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ELECTRON TRANSMISSION THROUGH A FULLY IONIZED GAS

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August 20, 1973

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ELECTRON TRANSMISSION THROUGH A FULLY IONIZED GAS^{*}

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INTRODUCTION

The mutual interaction of charged particles obeying an inverse-square scattering law is fundamental to the description of the plasma state. The interaction has been set down as a Fokker-Planck equation by Rosenbluth, MacDonald and Judd (RMJ).¹ Numerical methods have been applied to this equation for calculating the steady-state condition of a proposed mirror fusion reactor,² where the plasma build-up and compensation for end losses is provided by intense neutral beam injection. The early calculation of plasma conductivity by Spitzer, et al³ made use of a solution of this equation.

The present note sets down the form, together with approximate solutions, of the RMJ equation which applies approximately to the passage

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of fast electrons through a plasma. In a practical situation one can imagine a feeble planar beam of mono-energetic electrons (the "test particles") shot into a plasma normal to a planar plasma boundary, and analyzed in velocity beyond a second parallel boundary. Since the electrons would be accelerated through a sheath prior to entering the plasma (and would be decelerated by presumably the same energy upon exit), one may simplify the analysis to a considerable extent by taking the test particle speed to be several times greater than the thermal speed of the plasma electrons (and thus of the plasma ions). Of principle interest here is the "transmission coefficient", defined as that fraction of the incident beam distribution function which traverses the plasma and falls within the resolution of the analyzer in the same energy and momentum channel as does the entire test particle beam in the absence of the plasma.

We shall assume that the test-particle beam is feeble enough not to disturb the equilibrium properties of the target plasma. Furthermore, since the test particle speed exceeds the electron thermal speed, it is safe to assume that collective plasma modes will not influence the test particle.

THE GOVERNING EQUATION FOR THE TEST PARTICLES

Our starting point is the RMJ form of the Fokker-Planck collision term (Eq. 31, Ref. 1), specialized for the case of field particles with isotropic distributions.

$$\begin{aligned}
 \Gamma^{-1}(\partial f / \partial t)_c = & -\frac{1}{v^2} \frac{\partial}{\partial v} \left[\left(v^2 \frac{\partial h}{\partial v} + \frac{\partial g}{\partial v} \right) f \right] \\
 & + \frac{1}{2v^2} \frac{\partial^2}{\partial v^2} \left[\left(v^2 \frac{\partial g^2}{\partial v^2} \right) f \right] \\
 & + \frac{1}{2v^3} \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right].
 \end{aligned} \tag{1}$$

Here $f(x, \mu, v)$ is the distribution function for the test particles, assumed to depend only upon the single space coordinate x , and only upon the test particle speed v and angle $\cos^{-1} \mu$ of $\underline{v}$ relative to the x -axis. The constant $\Gamma = 4\pi e^4 / m^2$, and ²

$$\begin{aligned}
 g(v) = 4\pi \sum_{\alpha} \ell n \Lambda_{e\alpha} \left[v \int_0^v dv' v'^2 f_{\alpha}(v') \left(1 + \frac{1}{3} \frac{v'^2}{v^2} \right) \right. \\
 \left. + \int_v^{\infty} dv' v'^3 f_{\alpha}(v') \left(1 + \frac{1}{3} \frac{v^2}{v'^2} \right) \right]
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 h(v) = 4\pi \sum_{\alpha} \left(\frac{m+m_{\alpha}}{m_{\alpha}} \right) \ell n \Lambda_{e\alpha} \left[\frac{1}{v} \int_0^v dv' v'^2 f_{\alpha}(v') \right. \\
 \left. + \int_v^{\infty} dv' v' f_{\alpha}(v') \right].
 \end{aligned} \tag{3}$$

