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PERTURBATION SERIES APPROACH TO NUCLEAR MATTER*

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by

George A. Baker, Jr.[†]

Baker Laboratory

Cornell University

Ithaca, N. Y. 14850

and

Applied Mathematics Department

Brookhaven National Laboratory

Upton, N. Y. 11973

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Before one begins the actual computation of the ground state energy of infinite nuclear matter, it is desirable to know as much as possible about its nature. Since most of the technical aspects of these calculations are being adequately dealt with in other lectures presented here, we will confine ourselves mainly to a general overview of the structure of the ground-state energy function and its attendant perturbation expansion.¹ Most of the difficulties encountered in the computations can be traced to real physical effects which have forced us repeatedly to refine our mathematical formalism.

Before the middle 1950's it was thought that any use of perturbation theory in terms of potential strengths was impossible because the size of successive terms increased as powers of the number of particles. There was a plausible explanation of this failure also. Namely that for a simple, one-signed, attractive potential, nuclear collapse occurs so that the energy per particle was very large in proportion to the system size. Brueckner² showed however that in the perturbation series terms for the energy per particle, the parts which increased with system size canceled. Yet the nuclear collapse problem remained, because the perturbation series turned out to be only an asymptotic one rather than a convergent one. This problem should be thought of as a caution warning, since valuable information can frequently be obtained from an asymptotic series.

A further technical problem with a physical basis is the effect of a repulsive core. We know that a hard, or even infinitely hard, core has the effect of simply excluding the wave function from a region of space, and, if the density is not too high, it has only a finite effect on the energy. The technical problem of how to compute a finite effect from an infinitely strong force was again solved by Brueckner² by rearranging the potential strength

perturbation series so as to group into a single term, the sum of any number of successive interactions between the same pair of particles.

It was soon noticed³ that in the presence of an attractive force outside a hard-core, that singularities occur in the solution of the equations for the first, or ladder, term in the rearranged series. The problem here was traced to the physical phenomenon of superfluidity or superconductivity which can occur via Cooper pairs near the fermi surface for some types of potentials. The sort of potential required to produce a ground state of that structure is one with spacelike oscillations, which are probably absent in the nucleon potentials. The numerical expedient of Brueckner and Gammel⁴ to avoid a numerically singular denominator, or the more systematic series rearrangement of Baker and Kahane⁵ serve to eliminate this problem with almost no practical effect on the numerical content of the theory.

There is one more important physical effect which must be considered before the computation of the ground state energy is feasible. It has been evident for a long time that nuclei share many of the properties of a liquid drop. The successful aspects of the Bohr liquid drop model have long been recognized to be the bulk properties, surface energies, separation energies, etc. These properties are, of course, just the ones relevant to the present discussion. The highly developed theory of the interaction of matter in bulk teaches us that as we change the external variables like pressure, density, and temperature which govern the behavior of the system, that the liquid state can be made to disappear either continuously or discontinuously depending on the way the variables are varied. In Fig. 1 we show a sketch of a typical pressure-volume diagram for a liquid-vapor system. We have plotted several isotherms. As you know when we decrease the pressure for temperature less than a critical temperature, the volume changes discontinuously when the state of the system

reaches the dashed line, or phase boundary. This change in volume is a phase transition such as the vacuum boiling of water. If we control the volume instead of the pressure, such as by closing off the vacuum line in the above example, we can produce a system with two phases, liquid and vapor, in equilibrium. This region is inside the dashed line. The point at the top of the two phase region marked with a dot is the critical point at which the difference between the liquid and the vapor disappears. Here the fluctuations become very large and the compressibility diverges. It is a point where many of the thermodynamic properties have analytic singularities.

If we extend figure 1 by adding the potential interaction strength as a third variable we find, for nuclear type potentials, that the critical point moves to lower and lower temperatures as the strength decreases. In Fig. 2 we show the intersection of the phase surface with the $T = 0$ surface as a function of k_F and λ , interaction strength. The critical point singularity is now at C. For a range of interaction strengths indicated by the dashed lines, a vapor and a liquid coexist under positive pressures. For stronger interactions, the system is self-binding, that is zero external pressure is required, and the vapor phase becomes the vacuum. The location of nuclear matter, in this figure is at some point S on the liquid coexistence curve, L. The region above the solid curve is the two phase region.

One valid way to calculate the energy is now evident from Fig. 2. As we know, the exact energy for $\lambda = 0$ and any density, we can attempt to compute the energy at S by fixing the density and summing the perturbation series in λ .

There are several further popular reorganizations of the perturbation series which do not seem likely to be successful. Methods of these types were certainly not successful for the simpler classical case. One procedure,

through excellent for systems with repulsive forces only, is the hole-line approach. It is certainly true that in low order it does select correctly the most significant diagrams. It is however qualitatively something like an expansion in $(k_F^3 \lambda)$ as the inclusion of a new hole adds a factor of the interaction strength and another integral over the fermi sea. We have replotted Fig. 2 in Fig. 3 in an orthogonal coordinated system in which the parameter $(k_F^3 \lambda)$ increases vertically. The perturbation calculation in this parameter moves up the vertical line through the two-phase region where, to put it mildly, the danger of going wrong is very great.

The reason that this problem was not observed numerically before this time in the many calculations done using this approach has now been found by Baker and Gammel.⁶ In the numerical calculations involved, an additional rearrangement of the perturbation series was made called the Brandow choice⁷ of the intermediate state energies. In this case one uses self-consistent energies for the holes in intermediate states, but not for the particles. For a net attractive force, this choice leads to an effective gap in the single-particle energy spectrum at the fermi surface, and thus greatly reduces the contributions from higher diagrams. The effect is the opposite for a repulsive force and the procedure fails altogether. What Baker and Gammel⁶ found was that, in going from $\lambda = 0$ to full strength, certain singularities, zeros of the denominator of the Green's function, cross the contour of integration over intermediate state momentum. In the Brandow choice procedure this effect is not taken account of. The net effect is that one winds up on the wrong Riemann sheet of the ground-state energy function so that one is not calculating the answer to the physical problem. Thus although on the physical Riemann sheet, one can not expect success with that method, on the unphysical sheet it

was very successful.

In conclusion the calculation of the ground state energy of nuclear matter seems possible along a path of fixed density. It is not an easy problem for realistic potentials, but methods seem to be available that should stand a reasonable chance of producing a fairly accurate result.

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FIGURE CAPTIONS

Figure 1. Sketch of a typical pressure-volume diagram for a liquid vapor system. The dashed line is the coexistence curve.

Figure 2. The solid curve is the coexistence curve for a many-fermion system at zero temperature. It consists of three parts. Part L which runs right from point C is the liquid coexistence curve. Part G which runs from B to C is the gas coexistence curve, and part V which runs vertically up from B is the gas coexistence curve when the gas phase is a vacuum. Point C is the critical point. Point Z is where the energy per particle in the liquid phase on the coexistence curve crosses zero. Point S is a typical saturation point corresponding to a zero slope in the energy per particle vs k_F curve. The region above the coexistence curve is the two phase region and that below is the phase region.

Figure 3. A redrawn version Fig. 2 in terms of different variables. The two phase region is to the right and the one phase region to the left.

Figure 1.

Figure 2.

Figure 3.