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QUARTERLY PROGRESS REPORT NO. 10

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

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Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

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U.S. Atomic Energy Commission

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I. INTRODUCTION

The purpose of this research is to investigate the ability of a short pulse of electron cyclotron resonance radiation to produce an anisotropic energy distribution of electrons in a plasma, and the instabilities and waves produced by this anisotropy. In particular, the nature of the r.f. heating mechanism and the behavior of the radiation emitted from the plasma will be considered.

This report covers the period from 15 June 1969 until 14 September 1969. The electron cyclotron resonance device shown in Figure 1 was operated both in a short rapidly repeating mode, and a long (up to 1 millisecond) mode to obtain measurements of free-free bremsstrahlung, wall bremsstrahlung, plasma diamagnetism, energetic electron flux and electron density in the discharge.

Theoretical work on developing an expression for the free-free bremsstrahlung from a plasma containing an anisotropic electron energy distribution was continued. The two areas of interest currently under investigation are: (1) to write an energy distribution function for electrons in a plasma when the energy distribution of electrons is anisotropic and some of the electrons have relativistic velocities; (2) using this energy distribution function, calculate the free-free bremsstrahlung emission pattern from the plasma.

The plasma simulation work continued with computations for the conditions of electron cyclotron resonance, periodic boundary conditions, r.f. confinement (no magnetic field) and cold plasma (zero-temperature models).

II. MACHINE OPERATION

The magnet cooling system was modified by the relocation of the high pressure pump in the water line outside the building for noise purposes. The inlet pressure remained at 225 psi. The long pulser is still under test but should be operational soon.

Figure 1

DIAGRAM OF EXPERIMENTAL APPARATUS

III. EXPERIMENTAL RESULTS

In order to obtain more accurate measurements of the plasma diamagnetism, it was decided upon to make a new electrostatically shielded diamagnetic loop. This eliminates the pick-up of particle current which interferes with the diamagnetic signal.

The loop consists of 16 turns of #14 copper wire encased in 2850 ft. epoxy coating. The shield was fabricated from type 302 stainless steel foil, .001 inch thick. This thickness was chosen for its ease in handling and durability while still producing effective shielding. The time constant of the shield was calculated to be 4.58×10^{-7} seconds which still allows for detection of rapidly varying fields. (See response curve, Fig. 2.)

Before the foil could be used, it was necessary to anneal it so as to eliminate any magnetism produced by the cold rolling of the stainless steel. The foil was heated with a torch until it was red hot and allowed to cool in air. This eliminated any measurable magnetism, but left a chromium oxide deposit which was unacceptable for a vacuum system since it is a source of severe outgassing. To remove this, the foil was then cleaned by successive immersions into baths of nitric and hydrochloric acid, nitric acid, and distilled water as specified by Rosebury¹. Finally, the foil was degreased with a normal cleaning process.

Two annealed and cleaned stainless steel strips were then spot welded into cylinders of radii equal to the inside and outside radii of the epoxy coated loop. These were then placed around the loop and their edges spot welded together to form the shield. The leads from the loop were then shielded with a piece of stainless steel tubing which was epoxied to the loop. The tube was bent in an "L" shape so as to allow it to be brought out of the

Response of shielded diamagnetic loop
 V_{Loop} (induced) vs. frequency for $V_{\text{in}} = 5$ volts.
 V_{in} = amplitude of audio osc. voltage used to
induce V_{Loop} .

Figure 2

machine through a quick vacuum coupling. The results from this loop show that it indeed shields well and presents a much clearer diamagnetic signal than the unshielded loop. The loop is shown in Figure 3.

An experimental analysis is being made to determine the time dependence of the anisotropy of the electron energy distribution. The measurements are being made in the afterglow of the plasma, immediately following the second pulse of the two pulse code system developed last year.

The parallel energies are determined with an $\vec{E} \times \vec{B}$ analyzer similar to that developed for measurement of electron energies by Perkins and Barr². Experimental verification was made of the previously determined theoretically evaluated value for detector threshold energy--6000 eV. As can be seen from Fig. 4, data for two energy levels taken on separate days shows threshold values in the neighborhood of the theoretical energy.