The summation is over plasma species, $\alpha = e, i$; the ions are singly ionized; and²

$$\ln \Lambda_{e\alpha} = \ln \left\{ \left(\frac{m\alpha}{m+m_\alpha} \right) \frac{4\pi\lambda_D}{h} \left(\frac{2\bar{E}}{m} \right)^{1/2} \right\} - \frac{1}{2}$$

where λ_D is the plasma Debye length, h is Planck's constant and $\bar{E}$ is the average energy of the test particles. Variations with $\bar{E}$ of $\ln \Lambda_{e\alpha}$ as $f(v)$ changes will be neglected. We shall evaluate Eqs. (2) and (3) by assuming the plasma ions to be cold and fixed, and the plasma electrons to have a Maxwellian velocity distribution with thermal speed v_e . Since the test particle speed v is much greater than v_e , we find

$$g(v) \simeq Nv \left[\ln \Lambda_{ei} + \left(1 + \frac{1}{3} \frac{v_e^2}{v^2} \right) \ln \Lambda_{ee} \right] \quad (4)$$

$$h(v) \simeq \frac{N}{v} [\ln \Lambda_{ei} + 2 \ln \Lambda_{ee}]. \quad (5)$$

Inserting Eqs. (4) and (5) into Eq. (1) gives

$$\begin{aligned} (N\Gamma)^{-1} (\partial f / \partial t)_c = & \frac{1}{v^2} \frac{\partial}{\partial v} \left[\left(1 - \frac{1}{3} \frac{v_e^2}{v^2} \right) f \right] \ln \Lambda_{ee} \\ & + \frac{1}{3v^2} \frac{\partial^2}{\partial v^2} \left(\frac{v_e^2}{v} f \right) \ln \Lambda_{ee} \\ & + \frac{1}{2v^3} [\ln \Lambda_{ei} + \ln \Lambda_{ee}] \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right]. \quad (6) \end{aligned}$$

We wish to solve the steady-state Boltzmann equation for $f(v, x, \mu)$ using Eq. (6) as the collision term

$$v\mu \frac{\partial f(v, \mu, x)}{\partial x} = \left(\frac{\partial f}{\partial t} \right)_c \quad (7)$$

Eq. (7) will be solved in various limiting forms.

SCATTERING BY FIXED IONS

As a mathematical warm-up, we first calculate $f(v, \mu, x)$ taking into account only ion scattering. This is physically equivalent to electron test particles interacting in a plasma made up only of positive and negative ions.⁴ Eq. (6) simplifies to

$$\lambda_{ei}\mu \frac{\partial f(x, \mu)}{\partial x} = \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right] \quad (8)$$

where $\lambda_{ei} = m^2 v^4 / 4\pi N e^2 \ln \Lambda_{ei}$ is the electron-ion mean-free-path for electrons of speed v . According to Eq. (8), v is constant; the ions only deflect--but do not slow down--the test particles. The numerical value of λ_{ei} is $3.84 \times 10^{12} E^2 / N \ln \Lambda_{ei}$ cm, where E is the test particle energy in eV, and N is the plasma density in cm^{-3} .

Eq. (8) separates as $f(x, \mu) = X(x)M(\mu)$ and

$$\lambda_{ei} \frac{\partial X}{\partial x} + m(m+1)X = 0 \quad (9)$$

$$\frac{1}{\mu} \frac{\partial}{\partial \mu} \left\{ (1-\mu^2) M \right\} + m(m+1)M = 0, \quad (10)$$

where $m(m+1)$ is the separation constant. Eq. (9) has the solution

$$X(x) = \text{const.} \exp[-m(m+1)x/\lambda_{ef}]. \quad (11)$$

Eq. (10) is not a standard form, but would be Legendre's equation were it not for the μ^{-1} multiplying the first term. Since we anticipate that our solution will be of interest mainly in the forward direction where $\mu = 1$, we see that $\mu^{-1} = 1$. Then

$$M(\mu) = \text{const.} P_m(\mu)$$

where $P_m(\mu)$ is the Legendre polynomial of order m . The general solution of Eq. (8) is then approximately

$$f(x, v, \mu) = \sum_{m=0}^{\infty} C_m P_m(\mu) \exp[-m(m+1)x/\lambda_{ef}]. \quad (12)$$

