequency is $\omega_0 = eH_0/mc$. The applied electric field is E_1 at the $g-2$ frequency $\omega_1 = [(g-2)/2][eH_0/mc]$. The maximum rate of rotation of the spin due to E_1 is

$$\frac{d\theta}{dt} = \left(\frac{g-1}{2} \right) \frac{veE_1}{mc^2}$$

Since E_1 is a sinusoidally varying field, the average initial rate of rotation is one half of this

$$\frac{\overline{d\theta}}{dt} = \frac{1}{4} \frac{veE_1}{mc^2} = \frac{1}{4} \omega_0 \frac{R_0 e E_1}{mc^2} \quad g \approx 2$$

In quantum mechanical terms the initial increase in the other spin state is given by

$$|a_2|^2 = \frac{1}{64} (\omega_0 t)^2 \left(\frac{eE_1 R_0}{mc^2} \right)^2$$

A difficulty in this experiment is that the trapping electric field in the direction of the magnetic field has an associated radial variation of radial field given by $\partial E_r / \partial r = \frac{1}{2} \partial E_z / \partial z$. This produces a change in the orbital frequency but no change in the rate of rotation of the spin axis, so that the $g-2$ frequency is very sensitive to the trapping electric field. The fractional change in the orbital frequency is given by

$$\frac{\delta\omega}{\omega_0} = \frac{\lambda_0^2 e}{8\pi^2 (mc^2)} \left(\frac{\partial E_z}{\partial z} \right)$$

λ_0 is the wavelength of electromagnetic radiation of the orbital frequency. It is seen that the change in frequency is

independent of electron energy so it can be corrected for, if $\partial E_z / \partial z$ is sufficiently well known, and that the change in frequency can be greatly reduced by making the magnetic field very high and hence λ_0 small. For an estimate of the effect we assume a field of 50 kG produced by a superconductor, and that we wish to trap electrons with longitudinal velocities corresponding to 0.5 eV energy in a total length of 2 mm. If the electrodes are located 1 cm from the interaction region the potential required will be about 50 volts and $\partial E_z / \partial z$ is 100 v/cm². $\delta\omega/\omega_0$ is then calculated to be about 10^{-7} . If it is desired to measure the anomalous part of the magnetic moment to an accuracy of one part in 10^6 it is necessary to know $\delta\omega/\omega_0$ to about one part in 10^9 . This requires that the potential be known to an accuracy of about 0.5 volt, and the shape of the elements to an accuracy of a few mils.

The space charge of the trapped electrons will produce a $\partial E_r / \partial r$ which is non-uniform and unknown. To estimate this effect we assume $\partial E_r / \partial r = -\frac{1}{2} \partial E_z / \partial z = \rho / 4\epsilon_0$. Then for $\partial E_z / \partial z$ of 1 v/cm² = 10^4 v/m², $\rho = 2 \times 10^{-7}$ coulombs/m³ = 1.5×10^3 electrons/mm³, which gives a limit of 1.5×10^4 electrons in a volume of about 10 mm³.

To estimate the radio frequency field required to produce significant depolarization we set $\theta = 1$. Then

$$E_1 = \frac{4 mc^2}{vet}$$

If we assume a magnetic field of 50 kG, such that $f_1 = 1.5 \times 10^8 \text{ sec}^{-1}$, an orbital kinetic energy $T_0 = 0.5 \text{ eV}$, then to obtain an accuracy of about one part in a million in $(g-2)/g$, we require a τ of about $6 \times 10^{-3} \text{ sec}$. Then the electric field is calculated to be 15 v/cm.

A possible difficulty is the longitudinal motion produced by the radio frequency field E_1 . This is given by

$$X = \pm \frac{E_1 e}{m \omega_1^2} = \pm 3.0 \times 10^{-2} \text{ cm}$$

This is quite small and would allow the electrons to be trapped in a length of 2 millimeters. The exact orbital energy of the electrons is not important as the $g-2$ frequency is independent of the electron orbital energy.