Measurement of the parallel energy during the times from one to ten microseconds after the second pulse indicate that for the case of the mirror ratio = 1.02 (with magnetic field and gas pressure set for maximum detector signal) the parallel energy remains approximately equal to 500 eV over this time period.

The perpendicular energy density was determined from the plasma diamagnetism measured with the previously described shielded diamagnetic loop recently placed in the system. For the above case the diamagnetic signal gave a peak energy density of 2×10^{11} eV/cm³. The electron density was measured by observing the fundamental cavity mode frequency shift due to the plasma and utilizing an analysis method developed by Kuckes³. The perpendicular energy as a function of time was then obtained and is shown plotted along with the parallel energy in Fig. 5.

DIAMAGNETIC LOOP

Figure 3

Figure 4

Experimental Verification of Theoretically
Determined Threshold Energy of Detector for
ExB Analyzer

ANISOTROPY

Figure 5

The decay time constant of the perpendicular energy from Fig. 5 is approximately 0.5 μ s.

Measurements similar to the above are planned for various mirror ratios, magnetic field values, gas pressure, and spacing of the two power pulses.

IV. THEORETICAL RESULTS

Bremsstrahlung

Introduction: A variety of anisotropic energy distribution functions for relativistic electrons in a plasma have been investigated. These functions describe the number of electrons of various energies incident in any given direction on a point charge scattering center. The energy distribution function is used as a weighting function for the Heitler⁴ differential cross section for relativistic, quantum mechanical, free-free, electron-ion bremsstrahlung. Multiple integration techniques as previously reported⁵ of this weighted differential cross section yield the bremsstrahlung intensity at any particular angle. The range of integration of incident electron energy has been extended to include all relativistic electrons which contribute significantly to the bremsstrahlung.

Energy Distributions: A spherical coordinate geometry is used. The plasma is assumed to be in a d.c. magnetic field directed in the z direction with a symmetrical distribution of electron momenta in the ϕ direction. Normalization (within a factor of 2π) is accomplished by numerical integration of equation (1).

$$1 = A \int_0^\infty \int_0^\pi f(p, \theta) p^2 \sin \theta \, dp \, d\theta \quad (1)$$

where:

θ is the polar angle in the momentum space of the electron,

p is the electron momentum,

$f(p, \theta)$ is the assumed distribution function, and

A is the normalization constant which is determined by this equation.

The following distribution functions have been normalized:

$$f_1(p, \theta) = \exp(-E \left(\frac{\cos^2 \theta}{kT_{\parallel}} + \frac{\sin^2 \theta}{kT_{\perp}} \right))$$

where:

$E = \sqrt{p^2 + m^2}$ is the total energy of the electron,

p is the electron momentum times C , the velocity of light,

m is the electron mass times C^2 ,

kT_H is the characteristic temperature of the electrons moving parallel to the z axis, and

$kT_\perp$ is the characteristic temperature of the electrons moving perpendicular to the z axis.

$$f_2(p, \theta) = \exp(-E/(kT_H \cos^2 \theta + kT_\perp \sin^2 \theta))$$

$$f_3(p, \theta) = \exp\left\{-\left(\left(\frac{kT_\perp}{kT_H} \cos^2 \theta + \sin^2 \theta\right) p^2 + m^2\right)^{1/2} / kT_\perp\right\}$$

$$f_4(p, \theta) = E^{-4} (\cos^2 \theta / kT_H + \sin^2 \theta / kT_\perp)^{-4}$$

$$f_5(p, \theta) = E^{-6} (\cos^2 \theta / kT_H + \sin^2 \theta / kT_\perp)^{-6}$$

$$f_6(p, \theta) = E^{-8} (\cos^2 \theta / kT_H + \sin^2 \theta / kT_\perp)^{-8}$$

$$f_7(p, \theta) = \exp(-E/kT_H)$$

$$f_8(p, \theta) = \exp(-E/kT_\perp)$$

$$f_9(p, \theta) = B \exp(-E/kT_H) + \sin^2 \theta \exp(-E/kT_\perp)$$

$$f_{10}(p, \theta) = \sin^2 \theta \exp(-E/kT_\perp)$$

$$f_{11}(p, \theta) = \cos^2 \theta \exp(-E/kT_H) + \sin^2 \theta \exp(-E/kT_\perp)$$

f_1 is the Harris distribution⁶ derived as previously reported⁵.

