Two Pion Intermediate States in the Decay $K_S^0 \rightarrow 2\gamma^*$

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ABSTRACT

The unitarity relations for the two pion intermediate state in the decay $K_S^0 \rightarrow 2\gamma$ are solved using a recently derived form for the amplitude for the process $2\gamma \rightarrow 2\pi$, which keeps the exact one pion-exchange contribution, but neglects the left hand cut and inelastic cuts in the $\pi-\pi$ scattering amplitude. The resulting amplitude shows no direct enhancement of the decay rate from strong interactions and the rate could be zero if the cross section for $2\gamma \rightarrow 2\pi$ were zero at the K mass. A plausible choice of the amplitude for $2\gamma \rightarrow 2\pi$ gives a contribution which has exactly the magnitude that would be calculated using perturbation theory for point-like pions with no strong interactions, but is shifted by the $\pi-\pi$ scattering phase leading to a unitarity lower bound that is smaller by about a factor of two.

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The successes of current algebra and PCAC in describing processes involving low energy pions have dynamical implications. The possibility of fixing the low energy $\pi-\pi$ scattering amplitude to known small values¹ and the requirement for PCAC that the amplitudes vary as slowly as possible, suggest

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that most of the variation of the π - π partial wave amplitudes at energies below about .9 GeV should come from the elastic cut and possible resonance poles. It should, therefore, be a good approximation to treat the elastic cut exactly in a parametric form which allows resonance poles and to neglect the variation due to other singularities. The neglected singularities are the left hand cuts due to two and more pion exchange and the multipion production thresholds on the right. Both sets of singularities are expected from current algebra and phase space to have small low energy discontinuities; the left hand cuts are, in addition, on the other side of the point where the amplitude is fixed by the current algebra low energy theorem. This approximation is a parametric form corresponding to an infinite sum of "bubble" graphs with vertex functions having the lowest possible momentum dependence (this last from the PCAC smoothness requirement). It provides a good description² of π - π scattering below .9 GeV. It has also been used³ to explain the shape of the ρ as observed in e^+e^- annihilation. Recently, a form for the scattering amplitude for $\gamma\gamma \rightarrow \pi\pi$ has been written^{4,5} in an equivalent approximation in which the one pion exchange left hand cut and the two pion intermediate state unitarity cut are both treated exactly, but multipion exchange left hand cuts and multipion production thresholds are neglected. This again corresponds to an infinite sum of "bubble" graphs. Here, as in the case of the e^+e^- annihilation to two pions, gauge invariance is sufficient to determine the low energy limit of the amplitude and a general solution can be given without reference to the current algebra constraints (though one loses thereby, the principal argument for neglecting the other singularities). Unlike the e^+e^- annihilation, there does not seem to be as compelling an argument

for minimal momentum dependence of the vertices (this is discussed in Appendix A of reference 4) and the solutions of references 4 and 5 are written in terms of an undetermined entire function. The amplitude, and thus this function, should be measurable in forthcoming colliding beam experiments on the process $e^+e^- \rightarrow e^+e^-\pi\pi$.

In this note I give the solution in the same approximation for the $K_S^0 \rightarrow 2\gamma$ amplitude, using the previously mentioned result for the $\gamma\gamma \rightarrow \pi\pi$ amplitude. This amplitude is of current interest because of an apparent anomaly in the decay $K_L^0 \rightarrow 2\mu$ ⁶ and suggestions that this is connected with CP violating effects⁷; however, even if the anomaly in the K_L^0 decay should disappear, the process $K_S^0 \rightarrow 2\gamma$ should be of considerable experimental interest since, as the subsequent analysis will show, purely strong and electromagnetic effects can be separated out and the decay can be used to test models for the weak interactions.

In the subsequent development, I shall assume both the $\Delta I = 1/2$ rule, so that the two pions are in an $I=0$ state, and CP invariance. Throughout I shall use the notation and normalization conventions of references 2 and 4. Unitarity with two pion intermediate states gives relations among the following amplitudes, $t_0(s)$, the $I=0$ s-wave $\pi\pi$ scattering amplitude, $f^{I=0}(s)$, the $I=0$ s-wave scattering amplitude for $\gamma\gamma \rightarrow \pi\pi$, $A_{\pi\pi}(s)$, the decay amplitude for $K_S^0 \rightarrow 2\pi$, and $A_{\gamma\gamma}$, the decay amplitude for $K_S^0 \rightarrow 2\gamma$. These are,