To obtain the value of $(g-2)/g$, the g frequency would also be measured. This would be done by applying a radio frequency field at the g frequency, with the magnetic component of the radio frequency field normal to the magnetic trapping field, and observing the exact frequency at which depolarization occurs. The rate of rotation of the spin axis due to the applied radio frequency field is given by

$$\frac{d\theta}{dt} = \frac{1}{2} \frac{eH_2}{mc} = \frac{1}{2} \frac{eE_2}{cm}$$

We make the same assumptions used in estimating the field required at the $g-2$ frequency. $B_0 = 50 \text{ kG}$, $\theta = 1$, $f_2 \approx 1.5 \times 10^{11} \text{ sec}^{-1}$, $\tau \approx 6 \times 10^{-6} \text{ sec}$.

$$E_2 = \frac{2mc}{ct} \approx 5.0 \text{ v/cm.}$$

The very small fields required for depolarization at the g-2 and g frequencies show that the fields could be obtained without a resonant cavity about the trapping region. A field of 5 v/cm represents a power flow of only 3.3×10^{-2} watts/cm². Since it is possible to focus electromagnetic radiation to a size about equal to its wavelength, the lower limit on the power required at the g wavelength of 2 mm is 1.3×10^{-3} watts. It appears possible to obtain an amount of power large compared to this from the little storage ring (CEAL-1032), in a single line at 2 mm wavelength, since all the electrons in the ring can certainly be made to radiate coherently at this wavelength.

Since the g frequency varies inversely with the total energy of the electron, in order to obtain the accuracy of one part per million it would be necessary to keep the kinetic energy of the electrons to 0.5 eV or less. The kinetic energy due to orbital motion can be radiation damped to small values as the radiation damping time is about 0.1 seconds. It is possible to ensure that all electrons have longitudinal velocity less than that corresponding to 0.5 eV, by having a device which eliminates all electrons outside the 2 mm region.

In order to obtain the accuracy of one part per million, the magnetic field in the region of the trapped electrons must be uniform to about this accuracy. However the variations in frequency to first order variations in the magnetic field average

to zero due to the motion of the trapped electrons. It should not be very difficult to obtain second order variations of less than one part per million if the electrons are trapped in a region of 2 millimeters, since the size of the superconducting solenoid will be very much larger.

Since the trapping time is long, and the volume occupied by the trapped electrons is very small it will not be necessary to trap the polarized hydrogen atoms in a cavity with teflon walls as for the case of making polarized electrons in quantity. To obtain the polarized electrons it will merely be necessary to direct a beam of polarized hydrogen atoms through the volume occupied by the trapped electrons.

The method of doing the experiment would be to obtain electrons from a thermionic cathode and to trap them in 2 mm volume of the magnetic field by pulsing electrodes. A beam of hydrogen atoms with polarized electrons from a source as used in the hydrogen maser will then be directed into the region of the trapped electrons and will produce polarized electrons trapped in the magnetic field, by spin exchange collisions. This will require less than one second.

To measure the $g-2$ frequency, the proper frequency in the region of 150 mc will be applied to electrodes located near the volume of trapped, polarized electrons, until the proper frequency is found which somewhat depolarizes the electrons. The polarization will be measured by Mott scattering after the

electrons have been extracted from the magnetic field. This will be done by accelerating the electrons to high voltage and directing them through small holes in an iron plate which terminates the magnetic field, as described in CEAL-1016.

To measure the g frequency, the electrons will be allowed to remain in the magnetic bottle a fraction of a second after they have been polarized, so that radiation loss will reduce the kinetic energy to a fraction of an electron volt. Then the g frequency of about 1.5×10^{11} cycles/sec will be applied and the exact frequency determined which somewhat depolarizes the electrons. The polarization will be measured by Mott scattering, in the same manner as for the $g-2$ frequency.

From the ratio between the $g-2$ frequency and the g frequency, the anomalous part of the magnetic moment of the electron is obtained. It should be possible to measure this to accuracy of one part in 10^6 , thereby measuring the g value of the electron to one part in 10^9 .