f_2 is derived as follows: consider the Maxwell-Boltzmann distribution function for relativistic electrons given in equation (2).

$$f d^3 p = \exp(-\sqrt{p^2 + m^2}/kT) p^2 \sin\theta d\theta dp d\phi \quad (2)$$

By partial integration kT is found to be proportional to the average value of $v^2/\sqrt{1-v^2}$ (v is the velocity of an electron in the distribution function). v^2 may be separated into components as follows:

$$\begin{aligned} kT &= D(\overline{v^2/\sqrt{1-v^2}}) \\ &= D(\overline{v_H^2/\sqrt{1-v^2}}) + D(\overline{v_I^2/\sqrt{1-v^2}}) \\ &= D(\overline{v^2 \cos^2\theta/\sqrt{1-v^2}}) + D(\overline{v^2 \sin^2\theta/\sqrt{1-v^2}}) \\ &= D(\overline{v^2/\sqrt{1-v^2}}) \cos^2\theta + D(\overline{v^2/\sqrt{1-v^2}}) \sin^2\theta \end{aligned} \quad (3)$$

The last step is taken holding θ constant (its veracity may be seen immediately from equation (3)). Now kT is replaced by an anisotropic temperature function with the property that at $\theta=0$, $kT=kT_H$ and at $\theta=90^\circ$, $kT=kT_I$. Thus:

$$kT = kT_H \cos^2\theta + kT_I \sin^2\theta$$

where:

$$\begin{aligned} kT_H &= D(\overline{v^2/\sqrt{1-v^2}}) \text{ for } \theta=0 \text{ and} \\ kT_I &= D(\overline{v^2/\sqrt{1-v^2}}) \text{ for } \theta=90^\circ. \end{aligned}$$

f_2 is then $\exp(-E/(kT_H \cos^2\theta + kT_I \sin^2\theta))$

f_3 is derived by a Taylor expansion of the function:

$$f_3 = \exp(-\sqrt{a^2 p_H^2 + p_I^2 + m^2}/kT)$$

as p_H^2 is small relative to $p_I^2 + m^2$

$$\begin{aligned} f_3 &= \exp[(-\sqrt{p_I^2 + m^2}/kT) (\sqrt{1 + a^2 p_H^2/(p_I^2 + m^2)})] \\ &\approx \exp[(-\sqrt{p_I^2 + m^2}/kT) (1 + a^2 p_H^2/2(p_I^2 + m^2) + \dots)] \end{aligned}$$

$$\text{At } \theta=0, f_3 = \exp[(-m/kT)(1+a p_H^2/2m^2+\dots)]$$

$$= C \exp(-a p_H^2/2m kT),$$

which corresponds to the classical distribution:

$$C_1 \exp(-p_H^2/2m kT_H),$$

so $a=T/T_H$.

$$\text{At } \theta=90^\circ, f_3 = \exp(-\sqrt{p_L^2+m^2}/kT),$$

which corresponds to the relativistic distribution:

$$C_2 \exp(-\sqrt{p_L^2+m^2}/kT_L),$$

so $T=T_L$ and $a=T/T_H=T_L/T_H$.

$$p_H=p \cos\theta \quad p_L=p \sin\theta.$$

$$\text{Thus } f_3 = \exp(-(\frac{T_L}{T_H} p^2 \cos^2\theta + p^2 \sin^2\theta + m^2)^{1/2}/kT_L)$$

f_4, f_5 and f_6 are simple power functions of an anisotropic expression for kT divided by the total energy. Since the volume element in momentum space is proportional to p^2 , the lowest power considered for E was the fourth power to insure swift convergence.

f_7 and f_8 are isotropic energy distributions.