$$\text{Im } t_0(s) = \left(\frac{s - 4m_\pi^2}{4s} \right)^{1/2} |t_0|^2, \quad s \geq 4m_\pi^2 \quad (1)$$

$$\text{Im } f^{I=0}(s) = \left(\frac{s - 4m_\pi^2}{4s} \right)^{1/2} f^{I=0*}(s) t_0(s), \quad s \geq 4m_\pi^2 \quad (2)$$

$$\text{Im } A_{\pi\pi}(s) = \left(\frac{s - 4m_\pi^2}{4s} \right)^{1/2} A_{\pi\pi}^*(s) t_0(s), \quad s \geq 4m_\pi^2, \quad (3)$$

and

$$\text{Im } A_{\gamma\gamma}(s) = \frac{\alpha}{2} \left(\frac{s - 4m_\pi^2}{4s} \right)^{1/2} A_{\pi\pi}^*(s) f^{I=0}(s), \quad s \geq 4m_\pi^2, \quad (4)$$

The amplitudes $A_{\pi\pi}(s)$ and $A_{\gamma\gamma}(s)$ are analytic on the left; the amplitude $t_0(s)$

I shall assume to be analytic on the left; however, I shall assume that the amplitude $f^{I=0}(s)$ has the left hand singularity,

$$\text{Im } f^{I=0}(s) = \text{Im } f_0, \quad s \leq 0 \quad (5)$$

where f_0 is the s-wave $I=0$ Born amplitude for $\gamma\gamma \rightarrow \pi\pi$,

$$f_0 = \frac{1 - \beta^2}{\beta} \ln \frac{1 + \beta}{1 - \beta} \quad (6)$$

with β the c.m. pion velocity $\left(\frac{s - 4m_\pi^2}{s} \right)^{1/2}$. These last two assumptions were discussed earlier and correspond to the neglect of multipion exchange singularities. Implicit in equations 1 - 4 is the neglect of multipion production thresholds. The most general solution to equation 3, using eq. (1), is

$$A_{\pi\pi}(s) = \bar{A}_{\pi\pi}(s) t_0(s) \quad (7)$$

where $\bar{A}_{nn}$ is an arbitrary real entire function. Thus the most general solution to equation (4) using equations (1), (2), and (5), is, obviously,

$$H_{gg}(s) = \frac{\pi}{2} A_{nn}(s) t_0^{-1}(s) (f^{\bar{I}=0}(s) - f_c(s)) + \bar{A}_{gg} \quad (8)$$

where $\bar{A}_{gg}$ is an arbitrary real analytic function, ($\bar{A}_{gg}$ can have poles where t_0 has zeroes).

It is illuminating to rewrite equation (8) making use of the solutions for t_0 and $f^{\bar{I}=0}$. The most general solution to equation (1) is

$$t_0^{-1}(s) = g(s) + h(s) \quad (9)$$

where $g(s)$ is an arbitrary meromorphic function, and h is a function which satisfies

$$\bar{I}_{nn} h(s) = - \left(\frac{s - 4m_n^2}{4s} \right)^{1/2}, \quad s \geq 4m_n^2 \quad (10)$$

but has no other singularities. The function $h(s)$ may be written

$$h(s) = -i \left(\frac{s - 4m_n^2}{4s} \right)^{1/2} + h_1(s) \quad (11)$$

with

$$h_1(s) = \frac{1}{n} \left(\frac{s - 4m_n^2}{s} \right)^{1/2} \ln \left[\frac{s^{1/2} + (s - 4m_n^2)^{1/2}}{2m_n} \right] \quad (12)$$

A useful relation is

$$f_c(s) = \frac{8\pi m_\pi^2}{(s - 4m_\pi^2)} h_1(s) \quad (13)$$

The general solution for $f^{I=0}$ from equation (2) and (5) is slightly more complicated, but can be found following the same route taken in solving equation (1)⁴. It is

$$f^{I=0}(s) = t_c \left[\frac{s - 4m_\pi^2}{16\pi^2} f_c^2(s) + s(s) f_c(s) + \frac{m_\pi^2}{s} - \frac{1}{m} + s \bar{R}(s) \right] \quad (14)$$

where $\bar{R}(s)$ is an arbitrary real function, analytic except for possible poles at zeroes of t_0 . Equation (14) clearly satisfies equation (2) and it is simple, using the relation (13), to show that it also satisfies equation (5).

