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RF BEAM DYNAMICS
in
THE CEA STORAGE EXPERIMENT

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August 17, 1967

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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CAMBRIDGE ELECTRON ACCELERATOR

CAMBRIDGE 38, MASSACHUSETTS

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The radiofrequency accelerating system of the Cambridge Electron Accelerator (CEA) has 16 double cavities which are symmetrically distributed around the circumference and are strongly coupled by waveguides.¹ It is a high Q system ($Q = 12,500$) with an infinite number of resonant modes, but the accelerating mode, called the π mode, is separated from the closest undesired mode by approximately 3.5 bandwidths. (With an orbital frequency $f_o = 1.32138$ MHz, and a harmonic number $k = 360$, the π mode resonant frequency f is 475.700 MHz). Because the modes are well separated we can represent the rf system as a single resonant cavity. This equivalent cavity is fed through an isolator by the rf amplifier, and is loaded by the beam of circulating particles. A further characteristic of the rf system is its high shunt impedance, ($R_{po} = 100$ M Ω). High shunt impedance is advantageous for reasons of rf power economy when high beam intensity is not required.

When the beam intensity becomes large (> 1 ma), the beam-induced voltage must be taken into consideration. The following

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set of differential equations relates the beam and the rf system, i.e., synchrotron oscillations and cavity excitation.²

$$\begin{aligned}
 -\frac{dV}{dt} - \alpha V + \alpha V_g \cos \phi_v - \alpha V_B \sin \phi_B &= \\
 = F_1\left(\frac{dV}{dt}, V, \phi_v, \phi_B, t; \alpha, V_g, V_B\right) &= 0
 \end{aligned} \tag{1a}$$

$$\begin{aligned}
 -\frac{d\phi_v}{dt} - \alpha \tan \phi_Y - \alpha \frac{V_g}{V} \sin \phi_v + \alpha \frac{V_B}{V} \cos \phi_B &= \\
 = F_2\left(\frac{d\phi_v}{dt}, V, \phi_v, \phi_B, t; \alpha, V_g, V_B, \phi_Y\right) &= 0
 \end{aligned} \tag{1b}$$

$$\begin{aligned}
 -\frac{d\phi_B}{dt} - \frac{2 f_o \alpha' k \Delta U}{U_o} + \alpha \tan \phi_Y + \alpha \frac{V_g}{V} \sin \phi_v - \alpha \frac{V_B}{V} \cos \phi_B &= \\
 = F_3\left(\frac{d\phi_B}{dt}, \phi_B, \Delta U, V, \phi_v, t; f_o, U_o, \alpha, V_g, V_B, \phi_Y\right) &= 0
 \end{aligned} \tag{1c}$$

$$\begin{aligned}
 -\frac{d\Delta U}{dt} - f_o (V \sin \phi_B - V_R) &= \\
 = F_4\left(\frac{d\Delta U}{dt}, V, \phi_B, t; f_o, V_R\right) &= 0
 \end{aligned} \tag{1d}$$

where:

V = amplitude of the rf voltage across the
equivalent cavity, i.e., voltage/turn;

$$v = V \exp(i\omega t) / \sqrt{2}$$

$$\alpha = \pi f / Q = w / 2Q$$

η = proportionality constant < 1 (for calculation $\eta=1$)

$V_g = I_g R_p$ = amplitude of the voltage produced by the rf amplifier on the cavity when $I_B = 0$ and when the system is tuned to resonance;

$$v_g = V_g \exp[i(\omega t - \phi_v)]/\sqrt{2} = i_g R_p$$

$V_B = \eta \times I_B \times R_p$ = amplitude of beam-induced voltage when the system is tuned to resonance and $v_g = 0$

ϕ_v = phase angle between v and i_g

$\phi_Y = \tan^{-1} (2Q\Delta f/f)$, where Δf is the detuning of the cavity when $I_B = 0$

$(\pi/2 - \phi_B)$ = phase angle between the beam current I_B and the rf voltage V

ΔU = energy variation of the particles from the synchronous energy

$$V_R = V_{\text{exit}} + V_{\text{radiat}} = \tau \frac{dV_o}{dt} + \frac{8.84 \times 10^4 U_o^4}{r_o} [\text{GeV}]$$