f_9 is primarily an isotropic distribution function with characteristic temperature kT_H combined with a high energy tail of characteristic temperature kT_L weighted by $\sin^2\theta$. Thus parallel velocities remain characterized by kT_H while perpendicular velocities are affected by kT_L .

f_{10} is the same as f_9 without the background isotropic distribution.

f_{11} adds a $\cos^2\theta$ term so that when $T_H=T_L$ the distribution function will reduce to the isotropic case.

kT_H is taken as 3.1 keV. kT_L is 31.0 keV (equal to 10 times kT_H). f_9, f_{10} and f_{11} are identical since $\exp(-E/kT_H)$ is negligible when compared to $\exp(-E/kT_L)$.

A function g_1 is defined as $\int_0^\pi f_1 p^2 \sin\theta \, d\theta$. Graphs of $\log g_1$, $\log g_2$, and $\log g_3$ versus p are shown in Figure 6. $\log g_1$ and $\log g_2$ are parallel on this semilog plot which means that:

$$\log g_1 + C = \log g_2 \quad \text{or} \quad g_1 = a g_2.$$

The values of the normalization constants for the above distribution functions, f_i , are given in Table I.

In this phase of theoretical research on distribution functions work continues on three subjects: 1) checking and refining the computer program for numerical integration of the f_i to yield the normalization constants, A_i ; 2) extending the integration range to higher values of p ; and 3) studying the distribution function characteristics to decide which functions best describe a laboratory plasma.

Bremsstrahlung Intensity: Multiple integration of the Heitler differential cross section for bremsstrahlung weighted by some of the electron energy distribution functions described in the previous section has been carried out. The results, using f_1 , f_2 , f_3 and f_4 , of bremsstrahlung intensity versus photon energy are shown in Figure 7. The f_i were not normalized, and the photon intensity scales are at different powers of 10 for different f_i .

General observations from the data producing these graphs include:

- 1) for low energy incident electrons with $T_\perp = 10 T_\parallel = 31.0 \text{ keV}$, the photon intensity perpendicular to the magnetic field is less than the photon intensity parallel to the field; 2) however, on including electrons of relativistic energies, the perpendicular x-ray intensity exceeds the parallel; and 3) also the characteristic "temperature" obtained from the slope of x-ray intensity versus photon energy, is greater for the x-rays in the perpendicular direction compared to those in the parallel direction. This is seen

g_i vs Electron Momentum

Figure 6

Table I

Normalization constants, A_i

$$A_i = 1 / \int_0^{\infty} \int_0^{\pi} f_i p^2 \sin \theta \, d\theta \, dp$$

$$A_1 = .189 \times 10^9$$

$$A_2 = .620 \times 10^8$$

$$A_3 = .415 \times 10^8$$

A_4 integration to higher values of p required

$$A_5 = \text{about } .27 \times 10^8$$

$$A_6 = .16 \times 10^4$$

$$A_7 = .205 \times 10^{74}$$

$$A_8 = .219 \times 10^8$$

$$A_9 = .328 \times 10^8$$

$$A_{10} = .328 \times 10^8$$

$$A_{11} = .328 \times 10^8$$

as the curves for perpendicular x-ray intensity and for parallel x-ray intensity diverge on proceeding toward higher values of photon energy.

In this phase of theoretical research on bremsstrahlung intensity work is in progress to utilize the normalized distribution functions, described in the previous topic of theoretical research, in order to compare the bremsstrahlung intensities predicted by these different distribution functions on the same relative scale. Also the same problems of integration techniques encountered in the previous topic of distribution functions are present in this more involved problem of calculating bremsstrahlung intensities.

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VISITS OF LABORATORY STAFF, PUBLICATIONS AND CONFERENCE PAPERS

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FIGURE CAPTIONS

Figure 1 Diagram of experimental apparatus

Figure 2 Response of shielded diamagnetic loop

Figure 3 Diamagnetic loop

Figure 4 Experimental verification of theoretically determined threshold energy of detector for $\vec{E} \times \vec{B}$ analyzer

Figure 5 Anisotropy

Figure 6 g_1 vs electron momentum

Figure 7 Intensity vs photon energy