Now these expressions may be substituted in the solution $A_{\gamma\gamma}$, equation (8), giving

$$A_{\gamma\gamma} = \frac{\alpha}{2} A_{\pi\pi}(s) \left[\frac{h_1}{2} f_c - h t_c + \frac{m_\pi^2}{s} - \frac{1}{m} + s \bar{R}(s) \right] + \bar{A}_{\gamma\gamma}(s) \quad (15)$$

This may be rewritten as

$$A_{\gamma\gamma} = \frac{\alpha}{2} A_{\pi\pi}(s) \left(F(s) - \frac{1}{m} + s \bar{R}(s) \right) + \bar{A}_{\gamma\gamma}(s) \quad (16)$$

where $\frac{\alpha}{2} |A_{\pi\pi}(m_K^2)| F(s)$ is the amplitude for $K_S^0 \rightarrow \gamma\gamma$ which would come

from perturbation theory assuming point-like pions with no strong interactions⁸.
 $F(s)$ has the numerical magnitude,

$$\frac{\omega^2}{4} |F(s)|^2 = \frac{\Gamma(K_S^0 \rightarrow \pi^+ \pi^-)}{\Gamma(K_S^0 \rightarrow \pi^+ \pi^-)} \Big|_{\text{P.T. Theory}} \approx 2.5 \times 10^{-2} \quad (17)$$

Equation (16) is the principal result of this paper. It is noteworthy in that the purely strong interaction effects appear solely in the phase of $A_{\pi\pi}$, and the purely electromagnetic effects appear only in the function $\bar{R}(s)$ which will be measured in colliding beam experiments, so the only remaining undetermined function is $\bar{A}_{\gamma\gamma}$, whose measurement can thus provide a test of weak interaction models⁹.

In the absence of information about $\bar{R}(s)$ and $\bar{A}_{\gamma\gamma}(s)$ it is impossible to proceed further. To give an idea of the sorts of numbers involved, I shall make the common assumption that $f^{I=0}(s)$ and $A_{\gamma\gamma}(s)$ satisfy unsubtracted dispersion relations.¹⁰ Then $\bar{R}(s)$ and $\bar{A}_{\gamma\gamma}(s)$ will both be zero except for possible poles at zeroes of t_0 . In that case, the magnitude of $A_{\gamma\gamma}(M_K^2)$ will be just the perturbation theory magnitude except for the contribution of the possible poles, and the numerically small $-1/\pi$ term in the brackets. For the case where the pole appears at the current algebra zero the pole gives a small contribution almost exactly cancelling the $-1/\pi$ term, and the result is just the perturbation theory result, with the phase shifted by the $\pi-\pi$ scattering phase appearing in $A_{\pi\pi}$. If only the scattering amplitude $f^{I=0}(s)$ is assumed to satisfy an unsubtracted dispersion relation, and the imaginary part of

$A_{YY}(s)$ is calculated with $\bar{R}(s) = 0$ (or with small pole terms) to give a lower bound to $\Gamma(K_s^0 \rightarrow \pi\pi)$, then, because $F(s)$ is largely imaginary and the $\pi-\pi$, $I=0$ scattering phase is near 45° , the lower bound is smaller than the perturbation theory result by about a factor of two¹¹.

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References and Footnotes

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8. This gives a dispersion theoretic calculation of $F(s)$ and is the simplest method of resolving the perturbation theory ambiguities. That is because the function $F(s)$ has the correct absorptive part for the two pion loop and satisfies an unsubtracted dispersion relation. Other approaches are summarized by K. Nishijima, Fields and Particles, (Benjamin, New York, 1969) and F. J. Yndurain, Prog. Theor. Phys. 46, 990 (1971). (This second reference points out a mistake in Nishijima's treatment.)
9. It is not strictly true that the strong and electromagnetic interactions put no constraint on $\bar{A}_{YY}$ since $\bar{A}_{YY}$ must have a pole to compensate any pole which appears in $\bar{R}(s)$. This will happen for example, at the current algebra zero of t_0 . However, for this example the contribution of the poles is small.

10. This assumption seems dubious in the framework of an approximation in which certain singularities are neglected because their low-lying discontinuities are small.
11. There have been two recent estimates of the two pion contribution to this decay in addition to the perturbation theory calculation mentioned in reference 8. These are B. Martin, E. deRafael, and J. Smith, Phys. Rev. D2, 179 (1970) and Y. Kohara (to be published). Both of these papers report some enhancement of the decay from strong interaction effects; both assume unsubtracted dispersion relations and neglect multipion exchange and production, but both give $f^{I=0}(s)$ the incorrect discontinuity across the left hand cut of t_0 . $\text{Im } \hat{F}_\pi$ is some entire real function. This, I believe, is the reason for their differing with the present work.