V_{exit} = voltage needed for acceleration

V_{radiat} = voltage needed to offset the radiation losses

$$\tau = \frac{1}{f_o}$$

$U_o = U_{\text{max}} (1 - A \cos \Omega t) / (1 + A)$ = the particle energy

$$A = (U_{\text{max}} - U_{\text{inj}}) / (U_{\text{max}} + U_{\text{inj}})$$

U_{max} = maximum particle energy

U_{inj} = injection energy

$$\Omega = 2\pi(60 \text{ sec}^{-1})$$

$$\alpha' = \text{momentum compaction factor} = (\frac{\Delta C}{C}) / (\frac{\Delta p}{p}) = 0.0292$$

The CEA is to be utilized in an electron-positron storage experiment.³ In this experiment, the positron beam (up to 50 ma) will be obtained by multicycle injection. It will be electrostatically lifted to make room for subsequently injected electrons (also 50 ma); then both beams will be accelerated to three GeV and diverted into the bypass (BP) interaction region.

Multicycle injection requires amplitude and frequency modulation of the accelerating voltage. The amplitude is determined by the acceleration requirements and by the radiation losses. We prefer even higher acceleration voltage to prevent high energy beam losses due to quantum fluctuation; in fact we use the full voltage capability of our rf power supply (75 kW average). At low energy (i.e., injection energy) we choose much lower voltage to minimize the horizontal beam size.⁴ We propose to use sinusoidal amplitude modulation:

$$V(t) = (V_{\max} + V_{\text{inj}})(1 - V_{\max} - V_{\text{inj}})\cos\Omega t / ((V_{\max} + V_{\text{inj}}))/2$$

with $V_{\max} = 2.10$ megv/turn and $V_{\text{inj}} = 1.0$ megv/turn.

Frequency modulation is required to compensate for the change in resonant frequency as influenced by increasing beam intensity and by the suggested amplitude modulation. To prevent the excitation of undesired modes, the maximum frequency shift is held within 100 kHz.

Next we tried to estimate how the different parameters of the rf system should be adjusted to provide the proposed energy ($U_{\max} = 3\text{GeV}$) and beam intensity (100 ma). Because we

will inject small bursts of particles ($\sim 100 \mu\text{a}$) and because the decay rate of the rf voltage is small compared with the period of the magnet cycling, we are justified in using the steady state solution of equations (1a) and (1b)

$$\left(\frac{dV}{dt} = \frac{d\phi_v}{dt} = 0 \right)$$

The average power needed for excitation of the rf system for beam acceleration equals:

$$P_{\text{acc}} = \text{Re}(v i_g) = \frac{\eta V I_B \sin \phi_B}{2} + \frac{V^2}{2R_p} = P_{\text{fwd}} \left(1 - \frac{P_{\text{rev}}}{P_{\text{fwd}}} \right) \quad (2)$$

Where: $V = V_R \sin \phi_B$

P_{fwd} = forward power

P_{rev} = reverse power

η = proportionality factor < 1 (for calculation $\eta=1$)

Because we desire to minimize the power consumption, we choose a ratio $P_{\text{rev}}/P_{\text{fwd}}$ equal or close to zero. The ratio

$$\frac{P_{\text{rev}}}{P_{\text{fwd}}} = \frac{(1-\gamma^2)^2 + (2\gamma \sin \phi_v)^2}{(1+\gamma^2 + 2\gamma \cos \phi_v)^2} \quad (3)$$

It is a function of tuning (i.e., of the phase angle ϕ_v) and matching (i.e., of the quantity $\gamma = \Gamma/G_0$).

The phase angle ϕ_v is computed from (1a)(1b):

$$\phi_v = \tan^{-1} \left(\frac{V_B \cos \phi_B - V \tan \phi_Y}{V_B \sin \phi_B + V} \right)$$

By employing FM during multicycle injection, the angle ϕ_v can be minimized for most of the acceleration cycle. It is made zero at maximum field intensity, but at injection the system

cannot be tuned until the full beam intensity is reached. At this time the FM shift is $\Delta f = fV_B \cos\phi_B / 2QV_{inj}$ and determines the resonant frequency of the rf system. We plan, of course, to retune continuously while the beam is in the bypass and is gradually wasting away.

Let us call the normalized admittance of the system $\gamma = \Gamma/G$. Then the instantaneous admittance is $Y_{ai} = i_g/v = \Gamma e^{-i\phi} v$. $G_o = V_B \sin\phi_B + V_{max}/R_p \times V_{max}$ where G_o is the admittance of the system when the beam is in the BP and the system is tuned.

We calculated P_{fwd} , P_{rev} , Δf , for injection and for maximum field intensity. The results are shown on Figure 1.