Eq. (12) must satisfy a boundary value at $x = 0$, where we assume

$$f(0, v, \mu) = f_0(v) \delta(\mu-1) \quad (13)$$

where for a mono-energetic test particle beam

$$f_0(v) = \frac{N}{4\pi v_0^2} \delta(v-v_0).$$

Setting $x = 0$ in Eq. (12), multiplying it and Eq. (13) by $P_n(\mu)d\mu$ and integrating from -1 to 1 gives $C_m = (m + 1/2)$ for $m = n$, $C_m = 0$ otherwise thus

$$f(x, v, \mu) = f_0(v) \sum_{m=0}^{\infty} (m + 1/2) P_m(\mu) \exp[-m(m+1)x/\lambda_{ei}]. \quad (14)$$

Eq. (14) has two properties worthy of note. For one thing, the minimum value of $f(x, v, \mu = 1)$ is given by the $m = 0$ term and has the value $f_0(v)/2$. This is the limit of the full expression, for $\mu = 1$, as $x/\lambda_{ei} \rightarrow \infty$; after a great many small deflections the distribution is isotropic, and remains mono-energetic, with half the particles ending up on each boundary. The second point to note is the divergent character of Eq. (14) as $x/\lambda_{ei} \rightarrow 0$. This property was built into the equation when it was fitted to a delta-function in velocity at $x = 0$. The quantity which is measured physically is the integral of $f(v, \mu, x = L)$ over the angular apperture $A(\mu)$ of the detector, where $A(\mu)$ is peaked at $\mu = 1$. The transmission coefficient T is then

$$T = \frac{\sum_{m=0}^{\infty} (m + 1/2) \exp[-m(m+1)L/\lambda_{ei}] \int_{-1}^1 d\mu A(\mu) P_m(\mu)}{\int_{-1}^1 d\mu A(\mu)}. \quad (15)$$

One observes that T is never an exponentially decaying function of L/λ_{ei} . For $L/\lambda_{ei} \gg 1$, where the terms for $m > 1$ are insignificant, it scales as

$$T \sim \frac{1}{2} + \frac{3}{2} \exp(-2L/\lambda_{ei}) + \frac{5}{2} \exp(-6L/\lambda_{ei}) + \dots \quad (16)$$

SCATTERING BY COLD ELECTRONS AND FIXED IONS

We consider the limit $v_e \ll v$ in Eq. (6), and approximate $\ln \Lambda_{ei} = \ln \Lambda_{ee}$, to find for Eq. (7)

$$\mu \lambda(v) \frac{\partial f}{\partial x} = v \frac{\partial f}{\partial v} + \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right]. \quad (17)$$

The equation is made dimensionless by writing it in terms of the variable $u = v/v_0$. Furthermore, we set $\mu = 1$ on the left-hand side, as above

$$\lambda_0 u^4 \frac{\partial f}{\partial x} = u \frac{\partial f}{\partial u} + \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right] \quad (18)$$

where $\lambda_0 = \lambda(v_0)$. The eigenfunctions in μ are recognized as Legendre polynomials, so we write

$$f(x, u, \mu) = \sum_m P_m(\mu) f_m(x, u)$$

and Eq. (18) becomes

$$\lambda_0 \frac{\partial f_m}{\partial x} = u^{-3} \frac{\partial f_m}{\partial u} - m(m+1) u^{-4} f_m. \quad (19)$$

Let us Fourier transform the equation in x .

$$f_{km}(u) = \int_{-\infty}^{\infty} dx e^{ikx} f_m(x, u)$$

$$f_m(x, u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{-ikx} f_{km}(u). \quad (20)$$

We obtain

$$[ik\lambda_0 u^3 + m(m+1)u^{-1}]f_{km}(u) = \frac{\partial f_{km}(u)}{\partial u}$$

with the solution

$$f_{km}(u) = \text{const. } u^{m(m+1)} \exp(ik\lambda_0 u^4/4).$$

The general solution of Eq. (18) is then

$$f(x, u, \mu) = \sum_m P_m(\mu) u^{m(m+1)} \int_{-\infty}^{\infty} dk \exp[-ik(x - \lambda_0 u^4/4)] c_m(k) \quad (21)$$