APPENDIX

Quantum derivation of depolarization due to $(g-2)/2$ field

Quantum states are a_m and a_k with $|a_m| = 1$, $|a_k| = 0$ at $t = 0$

Sinusoidal perturbing field E_1 at $\omega_1 = (E_k - E_m)/\hbar$

$$|a_k(t)| = \frac{H'_{kmt}}{2\hbar} \text{ for } |a_k(t)| \ll 1$$

$$H' = \left(\frac{g-1}{2} \right) \frac{e\hbar}{mc} \frac{\underline{\sigma} \cdot \underline{v} \times \underline{E}_1}{c} \approx \frac{1}{2} \frac{e\hbar}{mc} \frac{\underline{\sigma} \cdot \underline{v} \times \underline{E}_1}{c}$$

$$H' = \frac{1}{2} \cdot \frac{e\hbar}{mc} \cdot E_1 \cdot \frac{\hbar}{mc} \left[- \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \frac{\partial}{\partial x} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \frac{\partial}{\partial y} \right]$$

Following Landau and Lifshitz, take wavefunctions

$$\psi_m = N_m e^{\frac{i}{\hbar}(p_x x + p_z z)} e^{-eH_0(y-y_0)^2/2c\hbar} H_n \left[\left(\frac{eH_0}{c\hbar} \right)^{1/2} (y-y_0) \right] \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$\psi_k = N_k e^{\frac{i}{\hbar}(p_x x + p_z z)} e^{-eH_0(y-y_0)^2/2c\hbar} H_{n+1} \left[\left(\frac{eH_0}{c\hbar} \right)^{1/2} (y-y_0) \right] \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

N_m , N_k are normalizing factors.

With energies

$$E_m = \left(n + \frac{1}{2} \right) \frac{e\hbar H_0}{mc} + p_z^2/2m + \frac{g}{2} \frac{e\hbar H_0}{mc}$$

$$E_k = \left(n + \frac{3}{2} \right) \frac{e\hbar H_0}{mc} + p_z^2/2m - \frac{g}{2} \frac{e\hbar H_0}{mc}$$

$$E_k - E_m = - \left(\frac{g-2}{2} \right) \frac{e\hbar H_0}{mc}$$

Neglect $e^{\frac{i}{\hbar}(p_x x + p_z z)}$ and consider only y coordinate

$$H'_{km} = H'_{mk} = \int_{-\infty}^{\infty} \bar{\psi}_k H' \psi_m dy$$

$$H'_{km} = \frac{1}{2} \cdot \frac{e\hbar}{mc} \cdot E_1 \cdot \frac{\hbar}{mc} \cdot \frac{cH_0}{c\hbar} \int_{-\infty}^{\infty} \bar{\psi}_k y \psi_m dy$$

$$\int_{-\infty}^{\infty} \bar{\psi}_k y \psi_m dy = \left(\frac{c\hbar}{eH_0} \right)^{\frac{1}{2}} \left(\frac{n}{2} \right)^{\frac{1}{2}} \quad (\text{From Shiff})$$

$$H'_{km} = \frac{1}{2} \frac{e\hbar}{mc} E_1 \frac{\hbar}{mc} \left(\frac{eH_0}{c\hbar} \right)^{\frac{1}{2}} \left(\frac{n}{2} \right)^{\frac{1}{2}}$$

$$n = \frac{T}{\hbar\omega_0} = \frac{1}{2} \frac{R_0^2 \omega_0 m}{\hbar}$$

$$\begin{aligned} |a_k(t)|^2 &= \frac{t^2}{4\hbar^2} \cdot \frac{1}{4} \left(\frac{e\hbar}{mc} \right)^2 \cdot E_1^2 \cdot \left(\frac{\hbar}{mc} \right)^2 \cdot \frac{eH_0}{c\hbar} \cdot \frac{1}{4} \frac{R_0^2 \omega_0 m}{\hbar} \\ &= \frac{1}{64} (\omega_0 t)^2 \left(\frac{eE_1 R_0}{mc^2} \right)^2 \end{aligned}$$

In agreement with classical calculation.

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