After considering the available rf power and the Δf allowed we concluded that a shunt impedance (R_p) of 40 megohms is about optimum. This can be achieved by disconnecting three of the 16 cavities and by reversing the phase of two of the remaining 13 cavities. The shunt impedance when n cavities are reversed is $R_p = \frac{(13-2n)^2}{13} \times \frac{13R_{po}}{16}$

Calculation indicates that if we use the conditions shown in Figure 1, stable operation will result. The stability check is based on the requirement $0 < V_B \sin 2\phi_Y < 2V \cos\phi_B^2$ which is deduced from the small signal oscillations.

Switching the beam into BP results in a 30% increase in the radiation losses which in turn causes a detuning accompanied by a transient that persists until the new equilibrium is reached. The maximum rate of change of oscillator frequency is 100 kHz/msec; therefore the voltage will decay to an intermediate steady state condition before the frequency can be changed by 10 kHz required for retuning.

We have calculated this new steady-state voltage, phase angles, etc., using equations (1a)(1b)(1c) and (1d) with

$$\frac{dV}{dt} = \frac{d\phi_v}{dt} = \frac{d\phi_B}{dt} = \frac{d\Delta U}{dt} = 0$$

The quantities V_B , $\tan\phi_y$ and V_g were kept constant. We found that an equilibrium exists if and only if $V > 2.09$ Mv/turn.

To determine whether the equilibrium is stable with respect to V_R , we assumed a small change in V_R . Expanding the set of equations (1) around the equilibrium point for the changed value of V_R , we obtained the following set of equations:

$$\frac{\delta F_1}{\delta V} \dot{q} + \frac{\delta F_1}{\delta V} q + \frac{\delta F_1}{\delta \phi_v} p + \frac{\delta F_1}{\delta \phi_B} s + \frac{\delta F_1}{\delta V_R} = 0$$

$$\frac{\delta F_2}{\delta \phi_v} \dot{p} + \frac{\delta F_2}{\delta \phi_v} p + \frac{\delta F_2}{\delta V} q + \frac{\delta F_2}{\delta \phi_B} s + \frac{\delta F_2}{\delta V_R} = 0$$

$$\frac{\delta F_3}{\delta \phi_B} \dot{s} + \frac{\delta F_3}{\delta \phi_B} s + \frac{\delta F_3}{\delta \phi_v} p + \frac{\delta F_3}{\delta V} q + \frac{\delta F_3}{\delta \Delta U} r + \frac{\delta F_3}{\delta V_R} = 0$$

$$\frac{\delta F_4}{\delta (\Delta U)} \dot{r} + \frac{\delta F_4}{\delta V} q + \frac{\delta F_4}{\delta \phi_B} s + \frac{\delta F_4}{\delta V_R} = 0$$

With

$$\frac{\delta V}{\delta V_R} = \frac{d}{dt} \frac{\delta V}{\delta V_R} = \dot{q}, \quad \frac{\delta V}{\delta V_R} = q;$$

$$\frac{\delta \phi_v}{\delta V_R} = \dot{p}, \quad \frac{\delta \phi_v}{\delta V_R} = p; \quad \frac{\delta \phi_B}{\delta V_R} = \dot{s}, \quad \frac{\delta \phi_B}{\delta V_R} = s;$$

$$\frac{\delta (\Delta U)}{\delta V_R} = \dot{r}, \quad \frac{\delta \Delta U}{\delta V_R} = r$$

The coefficients are constants and were evaluated for the new equilibrium. In each equation the last term on the left

is zero. The solutions of the above set of equations are of the type $e^{\lambda t}$. There are two different complex conjugate roots (with negative real parts) of the characteristic equations. This implies stable oscillatory motion in the multidimensional phase space. We hope that, when time permits, we will be able to study larger perturbations and find whether the condition continues to be stable.

Conclusion:

With our presently available rf power it appears possible to obtain 100 ma of stored beam at 3 GeV and maintain it during an extended interaction period. During multicycling the reflected power will be high, but it can be absorbed by the isolator. The calculations indicate that the shunt impedance should be decreased to about 40 m Ω , and AM and FM are required during multicycling. The proper AM to give the proposed voltages can be determined by considering the net forward power (i.e., P_{acc}), and the FM required to minimize P_{rev} can be determined by monitoring P_{fwd}/P_{rev} with a phase sensitive device.

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Attachments

ENDNOTES

¹Robinson, K. W., IRE Transactions, MTT-8, 393 (1960)

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FIGURE 1.

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Figure 1b

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