We match boundary conditions at $x = 0$

$$f_0 \delta(u-1) \delta(\mu-1) = \sum_m P_m(\mu) u^{m(m+1)} \int_{-\infty}^{\infty} dk \exp[ik\lambda_0 u^4/4] c_m(k).$$

Multiply both sides by $P_n(\mu)d\mu$ and integrate μ from -1 to 1,

$$f_0 \left(m + \frac{1}{2}\right) \delta(u-1) = u^{m(m+1)} \int_{-\infty}^{\infty} dk \exp[ik\lambda_0 u^4/4] C_m(k).$$

The functions $C_m(k)$ satisfy a Fourier transform, with transform variable $\lambda_0 u^4/4 \equiv \zeta$. Thus

$$\begin{aligned} C_m(k) &= f_0 \left(m + \frac{1}{2}\right) \lambda_0 \int_{-\infty}^{\infty} d\zeta e^{-i\zeta k} \left(\frac{4\zeta}{\lambda_0}\right)^{\frac{3-m(m+1)}{4}} \delta\left(\zeta - \frac{\lambda_0}{4}\right) \\ &= f_0 \left(m + \frac{1}{2}\right) \lambda_0 \exp(-ik\lambda_0/4). \end{aligned} \quad (22)$$

Yielding, as the solution to Eq. (18)

$$\begin{aligned} f(x, u, \mu) &= \frac{1}{2\pi} f_0 \lambda_0 \sum_m \left(m + \frac{1}{2}\right) P_m(\mu) u^{m(m+1)} \\ &\quad \int_{-\infty}^{\infty} dk \exp\left[-ik\left(x - \lambda_0 \frac{u^4}{4} + \frac{\lambda_0}{4}\right)\right] \\ &= \frac{2}{\pi} f_0 \left[\frac{1}{(1-u^4) + \frac{4x}{\lambda_0}} \right] \sum_m \left(m + \frac{1}{2}\right) P_m(\mu) u^{m(m+1)}. \end{aligned} \quad (23)$$

The transmission coefficient is obtained by integrating the equation over the velocity and angular aperture function of the detector. The x -dependence goes like λ_0/x . For $x \gg \lambda_0$, $u \neq 1$, the sum converges rather rapidly:

$$f(x, u < 1, \mu = 1) \sim \frac{f_0 \lambda_0}{2\pi x} \left[\frac{1}{2} + \frac{3}{2} u^2 + \frac{5}{2} u^6 + \frac{7}{2} u^{12} + \dots \right].$$

For $u = 1$, the distribution shrinks like λ_0/x as the electrons percolate down in velocity, slowing by energy transfer to the cold plasma electrons. The distribution remains discontinuous at $v = v_0^+$, however, since Eq. (17) does not allow any electrons to move to speeds above v_0 .

SCATTERING BY WARM ELECTRONS AND FIXED IONS

To judge the effects of plasma electron thermal motions, we retain the terms in v_e^2/v^2 to find

$$\begin{aligned} \mu \lambda_e u^4 \frac{\partial f}{\partial x} = u \frac{\partial}{\partial u} \left[\left(1 - \frac{2}{3u^2} \right) f + \frac{1}{3u} \frac{\partial f}{\partial u} \right] \\ + \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right], \end{aligned} \quad (24)$$

where $u \equiv v/v_e$ and $\lambda_e = m^2 v_e^4 / 4\pi n e^2 \ln \Lambda_{ei}$. As above, we proceed to solve Eq. (24) by overlooking the factor μ on the left-hand-side, since we are principally interested in f near $\mu = 1$. As in the previous section, we Fourier transform $f(x, u, \mu)$ in x . Thus

$$\begin{aligned} i k \lambda_e u^4 f_k = u \frac{\partial}{\partial u} \left[\left(1 - \frac{2}{3u^2} \right) f_k + \frac{1}{3u} \frac{\partial f_k}{\partial u} \right] \\ = \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f_k}{\partial \mu} \right]. \end{aligned} \quad (25)$$

This equation separates as $f_k = U(u)M(\mu)$, where

$$\begin{aligned} ik\lambda_e u^4 - \frac{u}{U} \frac{\partial}{\partial u} \left[\left(1 - \frac{2}{3u^2} \right) U + \frac{1}{3u} \frac{\partial U}{\partial u} \right] \\ = \frac{1}{M} \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial M}{\partial \mu} \right] \\ = -m(m+1). \end{aligned}$$

As above, $M(\mu) = P_m(\mu)$. The equation satisfied by $U(u)$ is

$$u^2 \frac{\partial^2 U}{\partial u^2} + 3(u^2-1)u \frac{\partial U}{\partial u} - [i3k\lambda_e u^6 + 3m(m+1)u^2 - 4]U = 0. \quad (26)$$

This equation is not a standard form. We solve it approximately by manipulating it into a form resembling Bessel's differential equation:⁵

$$z^2 \frac{\partial^2 Z}{\partial z^2} + \left(\frac{3}{2} z - 1 \right) z \frac{\partial Z}{\partial z} + \left[\frac{9}{4} \alpha^2 z^3 + \frac{9}{16} z^2 - \frac{3}{4} z + 1 - \frac{9}{4} v^2 \right] Z = 0. \quad (27)$$

Eq. (26) can be transformed to resemble Eq. (27) by setting $u^2 = z$. The coefficient of U which turns up is $-i3k\lambda_e/4 z^3 - 3/4 m(m+1)z + 4$. We make the identification $\alpha^2 = -ik\lambda_e/3$, and

$$v^2 = \frac{4}{3} m(m+1)z_0 + \frac{1}{4} z_0^2 - \frac{1}{3} z_0 - \frac{4}{3}$$

where some z 's are set equal to $z_0 = (u_0/u_e)^2$. It is not expected that this would yield serious error in the final result provided u/u_0 is close to unity. The solution to Eq. (27) is

$$z \exp\left(-\frac{3}{4} z\right) J_\nu(\alpha z^{3/2}),$$

so that

$$U(u) = u^2 \exp\left(-\frac{3}{4} u^2\right) J_\nu\left[\left(-\frac{ik\lambda_e}{3}\right)^{1/2} u^3\right].$$

The general solution for Eq. (24) (subject to the approximations stated above) is

$$f(x, u, \mu) = \frac{1}{2\pi} u^2 \exp\left(-\frac{3}{4} u^2\right) \sum_m P_m(\mu) \int_{-\infty}^{\infty} dk e^{-ikx} C_m(k) J_\nu\left[\left(-\frac{ik\lambda_e}{3}\right)^{1/2} u^3\right]. \quad (28)$$

We determine the functions $C_m(k)$ by matching to the boundary value at $x = 0$,

$$f_0 \delta(u-u_0) \delta(\mu-1) = \frac{1}{2\pi} u^2 \exp\left(-\frac{3}{4} u^2\right) \sum_m P_m(\mu) \int_{-\infty}^{\infty} dk C_m(k) J_\nu(\zeta k^{1/2})$$

where $\zeta = (-i \lambda_e/3)^{1/2} u^3$. We change variables $k^{1/2} = y$, multiply both sides by $P_n(\mu)d\mu$ and integrate on μ from -1 to 1. Then

$$\begin{aligned} & \pi\left(m + \frac{1}{2}\right) f_0 \delta(\zeta - \zeta_0) \zeta^{-2/3} \left(-i \frac{\lambda_e}{3}\right)^{1/3} \exp\left[\frac{3}{4} \left(-i \frac{\lambda_e}{3}\right)^{1/3} \zeta^{2/3}\right] \\ &= \int_{-\infty}^{\infty} dt \, t \, C_m(t^2) J_\nu(\zeta t). \end{aligned}$$

It appears that the $C_m(t^2)$ functions satisfy a Hankel transform.⁶ Therefore

$$\begin{aligned} C_m(t^2) &= \pi\left(m + \frac{1}{2}\right) \left(-i \frac{\lambda_e}{3}\right)^{1/3} \int d\zeta \, \zeta \, \zeta^{-2/3} \delta(\zeta - \zeta_0) \\ &\quad \exp\left[\frac{3}{4} \left(-i \frac{\lambda_e}{3}\right)^{1/3} \zeta^{2/3}\right] J_\nu(\zeta t) \\ &= \pi\left(m + \frac{1}{2}\right) \left(-i \frac{\lambda_e}{3}\right)^{1/3} \zeta_0^{1/3} \exp\left[\frac{3}{4} \left(-i \frac{\lambda_e}{3}\right)^{1/3} \zeta_0^{2/3}\right] J_\nu(\zeta_0 t). \end{aligned}$$

or

$$C_m(k) = \pi\left(m + \frac{1}{2}\right) \frac{\lambda_e}{3} u_0^{-2} \exp\left(\frac{3}{4} u_0^2\right) J_\nu\left[\left(-i k \lambda_e\right)^{1/2} u_0^3\right].$$

Therefore the approximate solution to Eq. (24) is

$$f(x, \mu, u) = \frac{1}{2} f_0 \left(\frac{u}{u_0} \right)^2 \exp \left[\frac{3}{4} (u_0^2 - u^2) \right] \frac{i\lambda_e}{3} \sum_m \left(m + \frac{1}{2} \right) P_m(\mu) \cdot \int_{-\infty}^{\infty} dk e^{-ikx} J_\nu \left[\left(-\frac{ik\lambda_e}{3} \right)^{1/2} u_0^3 \right] J_\nu \left[\left(-\frac{ik\lambda_e}{3} \right)^{1/2} u^3 \right] \quad (29)$$

The integral over k follows using the substitution $ik = \omega^2$ to yield:

$$f(x, \mu, u) = \frac{1}{2} f_0 \left(\frac{u}{u_0} \right)^2 \frac{\lambda}{3x} \exp \left\{ \frac{3}{4} (u_0^2 - u^2) - \frac{(u_0^6 - u^6)\lambda_e}{12x} \right\} \cdot \sum_m \left(m + \frac{1}{2} \right) P_m(\mu) I_\nu \left(\frac{u^3 u_0^3 \lambda_e}{6x} \right) \quad (30)$$

where $u < u_0$, and $\nu^2 = 4/3 m(m+1)u_0^2 + 1/4 u_0^4 - 1/3 u_0^2 - 4/3$.

For $\mu = 1$, $u = u_0$, Eq. (30) yields

$$f(x, \mu=1, u_0) = \frac{1}{2} f_0 \frac{\lambda_e}{3x} \sum_m \left(m + \frac{1}{2} \right) I_\nu \left(\frac{u_0^3 u_0^3 \lambda_e}{6x} \right).$$

Taking $\nu = u_0^2/2$, and $x \gg u_0^6 \lambda_e / 6$, one finds

$$f(x, 1, u_0) \sim \left(\frac{\lambda_e}{x} \right)^{u_0^2 + 2/2}.$$

For given detector resolution, Eq. (30) can be used to find the transmission coefficient.

SUMMARY

The Rosenbluth, MacDonald, and Judd form of the Fokker-Planck collision term has been applied to obtain approximate steady-state distribution functions for fast electrons penetrating a plasma. The plasma is represented in three model cases as: (a) made up of fixed positive and negative ions; (b) made up of fixed ions, and of cold electrons, but with electron thermal speed much less than test particle speed. The formalism can also be extended to give analytic forms for the distribution function of ions as test particles, provided the test particle speed is much less than the plasma electron thermal speed, and much greater than the plasma ion thermal speed.

Aside from direct application to experiments on electron transmission through plasma, these solutions could have use in checking numerical solutions in various limiting cases.

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